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Push Back Design in Two-element Deposits Incorporating Grade Uncertainty

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ABSTRACT

Push back design as a complex task, is one of the major steps in the open pit mines planning. Push backs can be generated by varying economic factors such as commodity price, mining cost, processing cost, etc. Another important issue in generating push backs is grade uncertainty, which can cause the problem be more complex. Conventional methods of push back design ignore grade uncertainty. To overcome this, "Grade Parameterization using Variance Algorithm" (GPVA) can be implemented. In this paper, an attempt was made to utilize GPVA in a hypothetical two-element deposit with the aim of minimization grade uncertainty effect on push backs design. Finally, the same example was solved using Whittle algorithm, the results indicate the superiority of the GPVA.

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1. INTRODUCTION

Planning long-term production in the open pit mines is one of the effective issues on mine liquidity. Also, long-term planning is a guideline for meeting short-term and mid-term objectives. In order to do long-term production planning, the ending area of the mine is divided into several growing pits or nests; then, these pits undergo some technical considerations about some parameters and are turned into pushbacks. Then, production planning is carried out on every one of the pushbacks so that the mine's comparative production plan can be achieved. In fact, the objective of pushbacks design is a presentation of long-term guidelines for the order of mining blocks of minerals in time and reaching the present-time maximum net value.

Design and planning of conventional mine production is a staged method to take into account the input parameters of one-element. Whereas in the two-element deposits, the considered parameters that make the project economical are related to the two-elements added to the complexity of the problem. The main reason for this complexity are an increase of the inputs and consequently increase of error in design because if the input parameters are considered to be certain, the carried out design will not be real, and the nature of uncertainty of input parameters in design has to be taken into account.

Dimitrakopoulos [1] classified the uncertainties of mining projects as follows:

- Uncertainty of the ore body model and related in situ grade variability and material type distribution.
- Uncertainty of technical mining specifications such as slope constraints, excavation capacities, etc.
- Uncertainty of economic issues including capital and operating costs, and commodity prices.

Grade uncertainty is among the most important factors causing discrepancy between targeted net present value and real situation.

Valee [2] reported that in the first years of operation after start-up, 60% of the surveyed mines had an average rate of production less than 70% of the designed capacity.

Since 1960, numerous algorithms have been introduced to design push backs according to the best ore in certain conditions. Lerchs and Grossman [3] used parameterization technique for designing push backs. Gershon [4] offered a method by which the ore quality and spatial condition of blocks were investigated for designing long-term production planning. Numerous

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researchers used parametric model for designing push backs [5-8]. Ramazan and Dagdelen [9] used waste minimization for designing push backs.

Also, a lot of researchers have studied pushback design and considering of uncertainty in long-term planning. They have tried to minimize the existing defects in the literature. Some of their studies are presented below:

Rahmanpour and Osanloo [10] used stimulation methods to plan production under grade uncertainty in a gold mine in Iran. The objective of this study was first to control quantity and quality of the factory input, and secondly to decrease distraction of the mine's short-term production planning from the objectives of the long-term plans and also from optimization of the production plan.

Smith [11] has used stochastic planning to plan long-term production in open pit mines under grade uncertainty conditions. The objective of the presented model has been determined as to maximize NPV and to simultaneously minimize distraction from production objectives.

Dimitrakopoulos and Ramazan [12] presented a linear planning model for risk-based planning in mines. The main point they considered in their modeling is that some parts of the ore deposit where grade had higher certainty be extracted in the earliest years of the mine's lifetime, and the parts with more uncertainty be extracted later when more information about them is achieved.

Gholamnejad et al. [13] evaluated long-term production planning under grade uncertainty. They presented a creative pattern whereby the use of a statistical logic, the grade variance of every block, with one coefficient, will be less than the average grade of the block. In this algorithm, a technique called "Grade Parameterization using Variance Algorithm" (GPVA) was applied. In this method, in addition to considering grade uncertainty, mining progresses were also planned.

Dimitrakopoulos and Ramazan [14] presented a model based on stochastic planning of integer numbers for optimization of production planning under grade uncertainty. In the presented model from the simulation studies too, probably ore deposit has been used, and the objective of this model, in addition to maximizing the present net value, has been to minimize distraction from production plan. The presented model was evaluated for two deposits of gold and copper, and at last, the achieved results were compared with the results achieved from traditional models. The results of this comparison showed that the presented model against the traditional model has had 10% increase for gold deposit and 25% increase for the copper deposit in their present net value.

Gholamnejad et al. [15] presented planning model of integer programing for push backs design under grade uncertainty in the open pit mines. In this model, the function of the blocks grade possibility distribution was considered as a stochastic input. Then, by the use of stochastic planning, its certain equivalent model that was a non-linear model was achieved. Considering the existing problems in solving non-linear problems in big dimensions, the non-linear model was made closer to a linear model. The objective of the presented model is to maximize the present net value.

