



Quality Factor of Free In-plane Vibration of a Fully Clamped Rectangular Micro-plate

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ABSTRACT

Thermo-elastic damping is a significant dissipation mechanism in high quality factor microstructures. In this paper, thermo-elastic damping of the in-plane vibration of fully clamped rectangular micro-plates has been studied. The governing equation of the micro-plate motion and heat conduction equation were derived. Then, The Galerkin method has been used to solve the coupled heat-displacement equations. Eventually, considering the micro-plate of various materials, the effects of geometrical parameters including the length and width of micro-plate and also ambient temperature on the thermo-elastic damping quality factor have been investigated.

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1. INTRODUCTION

Recently, Micro Electro Mechanical Systems (MEMS) have drawn many attentions due to the advantages of favorable scaling property, low energy consumption, high sensitivity, high mechanical resonance frequencies and high design flexibility. Micro-waveguides, micro-resonators, micro-mirrors, micro-switches and micro-pumps are essential parts in many MEMS devices such as charge detectors, radio frequency (RF) filters, mass flow sensors and many others. Micro-plates are one of the essential structures widely used in resonantly actuated micro-structures in which the sensitivity and resolution are underlying issues [1]. Thus, designing micro-resonators with high quality factor is of a great necessity.

The dissipation mechanisms contributing in the decline of the quality factor can be classified into extrinsic and intrinsic losses in which intrinsic losses such as Thermo-Elastic Damping (TED) cannot be controlled as easily as extrinsic losses such as air damping [1]. TED is a significant loss mechanism in

flexural resonators [1] which was discovered by Zener [2, 3]. TED, as a source of mechanical thermal noise, is a contributing factor in the decreasing of quality factor and consequently increasing of energy consumption. Duwel et al. [4] and Evoy et al. [5] have experimentally shown that TED is a major damping source in MEMS and NEMS. Lifshitz and Roukes derived an analytical expression for quality factor (QF) of TED in micro-beams and studied the effect of different geometrical parameters on it [6]. Sun et al. [7] and Sharma et al. [8] established the equation of the coupled thermo-elastic case for axi-symmetric out-of-plane vibrations of a circular micro-plate in order to study TED in micro-plates. TED of transversal vibrations of micro-beams and micro-plates have been discussed in the literature by Rezazadeh et al. [9], Najafi et al. [10], Maroofi et al. [11] and many others. Furthermore, Rezazadeh et al. [12] studied TED in a micro-beam resonator based on modified couple stress theory and Zhong et al. [13] investigated TED in a micro-plate resonator using modified couple stress. The mentioned theory is utilized to capture the size effect where there is a considerable internal length scale parameter [14, 15].

The natural frequencies of in-plane vibrations are much higher than those of transverse vibrations,

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however, in-plane vibrations occur where structure is made of piezoelectric or magnetostrictive materials subjected to electric or magnetic field [16, 17]. In-plane vibrations appeared in the design of ship hulls [18] and also in the outer sheets of the sandwich panels while the assembly itself undergoes lateral vibration [19]. Bardell et al. [20] studied in-plane vibration frequencies in simply supported plates, clamped and free plates using Rayleigh-Ritz method. Seok et al. [21] used the equations of plane stress including in-plane inertia to treat the problem of the free vibrations of a thin relatively short cantilever plate and presented the dispersion curves for in-plane flexure of rectangular plates. Gorman [22] obtained accurate analytical-type solutions by means of the superposition method for the free in-plane vibration of fully clamped rectangular plates. In other work, Gorman employed the superposition method to analyze the effects of elastic edge support on the in-plane free vibration frequencies and mode shapes of rectangular plates [23]. He also obtained accurate analytical-type solutions for the free in-plane vibration eigenvalues and mode shapes of fully clamped orthotropic rectangular plates [23]. Gorman presented exact solutions for the free in-plane vibration of the rectangular plates with two opposite edges simply supported [24]. Andrianov et al. [25] analyzed natural in-plane vibration of rectangular plates using homology perturbation approach. Hyde et al. [26] investigated free in-plane vibration of rectangular plates undergoing plane stress deformation through Ritz discrimination of the Rayleigh quotient.

