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An Investigation on the Effects of Gas Pressure Drop in Heat Exchangers on Dynamics of a Free Piston Stirling Engine

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ABSTRACT

This paper is devoted to study the effects of pressure drop in heat exchangers on the dynamics of a free piston Stirling engine. First, the dynamic equations governing the pistons as well as the gas pressure equations for hot and cold spaces of the engine are extracted. Then, by substituting the obtained pressure equations into the dynamic relationships the final nonlinear dynamic equations governing the free piston Stirling engine are acquired. Next, effects of the gas pressure drop in heat exchangers on maximum strokes of the pistons and their velocities and accelerations are investigated. Furthermore, influences of pressure drop increase in the heat exchangers on maximum and minimum gas volume and pressure in both hot and cold spaces are evaluated. Finally, the trend of variations of work and power corresponding to the increase of pressure drop in the heat exchangers are studied. Based on the obtained results of this paper, the assumption of uniform pressure distribution in the engine cylinder (as used in the Schmidt theory) causes some errors in predicting the dynamic behavior of the free piston hot-air engines. Besides, the increase of pressure drop in the heat exchangers results in deteriorating the dynamic performance of the engine.

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NOMENCLATURE

A	Cross sectional area of the piston and displacer (m^2)	m_d	Mass of displacer (kg)
A_r	Cross sectional area of the displacer rod (m^2)	m_e	Mass of the gas in the hot space (kg)
a_x	Accelerartion of power piston (m/s^2)	m_p	Mass of the power piston (kg)
a_y	Accelerartion of displacer piston (m/s^2)	p_c	Pressure in the cold space(N/m^2)
d^p	Particle diameter (μm)	p_e	Pressure in the hot space(N/m^2)
f_k^{eq}	Equilibrium distribution function	P_0	Initial pressure of working gas(N/m^2)
g	Gravity(m/s^2)	T_e	Gas temperature in expansion space (K)
b_d	Damping coefficient of the displacer piston ($N\ s\ m^{-1}$)	T_c	Gas temperature in compression space (K)
b_p	Damping coefficient of the power piston ($N\ s\ m^{-1}$)	V_e	Volume of the expansion space (m^3)
d_h	Hydrolic dynamic (m)	V_{eo}	Initial volume of expansion space (m^3)
f	Reynolds friction factor	V_c	Volume of compression space (m^3)
G	Gas mass flux(kg/m^2s)	V_{co}	Initial volume of compression space (m^3)
K_d	Spring stiffness of displacer ($N\ m^{-1}$)	v_x	Displacer piston velocity(m/s)
K_p	Spring stiffness of power piston ($N\ m^{-1}$)	v_y	power piston velocity(m/s)
m	Total mass of the gas in the engine (kg)	x	Displacer piston displacement (m)
m_c	Mass of the gas in the cold space (kg)	y	Power piston displacement (m)

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1. INTRODUCTION

In the modern world, efficient utilization of the various new forms of energy has evolved into one of the concerns for researchers. This matter has caused the scientists endeavors towards presenting innovative and more efficient methods for the usage of renewable energies to increase day by day. Solar Stirling engines are one of the ways that can be put to use to convert solar energy to other forms of energy. Solar Stirling engines consist of the two general classes of kinematic and dynamic engines [1]. The dynamic part of the Solar Stirling engines is categorized into the two main groups of fluidyne and free piston [2].The structure of free piston Stirling engines is in a way that unlike the kinematic type, pistons are not connected to each other by links. This issue has engendered various complexities in the manufacturing and commission of this type of engines. The stated complexities have continuously encouraged researchers to strive toward improving the presented theories.

The earliest presented theory for the Stirling engines is Schmidt's theory. Schmidt's theory is based on linear relations. In this theory, it is assumed that the pressure changes in the hot and cold spaces are equal. Additionally, the pistons movements are assumed to be on a sinusoidal pattern [3-6].

The fundamentals and basis of Schmidt's theory are based on kinematic engines and this has perpetuated the inability to have more accurate analyses on hot-air free-piston engines. Therefore, researchers have constructed various theories in order to better analyze and design this type of engines. The presented theories can be classified into 2 general categories, namely, linear and nonlinear theories. In the linear class, unlike Schmidt's theory, the stability and instability of the pistons movement has been discussed. Riofrio et al. [7] have inspected and analyzed free-piston Stirling engines. During this research, the behavior of power and displacer pistons has been analyzed based on the instability law of energy-producing systems. Besides, in this research, the instantaneous gas pressure was considered as constant throughout the cylinder. The achieved result indicates that the increased instability of the pistons movement increases the engine efficiency. Zare et al. [8] applied a genetic algorithm (GA) to analyze a free piston Stirling engine based on a desired frequency and linearization of the dynamic equations around an equilibrium state. Moreover, in this investigation the gas pressure was assumed to be the same in both hot and cold spaces. Then, they selected positions of closed-loop poles of the engine system such that the desired frequency was acquired. Unknown design parameters of the engine were thus found via an optimization scheme using GA. In the sequel, the proposed frequency-based design technique and the modeling approach were validated experimentally.