Lamghari and Dimitrakopoulos [16] used Tabu Search Algorithm for push back design in open pit mines in the uncertain conditions. This method is classified as one of the meta-heuristic methods. One of the features of this method is its capability in solving big problems. This model, either, like other meta-heuristic models, might not lead to optimized solutions.

In 2013, another model was presented by Benndrof and Dimitrakopoulos [17] for long-term production planning in the Yandi iron ore mines in Australia. In this study, by the use of planning stochastic integer programing, by taking into account the grade uncertainty, the mine's long-term production planning is optimized. The objective function of this model is presented as some objectives and includes optimization of the pit's economic parameters, minimization of distraction from production purposes including tonnage, mineral quality, and also minimization of mining costs. In this model, the grade uncertainty in the production planning of the multi-element mines have been inserted which is one of the advantages of this model. Also, this model has been used in a real case and the achieved results have led to increasing the present net value of the project.

Dimitrakopoulos and Goodfellow [18] presented an algorithm for considering grade uncertainty in pushbacks design for multi-element deposits. The objective of the presented model is to consider grade uncertainty while minimizing the risk originated from the quantity of sent material to different destinations in designing the pushbacks. The presented model in one of the mines in Chile was evaluated and the results showed 35-61% of risk decrease in the quantities of the sent materials to different destinations.

In majority of the aforementioned researches, grade uncertainty effect was investigated only for one element deposits, which can result in an inaccurate push back design. In the present paper, to overcome this shortcoming, a new algorithm which is a development of the model proposed by Gholamnejad in 2007 for one element deposits, is proposed for two-element deposits.

2. PUSH BACK DESIGN METHODS

Whittle method is used for long-term production planning of open pit mines [19]. In this method, seven effective factors on the block value are included:

T_{Oi}: The total amount of ore in the block i.

G_i: Estimated grade of the i[th] block.
R: The proportion of the product recovered by processing the ore.
P: The price obtainable per unit of product less any delivery costs.
C_p: The extra cost per ton of mining and processing of a block as ore rather than treating it as waste
T_{Ri}: The total amount of rock (ore and waste) in the i[th] block.
Here, the block's economic value is calculated as follows:

$$BEV_i = \left(T_{Oi} * R * \bar{G}_i * P\right) - (T_{Oi} * C_p) - \left(T_{Ri} * C_m\right) \tag{1}$$

Whittle reduced the number of economic variables by dividing Equation (1) by C_m:

$$\frac{BEV_i}{C_m} = \left(T_{Oi} * R * \bar{G}_i * \frac{P}{C_m}\right) - (T_{Oi} * \frac{C_p}{C_m}) - \left(T_{Ri}\right) \tag{2}$$

$$V_i = \left(T_{Oi} * R * \bar{G}_i * \lambda\right) - (T_{Oi} * \theta) - \left(T_{Ri}\right) \tag{3}$$

In Equation (3), V_i is the value generated per unit mining cost, λ is the ratio of block price to the block mining cost and θ is the ratio of processing cost to mining cost. It is noted that θ is constant but λ is varying [19].

An innovative model GPVA was introduced by Gholamnejad et al. for long-term production planning under grade uncertainty (Equation (4)) [13].

$$BEV_i^{\,l} = TO_i * \left[\left(\bar{G}_i - n_l \sigma_i\right) * R * (P - r) - C_r\right] - \left(TR_i * C_m\right) \tag{4}$$

where: R is the recovery, $\bar{G}$ is the average grade of extracted block, σ_i is the grade variance of the i[th] block, P is the price, C_r is the milling cost, C_m is the mining cost, TO_i is the ore block tonnage, TR_i is the block tonnage, r is the refining cost, and n_1 is a constant number whose variation results in generating different pits. That it is calculated from Equation (5) [13]:

$$n_l \in \left[\left(0 \le n_l \le 1.96\right) \cap \left(n_l \le \frac{\bar{G}_i}{\sigma_i}\right)\right] \tag{5}$$

For example, if the average grade of a block (Kriging estimation) is 36% and its grade variance is 12%, n_1 allowable range is calculated as follows:

$$n \in \left[(0 \le n \le 1.96) \cap (n \le \frac{36}{12})\right] =$$
$$\left[(0 \le n \le 1.96) \cap (n \le 3)\right]$$
$$\Rightarrow 0 \le n \le 1.96$$