In this paper, thermo-elastic damping of in-plane vibrations of a fully clamped rectangular micro-plate under electrostatic actuation is investigated. The Galerkin method and complex-frequency approach are applied to solve the coupled equations of the plate motion and heat conduction. Furthermore, it aims to determine how the length, width and ambient temperature influence the Q_{TED} .

2. MODEL DESCRIPTION AND PROBLEM FORMULATION

Figure 1 depicts a fully clamped rectangular micro-plate in which L , b and h are the length, width, and thickness of the plate, respectively.

2.1. Stress and Strain Fields Mechanical and thermal effects result in a general strain field as [27]:

$$e_{ij} = e_{ij}^{(M)} + e_{ij}^{(T)} \quad (1)$$

Stress in terms of strain can be written as:

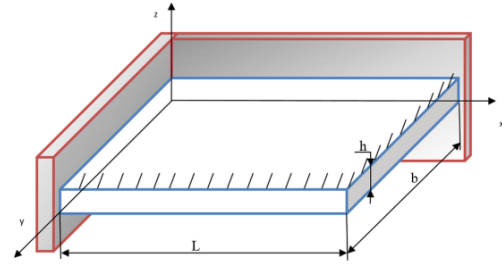


Figure 1. Schematic of a fully clamped micro-plate

$$\sigma_{ij} = C_{ijkl} e_{kl} - \beta_{ij} (T - T_0) \quad (2)$$

where, C_{ijkl} is a fourth-order elasticity tensor [28] and β_{ij} is a second-order tensor containing thermo-elastic moduli [29]. Thus, Equation (2) can be expressed as:

$$\sigma_{ij} = \lambda e_{kk} \delta_{ij} + 2\mu e_{ij} - (3\lambda + 2\mu)\alpha(T - T_0)\delta_{ij} \quad (3)$$

in which λ , μ , α and δ are Lamé's constant, shear modulus, thermal expansion coefficient and Kronecker delta, respectively. Equation (3) in terms of \mathbf{E} and \mathbf{V} can be rewritten as follows:

$$\sigma_{ij} = \frac{E}{1+\nu} e_{ij} + \left(\frac{E\nu}{(1+\nu)(1-2\nu)} \right) e_{kk} \delta_{ij} - \left(\frac{E}{1-2\nu} \right) \alpha(T - T_0)\delta_{ij} \quad (4)$$

2.2. Motion Equation The governing equation of motion is expressed as [30]:

$$\sigma_{ij,j} + \rho b_i = \rho a_i \quad (5)$$

where ρ , b_i and a_i are the mass density, body force, and acceleration vector, respectively. The equation of motion by ignoring the body force can be written as:

$$\lambda u_{k,ki} + \mu(u_{i,jj} + u_{j,ij}) - (3\lambda + 2\mu)\alpha(T - T_0)_{,i} = \rho \ddot{u}_i \quad (6)$$

2.3. Heat Equation Heat conduction equation with no sources is defined as [29]:

$$kT_{,ii} = \rho c \dot{T} + (3\lambda + 2\mu)\alpha T_0 \dot{e}_{ii} \quad (7)$$

where k and c are the thermal conductivity and the specific heat at constant pressure, respectively. Rewriting Equation (7) in terms of displacement results in the following equation:

$$kT_{,ii} = \rho c \dot{T} + (3\lambda + 2\mu)\alpha T_0 \dot{\mu}_{i,i} \tag{1}$$

Equations (6) and (8) are a system of coupled equations. Considering plane stress condition due to the thickness of the plate we have:

$$\begin{aligned} e_z &= \frac{\partial w}{\partial z} = -\frac{\nu}{E}(\sigma_x + \sigma_y) \\ &= -\frac{\nu}{1-\nu}(e_x + e_y) \\ &= -\frac{\nu}{1-\nu}\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) \end{aligned} \tag{9}$$

