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A Single Machine Capacitated Production Planning Problem Under Uncertainty: A Grey Linear Programming Approach

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ABSTRACT

The production planning is an important problem in most of manufacturing systems in practice. Unlike many researches existing in literature, this problem encounters with great uncertainties in parameters and input data. In this paper, a single machine capacitated production planning problem is considered and a linear programming formulation is presented. The production costs are assumed to be uncertain parameters. To handle the uncertainties in the model, the grey systems theory is employed and the concept of grey numbers is incorporated into an optimization framework. In such systems, the uncertain parameters with unknown distributions can be handled by grey numbers. The grey linear programming (GLP) is a development of the classical linear programming which allows uncertainty to be effectively communicated into the optimization process. Finally, the uncertain problem is transformed into a GLP, and is solved by two linear deterministic sub-models.

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1. INTRODUCTION

The increasing market competition has forced manufacturers to design their supply chain more efficiently. The production planning is a type of mid-term planning in a supply chain which have a great influence on firm's performance. This problem aims at determining production quantity, inventory size and workforce level within a mid-term time horizon. It is usually possible to ignore detailed material and resource requirements for products in this type of problems. It is one of the critical areas of supply chain planning which enables production managers and planners to update plan frequently, and compensate disruptions in product demand, costs, capacity and material supply. Ashayeri and Selen [1] considered a strategic production planning in a pharmaceutical case. Moreover, Wang and Liang [2] discussed a multi-product fuzzy production planning problem. Additionally, Wang and Liang [3] analyzed a multi-objective version of problem with imprecise demand, by using an interactive possibilistic programming method. Recently, Sturm et al., [4] studied the inter-relation of production planning and thermal energy. Jain and Palekar [5] studied dissimilar machines and production lines in a configuration-based production planning. Jamalnia and Soukhakian [6] considered a similar problem with fuzzy theory for multi-objective nonlinear production planning problem. Zhang et al. [7] developed the capacity expansion and multiple centers for this problem and designed a hybrid heuristic. Ghasemi Yaghin et al. [8] devised a multi-period, multi-product fuzzy multi-objective model of the problem. Hong et al., [9] studied a dual-mode production planning with mission constraints, single product and two optional technologies, and solved the model by using a dynamic programming algorithm on the basis of structural properties of problem. Liang et al., [10] suggested a column generation and decomposition method for a multi-item lot sizing production planning and facility location problem with backlogging to achieve feasible solutions. Vargas et al., [11] proposed a hierarchical collaborative production planning problem and then discussed a new approach based on inter-enterprise architecture to address the problem. Beemsterboer et al., [12] suggested a production planning problem of make-to-order/make-to-stock and analyzed optimal policies. Moreover, we can

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find more recent planning problems in literature with different assumptions and structures like [13, 14].

The majority of research has studied the optimization of production plans under deterministic assumptions. However, the uncertainties existing in real-world can influence the performance of production plans and schedules, outstandingly. Hence, in this paper, we considered a capacitated production planning problem with a single machine under uncertain cost parameters and customer demand. Then, in order to address the uncertainties in the model, the grey systems theory, as a useful and efficient tool, is employed and the grey linear programming is adapted for the problem.

The remainder of this paper is organized as follows. Section 2 presents the mathematical model of suggested problem. The grey linear programming approach is presented in Section 3. Section 4 describes a numerical analysis. Finally, Section 5 concludes paper.

2. DEFINITION AND MODELING

In this section, the problem of single machine production planning is formulated mathematically. Although production costs are assumed to be uncertain, we first formulate the deterministic equivalent of the problem. For this purpose, the parameters and variables used in the model are introduced below. The aim of the problem is to minimize the total cost of system TC as follows.