3. MODIFICATION OF GPVA FOR BEING APPLICABLE IN TWO-ELEMENT DEPOSITS

Block economic value (BEV) is a primary and essential need for push back design. Previous methods of constructing BEV are appropriate only for one-element deposits. The only method which is applicable for multi-element deposits is that of Osanloo and Ataei [20] model. In this model, an equivalent factor (f_{eq}) was defined by which BEV of multi-element deposits could be determined. In this way, BEV for two-element deposits can be calculated as follows: (Equation (6)):

$$BEV = TO * \left[R_1 (P_1 - r_1)\left(\bar{G}_1 + f_{eq} \bar{G}_2\right) - C_r\right] - (TR * C_m) \tag{6}$$

$$f_{eq} = \frac{R_2 (P_2 - r_2)}{R_1 (P_1 - r_1)} \tag{7}$$

In Equation (6), $\bar{G}_1$is grade of first element and $\bar{G}_2$ is grade of the second element.
For developing the new model, GPVA logic was incorporated in the BEV equation (Equation (6)) proposed by Osanloo and Ataei. In this way, one can study the effect of grade uncertainty into push back design for two–element deposits. (Equation (11))

$$BEV_i^{\,l} = TO * \left[R_1 (P_1 - r_1)\begin{pmatrix}\left(\bar{G}_{1i} - n_l \sigma_{1i}\right) + \\ f_{eq}\left(\bar{G}_{2i} - n_l \sigma_{2i}\right)\end{pmatrix} - C_r\right] - \left(TR * C_m\right) \tag{8}$$

$$\begin{aligned}
&\to \to \left(\bar{G}_{1i} - n_l \sigma_{1i}\right) + f_{eq}\left(\bar{G}_{2i} - n_l \sigma_{2i}\right) \\
&\to \to \left(\bar{G}_{1i} + f_{eq} \bar{G}_{2i}\right) - n_l (\sigma_{1i} + f_{eq} \sigma_{2i}) \\
&\to \to \left(\bar{G}_{1i} + f_{eq} \bar{G}_{2i}\right) = \bar{G}_{eq}
\end{aligned} \tag{9}$$

$$\to \to \ (\sigma_{1i} + f_{eq} \sigma_{2i}) = \sigma_{eq} \tag{10}$$

where: $\bar{G}_{eq}$ is the equivalent grade and σ_{eq} is the equivalent grade variance.

$$BEV_i^{\,l} = TO_i * \left[R_1 (P_1 - r_1)\left(\left(\bar{G}_{eq}\right)_i - n_l (\sigma_{eq})_i\right) - C_r\right] - \left(TR_i * C_m\right) \tag{11}$$

The process of grade uncertainty involvement in two-element deposits using the proposed method in this study is schematically shown in Figure 1.

4. APPLICATION OF THE PROPOSED MODEL IN A HYPOTHETICAL DEPOSIT

A hypothetical two-dimensional cross-section was taken into account for a lead and zinc deposit. Figures 2 and 3 show the grade block model for lead and zinc, respectively. Also, Figures 4 and 5 show their standard deviation (variance square root). Table 1 shows the input data parameters to calculate the economic value of each block.

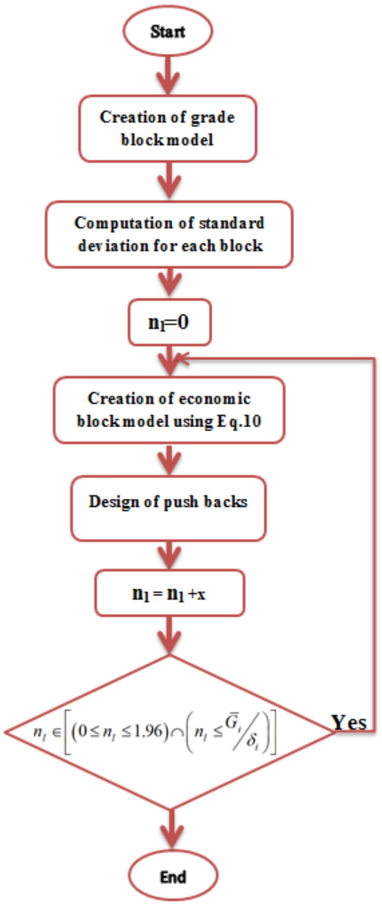

Figure1. Push back design process using modified GPVA

TABLE 1. Input data parameters

Description	Lead	Zinc	Unit
Recovery	85	90	%
Price (2015)	1787.82	1931.68	$/ton
Mining Cost	10	10	$/ton
Milling Cost	15	15	$/ton
Refining cost	100	100	$/ton
Discount rate	12	12	%

According to the economic parameters in Table 1, the equivalent factor is calculated using Equation (7):

$$f_{eq} = \frac{R_2(P_2 - r_2)}{R_1(P_1 - r_1)}$$

$$f_{eq} = \frac{0.85(1787.82 - 100)}{0.9(1931.68 - 100)} = 0.87$$

Using equivalent factor, Equations (9), and (10); equivalent grades and equivalent standard deviations are calculated for each block as shown in Figures 6 and 7.