It is assumed that the plate can only vibrate in x and y directions, hence the terms $\partial^2 u / \partial z^2$ and $\partial^2 v / \partial z^2$ are eliminated; it means that plate remains at a right angle in x and y directions or $u = u(x, y, t)$ and $v = v(x, y, t)$. Heat doesn't have enough time to transfer in z direction because the vibrations are not steady, which means: $T = T(x, y, t)$. Simplifying and transforming Equations (6) and (1) into non-dimensional form result in:

$$\begin{aligned} \frac{\partial^2 \hat{u}}{\partial \hat{x}^2} + B_1 \frac{\partial^2 \hat{u}}{\partial \hat{y}^2} + B_2 \frac{\partial^2 \hat{v}}{\partial \hat{x} \partial \hat{y}} \\ - B_3 \frac{\partial \hat{\theta}}{\partial \hat{x}} = \frac{\partial^2 \hat{u}}{\partial \hat{t}^2} \\ \frac{\partial^2 \hat{v}}{\partial \hat{x}^2} + B_4 \frac{\partial^2 \hat{v}}{\partial \hat{y}^2} + B_5 \frac{\partial^2 \hat{u}}{\partial \hat{x} \partial \hat{y}} \\ - B_6 \frac{\partial \hat{\theta}}{\partial \hat{y}} = \hat{\rho} \frac{\partial^2 \hat{v}}{\partial \hat{t}^2} \\ \frac{\partial^2 \hat{\theta}}{\partial \hat{x}^2} + B_7 \frac{\partial^2 \hat{\theta}}{\partial \hat{y}^2} - B_8 \frac{\partial^2 \hat{u}}{\partial \hat{t} \partial \hat{x}} \\ - B_8 \frac{\partial^2 \hat{v}}{\partial \hat{t} \partial \hat{y}} - B_9 \frac{\partial \hat{\theta}}{\partial \hat{t}} = 0 \end{aligned} \tag{10}$$

where:

$$\begin{aligned} B_1 &= \frac{(1-\nu)l^2}{(2-\nu)b^2}, B_2 = \frac{1}{2-\nu} \\ B_3 &= \frac{2(1-\nu)(1+\nu)\alpha T_0}{(2-\nu)(1-2\nu)}, B_4 = \frac{(2-\nu)l^2}{(1-\nu)b^2} \\ B_5 &= \frac{l^2}{(1-\nu)b^2}, B_6 = \frac{2(1+\nu)\alpha T_0 l^2}{(1-2\nu)b^2} \\ B_7 &= \frac{l^2}{b^2}, B_8 = \frac{\alpha l}{k} \sqrt{\frac{E^3(2-\nu)}{2\rho(1-\nu)^3(1+\nu)}} \\ B_9 &= \frac{cl}{k} \sqrt{\frac{\rho E(2-\nu)}{2(1-\nu)(1+\nu)}}, \hat{\rho} = \frac{2-\nu}{1-\nu} \end{aligned} \tag{11}$$

The non-dimensional parameters of Equation (10) are introduced as:

$$\begin{aligned} \hat{u} &= \frac{u}{l}, \hat{v} = \frac{v}{b}, \hat{y} = \frac{y}{b}, \hat{x} = \frac{x}{l}, \\ \hat{\theta} &= \frac{\theta}{T_0}, \hat{t}_0^2 = \frac{2\rho l^2(1+\nu)(1-\nu)}{E(2-\nu)}, \end{aligned} \tag{12}$$

where $\theta = T - T_0$.

2. 4. Solving the Governing Equations

The Galerkin method is applied to solve Equation (10). Thereby, it can be approximated in terms of linear combinations of a finite number of shape functions and time-dependent coefficients [31, 32].

$$\begin{aligned} \hat{u}(\hat{x}, \hat{y}, \hat{t}) &= \sum_{n=1}^N \sum_{m=1}^M \phi_n(\hat{x}) \phi_m(\hat{y}) a_{nm}(\hat{t}) \\ \hat{v}(\hat{x}, \hat{y}, \hat{t}) &= \sum_{k=1}^K \sum_{f=1}^F \psi_k(\hat{x}) \psi_f(\hat{y}) b_{kf}(\hat{t}) \\ \hat{\theta}(\hat{x}, \hat{y}, \hat{t}) &= \sum_{e=1}^E \sum_{r=1}^R \eta_e(\hat{x}) \eta_r(\hat{y}) c_{er}(\hat{t}) \end{aligned} \tag{13}$$