The second method for analyzing FPSEs is the nonlinear method. In this method, the behavior of some of the composing elements in the FPSEs is assumed to be nonlinear. A subject of great importance occurring while using the nonlinear method is the issue of limited cycle. On the other hand, in the recent nonlinear analysis the effect of gas pressure drop between the hot and cold spaces was not considered. Karabulut [9] conducted an analyticl study on the FPSEs behavior based on nonlinear relations. The performed analysis is conducted on the basis of the two hypotheses of closed and open thermodynamic cycles. It should be noted that on the performed research the terms of the limited cycle existence are neglected. Tavakolpour-Saleh et al. [10] applied a perturbation method to investigate of a FPSE possessing nonlinear springs. The technique of multiple-scale was employed in this investigation to achieve the final solutions of the governing nonlinear differential equations.

Until now, a grand number of researches have been conducted on the hot air engines. Tavakolpour et al. [11] proceeded towards the construction of a low-temperature hot-air engine where the cold source temperature is $20^{\circ C}$ and the hot source temperature is $100^{\circ C}$. This engine is equipped with a flat solar collector and is designed and created with two gamma cylinders. The mathematical model is extracted on the basis of limited dimension thermodynamic fundamentals and dimensionless variables where the effects of regeneration efficiency and the real gas temperature in the expansion and compression spaces are considered. Furthermore, the calculation of work and phase angle is performed based on Schmidt's theory. Jokar and Tavakolpour-Saleh [12] constructed an intelligent solar Stirling engine. This engine consists of a controllable displacer piston and a liquid power piston which pumps the water up to a designated height utilizing solar energy. During this research, the desired velocity for the displacer piston is controlled by an active control unit. Tavakolpour and Jokar [13] proceeded to control an intelligent solar Stirling pump using a neural network. Penswick et al. [14] studied and analyzed NASA models constructed based on each engine's parameters and proceeded to evaluate the power and work for free-piston engines afterwards. Minassians and Sanders [15] extracted the governing thermodynamic and dynamic equations on a hot-air engine and have analyzed the achieved linear equations afterwards. Subsequently, they proceeded to design and create 3-phased free-piston hot-air diaphragmatic engines in moderate temperature and followed it by comparing the experimental and analytical data.

Based on the outlined literature there was no published paper in which a dynamic model of a FPSE is presented taking into account the effect of gas pressure drop between the hot and cold spaces. In this research, unlike the common theories regarding the FPSEs analysis such as Schmidt's theory, the fluid pressures in both hot and cold spaces of the engine are not considered to be equal. In other words, the pressure drop that occurs between the hot and cold spaces of the engine as a result of the gas transfer between the two regions has a direct effect on the dynamical performance of the engine. The consideration of this pressure drop in the governing equations engenders a more nonlinear behavior in these engines. Nevertheless, during the following research, the equations governing the power and displacer pistons movement are extracted first and the effect of the existing pressure difference between the cold and hot spaces is accounted for in the dynamic equations. Next, the effect of pressure variations between the cold and hot regions on the engine dynamical performance is considered. It is important to note that during this analysis, a stable limit cycle is considered to be existed for the power and displacer pistons movement.

2. FREE PISTON STIRLING ENGINE

FPSEs generally include the displacer and power pistons, a cylinder and a set of springs (Figure 1). The displacer piston function is to transport the gas inside the cylinder from the cold space to the hot space and vice-versa. Moreover, in the FPSEs, the function of power generation is handled by the power piston. It should be noted that the FPSEs operate on Stirling thermodynamic cycle (Figure 2). The operation principle of a Stirling cycle is as follows (Figure 2) [16]:

-An isothermal process where heat is transferred from the heat source to the working fluid (Process 2-1).

-An isochoric process where heat is transferred from the working fluid to the regenerator (Process 3-2).

-An isothermal process where heat is transferred from the working fluid to the heat sink (Process 4-3).

-An isochoric process where heat is transferred from the regenerator to the working fluid (Process 1-4).