According to the equivalent grades, economic parameters and two-dimensional Lerchs and Grossman method, final pit limit is determined, as shown in Figure 8.

In the next stage, push backs are calculated using Whittle for two-element deposits, as shown in Figure 9.

0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	1.1	1.7	1.9	1.9	3.6	4.1	4.1	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	1.2	1.3	1.8	2	2	4	4.9	5.5	3.2	3.6	4.3	3.9	0	0	0	0	0	0	0
0	0	0	0	0	0	1.1	1.3	1.3	2	1.9	3.6	5.7	4.9	3.7	5.8	6.4	4.9	3.6	0	0	0	0	0	0
0	0	0	0	0	0	1.4	1.2	1	1.6	1.8	2.3	3.8	3	2.4	6.5	5.1	3.7	2.6	0	0	0	0	0	0
0	0	0	0	0	0	0	1	1	1.1	2.8	3.2	3.3	2.3	2.1	2.2	3.1	2.6	1.2	0.5	0	0	0	0	0
0	0	0	0	0	0	0	1	1.6	2	3.6	3.1	2.8	1.9	1.7	1.4	2.4	2.1	1.7	0.8	0	0	0	0	0
0	0	0	0	0	0	0	0	2.2	2.4	2.6	2.4	2.8	3	1.2	0	0	0	1.8	1.3	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	3.7	3.7	0	0	0	0	0	2.4	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

Figure 2. Grade Block Model for Zinc

0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0.6	1.2	1.4	1.4	3.1	3.6	3.6	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0.7	0.8	1.3	1.5	1.5	3.5	4.4	5	2.7	3.1	3.8	3.4	0	0	0	0	0	0	0
0	0	0	0	0	0	0.6	0.8	0.8	1.5	1.4	3.1	5.2	4.4	3.2	5.3	5.9	4.4	3.1	0	0	0	0	0	0
0	0	0	0	0	0	0.9	0.7	0.5	1.1	1.3	1.8	3.3	2.5	1.9	6	4.6	3.2	2.1	0	0	0	0	0	0
0	0	0	0	0	0	0	0.5	0.5	0.6	2.3	2.7	2.8	1.8	1.6	1.7	2.6	2.1	0.7	0	0	0	0	0	0
0	0	0	0	0	0	0	0.5	1.1	1.5	3.1	2.6	2.3	1.4	1.2	0.9	1.9	1.6	1.2	0.3	0	0	0	0	0
0	0	0	0	0	0	0	0	1.7	1.9	2.1	1.9	2.3	2.5	0.7	0	0	0	1.3	0.8	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	2.7	2.7	0	0	0	0	0	1.4	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

Figure 3. Grade Block Model for Lead

0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0.1	0.6	0.1	0.25	0.13	0.22	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0.12	0.11	0.2	0.74	0.12	0.15	0.1	0.2	0.2	0.12	0.1	0.2	0	0	0	0	0	0	0
0	0	0	0	0	0	0.4	0.38	0.37	0.5	0.8	0.2	0.1	0.11	0.12	0.19	0.2	0.2	0.3	0	0	0	0	0	0
0	0	0	0	0	0	0.6	0.5	0.4	0.5	0.65	0.74	0.6	0.15	0.12	0.21	0.2	0.2	0.2	0	0	0	0	0	0
0	0	0	0	0	0	0	0.6	0.6	0.62	0.74	0.11	0.15	0.14	0.18	0.17	0.2	0.2	0.3	0	0	0	0	0	0
0	0	0	0	0	0	0	0.1	0.9	0.8	0.5	0.6	0.7	0.2	0.8	0.6	0.4	0.2	0.6	0.12	0	0	0	0	0
0	0	0	0	0	0	0	0	0.13	0.9	0.9	1.1	0.9	0.2	0.17	0	0	0	0.9	0.8	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	1.2	1.3	0	0	0	0	0	1.2	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

Figure 4. Zinc Standard Deviation for each Block

0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0.15	0.5	0.25	0.12	0.12	0.16	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0.25	0.17	0.3	0.4	0.33	0.47	0.1	0.2	0.4	0.14	0.2	0.2	0	0	0	0	0	0	0
0	0	0	0	0	0	0.3	0.28	0.27	0.4	0.45	0.6	0.3	0.6	0.6	0.18	0.8	0.6	0.4	0	0	0	0	0	0
0	0	0	0	0	0	0.5	0.4	0.3	0.4	0.55	0.64	0.5	0.6	0.9	0.6	0.5	0.7	0.8	0	0	0	0	0	0
0	0	0	0	0	0	0	0.5	0.5	0.52	0.64	1	0.4	0.3	0.7	0.4	0.5	0.7	0.9	0	0	0	0	0	0
0	0	0	0	0	0	0	0.9	1	0.9	0.7	0.55	0.65	0.6	0.7	0.5	0.8	0.6	0.9	0.5	0	0	0	0	0
0	0	0	0	0	0	0	0	0.9	1.2	0.6	0.3	0.7	1	1.1	0	0	0	0.8	0.7	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	1.3	1.3	0	0	0	0	0	1.5	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

