

### International Journal of Engineering

Journal Homepage: www.ije.ir

# Introduction of Air Cushion to Reduce Seismic Demand in Cylindrical Water Storage Tanks

#### F. Ashrafzadeh, S. Tariverdilo\*, M.R. Sheidaii

Department of Civil Eng., Faculty of Eng., Urmia University, Urmia, Iran

#### PAPER INFO

#### ABSTRACT

Paper history: Received 20 December 2015 Received in revised form 29 March 2016 Accepted 14 April 2016

Keywords: Cylindrical Water Storage Tanks Air Cushion Seismic Response Boundary Condition Wall Pressure Water storage tanks not designed explicitly for seismic loading could require retrofit. One of the common ways of retrofit include some structural change in the lateral load resisting system that could be expensive and requires the tank to be out of service for relatively long time. This paper introduces a novel method to reduce seismic demand on tank's wall without structural intervention. This is done by employing air cushions adjacent to the wall. The paper investigates the effect of air cushion system on the seismic response of the cylindrical water storage tanks. While in tank without air cushion, the boundary condition adjacent to tank wall is kinematic with no control on the wall pressure, in the proposed method this boundary condition becomes kinetic, enabling control of dynamic fluid pressure on the tank walls. The response parameters of the tank is developed in terms of wall pressure, wave height, base shear, and overturning moment in cylindrical tanks of different sizes with and without air cushions under the far field and near source ground motions. The results demonstrate that the proposed method is an effective way to reduce sloshing force demand.

doi: 10.5829/idosi.ije.2016.29.04a.04

#### **1. INTRODUCTION**

Most of the existing water storage tanks have not been designed for hydrodynamic water pressure during seismic excitations. Adoption of traditional retrofit techniques, in addition to financial costs, requires shut down of the facility for long times that in most of the cases is not possible. This paper proposes application of air cushion on the existing tanks, which introduces flexibility to the tank wall and makes it possible to control sloshing induced water pressure on the tank walls. The proposed method is easy to apply and its financial costs is much smaller than traditional retrofit methods.

Hashemi et al. [1] used an analytical method to determine the dynamic response of flexible rectangular fluid containers. They found that there are significant differences in hydrodynamic pressure distribution on the walls of rigid and flexible tanks. Kianoush et al. [2] investigated the effect of earthquake frequency content on the response of rectangular flexible tanks. They looked into the response of tanks with different aspect ratios under vertical and horizontal ground motion excitations. The tank foundation is modeled as a linear elastic medium on six soil types. Considering the effect of soil structure interaction, they showed that the impulsive response is significantly dependent on the earthquake frequency content, while convective response is independent of the flexibility of the foundation.

Employing flexibility in the form of base isolation or wall flexibility, different researchers proposed different methods to reduce seismic demand in water storage tanks. Shekari et al. [3, 4] studied the seismic response of cylindrical base-isolated storage tanks. They investigated the slenderness effect and considered the response for long-period ground motion excitation. They concluded that for long period excitation, reduction or amplification of response parameters due to employment of base isolation largely depends on the tank's aspect ratio. Malhotra [5] proposed a new base isolation scheme for tanks, which is based on disconnecting the tank wall from its base and placing it

Please cite this article as: F. Ashrafzadeh, S. Tariverdilo, M.R. Sheidaii, Introduction of Air Cushion to Reduce Seismic Demand in Cylindrical Water Storage Tanks, International Journal of Engineering (IJE), TRANSACTIONS A: Basics Vol. 29, No. 4, (April 2016) 464-472

<sup>\*</sup>Corresponding Author's Email: *s.tariverdilo@urmia.ac.ir* (S. Tariverdilo)

on a flexible bearings. De Angelis et al. [6] experimentally and numerically investigated the effectiveness of base isolation for seismic protection of steel tanks with floating roof. They evaluated performance of high damping rubber bearings and also sliding isolators. The results show effectiveness of both types of base isolators.