3. MATHEMATICAL BACKGROUND

The first step to analyze the dynamic behavior of the FPSE is that the dynamical equations governing the pistons should be extracted. According to figure 1 and Newton's first law, the dynamical equations governing the power and displacer pistons are described as follow [1]:

$$M_{\mathrm{p}}\ddot{y} + b_{\mathrm{p}}\dot{y} + K_{\mathrm{p}}y = -(p_c - p_0)(A - A_{\mathrm{r}}) \quad (1)$$

$$M_{\mathrm{d}}\ddot{x} + b_{\mathrm{d}}\dot{x} + K_{\mathrm{d}}x = (p_e - p_{\mathrm{c}})A + (p_0 - p_{\mathrm{c}})A_{\mathrm{r}} \quad (2)$$

According to Figure 1, the instantaneous volumes of hot and cold regions are written as follows:

$$V_e = V_{e0} - Ax \quad (3)$$

$$V_c = V_{c0} - (A - A_r)(y - x) \quad (4)$$

By differentiation from Equations (3) and (4), the variations of hot and cold spaces volumes are achieved as follow:

$$\dot{V}_e = -A\dot{x} \quad (5)$$

$$\dot{V}_c = -(A - A_r)(\dot{y} - \dot{x}) \quad (6)$$

Unlike the assumptions in Schmidt's theory, throughout this research the pressure drop across the working fluid transportation between the hot and cold spaces of the engine are considered. The pressure drop is caused by the gas movement inside the regenerator. Accordingly, the frictional drag force created by the gas's movement can be described below [17]:

$$F' = \frac{2fG^2V_q}{d_h\rho R_e} \quad (7)$$

where the Reynolds number is equal to:

$$R_e = \frac{Gd_h}{\mu} \quad (8)$$

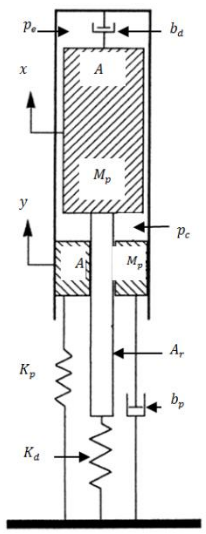

Figure 1. The view of a free-piston Stirling engine

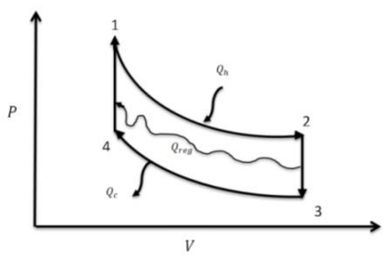

Figure 2. The Stirling cycle

The created frictional drag force is numerically equal and directionally opposite of the force created by the gas's pressure drop between the hot and cold spaces in the engine. Accordingly, the relation between the two forces can be described as:

$$F' + \Delta p A' = 0 \quad (9)$$

As a result, the pressure difference can be demonstrated as [16]:

$$\Delta p = -\frac{2fG^2V_q}{d_h \rho R_e A'} \quad (10)$$

Furthermore, Equation (10) can be illustrated as:

$$\Delta p = -R_q\dot{V}_q \quad (11)$$

where:

$$\dot{V}_q = \dot{V}_c - \dot{V}_e \quad (12)$$

It should be noted that R_q stands for all of the positive values in Equation (10).

Accordingly, the pressure relation between the two hot and cold regions can be determined as follow:

$$p_e = p_c + \Delta p \quad (13)$$

According to Equations (5), (6), (11) and (12) the pressure drop between the hot and cold spaces is obtained as follows:

$$\Delta p = R_q(-(A - A_r)(\dot{y} - \dot{x}) + A\dot{x}) \quad (14)$$

By assuming that the gas inside the cylinder behaves as an ideal gas, the gas mass inside the cold and hot spaces of the hot air engine can be acquired as:

$$m_e = \frac{p_e V_e}{RT_e} \quad (15)$$

$$m_c = \frac{p_c V_c}{RT_c} \quad (16)$$

Consequently, the total miss inside the cylinder is calculated using Equations (13), (15) and (16):

$$m = m_e + m_c = \frac{(p_c + \Delta p)V_e}{RT_e} + \frac{p_c V_c}{RT_c} \quad (17)$$

Accordingly, the working fluid pressure inside the cold region can be described as follow:

$$p_c = \left(m - \frac{\Delta p V_e}{RT_e}\right)\left(\frac{V_e}{RT_e} + \frac{V_c}{RT_c}\right)^{-1} \quad (18)$$

According to Equations (3) and (4), $m\left(\frac{V_e}{RT_e} + \frac{V_c}{RT_c}\right)$ can be rewritten as:

$$m\left(\frac{V_e}{RT_e} + \frac{V_c}{RT_c}\right)^{-1} = \frac{mRT_eT_c}{\alpha_1 x + \alpha_2 y + \alpha_3} \quad (19)$$

where

$$\alpha_1 = T_e(A - A_r) - AT_c \quad (20)$$

$$\alpha_2 = T_e(A - A_r) \quad (21)$$

$$\alpha_3 = V_{e0}T_c + V_{c0}T_e \quad (22)$$

Moreover, by substituting Equations (3), (4) and (12) in $\left(\frac{\Delta p V_e}{RT_e}\right)\left(\frac{V_e}{RT_e} + \frac{V_c}{RT_c}\right)^{-1}$ the second part of equation (18) can be written as:

$$\left(\frac{\Delta p V_e}{RT_e}\right)\left(\frac{V_e}{RT_e} + \frac{V_c}{RT_c}\right)^{-1} = \frac{\beta_1\dot{x} + \beta_2\dot{y} + \beta_3\dot{x}x + \beta_4\dot{y}x}{\alpha_1 x + \alpha_2 y + \alpha_3} \quad (23)$$

where:

$$\beta_1 = -R_q(2A - A_r)V_{e0}T_c \quad (24)$$

$$\beta_2 = R_q(A - A_r)V_{e0}T_c \quad (25)$$

$$\beta\_2 = R\_q\ (2A - A\_r\ )AT\_c \quad (26)$$

$$\beta_2 = -R_q(A - A_r)AT_c \quad (27)$$

According to Equations (19) and (23), the instantaneous gas pressure in the cold region can be written as below:

$$p_c = \frac{mRT_eT_c}{\alpha_1 x + \alpha_2 y + \alpha_3} \times \ (1 + \beta_1\dot{x} + \beta_2\dot{y} + \beta_3\dot{x}x + \beta_4\dot{y}x) \quad (28)$$

Beside, by accordance to Equation (13), the pressure variations in the hot space can be calculated as:

$$p_e = \frac{mRT_eT_c}{\alpha_1 x + \alpha_2 y + \alpha_3} \times (1 + \beta_1\dot{x} + \beta_2\dot{y} + \beta_3\dot{x}x + \beta_4\dot{y}x) + R_q(-(A - A_r)(\dot{y} - \dot{x}) + A\dot{x}) \quad (29)$$

The displacement, velocity and acceleration of the power and displacer pistons are equal to 0 while in static equilibrium:

$$\Delta p = R_q(-(A - A_r)(\dot{y} - \dot{x}) + A\dot{x}) = p_e - p_c = 0 \quad (30)$$

Accordingly, according to Equation (2) for the displacer piston in equilibrium we have:

$$A(p_c - p_e) + A_r(p_0 - p_c) = 0 \quad (31)$$

According to Equation (30), in static equilibrium, the pressure gradient is equal to zero. As a result, $(p_c - p_e)$ in Equation (31) is zero. Therefore, we can proceed to the following relation from Equation (31) in static equilibrium:

$$p_0 = p_c \quad (32)$$

Therefore, the initial pressure relation can be described as:

$$p_0 = \frac{mRT_eT_c}{V_{e0}T_c + V_{c0}T_e} \quad (33)$$

By substituting the achieved equations for the instantaneous gas pressure in the cold and hot spaces and also the equations governing the pressure gradient and initial pressure, the final dynamical equations governing the pistons (Equations (1) and (2)) can be rewritten as:

$$M_d\ddot{x} = R_q(A - A_r)A\dot{y} + \left(-R_q(2A - A_r)A - b_d\right)\dot{x} - \quad (34)$$

$$\left(\left(\frac{mRT_eT_cA_r}{M_d}\right)\frac{\alpha_3(1+\beta_1\dot{x}+\beta_2\dot{y}+\beta_3\dot{x}x+\beta_4\dot{y}x)}{\alpha_3(\alpha_1x+\alpha_2y+\alpha_3)}\right) - K_dx + \frac{mRT_eT_cA_r}{M_d}\left(\frac{1}{\alpha_3}\right)$$

$$M_p\ddot{y} = (A-A_r)mRT_eT_c(\left(\frac{1}{\alpha_3}\right) - \frac{\alpha_3(1+\beta_1\dot{x}+\beta_2\dot{y}+\beta_3\dot{x}x+\beta_4\dot{y}x)}{\alpha_3(\alpha_1x+\alpha_2y+\alpha_3)}) - K_py - b_p\dot{y} \quad (35)$$

4. SOLUTION METHOD OF DYNAMIC EQUATIONS

According to the nonlinear Equations (34) and (35), the numerical method of central difference [8] is recommended for achieving the pistons displacement, velocity and acceleration. The nonlinear dynamical equations for the system can be shown as:

$$[m]\ddot{\vec{x}} + [c]\dot{\vec{x}} + [k]\vec{x} = \vec{F} \quad (36)$$

where the matrices of mass, damping, spring stiffness and force can be described as:

$$[M] = \begin{bmatrix} M_d & 0 \\ 0 & M_p \end{bmatrix} \quad (37)$$

$$[c] = \begin{bmatrix} (R_q(2A-A_r)Ab_d) & -R_q(A-A_r)A \\ 0 & b_p \end{bmatrix} \quad (38)$$

$$[k] = \begin{bmatrix} K_d & 0 \\ 0 & K_p \end{bmatrix} \quad (39)$$

$$\vec{F} = \begin{bmatrix} -A_r\frac{mRT_eT_c}{M_d}\left(\left(\frac{1}{\alpha_3}\right)-\gamma\right) \\ -(A-A_r)mRT_eT_c\left(\left(\frac{1}{\alpha_3}\right)-\gamma\right) \end{bmatrix} \quad (40)$$

where, γ is equal to:

$$\gamma = \frac{\alpha_3(1+\beta_1\dot{x}+\beta_2\dot{y}+\beta_3\dot{x}x+\beta_4\dot{y}x)}{\alpha_3(\alpha_1x+\alpha_2y+\alpha_3)} \quad (41)$$