Figure 5. Lead Standard Deviation for each Block

0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	1.62	2.74	3.12	3.12	6.3	7.232	7.23	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	1.81	2	2.93	3.31	3.31	7.05	8.728	9.85	5.549	6.297	7.6	6.9	0	0	0	0	0	0	0
0	0	0	0	0	0	1.62	2	2	3.31	3.12	6.3	10.22	8.73	6.484	10.41	12	8.7	6.3	0	0	0	0	0	0
0	0	0	0	0	0	2.18	1.81	1.44	2.56	2.93	3.87	6.671	5.18	4.053	11.72	9.1	6.5	4.4	0	0	0	0	0	0
0	0	0	0	0	0	0	1.44	1.44	1.62	4.8	5.55	5.736	3.87	3.492	3.679	5.4	4.4	1.8	0.5	0	0	0	0	0
0	0	0	0	0	0	0	1.44	2.56	3.31	6.3	5.36	4.801	3.12	2.744	2.183	4.1	3.5	2.7	1.06	0	0	0	0	0
0	0	0	0	0	0	0	0	3.68	4.05	4.43	4.05	4.801	5.18	1.809	0	0	0	2.9	2	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	6.049	6.05	0	0	0	0	0	3.62	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

Figure 6. Equivalent Grades for each Block

0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0.23	1.04	0.32	0.35	0.23	0.359	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0.34	0.26	0.46	1.09	0.41	0.56	0.187	0.37	0.548	0.242	0.3	0.3	0	0	0	0	0	0	0
0	0	0	0	0	0	0.66	0.62	0.6	0.85	1.19	0.72	0.361	0.63	0.642	0.347	0.9	0.7	0.6	0	0	0	0	0	0
0	0	0	0	0	0	1.04	0.85	0.66	0.85	1.13	1.3	1.035	0.67	0.903	0.732	0.7	0.8	0.9	0	0	0	0	0	0
0	0	0	0	0	0	0	1.04	1.04	1.07	1.3	0.98	0.498	0.4	0.789	0.518	0.6	0.8	1.1	0	0	0	0	0	0
0	0	0	0	0	0	0	0.88	1.77	1.58	1.11	1.08	1.266	0.72	1.409	1.035	1.1	0.7	1.4	0.56	0	0	0	0	0
0	0	0	0	0	0	0	0	0.91	1.94	1.42	1.36	1.509	1.07	1.127	0	0	0	1.6	1.41	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	2.331	2.43	0	0	0	0	0	2.51	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

Figure 7. Equivalent Standard Deviation for each Block

0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	1.62	2.74	3.12	3.12	6.3	7.232	7.23	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	1.81	2	2.93	3.31	3.31	7.05	8.728	9.85	5.549	6.297	7.6	6.9	0	0	0	0	0	0	0
0	0	0	0	0	0	1.62	2	2	3.31	3.12	6.3	10.22	8.73	6.484	10.41	12	8.7	6.3	0	0	0	0	0	0
0	0	0	0	0	0	2.18	1.81	1.44	2.56	2.93	3.87	6.671	5.18	4.053	11.72	9.1	6.5	4.4	0	0	0	0	0	0
0	0	0	0	0	0	0	1.44	1.44	1.62	4.8	5.55	5.736	3.87	3.492	3.679	5.4	4.4	1.8	0.5	0	0	0	0	0
0	0	0	0	0	0	0	1.44	2.56	3.31	6.3	5.36	4.801	3.12	2.744	2.183	4.1	3.5	2.7	1.06	0	0	0	0	0
0	0	0	0	0	0	0	0	3.68	4.05	4.43	4.05	4.801	5.18	1.809	0	0	0	2.9	2	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	6.049	6.05	0	0	0	0	0	3.62	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

Figure 8. Final pit limit using two-dimensional Lerchs and Grossman method

0	0	7	6	6	4	3	1	1	1	1	1	1	1	1	1	1	2	2	2	2	3	5	5	0
0	0	0	7	6	6	4	3	1	1	1	1	1	1	1	1	2	2	2	2	3	5	5	0	0
0	0	0	0	7	6	6	4	3	1	1	1	1	1	1	2	2	2	2	3	5	5	0	0	0
0	0	0	0	0	7	6	6	4	3	1	1	1	1	2	2	2	2	3	5	5	0	0	0	0
0	0	0	0	0	0	7	6	6	4	3	1	1	2	2	2	2	3	5	5	0	0	0	0	0
0	0	0	0	0	0	0	7	6	6	4	3	2	2	2	3	3	5	5	2	0	0	0	0	0
0	0	0	0	0	0	0	0	7	6	6	4	3	3	3	5	5	5	0	3.62	0	0	0	0	0
0	0	0	0	0	0	0	0	0	7	6	6	4	4	5	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	7	7	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