By substituting Equation (13) into Equation (10), error functions are obtained as follows:

$$\begin{aligned} \sum_{n=1}^N \sum_{m=1}^M \phi_n''(\hat{x}) \phi_m(\hat{y}) a_{nm}(\hat{t}) \\ + B_1 \sum_{n=1}^N \sum_{m=1}^M \phi_n(\hat{x}) \phi_m''(\hat{y}) a_{nm}(\hat{t}) \\ + B_2 \sum_{k=1}^K \sum_{f=1}^F \psi_k'(\hat{x}) \psi_f'(\hat{y}) b_{kf}(\hat{t}) \\ - B_3 \sum_{e=1}^E \sum_{r=1}^R \eta_e'(\hat{x}) \eta_r(\hat{y}) c_{er}(\hat{t}) \\ - \sum_{n=1}^N \sum_{m=1}^M \phi_n(\hat{x}) \phi_m(\hat{y}) \ddot{a}_{nm}(\hat{t}) = \epsilon_1 \\ \sum_{k=1}^K \sum_{f=1}^F \psi_k''(\hat{x}) \psi_f(\hat{y}) b_{kf}(\hat{t}) \\ + B_4 \sum_{k=1}^K \sum_{f=1}^F \psi_k(\hat{x}) \psi_f''(\hat{y}) b_{kf}(\hat{t}) \\ + B_5 \sum_{n=1}^N \sum_{m=1}^M \phi_n'(\hat{x}) \phi_m'(\hat{y}) a_{nm}(\hat{t}) \\ - B_6 \sum_{e=1}^E \sum_{r=1}^R \eta_e(\hat{x}) \eta_r'(\hat{y}) c_{er}(\hat{t}) \\ - \hat{\rho} \sum_{k=1}^K \sum_{f=1}^F \psi_k(\hat{x}) \psi_f(\hat{y}) \ddot{b}_{kf}(\hat{t}) = \epsilon_2 \end{aligned} \tag{14}$$

$$\begin{aligned} & \sum_{e=1}^E \sum_{r=1}^R \eta_e''(\hat{x}) \eta_r(\hat{y}) c_{er}(\hat{t}) \\ & + B_7 \sum_{e=1}^E \sum_{r=1}^R \eta_e(\hat{x}) \eta_r''(\hat{y}) c_{er}(\hat{t}) \\ & - B_8 \sum_{n=1}^N \sum_{m=1}^M \phi_n'(\hat{x}) \phi_m(\hat{y}) \dot{a}_{nm}(\hat{t}) \\ & - B_8 \sum_{k=1}^K \sum_{f=1}^F \psi_k(\hat{x}) \psi_f'(\hat{y}) \dot{b}_{kf}(\hat{t}) \\ & - B_9 \sum_{e=1}^E \sum_{r=1}^R \eta_e(\hat{x}) \eta_r(\hat{y}) \dot{c}_{er}(\hat{t}) = \epsilon_3 \end{aligned}$$

Shape functions which satisfy the boundary conditions are considered as follows:

$$\begin{aligned} \varphi_n(\hat{x}) &= \sin(n\pi\hat{x}) \\ \psi_k(\hat{x}) &= \sin(k\pi\hat{x}) \\ \eta_e(\hat{x}) &= \sin(e\pi\hat{x}) \\ \phi_m(\hat{y}) &= \sin(m\pi\hat{y}) \\ \psi_f(\hat{y}) &= \sin(f\pi\hat{y}) \\ \eta_r(\hat{y}) &= \sin(r\pi\hat{y}) \end{aligned} \tag{15}$$

Weighted residual method of Galerkin is applied to Equation (14) as follow:

$$\begin{aligned} \int_0^1 \int_0^1 \phi_i(\hat{x}) \phi_j(\hat{y}) \epsilon_1 d\hat{x} d\hat{y} &= 0 \\ i = 1, \dots, N, \quad j = 1, \dots, M \\ \int_0^1 \int_0^1 \psi_q(\hat{x}) \psi_g(\hat{y}) \epsilon_2 d\hat{x} d\hat{y} &= 0 \\ q = 1, \dots, K, \quad g = 1, \dots, F \\ \int_0^1 \int_0^1 \eta_p(\hat{x}) \eta_c(\hat{y}) \epsilon_3 d\hat{x} d\hat{y} &= 0 \\ p = 1, \dots, E, \quad c = 1, \dots, R \end{aligned} \tag{16}$$

