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A Non-linear Integer Bi-level Programming Model for Competitive Facility Location of Distribution Centers

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ABSTRACT

The facility location problem is a strategic decision-making for a supply chain which determines the profitability and sustainability of its components. This paper deals with a scenario where two supply chains, consisting of a producer, a number of distribution centers and several retailers provided with similar products, compete to maintain their market shares by opening new distribution centers because of increasing demand. The competition problem is formulated as a non-linear integer bi-level mathematical model, where the upper level represents the decisions of the leader producer and the lower level administrates the decisions of the follower producer. It has been shown that even in small-scale problems, bi-level mathematical programming problems are strongly NP-hard, so an adapted bi-level ant colony algorithm with inter-level information sharing is developed to solve the problem.To evaluate the performance of the developed ant colony algorithm, the upper bound of the competitive facility location problem is determined by solving the upper-level problem as an integer linear programming model without considering the follower's decision. Comparing the computational results of the developed ant colony algorithm with those of the determined upper bounds shows the satisfactory capability of the proposed approach for solving even medium- and large-scale problems.

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1. INTRODUCTION

The facility location problem is a branch of operation research with great significance from both practical and combinatorial optimization perspectives. Facility location is an effective tool which easily facilitates its goal by reducing transportation costs and accelerating the rate of return of investment [1]. The classical location problem is concerned with determination of the location of a project to optimize the allocation of facilities to customers. Competitive facility location is a special case of the location problem where at least two decision-makers, simultaneously or consequently, start to seek maximum market shares to optimize their objective functions by opening new distribution centers, but not before giving due consideration to the strength of the competitors. The special case where only two competitors attempt to open their distribution centers is known in the literature as the bi-level competitive facility location (CFL) problem. The CFL problem was first introduced by Hotelling [2] and then developed extensively by other researchers. Tietz [3] developed the Hotelling's model by discussing the location of multiple facilities. Huff [4] studied the CFL problem obtaining the highest market share among competitors by defining a function for the attractiveness of facilities from the viewpoint of customers by considering parameters such as quality and distance. Hakimi [5] studied the competitive facility location problem where the leader first opens a certain number of facilities and the follower responds by opening its own facilities, and each customer satisfies his own demands from the nearest facility available in the market. Labbé and Hakimi [6] studied the competitive facility location problem at multiple markets in the context of cournot competition. Pal and Sarkar [7] developed cournot competition by allowing the location of multiple facilities and assuming that each producer can supply all markets. Aboolian et al. [8] studied competitive facility

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location with regard to the number of facilities, their locations and their types (product variety and capacity of the facility); they formulated the problem as a nonlinear integer programming model and obtained its solutions through two heuristic algorithms, the greedy algorithm and the steepest descent algorithm. Beresnev [9] followed a new approach by formulating the CFL problem as a bi-level programming model, and then presented a new method for determining the upper bound of the problem; in this model, both competitors were seeking to maximize their profits.

Saidani et al. [10] studied the facility location problem by considering the responses of competitors in the market and used Huff's attractiveness function to determine the market share. Ashtiani et al. [11] used robust programming to determine the optimum solution of the competitive facility location problem with the aim of maximizing market share for competitors, under assumption of an unknown number of follower centers. Beresnev [12] continued his research on the competitive facility location problem by developing a branch and bound method to determine an optimum solution. Rahmani and MirHassani [13] studied the competitive facility location problem as a bi-level mathematical programming model; using Lagrangian relaxation method they obtained acceptable solutions for their mathematical model; their results indicated that the proposed method was highly efficient. MirHassani et al. [14] used a modified particle swarm optimization algorithm to solve the competitive location problem and compared their results with the upper bound obtained from solving a mathematical model; based on the conducted analysis, their results showed the capability of the proposed meta-heuristic algorithm of obtaining high-quality solutions. Although the competitive location problem has been the subject of much research, analyzing the problem as a bi-level mathematical programming model is a somewhat neglected approach (Beresnev [12]), and few studies on this subject include the studies of Beresnev ([9, 12, 15, 16]), Rahmani and MirHassani [13] and MirHassani et al. [14].

Increase in consumption of foodstuff such as dairy products and prepared or semi-prepared food and even introduction of a new product by one or several producers can affect the market balance. Thus, when the level of production or the capacity of existing distribution centers cannot meet the market's demand, each producer, depending on its strength, attempts to survive in the competitive market by increasing the value of production or opening new distribution centers; on the other hand, in markets of products such as food and medicine, each customer may satisfy his own demand with more than one producer or distributer center, and each producer seeks to gain the maximum market share in order to keep its customers and increase their satisfaction level, supplying the demands of one or all of its customers through more than one distribution center; as these assumptions are not considered in studies similar to this research, our developed mathematical model can be used by numerous industries such as dairy manufacturers, pharmaceutical producers and cosmetic and healthcare industries.