According to the finite difference method, the matrix for the movement variations for the power and displacer pistons can be achieved as below [8]:

$$\begin{bmatrix}\vec{x}_{i+1}\\ \vec{y}_{i+1}\end{bmatrix} = \left[\frac{1}{(\Delta t)^2}[m] + \frac{1}{2\Delta t}[c]\right]^{-1}\left[\vec{F}_i - \left([k] - \frac{2}{(\Delta t)^2}[m]\right)\begin{bmatrix}\vec{x}_i\\ \vec{y}_i\end{bmatrix} - \left(\frac{1}{(\Delta t)^2}[m] - \frac{1}{2\Delta t}[c]\right)\begin{bmatrix}\vec{x}_{i-1}\\ \vec{y}_{i-1}\end{bmatrix}\right] \quad (42)$$

Accordingly, the difference equations for the relations (34) and (35) can be described as:

$$\vec{x}_{i+1} = \frac{\delta_1}{\delta_4}(A_r\frac{mRT_eT_c}{M_d}\left(\gamma_i - \left(\frac{1}{\alpha_3}\right)\right) + \left(\frac{2M_d}{(\Delta t)^2} - K_d\right)\vec{x}_i + \left(\frac{M_d-2R_q(2A-A_r)Ab_d}{(\Delta t)^2}\right)\vec{x}_{i-1} + \frac{R_q(A-A_r)A}{2\Delta t}\vec{y}_{i-1}) + \frac{\delta_2}{\delta_4}((A-A_r)mRT_eT_c\left(\gamma_i - \left(\frac{1}{\alpha_3}\right)\right) + \left(\frac{2M_p}{(\Delta t)^2} - K_p\right)\vec{y}_i + \left(\frac{2b_p}{\Delta t} - \frac{M_p}{(\Delta t)^2}\right)\vec{y}_{i-1}) \quad (43)$$

$$\vec{y}_{i+1} = \frac{\delta_3}{\delta_4}((A-A_r)mRT_eT_c\left(\gamma_i - \left(\frac{1}{\alpha_3}\right)\right) + \left(\frac{2M_p}{(\Delta t)^2} - K_p\right)\vec{y}_i + \left(\frac{2b_p}{\Delta t} - \frac{M_p}{(\Delta t)^2}\right)\vec{y}_{i-1}) \quad (44)$$

$$\delta_1 = \frac{M_p+b_p\Delta t}{(\Delta t)^2} \quad (45)$$

$$\delta_2 = \frac{R_q(A-A_r)A}{\Delta t} \quad (46)$$

$$\delta_3 = \frac{M_d+\Delta tR_q(2A-A_r)Ab_d}{\Delta t} \quad (47)$$

$$\delta_4 = \left(M_p + b_p\Delta t\right)(M_d+\Delta t)R_q(2A-A_r)Ab_p \quad (48)$$

Following this pattern, the velocities and accelerations for the pistons can be described in difference forms as below:

$$\begin{bmatrix}v_{x_{i+1}}\\ v_{y_{i+1}}\end{bmatrix} = \frac{1}{2\Delta t}[\vec{x}_{i+1} - \vec{x}_{i-1}] \quad (49)$$

$$\begin{bmatrix}a_{x_{i+1}}\\ a_{y_{i+1}}\end{bmatrix} = \frac{1}{(\Delta t)^2}[\vec{x}_{i+1} - 2\vec{x}_i + \vec{x}_{i-1}] \quad (50)$$

5. RESULTS AND DISCUSSION

One of the most important subject that has not yet been analyzed regarding the FPSEs is the effect of considering the pressure drop between the hot and cold spaces on the dynamics of these engines. Furthermore, according to the achieved mathematical model, the analysis of the effect of pressure drop in the heat exchangers on the dynamics of the FPSEs was conducted. Accordingly, by the variation of the pneumatic resistance R_q (Equation (11)) in the range of 0 to 2000, the displacement, velocity and acceleration of the power and displacer pistons were evaluated. Moreover, the changes in the amounts of volume and gas pressure in the hot and cold spaces caused by R_q variation can be analyzed. Finally, by changing the pneumatic resistance, the analysis of work and generated power has been conducted. It should be noted that the solution to Equations (34) and (35) has been performed using the finite difference numerical method [8]. The conducted analysis in this section is in a way that the limited cycle phenomenon is considered. Table 1 demonstrates the numerical values used in the simulation studies.