Dynamic interaction of a flexible container and sloshing liquid can also be used to control liquid sloshing. Anderson [7], Guzel et al. [8, 9] and Gradinscak [10] adopting numerical and experimental models, studied the potential of employing wall flexibility to reduce sloshing in flexible containers. Mousavi and Tariverdilo [11] applied flexible internal wall in water storage tanks as mass absorber to control seismic demand on external walls. Mahmoudi et al. [12] investigated using padded wall as a means to reduce sloshing pressure in rectangle tanks.

This paper investigates the effect of air cushion (placed adjacent to tank wall) on the dynamic response of water storage tanks. To assess the efficiency of the proposed method, different ground motion records including near source and far field records are considered in the simulations. Simulations also include tanks of different aspect ratios. In the following, first the mathematical formulation for the proposed retrofit method is derived and then the simulation results are presented.

#### **2. MATHEMATICAL MODEL**

To provide better assessment on the performance of the tank with air cushion, first the mathematical formulation for the tank with rigid wall is derived and then the formulation for the tank with padded wall will be given.

**2. 1. Response of Rigid Tank** Fluid flow in the case of incompressible and inviscid fluid could be described by a potential function satisfying the Laplace equation. In cylindrical coordinates, the Laplace equation takes the following form:

$$\nabla^2 \varphi = \frac{\partial^2 \varphi}{\partial r^2} + \frac{1}{r} \frac{\partial \varphi}{\partial r} + \frac{1}{r^2} \frac{\partial^2 \varphi}{\partial \theta^2} + \frac{\partial^2 \varphi}{\partial z^2} = 0$$
(1)

For rigid tank subjected to ground motion excitation, the boundary conditions will be (for notations see Figure 1):

$$\frac{\partial \varphi}{\partial z}\Big|_{z=0} = 0$$

$$\frac{\partial \varphi}{\partial r}\Big|_{r=R} = \dot{x}_g \cos\theta$$
(2)
$$\frac{\partial^2 \varphi}{\partial t^2} + g \left. \frac{\partial \varphi}{\partial z} \right|_{z=H} = 0$$

The first boundary condition satisfies the impermeability condition on the bottom surface of the tank, while the second one describes the lateral motion of its rigid wall, and finally the third equation is the free surface boundary condition at z=H. Note that the boundary condition on the tank wall is a kinematic boundary condition with no control on pressure.

Using the method of separation of variables, the analytical solution satisfying the first and second boundary conditions will be:

$$\varphi = [r\dot{x}_g + \sum_{k=1}^{\infty} A_k(t) J_1(\lambda_k r) \cosh(\lambda_k z)] \cos\theta$$
(3)

where  $J_1$  is Bessel function of the first kind of order one, and  $\lambda_i$  is obtained by evaluating the roots of  $J_1(\lambda_k R)=0$ . The modal amplitude  $A_k$  is determined by imposing free surface boundary condition, which leads to:

$$\sum_{k=1}^{\infty} [\cosh(\lambda_k H)\ddot{A}_k(t) + g\lambda_k \sinh(\lambda_k H)A_k(t)]J_1(\lambda_k r)$$

$$= -i\ddot{x}_a$$
(4)

Orthogonality of Bessel functions is employed to decouple these coupled differential equations, as:

$$\ddot{A}_k(t) + \Omega_k^2 A_k(t) = a_k \ddot{x}_g \tag{5}$$

where we have:

$$\Omega_{k}^{2} = g \lambda_{k} \tanh(\lambda_{k} H)$$

$$a_{k} = -\frac{\int_{0}^{R} r^{2} J_{1}(\lambda_{k} r) dr}{\int_{0}^{R} r J_{1}^{2}(\lambda_{k} r) \cosh(\lambda_{k} H) dr}$$
(6)

Here, the time evolution of the modal amplitude could be evaluated by numerical integration.