The main objective of the current study is to develop a mathematical model for the competitive facility location problem with the highest degree of adaptability to real-world applications; for example, each producer can spend a limited budget on expanding the distribution centers or production sites, which has not been considered by many researchers up to now. As a results of budget limitation, the producer can open only a few distribution centers among the set of candidate locations. In distribution of commodities such as dairy products, supplying a customer with only one distribution center may not be economically justified, so in this paper, we assume that each distribution center can cover more than one customer and each customer can satisfy his demand with more than one distribution center.

In view of the above points, the bi-level competitive facility location problem presented by Beresnev [12] will be studied and developed as a bi-level mathematical programming model through applying the following changes:

1. Facility location will be subject to budget constraint;
2. The two competitors have an initial market share and try to keep it by creating new centers;
3. New distribution centers only cover customers that are being covered by the current distribution centers of each producer (the new distribution centers of the leader are only used to supply the customers that are covered by the existing distribution centers of the leader, and vice versa);
4. Each customer can satisfy his demand from more than one distribution center of each producer (only the leader or the follower);
5. Each distribution center can cover more than one customer;
6. The profit gained from covering different customers with distribution centers may be different.

Further information about competitive facility location can be found in the works of Kress and Pesch [17] and Drezner [18].

2. BI-LEVEL MATHEMATICAL PROGRAMMING MODEL FOR COMPETITIVE FACILITY LOCATION PROBLEM

In this study, it is assumed that both competitors currently have several distribution centers, which are used to transfer the products to their customers. Each customer acquires his products through the distribution centers of the leader or the follower. Now, suppose that the two competitors start to open new distribution centers to deal with growth of market demand.

Suppose that the two competitors operate with a non-cooperative behavior based on game theory approach; the main feature of the non-cooperative games is that each player looks for his own benefit. The Nash and Stackelberg equilibriums are the most important methods used in many non-cooperative games. The Nash equilibrium is used when the players of a game choose their strategies simultaneously; but in a leader-follower scenario, the leader can act before the follower; in this case, the optimal strategy of each player can be determined through the Stackelberg equilibrium.

The Nash equilibrium applies when the players do not cooperate with each other and determine their decisions simultaneously (like in playing rock-paper-scissors). A Stackelberg game is used in a non-cooperative and sequential decision making process. In this game, one player acts as a leader and another plays as a follower. The leader first chooses his decision taking the follower's reaction into account, and then the follower sees this decision and selects his best decision [19]. The Stackelberg equilibrium consists of two concepts: the leader and the follower. This equilibrium is applicable when one of the players can move before the other players and play as the leader.

In other words, the leader has more power than the follower, and hence in this game, the leader makes the first decisions. Afterwards, the follower makes his own decisions according to the decisions of the leader. In a leader-follower environment, the follower chooses the best response to the decision of the leader, and the leader optimizes his objective function according to the follower's response.

Accordingly, the competitive location problem will be formulated as a non-linear integer bi-level programming model with regard to the following assumptions (It should be noted that bi-level programming is a representation of Stackelberg game).

1. Two supply chains each of them with one producer, multiple distribution centers and several retailers are considered;
2. The decision-making of the competitors is based on the Stackelberg game;
3. Each distribution center can cover more than one customer;
4. Each customer is covered only by leader or follower distribution centers;
5. The demand of each customer can be satisfied through more than one distribution center;
6. Distribution must be done through distribution centers, and direct shipping from producers to retailers is not allowed;
7. Each producer supplies only a part of the market.

First, the parameters of the problem are introduced, and following that, the bi-level formulation of the competitive facility location problem is presented.

Indices

L	*Indices for leader*
F	*Indices for follower*
j, k	*Indices for customer*
i, s	*Indices for distribution centers*

Parameters

n	*Number of potential locations;*
n_L	*Number of existent DCs of leader;*
n_F	*Number of existent DCs of follower;*
m	*Number of customers;*
S	*The maximum number of customers that each DC can serve;*
f_i	*Setup cost of* i^{th}*DC of leader;*
g_i	*Setup cost of* i^{th}*DC of follower;*
B_L	*Total budget of leader to open new DCs;*
B_F	*Total budget of follower to open new DCs;*
Φ_{ij}	*Net profit of delivered products from new DC i tocustomer j;*
$\widetilde{\Phi}_{ij}$	*Net profit of delivered products from existent leaders' DC i to customer j;*
$\breve{\Phi}_{ij}$	*Net profit of delivered products from existent followers' DC i to customer j;*