One of the important and essential facts in the design of the FPSEs is the prediction of the pistons movement stroke. The estimation of the pistons movement stroke is possible based on the dynamical models presented up to day. Accordingly, the effect of R_q variation in the range of 0 to 2000 on the maximum stroke length for the power and displacer pistons is analyzed in the first step (Figures 3 and 4).

TABLE 1. The numerical values for the engine's design parameters [1, 2]

Parameter value	Parameter
0.0017(m^2)	A
0.0001(m^2)	A_r
580(g)	M_p
210(g)	M_d
810(N/m)	K_d
1015(N/m)	K_P
700(K)	T_h
300(K)	T_k

According to Figure 3, by increasing the maximum R_q, the maximum stroke length for the power piston will decrease, such that by increasing the amount of pressure drop, the maximum stroke length for the power piston decreased from 0.008142 to 0.007294 meters. Additionally, R_q variation causes the maximum stroke length for the displacer piston to decrease from 0.021 to 0.016 m (Figure 4).

In Figures 5 and 6, the maximum velocity variations influenced by R_q changes in a working cycle for the FPSE is illustrated.

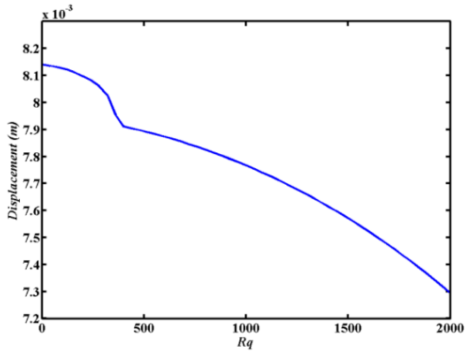

Figure 3. The effect of R_q on the maximum of power piston stroke

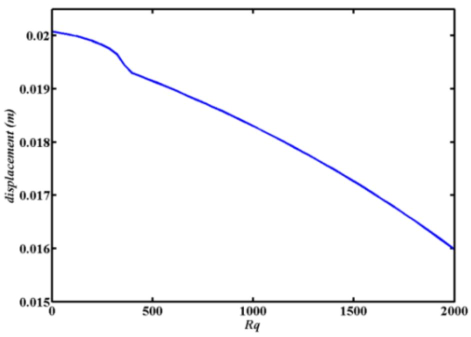

Figure 4. The effect of R_q on the maximum of displacer piston stroke

Besides, the maximum velocity for the power and displacer pistons has decreased from 1.7675 to 1.299 m/s and 2.0941 to 1.5887 m/s respectively.

According to Figures 7 and 8, the maximum acceleration for the power and displacer pistons has decreased under the influence of R_q variation. Based on this fact, the power piston acceleration has decreased from 98.24 to 77.48 m/s^2. Moreover, by increasing R_q, the displacer piston acceleration has changed from 137.26 to 101.76 m/s^2.