Figure 9. Push back design using Whittle algorithm

0	0	7	7	5	5	3	3	1	1	1	1	1	1	1	1	1	1	2	2	4	4	6	6	0
0	0	0	7	7	5	5	3	3	1	1	1	1	1	1	1	1	2	2	4	4	6	6	0	0
0	0	0	0	7	7	5	5	3	3	1	1	1	1	1	1	2	2	4	4	6	6	0	0	0
0	0	0	0	0	7	7	5	5	3	3	1	1	1	1	2	2	4	4	6	6	0	0	0	0
0	0	0	0	0	0	7	7	5	5	3	3	1	1	2	2	4	4	6	6	0	0	0	0	0
0	0	0	0	0	0	0	7	7	5	5	3	3	3	3	4	4	6	6	0	0	0	0	0	0
0	0	0	0	0	0	0	0	7	7	5	5	6	6	6	6	6	6	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	7	7	7	7	7	7	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	7	7	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

Figure 10. Push back design using modified GPVA

A total of 7 push backs are obtained using Whittle method. The numbers in each block show the push back number.

In the next stage, corresponding push backs are calculated using proposed method in two-element deposits, as shown in Figure 10.

In order to compare the risk of grade uncertainty for each of the above methods, we take into account a number which reflects the corresponding grade uncertainty for the total blocks of its pit. Standard deviation is an acceptable indicator for showing grade uncertainty for each block. To this end, "Push Back Risk Indicator" (PRI) is defined for each push back as follows [13]:

$$PRI_K = \sum_{i=1}^{n_k} \sigma_i^2 + \sum_{i=1}^{n_k} \sum_{j=1}^{n_k} \operatorname{cov}(i, j), i \neq j \tag{12}$$

where: PRI_K: Undiscounted risk indicator for each push back, K is the push back number, n_k is the total number of push backs, σ_i^2 is the variance of the ith block in each push back, and $\operatorname{cov}(i, j)$ is the covariance between block i and block j within the kth push back.

Since the mining risk in premier years is more critical than next, the risk in each push back must be discounted so that the discount rate is dependent on the life of each pit. "Discounted Push back Risk Indicator" (DPRI) is calculated using Equation (13) [13]:

$$DPRI = \sum_{K=1}^{K} \frac{PRI_k}{(1+i_k)^{t_k}} \tag{13}$$

where: t_k is the life of each push back in years, and i_k is the effective interest rate for the kth push back.

Assume that the annual mining capacity is 5 blocks and the interest rate is 12%, then Tables 2 and 3 show the push back characteristics using both mentioned methods:

TABLE 2. Push back Characteristics generated by Whittle Method

Push back No.	Ore Block Numbers	Waste Block Numbers	Push back life time (Yr)	PRI	*DPRI*	Average of equivalent grades	Net value (M$)	NPV (M$)
1	18	12	6	8.84	4.48	3.54	5.24	2.65
2	14	9	4.6	5.42	3.2	4.39	5.89	1.77
3	12	4	3.2	9.07	6.31	3.05	1.6	0.33
4	7	2	1.8	7.42	6.05	2.96	0.82	0.14
5	7	9	3.2	6.67	4.64	1.38	0.56	0.07
6	11	5	3.2	11.92	8.30	1.86	0.75	0.06
7	6	4	2	20.39	16.26	2.23	0.73	0.05
Total	**75**	**45**	**24**	**69.75**	**49.26**	**19.44**	**15.62**	**5.08**

TABLE 3. Push back Characteristics generated by the proposed method

Push back No.	Ore Block Numbers	Waste Block Numbers	Push back life time (Yr)	PRI	*DPRI*	Average of equivalent grades	Net value (M$)	NPV (M$)
1	17	13	6	4.41	2.23	3.71	5.71	2.89
2	6	4	2	2.46	1.96	5.21	3.56	1.43
3	12	2	2.8	9.61	6.99	3.03	1.16	0.34
4	6	6	2.4	2.75	2.10	3.30	2.01	0.45
5	10	4	2.8	7.31	5.32	2.13	0.84	0.13
6	9	9	3.6	9.51	6.32	1.72	0.82	0.09
7	15	7	4.4	33.7	20.46	2.30	1.50	0.10
Total	**75**	**45**	**24**	**69.75**	**45.40**	**21.43**	**15.62**	**5.45**

As shown in Tables 2 and 3, DPRI is 49.26 and 45.40 using Whittle and proposed methods, respectively, showing that designing push back gives lesser risk for long-term production planning using the proposed method (Figure 11).