Decision variable

X_j	$X_j = \begin{cases} 1 \; if \; leader \; open \; facility \; j \\ 0 \quad otherwise \end{cases}$
x_{ij}	$x_{ij} = \begin{cases} 1 \; if \; customer \; j \; covered \; by \; new \; facility \; i \; of \; leade \\ 0 \quad otherwise \end{cases}$
$\hat{x}_{ij}$	$\hat{x}_{ij} = \begin{cases} 1 \; if \; customer \; j \; covered \; by \; existent \; facility \; i \; of \; l \\ 0 \quad otherwise \end{cases}$
Y_j	$Y_j = \begin{cases} 1 \; if \; follower \; open \; facility \; j \\ 0 \quad otherwise \end{cases}$
y_{ij}	$y_{ij} = \begin{cases} 1 \; if \; customer \; j \; covered \; by \; new \; facility \; i \; of \; follo \\ 0 \quad otherwise \end{cases}$
$\hat{y}_{ij}$	$\hat{x}_{ij} = \begin{cases} 1 \; if \; customer \; j \; covered \; by \; existent \; facility \; i \; foll \\ 0 \quad otherwise \end{cases}$

According to the definitions of the parameters, a non-linear integer bi-level programming model of the competitive facility location problem is presented as follows:

$$\boldsymbol{Max}\left[\sum_{j=1}^{k}\left(\sum_{i=1}^{n}\sum_{s=1}^{n}\Phi_{ij}x_{ij}\left(1-y_{sj}^{*}\right)+\sum_{i=1}^{n_L}\left(\sum_{s=1}^{n_F}\widetilde{\Phi}_{ij}\hat{x}_{ij}\left(1-\hat{y}_{sj}^{*}\right)\right)\right)\right]-\sum_{i=1}^{n}f_i\,.X_i \quad (1)$$

s.t.

$$\sum_{i=1}^{n}f_i\,.X_i\le B_L\forall i \quad (2)$$

$$x_{ij}\le X_i \qquad \forall i,j \quad (3)$$

$$\sum_{i=1}^{n_L}\hat{x}_{ij}\le m \qquad \forall j \quad (4)$$

$$\sum_{j=1}^{m}\hat{x}_{ij}\le S \qquad \forall i \quad (5)$$

$$x_{ij}\le\ \hat{x}_{ik} \qquad \forall i,j,k \quad (6)$$

$$X_i\epsilon\{0,1\}; \qquad \forall i \quad (7)$$

$$x_{ij}\epsilon\{0,1\}; \qquad \forall i,j \quad (8)$$

$$\hat{x}_{ij}\epsilon\{0,1\}; \qquad \forall i,j \quad (9)$$

$$\boldsymbol{Max}\left[\sum_{j=1}^{k}\left\{\sum_{i=1}^{n}\Phi_{ij}y_{ij}+\sum_{i=1}^{n_F}\breve{\Phi}_{ij}\hat{y}_{ij}\right\}\right]-\left\{\sum_{i=1}^{n}g_i\,.Y_i\right\} \quad (10)$$

$s.t$

$$X_i+Y_i\le 1; \qquad \forall i \quad (11)$$

$$\sum_{i=1}^{n}g_i\,.Y_i\le B_F\ \forall j \quad (12)$$

$$y_{ij}\le Y_i \qquad \forall i,j \quad (13)$$

$$\hat{x}_{ij}+\hat{y}_{sj}=1 \qquad \forall i,j,s \quad (14)$$

$$y_{ij}\le\ \hat{y}_{ik} \qquad \forall i,j,k \quad (15)$$

$$\sum_{j=1}^{n_f}\hat{y}_{ij}\le m \qquad \forall k \quad (16)$$

$$\sum_{j=1}^{m}\hat{y}_{ij}\le S \qquad \forall j \quad (17)$$

$$Y_i\epsilon\{0,1\} \qquad \forall j \quad (18)$$

$$y_{ij}\epsilon\{0,1\} \qquad \forall i,j \quad (19)$$

$$\hat{y}_{ij}\epsilon\{0,1\}; \qquad \forall i,j \quad (20)$$

Equation (1) determines the objective function of the leader with respect to lost profit due to customer served by the new and existing distribution centers of the follower; the leader's budget constraint to open new centers is presented by Equation (2); constraint (3) ensures product distribution through opened distribution centers; constraints (4) indicates that the demand of each customer can be supplied by all of the distribution centers of the leader; constraint (5) indicates the maximum number of customers served by each of the existing distribution centers of the leader, Equation (6) states that new distribution centers deliver products only to customers who are covered by existing centers; Equations (7) to (9) represent the status of upper-level decision variables; Equation (10) states the objective function of the follower producer, which aims to maximize profit through opening new distribution centers and also through using existing distribution centers; Equation (11) ensures that at every candidate location, only the leader or follower can open a new distribution center; constraint (12) represents the budget constraint of the follower; constraint (13) acts like constraint (3) but at the upper level; customer segmentation for the two competitors based on existing distribution centers is stated by Equation (14), meaning that for each customer, the product will be delivered only by one of the two competitors; constraint (15) acts like constraint (6) but at the upper level; constraints (16) and (17) act like constraints (4) and (5); constraints (18)-(20) define lower-level decision variables just like constraints (6) -(8).