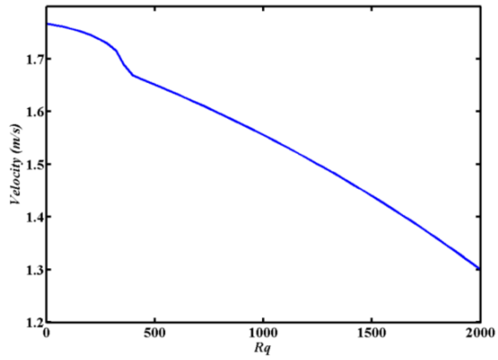

Figure 5. The effect of R_q on the maximum of power piston velocity

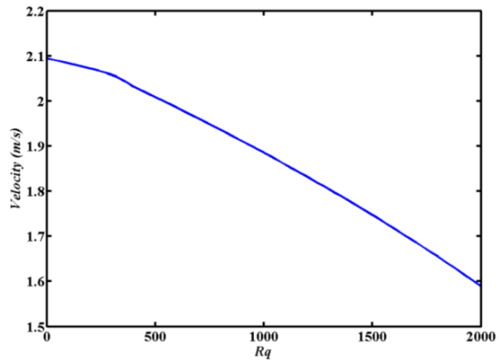

Figure 6. The effect of R_q on the maximum of displacer piston velocity

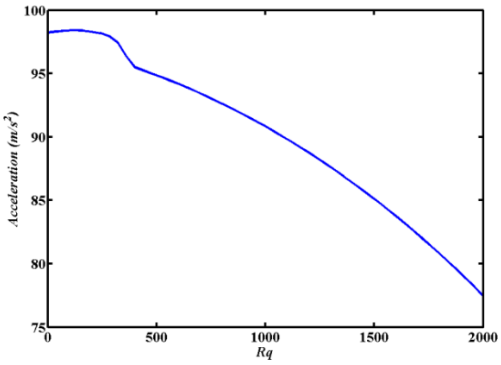

Figure 7. The effect of R_q on the maximum power piston acceleration

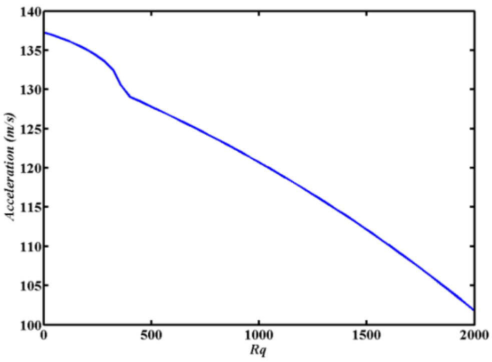

Figure 8. The effect of R_q's increase on the displacer piston's maximum acceleration

Influenced by R_q variation, the maximum pressure in the hot region has decreased from 1.46 to 1.16 bar (Figure 9).

On this basis, the amount of maximum pressure in the cold space has also decreased from 1.46 to 1.168 bar (Figure 10).

The minimum produced pressures in each working cycle for the hot and cold space are also increased from 0.5995 and 0.5995 to 0.6104 and 0.618 under the influence of R_q increase (Figures 9 and 10).

By increasing the amount of R_q, the maximum volume of the hot region has decreased from 0.000058729 to 0.00004657 m^3 (Figure 11). Furthermore, this variation has caused the minimum hot volume to increase from 0.000003244 to 0.00001366 m^3 (Figure 11). Influenced by R_q variation, the maximum cold space volume has decreased from 0.00004523 to 0.0000369 m^3 (Figure 12). Also, this fact has caused the minimum existing volume in the cold space to increase from 0.000002341 to 0.000004741 m^3 (Figure 12). This issue has caused the minimum available volume in the cold space to increase from 0.000002341 to 0.000004741 m^3 (Figure 12).

In the end, the increase in R_q amount has caused the produced work to change from 0.423 to 0.365 J (Figure 13). Based on this fact, the generated power in each working cycle has also decreased from 3.69 to 3.04 W (Figure 14).

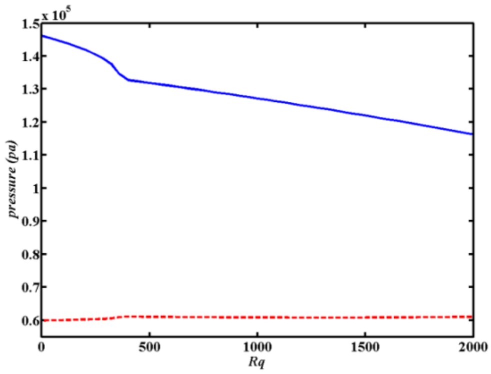

Figure 9. The effect of R_q on the maximum and minimum pressures of the hot space

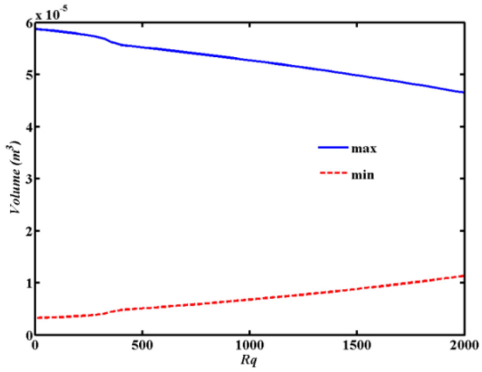

Figure 11. The effect of R_q on the maximum and minimum volumes of the hot space

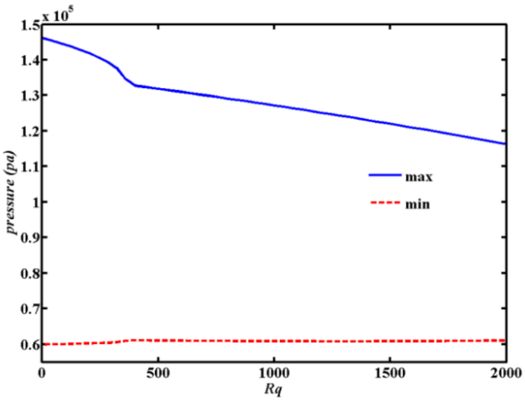

Figure 10. The effect of R_q on the maximum and minimum pressures of the cold space