5. CONCLUSION

In this article, the feasibility of application of the modified GPVA in push back design for a two-element deposit was studied. The main findings of this research work are as follows:

- Proposed method can create push backs with lesser risk compared to whittle method.
- Average sum of equivalent grades of each push back for the proposed method is higher than Whittle method (Figure 12).

Net value of the push back designed by the proposed method in the early years is higher than that of the Whittle method which result in improvement of the project overall NPV (Figure (13)).

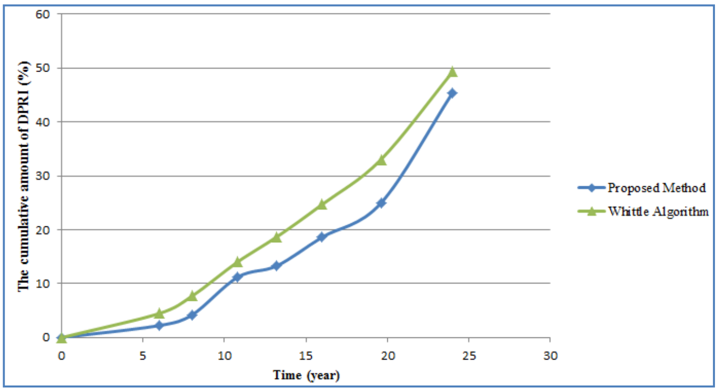

Figure 11. The cumulative DPRI vs time obtained from Whittle and modified GPVA

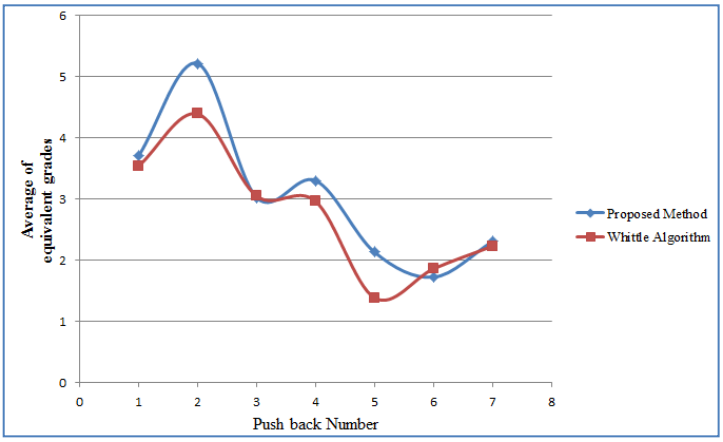

Figure 12. Average of the equivalent grades vs push back Number

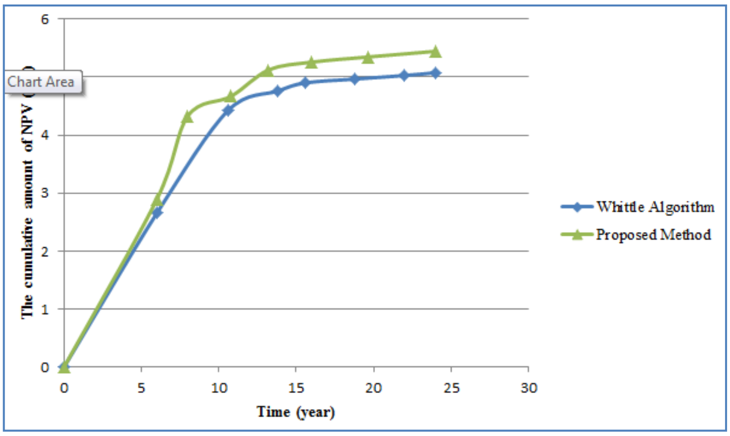

Figure 13. The cumulative NPV vs time obtained from Whittle and modified GPVA

6. REFERENCES

1. Dimitrakopoulos, R., "Conditional simulation algorithms for modelling orebody uncertainty in open pit optimisation", ***International Journal of Surface Mining, Reclamation and Environment***, Vol. 12, No. 4, (1998), 173-179.
2. Vallee, M., "Mineral resource+ engineering, economic and legal feasibility= ore reserve", ***CIM bulletin***, Vol. 93, No. 1038, (2000), 53-61.
3. Lerchs, H. and FI, G., "Optimum design of open-pit mines", in Operations Research, Inst Operations Research Management Sciences 901 Elkridge Landing Rd, Ste 400, Linthicum Hts, MD 21090-2909. Vol. 12, (1964), B59.
4. Gershon, M., "An open-pit production scheduler: Algorithm and implementation", ***Min. Eng.(Littleton, Colo.);(United States)***, Vol. 39, No. 8, (1987).
5. Dagdelen, K. and François-Bongarçon, D., "Towards the complete double parameterization of recovered reserves in open pit mining", in Proceedings of 17th international APCOM symposium., (1982), 288-296.
6. François-Bongarçon, D. and Guibal, D., "Algorithms for parameterizing reserves under different geometrical constraints", in Proc. 17th symposium on the application of computers and operations research in the mineral industries (APCOM),(New York: AIME, (1982), 297-309.
7. Wang, Q. and Sevim, H., "Alternative to parameterization in finding a series of maximum-metal pits for production planning", ***Mining Engineering***, Vol. 47, No. 2, (1995), 178-182.
8. Osanloo, M. and Ataei, M., "Using 2d lerchs and grossmann algorithm to design final pit limits of sungun copper deposit of iran", ***International Journal of Engineering***, Vol. 13, No. 4, (2000), 81-89.
9. Ramazan, S. and Dagdelen, K., "A new push back design algorithm in open pit mining", in Proceedings of 17th MPES conference, Calgary, Canada., (1998), 119-124.
10. Rahmanpour, M. and Osanloo, M., "Resilient decision making in open pit short-term production planning in presence of geologic uncertainty", ***International Journal of Engineering-Transactions A: Basics***, Vol. 29, No. 7, (2016), 1022.