In this model, the upper-level decision-maker determines his strategy, and then the lower-level decision-maker, knowing this strategy, determines his policy to optimize his own objective function, and lastly, the optimum response of the leader is determined based on the best response of the follower.

Jeroslow [20] proved that even in small scale, a bi-level mathematical programming problem is strongly NP-hard; thus, several heuristic and meta-heuristic methods have been developed to deal with the high complexity of bi-level mathematical programming problems.

Beresnev [12] used a branch-and-bound algorithm to solve the bi-level competitive facility location problem and introduced a technique to determine an upper bound for the problem. MirHassani et al. [14] used a modified version of particle swarm optimization algorithm to solve the competitive facility location problem as a bi-level mathematical programming model. Farvaresh and Sepehri [21] presented a branch-and-cut method by defining valid inequalities based on Steiner tree for the bi-level mathematical programming problem. Several researchers have used meta-heuristic techniques to solve bi-level mathematical programming; more information about numerous metaheuristic methods proposed to solve bi-level mathematical problems can be found in the work of El-Ghazali [22].

Considering that this study follows a meta-heuristic approach to solve the bi-level programming problem, in the following, several metaheuristic approaches developed for this purpose are briefly reviewed. The prominent metaheuristic-based solutions developed for bi-level mathematical programming problems can be classified into four categories:

I. Nested Sequential Approach In this category, the lower-level problem must be solved by an exact, heuristic or metaheuristic method based on the results generated for the upper-level problem, and the result of the lower-level problem must be used to re-solve the upper-level problem, and this process must be repeated until the stopping criteria are met.The main flaw of this approach is the complexity of the process, since for each solution obtained at each stage for the upper-level problem, the optimal solution of the lower-level problem must also be determined.

II. Single-level Transformation Approach In this approach, first, a technique such as Karush–Kuhn–Tucker conditions must be used to transform the bi-level mathematical programming problem to a single-level model, and it must then be solved by an exact, heuristic or metaheuristic method.

III. Multi-objective Approach In this category, the bi-level problem must be transformed into a single-level multi-objective model; linking between the pareto-optimal solution of a multi-objective problem and solution space of a bi-level problem is the most important part of the method. This technique has been used in only a few studies (El-Ghazali [22]).

IV. Co-evolutionary Approach In this approach, each level of the problem must be solved by a separate meta-heuristic algorithm; the information of the two levels must then be shared, and the process of parallel solution must continue until a termination criterion is achieved.The most important point in this approach is how to share information between two levels of the problem.This approach also requires a particular segment of memory to be dedicated to shared information. Figure 1 shows the general framework of this method.

In this study, the bi-level mathematical model is solved by a co-evolutionary approach in which both levels are solved by ant colony algorithm simultaneously.

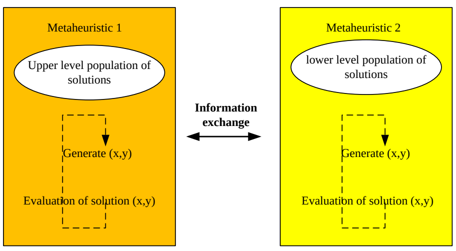

Figure 1. General framework of co-evolutionary solution to bi-level problem [22]

To evaluate the performance of the proposed algorithm, the upper bound of the problem is developed based on the approach described in Section 4, and the results of ant colony algorithm are compared with this bound.

3. THE PROPOSED ANT COLONY ALGORITHM

In this study, each level of the bi-level mathematical problem is solved by the ant colony algorithm proposed by Dorigo and Gambardella [23]. The discussed problem is a three-level supply chain consisting of a producer for each supply chain, a number of distribution centers and several retailers, which can all be represented on a network.The arcs between the first and second levels represent the products flowing from producers to distribution centers, and those between the second and third levels represent delivery of the products from distribution centers to retailers. It should be noted that in this network, direct shipment from producers to retailers is not allowed.The general structure of the used ant colony algorithm is described in the following:

Algorithm (1)

Step 0. Set all problem parameters (including the number of ants of the two competitors, the initial pheromone path, etc.)