Linearized Bernoulli's equation for incompressible fluid and irrotational flow reads as (Ibrahim [13]):

$$\rho \frac{\partial \varphi}{\partial t} + p + \rho gz = 0 \tag{7}$$

Neglecting hydrostatic pressure, we have:

$$p = -\rho \frac{\partial \varphi}{\partial t} \tag{8}$$

Now, integrating pressure on the tank's wall, it will be possible to drive time evolution of the shear and overturning moment on the wall:

$$V = \int_{0}^{H} \int_{0}^{2\pi} pRd\theta dz, \ M = \int_{0}^{H} \int_{0}^{2\pi} pzRd\theta dz$$
(9)

To calculate wave elevation, again we use linearized Bernoulli's equation, this time on the free surface, which gives wave elevation as:

$$h = \frac{1}{g} \frac{\partial \varphi}{\partial t} \tag{10}$$

**2. 2. Response of Tank with Air Cushion** This paper proposes adoption of air cushion system as a new technique to reduce sloshing introduced force on the tank walls. The air cushion includes a number of pressurized air packets kept in their positions adjacent to the tank wall (Figure 1). The simplest case would be the air cushion system only introducing stiffness. In this case, change in the volume of the air pocket develops the restoring force. Damping could be provided by introducing system's resisting air transfer between the packets.

The main concept of employing air cushion is to change kinematic boundary condition on the tank wall (Equation 2b) with no control on pressure, to kinetic boundary condition. This way, it will be possible to control sloshing introduced force on the tank wall. To simplify the mathematical treatment, we model the air cushion system as a linear combination of stiffness  $k_{ac}$  and damping  $c_{ac}$ . While the governing equation remains the same, the boundary conditions take the following form

$$\begin{aligned} \frac{\partial \varphi}{\partial z}\Big|_{z=0} &= 0\\ \rho \frac{\partial \varphi}{\partial t}\Big|_{r=R} &= c_{ac} \left(\frac{\partial \varphi}{\partial r}\Big|_{r=R} - \dot{x}_g \cos \theta\right) +\\ k_{ac} \left(\int_0^t \frac{\partial \varphi}{\partial r}\Big|_{r=R} dt - x \cos \theta\right) \end{aligned}$$
(11)  
$$\begin{aligned} \frac{\partial^2 \varphi}{\partial t^2} + g \frac{\partial \varphi}{\partial z}\Big|_{z=H} &= 0 \end{aligned}$$

Comparing Equations (11) and (2), it is evident that the first and third boundary conditions remain the same, but the second one is changing from a kinematic boundary condition to a kinetic one.

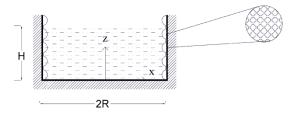


Figure 1. Air cushion padded on the wall of cylindrical tank

The second equation maintains force equilibrium between fluid pressure and air cushion that is a combination of stiffness and damping terms. To solve the resulting partial differential equation, the potential function could be decomposed into three components as:

$$\varphi(r,\theta,z,t) = \varphi_1(r,\theta,z,t) + \varphi_2(r,\theta,z,t) + \varphi_3(r,\theta,z,t)$$

$$\varphi_3(r,\theta,z,t)$$
(12)

With the following boundary conditions for each component:

$$\frac{\partial \varphi_{1}}{\partial z}\Big|_{z=0} = 0$$

$$\frac{\partial \varphi_{1}}{\partial r}\Big|_{r=R} = \dot{x}_{g} \cos\theta$$
(13)
$$\frac{\partial^{2} \varphi_{1}}{\partial t^{2}} + g \left. \frac{\partial \varphi_{1}}{\partial z} \right|_{z=H} = 0$$

and

$$\frac{\partial \varphi_2}{\partial z}\Big|_{z=0} = 0$$

$$\rho\left(\frac{\partial \varphi_1}{\partial t} + \frac{\partial \varphi_2}{\partial t} + \frac{\partial \varphi_3}{\partial t}\right) = c_{ac} \frac{\partial \varphi_2}{\partial r}\Big|_{r=R} + k_{ac} \int_0^t \frac{\partial \varphi_2}{\partial r}\Big|_{r=R} dt \qquad (14)$$

$$\frac{\partial \varphi_2}{\partial z}\Big|_{z=H} = 0$$

and

$$\frac{\partial \varphi_3}{\partial z}\Big|_{z=0} = 0$$

$$\frac{\partial \varphi_3}{\partial r}\Big|_{r=R} = 0$$

$$\frac{\partial^2 \varphi_3}{\partial t^2} + g \frac{\partial \varphi_3}{\partial z}\Big|_{r=H} = -\frac{\partial^2 \varphi_2}{\partial t^2}$$
(15)