11. Smith, M.L., "Integrating conditional simulation and stochastic programming: An application in production scheduling", ***Computer Application in the Materials Industries***, (2001), 203-208.
12. Dimitrakopoulos, R. and Ramazan, S., "Uncertainty based production scheduling in open pit mining", ***SME Transactions***, Vol. 316, (2004).
13. Gholamnejad, J. and Osanloo, M., "Incorporation of ore grade uncertainty into the push back design process", ***Journal of the South African Institute of Mining & Metallurgy***, Vol. 107, No. 3, (2007), 177-183.
14. Dimitrakopoulos, R. and Ramazan, S., "Stochastic integer programming for optimising long term production schedules of open pit mines: Methods, application and value of stochastic solutions", ***Mining Technology***, Vol. 117, No. 4, (2008), 155-160.
15. Gholamnejad, J., Osanloo, M. and Khorram, E., "A chance constrained integer programming model for open pit long-term production planning", ***International Journal of Engineering Transactions A: Basics***, Vol. 4, No. 21, (2008), 407-418.
16. Lamghari, A. and Dimitrakopoulos, R., "A diversified tabu search approach for the open-pit mine production scheduling problem with metal uncertainty", ***European Journal of Operational Research***, Vol. 222, No. 3, (2012), 642-652.
17. Benndorf, J. and Dimitrakopoulos, R., "Stochastic long-term production scheduling of iron ore deposits: Integrating joint multi-element geological uncertainty", ***Journal of Mining Science***, Vol. 49, No. 1, (2013), 68-81.
18. Goodfellow, R. and Dimitrakopoulos, R., "Algorithmic integration of geological uncertainty in pushback designs for complex multiprocess open pit mines", ***Mining Technology***, Vol. 122, No. 2, (2013), 67-77.
19. Whittle, J., "A decade of open pit mine planning and optimization-the craft of turning algorithms into packages", (1999).
20. Osanloo, M. and Ataei, M., "Using equivalent grade factors to find the optimum cut-off grades of multiple metal deposits", ***Minerals Engineering***, Vol. 16, No. 8, (2003), 771-776.

Push Back Design in Two-element Deposits Incorporating Grade Uncertainty

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چکیده

طراحی پوش‌بک به عنوان یک امر پیچیده، یکی از مراحل اصلی طراحی معادن روباز است، پوش‌بک‌ها را می‌توان با استفاده از فاکتورهای اقتصادی همچون قیمت ماده معدنی، هزینه معدنکاری، هزینه فرآوری و غیره ایجاد کرد. از مسائل مهم دیگر در ایجاد پوش‌بک‌ها عدم‌قطعیت عیار می‌باشد، که می‌تواند مسئله طراحی را پیچیده‌تر کند. در روش‌های سنتی طراحی پوش‌بک، عدم‌قطعیت عیار در نظر گرفته نمی‌شود. برای غلبه بر این مشکل، می‌توان الگوریتم پارامترسازی عیار با استفاده از واریانس (GPVA) را اجرا کرد. در این مقاله، تلاش شده تا در یک ذخیره دو عنصره فرضی با استفاده از روش GPVA ، طراحی پوش‌بک‌ها با هدف کمینه سازی اثر عدم‌قطعیت عیار صورت پذیرد. در نهایت، همین مسئله با استفاده از الگوریتم ویتل نیز حل شد که نتایج حاصله نشان‌دهنده برتری الگوریتم GPVA می‌باشد.

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