By substituting Equation (15) into Equation (16) and simplifying it for the first mode of displacements in x and y directions, the first mode of heat in y direction and the second mode of heat in x direction, results can be described as:

$$\begin{aligned} \ddot{a}_{11}(\hat{t}) + (1 + B_1)\pi^2 a_{11}(\hat{t}) \\ - \frac{8}{3} B_3 c_{21}(\hat{t}) &= 0 \\ \hat{\rho} \ddot{b}_{11}(\hat{t}) + (1 + B_4)\pi^2 b_{11}(\hat{t}) &= 0 \\ B_9 \dot{c}_{21}(\hat{t}) + (4 + B_7)\pi^2 c_{21}(\hat{t}) \\ + \frac{8}{3} B_8 \dot{a}_{11}(\hat{t}) &= 0 \end{aligned} \tag{17}$$

in which $a_{11}(\hat{t})$, $b_{11}(\hat{t})$ and $c_{21}(\hat{t})$ are introduced as:

$$\begin{aligned} a_{11}(\hat{t}) &= \alpha_{11} e^{st} \\ b_{11}(\hat{t}) &= \beta_{11} e^{st} \\ c_{21}(\hat{t}) &= \gamma_{21} e^{st} \end{aligned} \tag{18}$$

Substituting Equation (18) into Equation (17) gives:

$$\begin{bmatrix} s^2 + (1 + B_1)\pi^2 & 0 \\ 0 & \hat{\rho} s^2 + (1 + B_4)\pi^2 \\ \frac{8}{3} B_8 s & 0 \\ -\frac{8}{3} B_3 & \\ 0 & \\ B_9 s + (4 + B_7)\pi^2 \end{bmatrix} \begin{Bmatrix} \alpha \\ \beta \\ \gamma \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \end{Bmatrix} \tag{19}$$

Therefore, the complex frequencies of the system can be obtained by solving the following characteristic equation.

$$\det \begin{bmatrix} s^2 + (1 + B_1)\pi^2 & 0 \\ 0 & \hat{\rho} s^2 + (1 + B_4)\pi^2 \\ \frac{8}{3} B_8 s & 0 \\ -\frac{8}{3} B_3 & \\ 0 & \\ B_9 s + (4 + B_7)\pi^2 \end{bmatrix} = 0 \tag{20}$$

3. NUMERICAL RESULTS

The micro-plates with the specifications as given in Table 1 [33, 34] are considered for investigating the effects of length, width, ambient temperature and material properties on the quality factor.

According to complex frequency approach, The quality factors of thermo-elastic damping (Q_{TED}) for the cases in which ζ is small, can be achieved as follows [6]:

$$Q_{TED} = \frac{1}{2\zeta} \cong \frac{1}{2} \left| \frac{\text{Re}(\omega)}{\text{Im}(\omega)} \right| \tag{21}$$

In this study, the coefficient of linear thermal expansion is assumed to be constant.

Table 2 shows the evaluated frequency result for the first mode of displacements when there is no heat effect in vibrations which is in a good agreement with those of other researchers.

Figure 2 illustrates the quality factor versus length of micro-plate for the first mode of displacements at the constant ambient temperature ($T_0 = 300 \text{ K}$) for different material. It is clear from this figure that increment of the length of the micro-plates leads to a rise in the quality factor. Additionally, it can be seen that in the cases of Silicon-Carbide (SiC) and Si plates, increasing slope is much higher than the other materials.

TABLE 1. Material properties of micro-plates.