Until termination condition holds do:

Step 1. Given the number of leader (follower) ants, repeat the following steps:

Step 1. 1. Use the selection rule to generate an initial solution;

Step 1. 2. If this solution is not acceptable[2], go to step 1-3; otherwise, use algorithm (2) to convert it into an acceptable solution;

Step 1. 3. Add the resulting acceptable solution to the Tabu list[3];

Step 2. If the solution of the previous step pertains to the leader, repeat step 1 for the follower; otherwise, go to step 3;

Step 3. Alter the pheromone path with both local and global updating rules;

Step 4. If the termination condition does not hold, go to step 1; otherwise, go to step 5;

Step 5. Select the best solution;

Step 6. End.

---

[2]-Acceptable solution is a solution whose cost does not exceed the total available budget.

[3]-Tabu list includes those new leader or follower distribution centers that cannot be selected in the next iteration by either the follower or the leader.

1. 3. Selection Rule Artificial ants use a probability law inspired by the behavior of natural ants for consecutive selection of distribution centers for both competitors and construction of a solution with respect to data obtained via a pheromone path which is updated over time. Based on the system designed by Dorigo and Gambardella [23], each ant of the leader or the follower uses the following rule to select the new distribution center $\boldsymbol{j}$ with probability of $\boldsymbol{q_0^X (X = L\ or\ F)}$, where $\boldsymbol{F}$ signifies the follower and $\boldsymbol{L}$ signifies the leader.

$$j = argmax\left\{\tau_j \eta_j^{\beta}\right\} \tag{21}$$

where τ_j denotes the pheromone path, η_j represents the heuristic information of distribution center j and β is a value between 0 and 1, signifying the importance of η_j; in a particular iteration of the algorithm, distribution center j is selected with a probability of $(1 - q_0^X)$ according to the following probability distribution:

$$p_j = \frac{\tau_j \eta_j^{\beta}}{\sum_{j=1}^{N} \tau_j \eta_j^{\beta}} \tag{22}$$

Parameter η_j is determined based on the two rules of nearest and most profitable distribution center, as shown below.

Parameter φ_j is calculated for each potential location:

$$\varphi_j = \frac{profit_{ij}}{\sum profit_{ij}} \tag{23}$$

In addition, based on the rule of nearest location, parameter ψ_j is calculated as follows:

$$\psi_j = \frac{1/d_{ij}}{\sum 1/d_{ij}} \tag{24}$$

and finally:

$$\eta_j = Max\{\varphi_j, \psi_j\} \tag{25}$$

After generating solutions based on algorithm (1), the costs of some of the generated solutions may exceed the total available budget, so algorithms (2) is used to convert these solutions into acceptable solutions.

Algorithm (2)

Step 1. Select among the not-selected centers the one with the highest benefit-cost ratio for the leader (follower), and insert it in place of selected center with the highest cost.

Step 2. As long as cost is higher than budget, repeat step 1.

The following rule is used for local updating of pheromone in each iteration:

$$\tau_j = (1 - \rho_X)\tau_j + \rho_X \tau_0 \tag{26}$$

where ρ_X is considered for each competitor. The following relationship is used for global updating of the pheromone path based on the best found solution,which, at this stage, is the most profitable distribution center:

$$\tau_j = (1 - \rho_X)\tau_j + \rho_X \tau_0 \left({pr_j}/{pr_{best}}\right) \tag{27}$$

After the termination of both algorithms 1 and 2, a new ant colony algorithm is run to maximize the profits of product delivery.

Algorithm (3)

Step 0. Set all parameters for all new and existing distribution centers of the two competitors (number of ants, initial pheromone value, and probability of acceptance);

Step 1. Repeat the following steps for all new and existing leader (follower) centers and all ants in these centers;

Step 1. 1. Use the selection rule to generate an initial solution for the existing distribution centers (specifying the customers to which each distribution center is allowed to send the product);

Step 1. 2. Use the selection rule to generate a solution for new distribution centers only for customers selected in the previous step;

Step 1. 3. Add the resulting solution (selected customers) to the ban list;

Step 2. If the solution of the previous step pertains to the leader, repeat step 1 for the follower; otherwise, go to step 3;

Step 3. Alter the pheromone path with both local and global updating rules;

Step 4. If the termination condition does not hold, go to step 1; otherwise, go to step 5;

Step 5. Select the best solution (the objective function, new distribution centers and covered customers);

Step 6. End.

Local and global pheromone path updating rules are similar to those used for selection of new distribution centers but do not necessarily employ the parameters of the previous step.