It is easy to verify that the sum of  $\varphi_1, \varphi_2$  and  $\varphi_3$ satisfies the boundary conditions of Equation (11). The interesting point is that the boundary conditions of Equation (13) for  $\varphi_1$  are exactly the boundary conditions for  $\varphi$  in Equation (2). Therefore, the solution for  $\varphi_1$ and  $\varphi$  will be the same. This means that the sum of  $\varphi_2$ and  $\varphi_3$  simulates the difference between the rigid tank and tank employing air cushion. As stiffness coefficient  $k_{ac}$  increases,  $\varphi_2$  and  $\varphi_3$  tend to zero, and this means that the solution becomes closer to the solution for rigid tanks. The analytical solution for  $\varphi_2$  by imposing the first and third boundary conditions is

$$\varphi_2 = \sum_{j=1}^{\infty} B_j(t) I_1(\mu_j r) \cos\left(\mu_j z\right) \cos\theta$$
(16)

where  $I_1$  denotes modified Bessel function of the first kind of order one, and  $\mu_j$  equals  $j\pi/H$ . The analytical solution for  $\varphi_3$  imposing the first and the second boundary conditions is:

$$\varphi_3 = \sum_{k=1}^{\infty} C_k(t) J_1(\lambda_k r) \cosh\left(\lambda_k z\right) \cos\theta \tag{17}$$

The modal amplitudes are determined by imposing the remaining boundary conditions, which means:

$$\begin{aligned} &k_{ac} \sum_{j=1}^{\infty} \int B_{j}(t) I_{1}^{'}(\mu_{j}R) \cos\left(\mu_{j}z\right) dt + \\ &c_{ac} \sum_{j=1}^{\infty} B_{j}(t) I_{1}^{'}(\mu_{j}R) \cos\left(\mu_{j}z\right) = \\ & \left( R\ddot{x}_{g} + \sum_{k=1}^{\infty} \dot{A}_{k}(t) J_{1}(\lambda_{k}R) \cosh(\lambda_{k}z) + \right) \\ &\rho \left( \sum_{j=1}^{\infty} \dot{B}_{j}(t) I_{1}(\mu_{j}R) \cos\left(\mu_{j}z\right) + \right) \\ &\sum_{k=1}^{\infty} \dot{C}_{k}(t) J_{1}(\lambda_{k}R) \cosh(\lambda_{k}z) \\ &\sum_{k=1}^{\infty} \left( \ddot{C}_{k}(t) \cosh\left(\lambda_{k}H\right) + \\ &g \lambda_{k}C_{k}(t) \sinh\left(\lambda_{k}H\right) \right) J_{1}(\lambda_{k}r) = \\ &- \sum_{j=1}^{\infty} \ddot{B}_{j}(t) I_{1}(\mu_{j}r) \cos\left(\mu_{j}H\right) \end{aligned}$$
(18)

Multiplying the first equation by  $\cos(\mu_m z)$  and integrating from 0 to *H*, and at the same time multiplying the second equation by  $rJ_1(\lambda_n r)$  and integrating from 0 to *R*, we have:

$$\dot{B}_{m}(t) = -b_{m} \left( k_{ac} \int_{0}^{t} B_{m}(t) dt + c_{ac} B_{m}(t) \right) + \sum_{k=1}^{\infty} c_{mk} \left( \dot{A}_{k}(t) + \dot{C}_{k}(t) \right)$$
(19)

$$\ddot{C}_n(t) = -\Omega_n^2 C_n(t) + \sum_{j=1}^{\infty} d_{nj} \ddot{B}_j(t)$$

where we have:

$$b_{m} = -\frac{I_{1}'(\mu_{m}R)}{\rho I_{1}(\mu_{m}R)},$$

$$c_{mk} = -\frac{\int_{0}^{H} J_{1}(\lambda_{k}R)\cosh(\lambda_{k}z)\cos(\mu_{m}z)dz}{I_{1}(\mu_{m}R)\int_{0}^{H}\cos^{2}(\mu_{m}z)dz}$$
(20)

$$d_{nj} = -\frac{\cos(\mu_j H) \int\limits_0^{p} r I_1(\mu_j r) J_1(\lambda_n r) dr}{\cosh(\lambda_n H) \int\limits_0^{R} r J_1^2(\lambda_n r) dr}$$

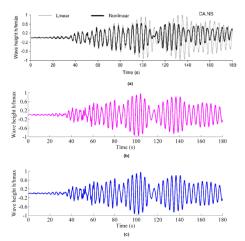
The time evolution of modal amplitudes are determined using numerical integration.