Parameters	Unit	Si	SiC	PolySilicon	Gold	Nickel
Young's modulus (E)	GPa	169	400	160	79	210
Poisson's ratio (ν)	---	0.28	0.185	0.22	0.44	0.31
Thermal conductivity (k)	w/mk	150	70	148	318	92
Density (ρ)	kg/m^3	2300	3200	2330	19320	8900
Specific heat at constant volume (C_v)	j/kgk	695	938	107	129	438
Coefficient of linear thermal expansion (α) $\times 10^{-6}$	k^{-1}	2.6	3	4.7	14.21	13

TABLE 2. Non-dimensional Natural frequency (ω) of micro-plate

Ref.	ω
Present	3.741
Bardell et al. [20]	3.555
Gorman [23]	3.555
Hyde et al. [26]	3.549
Dozio [35]	3.555
Du et al. [36]	3.554

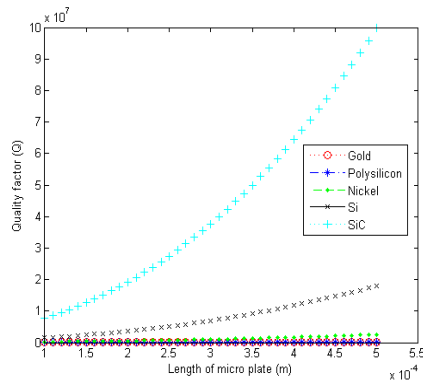


Figure 2. Thermo-elastic damping quality factor versus the length of the micro-plates of SiC, Si, nickel, polysilicon, and gold

The quality factors of thermo-elastic damping with respect to the variations of ambient temperature are depicted in Figure 1. As shown, increasing ambient temperature results in decreasing of Q_{TED} .

The variations of Q_{TED} for various values of plate widths are plotted in Figure 2 at the constant ambient temperature ($T_0 = 300 k$). It can be seen that as the width of the plate increases, the Q_{TED} decreases.

Figure 3, 6 and 7 depicted an attempt to magnify the lower region of Figures 2, 3, and 4 where Q_{TED} of gold and polysilicon are much lower than those of nickel, Si, and SiC and consequently not clear enough.

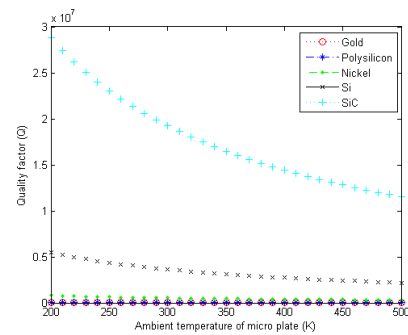


Figure 1. Effect of ambient temperatures on Q_{TED} of the micro-plates ($L=b=200\mu m$) of SiC, Si, nickel, polysilicon, and gold

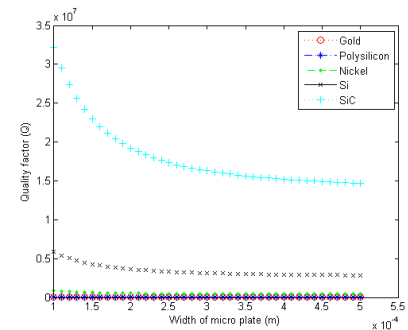


Figure 2. Thermo-elastic damping quality factor versus the width of the micro-plates of SiC, Si, nickel, polysilicon, and gold

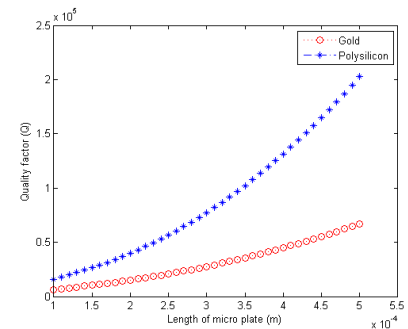


Figure 3. Q_{TED} versus the length of the micro-plates of polysilicon and gold

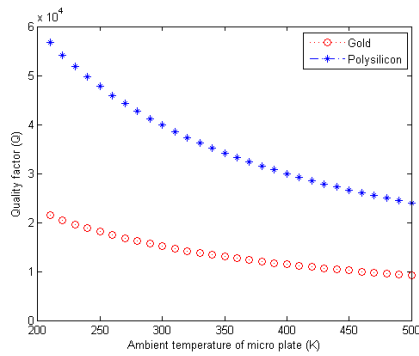


Figure 4. Effect of various ambient temperatures on Q_{TED} of the micro-plates ($L=b=200\mu m$) of polysilicon, and gold