Parameters

I	Number of products
T	Planning horizon
Q	Number of parts required for producing products
i	Product index $i = 1, \dots, I$
t	Time period index of $t = 1, \dots, T$
q	Part type index $q = 1, \dots, Q$
D_{it}	Demand for product i at period t
ρ	Working hours of a worker at each period
p_{it}	Regular time production cost of product type i at period t
o_{it}	Over time production cost of product type i at period t
c_{it}	Subcontracting cost of product type i in stage k of period t
h_{it}	Inventory cost of product type i at period t
b_{it}	Backorder cost of product type i at period t
ws_t	Salary cost of a worker in stage k of period t
wt_t	Cost to hire one worker at period t
wl_t	Cost to lay off one worker at period t
H_t^{max}	The maximum workforce level available for hiring at period t
W_t^{max}	The maximum overall workforce level in period t
L^{max}	The maximum workforce level can be laid off at each period
σ_i^q	Number of other purchased part type q that is needed for producing one unite of product i
U_t^q	Maximum available number of part type q at period t
υ_i	Capacity of machine that is needed for producing one unit of product i
τ_i	Capacity of machine that is used for setting up in producing one unit of product i
Ca_t	Total available capacity of machine in regular time in period t
β_t	A fraction of capacity of machine that is available in over time of in period t
C_t^{max}	Maximum number of products which can be subcontracted in period t
δ_i^{space}	Amount of warehouse space occupied by one unit of product i
P_t^{space}	Maximum warehouse space that is available in period t
δ_{it}	Worker hours required per unit of product i in period t
γ_t	The ratio of regular time working hours of a worker available for use in overtime in period t
M	A big positive number

Variables

P_{it}	Units of product type i produced in regular time in period
O_{it}	Units of product type i produced in over time in period t
C_{it}	Units of product type i subcontracted in period t
I_{it}	The inventory level of product type i in period t
B_{it}	The backorder level of product type i in period t
W_t	The overall workforce level in period t
H_t	The number of workers hired in period t
L_t	The number of workers laid off in period t
Y_{it}	A binary variable that indicates setup decision of product i on machine in period t

Constraint (2) indicates that total amount of products including regular time, overtime, subcontracting, and inventory storage should be equal to sum of current demand and backorders from previous periods. Constraint set (3) ensures that total amount of parts required for each product should not violate market capacity. Constraint (4) indicates that quantities of subcontracting cannot exceed maximum number of products which can be subcontracted, and constraint (5) shows that the required shortage space cannot exceed warehouse space available. Constraint (6) is related to capacity of machine and expresses that sum of processing times and setup times on machine should be less than regular time capacity. Moreover, constraint (7) ensures that processing time of overtime production cannot be greater than overtime capacity of machine. Constraint (8) determines number of workers at each period. Constraints (9) and (10) ensure that working hours in regular time and overtime cannot be greater than maximum time. Constraint (11) expresses that workforce should be less than a maximum level at each period. Constraint set (12) ensures that hired workforce in each period cannot violate maximum workforce level. Constraint (13) forces that number of laid off workers cannot violate a legal maximum number. Finally, constraints (14) give the decision variables in the model.

Since the cost parameters are uncertain in model (1-14), we cannot solve above linear model, and should consider a new approach which is capable of handling uncertainties. To this end, the grey programming, as an interval based decision making approach, will be introduced in following sections.

3. THEORY of GREY SYSTEMS

In this section, the concept of grey systems will be defined to be used in subsequent sections. A grey number $G^{\pm}$ is a number whose exact value is unknown, but a bounded interval within which the value lies is known [15]. A grey system is a system that includes information as grey numbers; and a grey decision is a decision making system for a grey system. A grey number $G^{\pm}$ is defined as an interval with known lower and upper limit s G^{+} and G^{-}, respectively, as $G^{\pm}=[G^{+},G^{-}]$. One of the main advantages of grey system theory is that it works even if the probability distribution and membership functions cannot be recognized [16]. A grey number $G^{\pm}$ becomes a deterministic number or white number when its upper and lower limits are equal $G^{+}=G^{-}$. Some useful information on this approach are summarized in sequel [17].

Definition 1. A grey number is defined as an interval with known upper and lower limits and unknown distribution information.

Definition 2. The whitened value of a grey number $G^{\pm}$ which is shown by $G_v^{\pm}$ is defined as a deterministic number with a value lying between upper and lower limits.

$$G^{-}\le G_v^{\pm}\le G^{+} \tag{15}$$

Definition 3. The mid-value of grey number $G^{\pm}$ which is shown by $G_m^{\pm}$ is defined as the mid-point value of upper and lower limits.

$$G_m^{\pm}=\frac{G^{-}+G^{+}}{2} \tag{16}$$

Definition 4. The width of grey number $G^{\pm}$ which is shown by $G_w^{\pm}$ is defined as the difference between upper and lower limits.