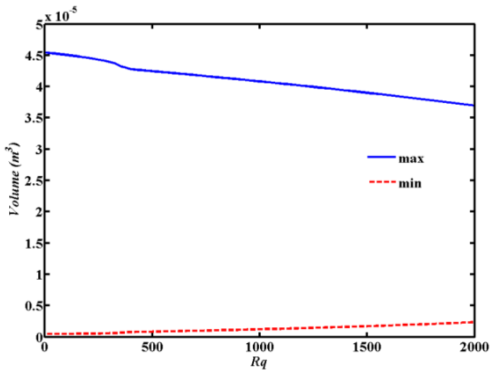

Figure 12. The effect of R_q on the maximum and minimum volumes of the cold space

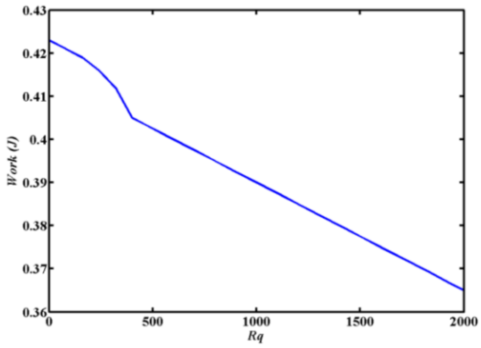

Figure 13. The effect of R_q on the generated work in each working cycle

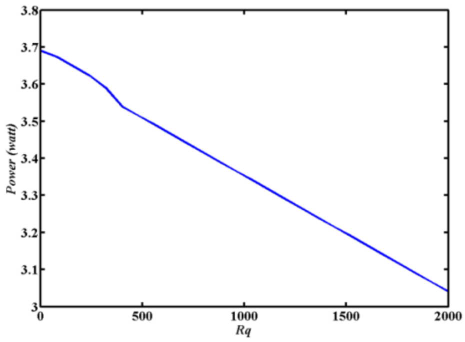

Figure 14. The effect of R_q on the generated power

6. CONCLUSION

Throughout the conducted research, the analysis of the effect of the pressure drop existence between the hot and cold regions on the performance of the engine has been conducted. The results indicate that by increasing the amount of R_q from 0 to 2000, the parameters which are effective on the engine performance such as the maximum stroke length for the displacer piston, work, velocity and acceleration for the pistons have decreased. Moreover, influenced by R_q variation, the maximum volume and pressure in the cold and hot spaces have continuously decreased. However, by increasing R_q amount from 0 to 2000, the minimum pressure in the cold and hot spaces has increased. On this basis, the increase in pressure drop between the two working spaces in the cylinder has caused the generated work and power in each working cycle to decrease. As a matter of fact, throughout the conducted researched unlike the earlier researches, the effect of an important parameter, namely, the pressure drop has been considered. The existing pressure drop which has been created by the gas motion inside the cylinder engenders a sort of negative work in the engine which decreases the engine performance. The consideration of the pressure drop between the cold and hot spaces causes the analysis of the FPSE's performance to become complex and the issue of the stable limited cycle existence analysis for the pistons motion to unearth. This issue causes the conducted analysis to be more accurate and trustworthy than the other analyses.

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An Investigation on the Effects of Gas Pressure Drop in Heat Exchangers on Dynamics of a Free Piston Stirling Engine

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چکیده

این مقاله به بررسی تاثیر افت فشار بین فضای گرم و سرد سیلندر بر دینامیک یک موتور استرلینگ پیستون آزاد اختصاص دارد. ابتدا معادلات دینامیکی حاکم بر پیستون‌ها در حالت خطی بیان شده و معادلات حاکم بر فشار گاز در فضاهای گرم و سرد موتور استخراج می گردند. سپس با جایگذاری معادلات فشار در روابط دینامیک پیستونها، مدل ریاضی غیر خطی حاکم بر دینامیک موتور هوای گرم پیستون آزاد ارائه می‌گردد. در ادامه اثر افت فشار در مبدلها بر ماکزیمم دامنه‌ی حرکت پیستونها و سرعت و شتاب آن‌ها مورد بررسی قرار می‌گیرد. همچنین اثر افزایش افت فشار در مبدلها بر مقادیر ماکزیمم و مینیمم فشار و حجم گاز در فضاهای گرم و سرد موتور مورد ارزیابی قرار گرفته است. در آخر روند تغییرات کار و توان تولید شده در موتور به ازاء افزایش افت فشار در مبدلها مورد تحلیل قرار خواهد گرفت. بر اساس نتایج بدست آمده از این تحقیق، فرض فشار یکسان گاز در فضاهای گرم و سرد موتورهای استرلینگ (که در تئوری تحلیلی اشمیت مورد استفاده قرار گرفته است) باعث به وجود آمدن خطا در پیش بینی رفتار دینامیکی موتورهای هوای گرم پیستون آزاد می‌گردد. همچنین افزایش افت فشار در مبدلها باعث ایجاد تاثیرات منفی بر عملکرد دینامیکی این موتورها شده است.

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