### 3. VERIFICATION OF MATHEMATICAL MODELLING

To verify derived solutions for tank with and without air cushion, we compare asymptotic solution of derived equations for tanks with air cushion with well-known solution for the rigid tanks. Rigid model of Hernandez et al. [14] is selected for the validation purpose. They investigated the sloshing response of a cylindrical tank with water height of 2.75 m and diameter of 11 m subjected to the 1985 Mexico ground motion record. Figure 2 compares the response history for wave elevation of Hernandez et al. results (grey line in Figure 2a, which represents linear response) with the response calculated employing derived formulation for the tank with rigid wall and wall with air cushion. As could be seen, there is good correlation between linear response of Hernandez model and the asymptotic solution for the tank with air cushion.

#### 4. MODEL TANKS AND GROUND MOTIONS

Sloshing effect is investigated in cylindrical tanks of different aspect ratios. To evaluate the performance of the proposed retrofit method, four tanks with different aspect ratios (or fundamental periods) are considered. Table 1 gives the dimensions of the assumed tanks and their natural periods. As could be seen, natural periods increase significantly for tanks with larger aspect ratios (D/H). As will be demonstrated, aspect ratio together with frequency content of the ground motion record will be main factors controlling the response of the tank to different excitations.



**Figure 2.** Wave elevation response history; a) Hernandez-Barrios's model, b) tank with rigid wall, c) padded tank with infinite stiffness used for air cushion.

**TABLE 1.** Dimension and natural periods of tanks considered in the study

Tank	H(m)	<i>D</i> ( <b>m</b> )	D/H	$T_1$ (sec)	$T_2$ (sec)	$T_3$ (sec)
A	5	5	1	2.33	1.37	1.09
В	5	10	2	3.39	1.94	1.54
С	5	20	4	5.48	2.76	2.17
D	5	30	6	7.74	3.46	2.67

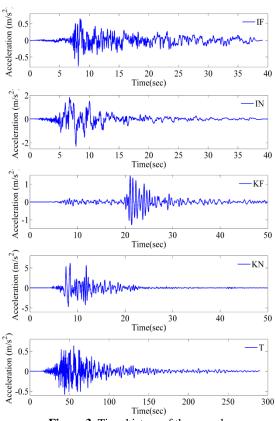
Three different types of ground motion records are considered in the study. These include near field, far field and far filed long period records. These are two records from Imperial Valley and Kobe earthquakes, including near field and far field records and one long period far field record from Tokachi-Oki earthquake are considered. All of the ground motions are recorded in soil type D in NEHRP classification [15]. Table 2 gives designation, component, date, magnitude, focal distance and peak ground acceleration (PGA) of the records.

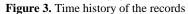
The time history and Fourier spectrum of the ground motions are shown in Figures 3 and 4. Larger energy content of Imperial Valley and Tokachi-Oki records in higher periods is evident in Figure 4. By this observation, larger response could be anticipated for tanks with larger fundamental periods (tanks with larger aspect ratio). Also note that, although Tokach-Oki record has relatively smaller peak ground acceleration (PGA), it has large energy content that is mainly due to long duration of this record compared to the other records.

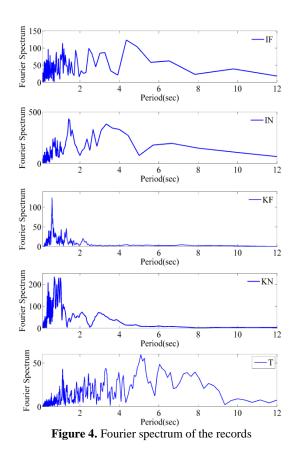
Pseudo acceleration and displacement response spectra of each record are also plotted in Figure 5. The natural periods of baseline case (tank B), are also marked in the figure.