$$G_w^{\pm}=G^{+}-G^{-} \tag{17}$$

Definition 5. For given grey numbers $G_1^{\pm}=[G_1^{-},G_1^{+}]$ and $G_2^{\pm}=[G_2^{-},G_2^{+}]$, the basic operations on these grey numbers are defined as follows.

$$G_1^{\pm}+G_2^{\pm}=[G_1^{-}+G_2^{-},G_1^{+}+G_2^{+}] \tag{18}$$

$$G_1^{\pm}-G_2^{\pm}=[G_1^{-}-G_2^{-},G_1^{+}-G_2^{+}] \tag{19}$$

Definition 6. For a given grey number $G^{\pm}$, we can tell that,

$$G^{\pm}\ge 0 \quad \text{iff} \quad G^{-}\ge 0 \ and \ G^{+}\ge 0, \tag{22}$$

$$G^{\pm}\le 0 \quad \text{iff} \quad G^{-}\le 0 \ and \ G^{+}\le 0. \tag{23}$$

Definition 7. For given grey numbers $G_1^{\pm}=[G_1^{-},G_1^{+}]$ and $G_2^{\pm}=[G_2^{-},G_2^{+}]$, we have,

$$G_1^{\pm}\le G_2^{\pm} \quad \text{iff} \quad G_1^{-}\le G_2^{-} \quad \text{and} \quad G_1^{+}\le G_2^{+} \tag{24}$$

Definition 8. A grey matrix $\boldsymbol{G}^{\pm}$ is defined as a matrix $[g_{ij}^{\pm}]_{n\times m}$ whose elements are grey numbers $g_{ij}^{\pm}$ for $i=1,2,\dots n$ and $j=1,2,\dots m$.

Definition 9. For given grey matrix $\boldsymbol{G}^{\pm}=[g_{ij}^{\pm}]_{n\times m}$ we have:

$$\boldsymbol{G}^{\pm}\ge 0 \quad \text{iff} \quad g_{ij}^{\pm}\ge 0 \quad \forall \ i=1,2,\dots n, \ j=1,2,\dots m \tag{25}$$

Definition 10. For a grey number $G^{\pm}$, the grey degree which is shown by $GD(G^{\pm})$is defined as follows:

$$GD(G^{\pm})=\left(\frac{G_w^{\pm}}{G_m^{\pm}}\right)\times 10\% \tag{26}$$

The above basic relations establish a new uncertain optimization approach as grey linear programming (GLP) which will be described in sequel.

4. GREY LINEAR PROGRAMMING (GLP)

The grey linear programming (GLP) is a method for decision making under uncertainty. It is a development of the traditional linear programming method. The GLP can be presented as following standard form:

$$Maximize\ f = \boldsymbol{c}^{\pm}x \text{ , Subject to } \boldsymbol{A}^{\pm}x \leq b,\ x \geq 0 \tag{27}$$

in which

$$\boldsymbol{c}^{\pm} = [c_1^{\pm}, c_2^{\pm}, \dots, c_m^{\pm}] \tag{28}$$

$$x^{\mathrm{T}} = (x_1, x_2, \dots, x_m) \tag{29}$$

$$b^{\mathrm{T}} = (b_1, b_2, \dots, b_n) \tag{30}$$

$$\boldsymbol{A}^{\pm} = [a_{ij}^{\pm}]\ \forall\ i = 1,2,\dots,n,\ j = 1,2,\dots,m \tag{31}$$

and

$$c_j^{\pm} = [c_j^{-}, c_j^{+}] \text{ and } a_{ij}^{\pm} = [a_{ij}^{-}, a_{ij}^{+}]\quad \forall\ i = 1,2,\dots,n,\ j = 1,2,\dots,m \tag{32}$$

The optimal solution of problem is also a grey number, since the parameters in the model (27) are grey.

$$f^{*\pm} = [f^{*-}, f^{*+}] \tag{33}$$

$$x_j^{\pm} = [x_j^{-}, x_j^{+}] \tag{34}$$

The aim of GLP is to obtain grey objective function $f^{*\pm}$, and grey decision variables $x_j^{\pm}$ of model (27) as uncertain interval outputs of decision procedure. To this end, let us first analyze the coefficients of variables in the objective function of our model (1-14). Since the standard form of the GLP has a maximization objective form, we convert the total cost TC (1) minimization into new maximization form $NTC = -TC$. Since all of the coefficients in NTC are negative, we can obtain upper and lower limits for objective function as follows:

$$f^{+} = c_1^{+}x_1^{-} + c_2^{+}x_2^{-} + \dots + c_m^{+}x_m^{-} \tag{35}$$

$$f^{-} = c_1^{-}x_1^{+} + c_2^{-}x_2^{+} + \dots + c_m^{-}x_m^{+} \tag{36}$$

According to f^{+}, the model constraints can be presented as:

$$a_{i1}^{+}x_1^{-} + a_{i2}^{+}x_2^{-} + \dots + a_{im}^{+}x_m^{-} \leq b_i \tag{37}$$

and according to f^{-}, the model constraints can be presented as:

$$a_{i1}^{-}x_1^{+} + a_{i2}^{-}x_2^{+} + \dots + a_{im}^{-}x_m^{+} \leq b_i \tag{38}$$

Therefore, main model (27) can be solved via two sub-models presented below.

$$\begin{aligned} &Maximize\ f^{-} = c_1^{-}x_1^{+} + c_2^{-}x_2^{+} + \dots + c_m^{-}x_m^{+} \\ &Subject\ to \\ &a_{i1}^{-}x_1^{+} + a_{i2}^{-}x_2^{+} + \dots + a_{im}^{-}x_m^{+} \leq b_i \\ &x_j^{+} \geq 0\quad \forall\ j = 1,2,\dots,m \end{aligned} \tag{39}$$

$$\begin{aligned} &Maximize\ f^{+} = c_1^{+}x_1^{-} + c_2^{+}x_2^{-} + \dots + c_m^{+}x_m^{-} \\ &Subject\ to \\ &a_{i1}^{+}x_1^{-} + a_{i2}^{+}x_2^{-} + \dots + a_{im}^{+}x_m^{-} \leq b_i \\ &x_j^{-} \leq x_{j\mathrm{opt}}^{+}\quad \forall\ j = 1,2,\dots,m \\ &x_j^{-} \geq 0\quad \forall\ j = 1,2,\dots,m \end{aligned} \tag{40}$$

The above models are conventional linear programming and can be readily solved. The lower limit of objective function f^{-}, and upper limit of variables $x_j^{+}(x_{j\mathrm{opt}}^{+})$ are obtained by model (39). Moreover, the upper limit of objective function f^{+}, and lower limit of variables $x_j^{-}(x_{j\mathrm{opt}}^{-})$ are obtained by model (40). Now, we have obtained the objective function and variables of the GLP as grey numbers $f^{\pm} = [f^{-}, f^{+}]$ and $x^{\pm} = [x^{-}, x^{+}]$.

5. GLP APPLICATIONS

Based on the results obtained in previous sections, the aim of current section is to handle the production planning model (1-14) as a GLP. As mentioned previously, the cost parameters in the model are assumed to be uncertain and lie on specific interval limits with unknown distribution patterns. To obtain the range of objective function and variables in the model, we should re-structure the model (1-4) as grey linear models (39) and (40). Since standard form of the GLP had a maximization objective form, we convert the total cost TC minimization into new maximization of form $NTC = -TC$. Since all of the coefficients in NTC are negative. The limits of objective function is customized as follows.

$$\begin{aligned} NTC^{-} = &- \textstyle\sum_{i=1}^{I}\sum_{t=1}^{T}(p_{it}^{-}P_{it}^{+} + o_{it}^{-}O_{it}^{+} + c_{it}^{-}C_{it}^{+}) - \\ &\textstyle\sum_{i=1}^{I}\sum_{t=1}^{T}(h_{it}^{-}I_{it}^{+}) - \sum_{i=1}^{I}\sum_{t=1}^{T}(b_{it}^{-}B_{it}^{+}) - \\ &\textstyle\sum_{t=1}^{T}(ws_t^{-}W_t^{+}) - \sum_{t=1}^{T}(wt_t^{-}H_t^{+}) - \sum_{t=1}^{T}(wl_t^{-}\ L_t^{+}) \end{aligned} \tag{41}$$

$$\begin{aligned} NTC^{+} = &- \textstyle\sum_{i=1}^{I}\sum_{t=1}^{T}(p_{it}^{+}P_{it}^{-} + o_{it}^{+}O_{it}^{-} + c_{it}^{+}C_{it}^{-}) - \\ &\textstyle\sum_{i=1}^{I}\sum_{t=1}^{T}(h_{it}^{+}I_{it}^{-}) - \sum_{i=1}^{I}\sum_{t=1}^{T}(b_{it}^{+}B_{it}^{-}) - \\ &\textstyle\sum_{t=1}^{T}(ws_t^{+}W_t^{-}) - \sum_{t=1}^{T}(wt_t^{+}H_t^{-}) - \sum_{t=1}^{T}(wl_t^{+}\ L_t^{-}) \end{aligned} \tag{42}$$