It should be mentioned that the profits gained from delivery through new and existing distribution centers are expressed as a $m \times n$ matrix, where m is the number of existing or new distribution centers of the two competitors, and n is the number of customers. Each distribution center covers only a certain number of customers, which is based on its profit threshold; in this study, this threshold is defined as $max\{min\, p_{ij}\}$ for all existing and new distribution centers of the two competitors.

4. DETERMINING AN UPPER BOUND FOR THE COMPETITIVE LOCATION PROBLEM

The optimality of solution of the proposed mathematical model is not clear, so to evaluate the efficiency of the proposed algorithm, an upper bound will be found, and the computational results will be compared with this value.

Theorem. The upper bound for the objective function of the competitive location problem can be determined by solving the following problem:

$$\boldsymbol{Max}\left[\sum_{j=1}^{k}\left(\sum_{i=1}^{n}\Phi_{ij}x_{ij}+\sum_{i=1}^{n_L}\widetilde{\Phi}_{ij}\hat{x}_{ij}\right)\right] - \sum_{j=1}^{n}f_j\,.X_j \tag{28}$$

$$\sum_{j=1}^{n}f_j\,.X_j \le B_L \forall j \tag{29}$$

$$x_{jk} \le X_j \qquad \forall j,k \tag{30}$$

$$\sum_{j=1}^{n_l}\hat{x}_{jk} \le m \qquad \forall k \tag{31}$$

$$\sum_{k=1}^{m}\hat{x}_{jk} \le S \qquad \forall j \tag{32}$$

$$x_{jk} \le \hat{x}_{lk} \qquad \forall k,j,l \tag{33}$$

$$X_j \epsilon \{0,1\}; \qquad \forall j \tag{34}$$

$$x_{ij} \epsilon \{0,1\}; \qquad \forall i,j \tag{35}$$

$$\hat{x}_{ij} \epsilon \{0,1\}; \qquad \forall i,j \tag{36}$$

Proof. Please see appendix A.

5. COMPUTATIONAL RESULTS

To evaluate the performance of the developed ant colony algorithm, 20 test problems were generated, and then, the upper bound of each problem was determined by the mathematical programming model presented in Section 4.

These values were then compared with the results of the developed ant colony algorithm. Table1 shows the obtained upper bound for the objective function of the leader-follower problem and the objective function values of the upper-level and lower-level problems calculated by ant colony algorithm.

The proposed ant colony algorithm was coded in VB 6.0, and was run ten times for each test problem on a computer with Core *i5* processor and 4GB RAM under 64-bit Windows 7 operating system.The upper bound of each instance problem was determined with Lingo 9.0 software.

In all instances, the profit of the existing and new distribution centers and the cost of setting up a new distribution center were generated randomly from the intervals [500,2500], [1000,3500], and [2000,4000] in that order.

TABLE 1. Computational results for test problems

Problem no.	**Upper level problem**				**Ant colony results**				
					Obj. of upper level problem		**Obj. of lower level problem**		CPU time(s)
	Upper Bound	integer	constraints	CPU time(s)	Max	Min	Max	Min	
1	14800	50	440	<1 S	7710	7710	2800	2800	<1 S
2	15290	60	672	<1 S	8060	8060	2800	2800	<1 S
3	16420	70	954	<1 S	10060	10060	7190	7190	<1 S
4	22110	96	1834	<1 S	12710	12710	4900	4900	<1 S
5	22310	96	1834	2	15715	15715	4950	4950	<1 S
6	29770	128	3224	2	25090	23770	9850	7520	<1 S
7	32250	144	4194	3	20670	20670	11450	10750	<1 S
8	77915	176	6518	3	34310	30800	14475	14175	1.23
9	145105	198	8230	4	45665	44260	15710	15710	1.6
10	163615	234	11816	5	46880	44585	28575	24385	1.87
11	347685	448	44370	7	69064	64403	21727	18875	4.26
12	334893	480	50912	10	60823	60263	39516	38305	4.49
13	402339	540	65346	12	70189	67025	39369	38559	4.94
14	558403	612	83880	26	67089	66252	41981	39277	5.54
15	627325	684	105374	24	83457	82345	36017	35970	8.62
16	672393	760	130760	46	96788	94302	45678	44900	13.45
17	737893	840	160482	606	11067	109870	65146	62789	33.98
18	770517	880	176904	673	115980	114567	77893	77456	41.5
19	801692	920	194126	719	121345	120890	82341	81706	73.65
20	822430*	924	195008	>3600	126789	124976	89612	87256	135.28