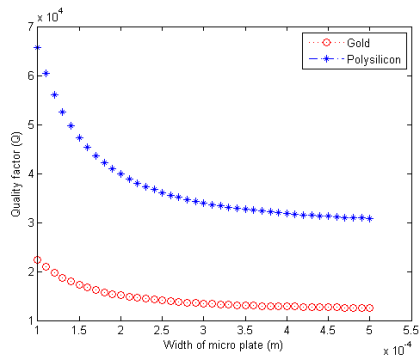


Figure 5. Thermo-elastic damping quality factor versus the width of the micro-plates of polysilicon and gold

Figure 8 depicts the first mode of displacement for fully clamped square micro-plate.

Table 3 presents the differences between Q_{TED} of in-plane and transversal vibrations for silicon fully clamped micro-plate under various temperatures. It can be seen that Q_{TED} s of in-plane vibrations are extremely higher than those of transversal vibration.

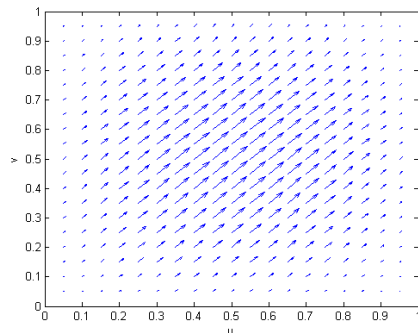


Figure 6. First mode of displacements of the micro-plate

TABLE 3. Differences between Q_{TED} in in-plane and transversal vibrations for silicon fully clamped micro-plate

T_0 (K)	Q_{TED} (in this paper)	Q_{TED} (Nayfeh, Younis [1])
100	213.5×10^5	$\approx 4.5 \times 10^5$
150	142.3×10^5	$\approx 3 \times 10^5$
300	71.1×10^5	$\approx 1.5 \times 10^5$

4. CONCLUSION

This paper studied in-plane vibration of a fully clamped rectangular micro-plate considering thermo-coupling effect. The governing equation of the micro-plate motion and heat conduction equation were extracted. Then, the non-dimensional forms of these equations were obtained. Afterwards, Galerkin method was applied to solve the equations. The complex fundamental Eigen frequency of the first mode of the micro-plate vibration was determined by solving the coupled ordinary equations and eventually quality factor of the system was achieved. The results indicate that Q_{TED} is decreased by increasing the width and ambient temperature of the micro-plate while it is increased by an increment of the length of the plate. Additionally, it is demonstrated that Q_{TED} of gold and poly-silicon plates change slightly over the width of the micro-plates in comparison with nickel, silicon, and Si-Carbide. Moreover, the obtained Q_{TED} of in-plane vibrations is presented in comparison to transversal vibrations and it is observed that Q_{TED} of in-plane vibrations is about 50 times higher than that of transversal vibrations.

It can be suggested that the structures with in-plane vibrations could be used instead of those with the transversal vibrations where the high value of the magnitude of vibration is not of require. The results offered to be employed in the design of MEMS waveguides and MEMS resonators with the high quality factor. Future work could be devoted to extending this study based on couple stress theory.

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Quality Factor of Free In-plane Vibration of a Fully Clamped Rectangular Micro-plate

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میرایی ترموالاستیک ، مکانیزم اتلافی قابل توجهی در میکروساختارهایی با فاکتور کیفیت بالا است. این مقاله میرایی ترموالاستیک ارتعاشات درون صفحه‌ای را در میکروصفحه مستطیلی کاملاً گیردار مورد مطالعه قرار می‌دهد. معادله حاکم بر ارتعاشات میکروصفحه مستطیلی و معادله هدایت حرارتی استخراج و روش گلرکین برای حل معادلات کوپل شده بکار گرفته شده‌است. سپس، برای میکروصفحات مسطیلی از جنس‌های مختلف، اثر ابعاد شامل طول و عرض و همچنین اثر دمای محیط بر فاکتور کیفیت مورد بررسی قرار گرفته‌است.

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