Therefore, the GLP can be presented for the problem as two following LP models:

$$\begin{aligned} &Maximize\ NTC^{-} \\ &Subject\ to \end{aligned} \tag{43}$$

$$\begin{aligned} &P_{it}^{+} + O_{it}^{+} + C_{it}^{+} + B_{it}^{+} - B_{i(t-1)}^{+} + I_{i(t-1)}^{+} - I_{it}^{+} \leq D_{it} \\ &i = 1,\dots,I,\ \ t = 1,\dots,T \end{aligned} \tag{44}$$

$$\begin{aligned} &-P_{it}^{+} - O_{it}^{+} - C_{it}^{+} - B_{it}^{+} + B_{i(t-1)}^{+} - I_{i(t-1)}^{+} + I_{it}^{+} \leq \\ &-D_{it} \\ &i = 1,\dots,I,\ \ t = 1,\dots,T \end{aligned} \tag{45}$$

$\sum_{i=1}^{I} \sigma_i^q (P_{it}^+ + O_{it}^+) \le U_t^q \;, \quad t = 1, \dots, T \;, \; q = 1, \dots, Q$ (46)

$\sum_{i=1}^{I} C_{it}^+ \le C_t^{max} \;, \quad t = 1, \dots, T$ (47)

$\sum_{i=1}^{I} (\delta_i^{space} I_{it}^+) \le P_t^{space} \;, \quad t = 1, \dots, T$ (48)

$\sum_{i=1}^{I} (\upsilon_i \; P_{it}^+ + \tau_i \; Y_{it}^+) \le Ca_t \;, \quad t = 1, \dots, T$ (49)

$\sum_{i=1}^{I} (\upsilon_i \; O_{it}^+) \le \beta_t \; Ca_t \;, \quad t = 1, \dots, T$ (50)

$W_t^+ \le W_{t-1}^+ + H_t^+ - L_t^+ \;, \quad t = 1, \dots, T$ (51)

$-W_t^+ \le -W_{t-1}^+ - H_t^+ + L_t^+ \;, \quad t = 1, \dots, T$ (52)

$\sum_{i=1}^{I} (\delta_{it} P_{it}^+) \le \rho W_t \;, \quad t = 1, \dots, T$ (53)

$\sum_{i=1}^{I} (\delta_{it} O_{it}^+) \le \rho \gamma_t \; W_t \;, \quad t = 1, \dots, T$ (54)

$W_t^+ \le W_t^{max} \;, \quad t = 1, \dots, T$ (55)

$H_t^+ \le H_t^{max} \;, \quad t = 1, \dots, T$ (56)

$L_t^+ \le L^{max} \;, \quad t = 1, \dots, T$ (57)

$P_{it}^+ \ge 0, O_{it}^+ \ge 0, C_{it}^+ \ge 0, I_{it}^+ \ge 0$

$B_{it}^+ \ge 0, W_t^+ \ge 0, H_t^+ \ge 0, L_t^+ \ge 0$ (58)

$Maximize \; NTC^+$

Subject to (59)

$P_{it}^- + O_{it}^- + C_{it}^- + B_{it}^- - B_{i(t-1)}^- + I_{i(t-1)}^- - I_{it}^- \le D_{it}$

$i = 1, \dots, I, \; t = 1, \dots, T$ (60)

$-P_{it}^- - O_{it}^- - C_{it}^- - B_{it}^- + B_{i(t-1)}^- - I_{i(t-1)}^- + I_{it}^- \le -D_{it}$

$i = 1, \dots, I, \; t = 1, \dots, T$ (61)

$\sum_{i=1}^{I} \sigma_i^q (P_{it}^- + O_{it}^-) \le U_t^q \;, \quad t = 1, \dots, T \;, \; q = 1, \dots, Q$ (62)

$\sum_{i=1}^{I} C_{it}^- \le C_t^{max} \;, \quad t = 1, \dots, T$ (63)