6. CONCLUSION

This paper deals with a location problem with two supply chains consisting of one producer, a number of distribution centers and several retailers, where each producer obtains a portion of the market share by delivering its products through distribution centers. The leader producer establishes new distribution centers to exploit the new market demand due to increase in market demand, and the follower reacts by opening its own new distribution centers. These two competitors choose new distribution centers among several potential locations based on the rules of Stackelberg game and their own strength. The problem was described as a bi-level mathematical model, and then, because of its high complexity, an adopted ant colony algorithm was developed to achieve high-quality solutions. Compared to similar researches, the bi-level mathematical model of this paper is more compatible with the real world because new distribution centers are opened based on budget constraint, such as what happens in real world, each distribution center in the model can serve more than one customer or retailer whilst each retailer could be supplied by more than one distributor.To evaluate the capability of the proposed ant colony algorithm, an upper bound was developed for the problem, and the results were compared with the upper bound; the comparison showed the high performance of the proposed approach to obtain high-quality and near-optimal solutions. Our suggestions for future research on this problem include examination of other problems such as relocating of one or more than one distribution center, development of better upper bounds with different approaches, use of different methods of product distribution such as vehicle routing and hybrid delivery approaches as well as other methods of transport that could make the problem more conforming with real-world applications.

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8. APPENDIX

Determination of the Upper Bound for the Problem

The upper bounds of the problem instances were determined by a method similar to the one proposed by Beresnev and Mel'nikov [15]. Each customer could be covered by both new and existing distribution centers of one of the two competitors.

We show each acceptable solution of the upper-level problem with an ordered triple of $X = (x_i, x_{ij}, \hat{x}_{ij})$, for which the lower-level problem will have an optimum solution in the form of $Y^* = (y_i^*, y_{ij}^*, \hat{y}_{ij}^*)$. In addition, we denote each acceptable solution of the follower problem with $\widetilde{Y} = (y_i, y_{ij}, \hat{y}_{ij})$. An admissible solution to the above leader-follower problem will be expressed as $(X, \widetilde{Y})$, the value of which can be determined by replacement in Equation (1) and is represented by $\Omega(X, \widetilde{Y})$. Also, the optimum solution of the problem is shown with $\Omega(X^*, \widetilde{Y}^*)$, and for each acceptable solution $\Omega(X, \widetilde{Y})$, the relationship $\Omega(X^*, \widetilde{Y}^*) \geq \Omega(X, \widetilde{Y})$ holds.

Supposing that the two competitors are not of equal strength, we assume that to maximize the profit, the follower producer selects the locations with lower importance to the leader; so a non-cooperative game will be played between the two competitors. Thus, for each arbitrary solution X, there is an optimum non-cooperative solution in the form of $\overline{Y}$ which applies in the relationship $\Omega(X, \widetilde{Y}) \geq \Omega(X, \overline{Y})$. According to the above definition, an acceptable non-cooperative solution is defined in the form of $(X, \overline{Y})$, and an optimum non-cooperative solution is defined in the form of $(X^*, \overline{Y}^*)$. We show the optimal value of the objective function with $\Omega(X^*, \overline{Y}^*)$.

Lemma 1. For each possible solution to the problem and for each customer, the following relationship is true:

$$\sum_{i=1}^{n} \Phi_{ij} x_{ij} \left(1 - \sum_{i=1}^{n} y_{ij}^*\right) + \sum_{i=1}^{n_L} \widetilde{\Phi}_{ij} \hat{x}_{ij} \left(1 - \sum_{i=1}^{n_F} \hat{y}_{ij}^*\right) \leq m\left(\max\{\Phi_{ij} x_{ij}\} + \max\{\widetilde{\Phi}_{lj} \hat{x}_{lj}\}\right)$$

Proof. If customer j is covered by at least one of the follower's centers, i.e. $\Phi_{ij} x_{ij} = 0$ and $\widetilde{\Phi}_{ij} \hat{x}_{ij} = 0$, the proof is completed, because one of the two equations $\sum_{i=1}^{n} y_{ij}^* = 1$ and $\sum_{i=1}^{n_F} \hat{y}_{ij}^* = 1$ or both of them will be true; suppose that for a set of candidate locations I_{new} and a set of existing locations I_{exist}, one of the equations $\Phi_{ij} x_{ij} > 0$ (for all $i \in I_{new}$) and $\widetilde{\Phi}_{i'j} \hat{x}_{i'j} > 0$ (for all $i' \in I_{exist}$) or both of them are true; in this case, we will have:

$$\sum_{i=1}^{n} \Phi_{ij} x_{ij} \left(1 - \sum_{i=1}^{n} y_{ij}^*\right) \leq m\left(\max_i \Phi_{ij} x_{ij}|_{x_{ij}=1}\right)$$
$$\leq m\left(\max_{i,l}\{\Phi_{ij} x_{ij}, \widetilde{\Phi}_{lj} \hat{x}_{lj}\}|_{\widehat{x_{lj}}+x_{ij} \geq 1}\right) \leq$$
$$m\left(\left(\max_i\{\Phi_{ij} x_{ij}\} + \max_l \{\widetilde{\Phi}_{ij} \hat{x}_{ij}\}\right)|_{\widehat{x_{lj}}+x_{ij} \geq 1}\right)$$
$$\sum_{i=1}^{n_l} \widetilde{\Phi}_{ij} \hat{x}_{ij} \left(1 - \sum_{i=1}^{n_F} \hat{y}_{ij}^*\right) \leq m\left(\max_{i,j} \widetilde{\Phi}_{ij} \hat{x}_{ij}|_{\widehat{x_{ij}}=1}\right)$$
$$\leq m\left(\max_{i,j}\{\Phi_{ij} x_{ij}, \widetilde{\Phi}_{ij} \hat{x}_{ij}\}|_{\widehat{x_{ij}}+x_{ij} \geq 1}\right)$$
$$\leq m\left(\left(\max_{i,j}\{\Phi_{ij} x_{ij}\} + \max_{i,j} \{\widetilde{\Phi}_{ij} \hat{x}_{ij}\}\right)|_{\widehat{x_{ij}}+x_{ij} \geq 1}\right)$$

If each customer j is covered by one of the existent or new distribution centers or both of them, the following equation is true:

$$\sum_{i=1}^{n} \Phi_{ij} x_{ij} \left(1 - \sum_{i=1}^{n} y_{ij}^*\right) + \sum_{i=1}^{n_L} \widetilde{\Phi}_{ij} \hat{x}_{ij} \left(1 - \sum_{i=1}^{n_F} \hat{y}_{ij}^*\right) \leq m\left(\max_i\{\Phi_{ij} x_{ij}\} + \max_i \{\widetilde{\Phi}_{ij} \hat{x}_{ij}\}\right)$$

The proof is complete, so the quantity of

$$\text{Max} \left(\sum_{j=1}^{k}\left(\max_i\{\Phi_{ij} x_{ij}\} + \max_s \{\widetilde{\Phi}_{sj} \hat{x}_{sj}\}\right)\right) - \sum_{j=1}^{n} f_j . X_j$$

is an upper bound for the optimal value of the objective function of competitive facility location.

A Non-linear Integer Bi-level Programming Model for Competitive Facility Location of Distribution Centers

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چکیده

مکانیابی تسهیلات یک تصمیم‌گیری استراتژیک برای زنجیره‌های تامین محسوب می‌شود و سودآوری و همچنین بقاء اعضاء زنجیره‌تامین را تضمین میکند. در این تحقیق دو زنجیره تامین سه سطحی رقیب شامل یک تولیدکننده در هر زنجیرهبه همراه تعدادی مرکز توزیع و چند خرده‌فروش در نظر گرفته شده است که هریک محصولات مشابهی را برای بازار هدف خود ارسال می‌کنند و به دلیل تغییر در سطح تقاضای بازار از نظر میزان تقاضا برای محصولات فعلی، دو رقیب تصمیم به افزایش مراکز توزیع خود می‌گیرند تا با جلوگیری از ایجاد کمبود، مشتریان و به دنبال آن سهم بازار را از دست ندهند. با در نظر گرفتن این نکته که دو رقیب از قدرت یکسان برخوردار نیستند رقابت برای انتخاب بهترین مکان‌ها از بین تعدادی مکان کاندیدا بعنوان مراکز توزیع جدید ایجاد خواهد می‌شود؛ مساله مکانیابی رقابتی توصیف شده بصورت یک مدل برنامه‌ریزی ریاضی دوسطحی گسسته فرموله میشود که سطح بالا نشان‌دهنده تصمیم‌گیری تولیدکننده رهبر و سطح پایین تصمیمات تولیدکننده پیرو را نشان می‌دهد. از آنجا که مسایل برنامه‌ریزی دوسطحی حتی در ابعاد کوچک NP-hard هستند، لذا با استفاده از رویکرد تبادل اطلاعات بین دوسطح تصمیم‌گیرنده، الگوریتم مورچگان دوسطحی برای حل مساله توسعه شده است. به منظور حصول اطمینان از کارآیی الگوریتم مورچگان توسعه یافته، مساله سطح بالا بصورت یک مدل برنامه‌ریزی خطی عدد صحیح و بدون در نظر گرفتن تصمیم‌گیری پیرو، حل می‌شود. مقایسه نتایج محاسباتی حاصل از الگوریتم مورچگان با کران بالای بدست آمده، نشان‌دهنده توانایی بالای رویکرد پیشنهادی در حل مسایل حتی با ابعاد متوسط و بزرگ می‌باشد.

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