$\sum_{i=1}^{I} (\delta_i^{space} I_{it}^-) \le P_t^{space} \;, \quad t = 1, \dots, T$ (64)

$\sum_{i=1}^{I} (\upsilon_i \; P_{it}^- + \tau_i \; Y_{it}^-) \le Ca_t \;, \quad t = 1, \dots, T$ (65)

$\sum_{i=1}^{I} (\upsilon_i \; O_{it}^-) \le \beta_t \; Ca_t \;, \quad t = 1, \dots, T$ (66)

$W_t^- \le W_{t-1}^- + H_t^- - L_t^- \;, \quad t = 1, \dots, T$ (67)

$-W_t^- \le -W_{t-1}^- - H_t^- + L_t^- \;, \quad t = 1, \dots, T$ (67)

$\sum_{i=1}^{I} (\delta_{it} P_{it}^-) \le \rho W_t \;, \quad t = 1, \dots, T$ (69)

$\sum_{i=1}^{I} (\delta_{it} O_{it}^-) \le \rho \gamma_t \; W_t \;, \quad t = 1, \dots, T$ (70)

$W_t^- \le W_t^{max} \;, \quad t = 1, \dots, T$ (71)

$H_t^- \le H_t^{max} \;, \quad t = 1, \dots, T$ (72)

$L_t^- \le L^{max} \;, \quad t = 1, \dots, T$ (73)

$P_{it}^- \le P_{it\text{opt}}^+, \; O_{it}^- \le O_{it\text{opt}}^+, \; C_{it}^- \le C_{it\text{opt}}^+, \; I_{it}^- \le I_{it\text{opt}}^+, \; B_{it}^- \le B_{it\text{opt}}^+, W_t^- \le W_{t\text{opt}}^+, \; H_t^- \le H_{t\text{opt}}^+, \; L_t^- \le L_{t\text{opt}}^+$ (74)

$P_{it}^- \ge 0, O_{it}^- \ge 0, C_{it}^- \ge 0, I_{it}^- \ge 0$

$B_{it}^- \ge 0, W_t^- \ge 0, H_t^- \ge 0, L_t^- \ge 0$ (75)

Therefore, the GLP model can be simply solved by solving above LP models using classical optimization solvers.

6. CONCLUSIONS

A set of customer demands from different products, in a single machine capacitated production planning, should be satisfied by producing in different periods in our model. In order to consider the real-world conditions, the cost coefficients, as effective parameters, were assumed to be uncertain. To address the problem described, a linear programming formulation was presented. To handle the uncertainties in the model, the grey systems theory is utilized and the concept of grey numbers is incorporated into optimization framework. The grey linear programming (GLP) as a development of the classical linear programming was used, and the uncertain problem transformed into a grey linear programming and then decomposed into two sub-models. As a result, the uncertain problem can be solved by solving linear deterministic sub-models.

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Uncertainty.

چکيده

برنامه‌ریزی تولید یکی از مسایل مهم در اکثر سیستم‌های تولیدی در عمل است. برخلاف اکثر تحقیقاتی که در ادبیات وجود دارند، این مسأله مواجه با عدم قطعیت زیادی در پارامترها و داده‌های ورودیش است. در تحقیق حاضر، یک مسأله برنامه‌ریزی تولید تک ماشینه با ظرفیت محدود مورد توجه قرار گرفته است و یک فرمول‌بندی برنامه‌ریزی خطی برای آن ارایه شده است. فرض شده است که هزینه‌های تولید پارامترهای غیرقطعی مسأله هستند. به منظور مواجهه با این عدم قطعیت‌ها در مدل مسأله، تئوری سیستم‌های خاکستری به کار گرفته و مفهوم اعداد خاکستری با یک رویکرد بهینه‌سازی ترکیب شد. در چنین سیستم‌هایی، پارامترهای غیرقطعی با توزیع نامعلوم را می‌توان توسط اعداد خاکستری مدل‌سازی کرد. برنامه‌ریزی خطی خاکستری، توسعه‌ی برنامه‌ریزی خطی کلاسیک است که اجازه می‌دهد عدم قطعیت به طور موثری در فرآیند بهینه‌سازی لحاظ گردد. نهایتاً، مسأله‌ی غیرقطعی به یک مسأله برنامه‌ریزی خطی خاکستری تبدیل شده و توسط دو زیرمدل خطی حل می‌شود.

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