 TABLE 2. The ground motions records used in the simulations

Desig nation	Record/ Componen	Date t	Magnitude	Distance (km)	PGA (m/s <sup>2</sup> )
IF	Imperial Valley/ CAL315	10/15/1979	6.5	23.8	0.76
IN	Imperial Valley/ ECC092	10/15/1979	6.5	7.3	2.31
KF	Kobe/ HIK090	1/16/1995	6.9	96	1.47
KN	Kobe/ KJM090	1/16/1995	6.9	1	5.87
Т	Tokachi/ EW	9/25/2003	8.3	139	0.72







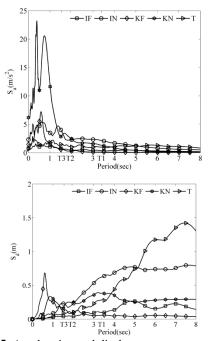


Figure 5. Acceleration and displacement response spectra of the records

#### **5. OPTIMUM AIR CUHESION STIFFNESS**

To evaluate optimum air cushion stiffness for different response parameters, Figure 6 depicts the evolution of the response parameters for increasing stiffness of the air cushion in tank B. The response parameters include wave elevation, and also the wall base overturning moment and shear. The response parameters are evaluated for the rigid tank and the tank with flexible wall. As is evident, it is possible to reduce seismic demand on all of the response parameters for air cushion stiffness of about 10 kN/m in all of the ground motions. Efficiency of the air cushion system degrades for larger stiffness values and as anticipated for larger stiffness values all of the response parameters asymptotically tend to that of rigid tank. For the intermediate stiffness values the response of tank with flexible wall is even larger than the rigid tank.

#### 6. PERFORMANCE OF AIR CUHESION

Evaluating the response parameters for stiffness of 10 kN/m (as the optimum air cushion stiffness for the tank B), Tables 3 and 4 gives the order of change in the response parameters for the tanks considered in the study. The reduction in the response parameters due to introduction of air cushion is quite remarkable for tanks A and B (tanks with small aspect ratio), and this efficiency decreases for tanks C and D (tanks with larger aspect ratios).

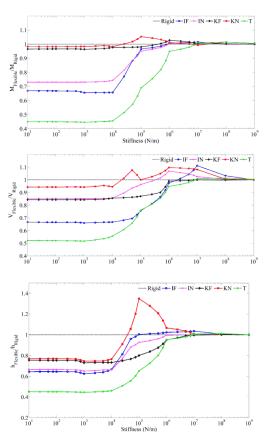


Figure 6. Evolution of base overturning moment, shear and wave elevation for tank B subjected to different ground motions

In other words, there is a noticeable reduction in the efficiency of the air cushion in the case of tanks with larger aspect ratio (D/H). This is partly due to higher natural periods for tanks C and D and change in the transfer function (as will be discussed in the next section), and also due to the use of the air cushion stiffness that is only optimized for tank B.

As expected, local response parameters (wave elevation) is more affected by the presence of air cushion rather than global response parameters (base shear or moment). This reveals that it is easier to change local response rather than the mean response parameters.

As will be explained in the next section, the reduction observed for different response parameters is primarily because of frequency shift in the fundamental period of the tank due to the use of air cushion. Note that the reduction in the response parameters below optimum stiffness is steady, which means that the reduction in the response parameters for this tank is not due to accidental variation of ordinates of the response spectrum, and on the other hand, the reduction is dependable and could be used for design purpose

**TABLE 3.** Effect of using air cushion on the response parameters.

Wave elevation (m)								
	Tank with rigid wall				Tank with flexible wall			
Tank	А	В	С	D	А	В	С	D
IF	0.21	0.27	0.38	0.20	0.10	0.17	0.34	0.17
IN	0.78	1.66	1.12	1.37	0.45	1.07	0.96	1.28
KF	0.14	0.09	0.09	0.11	0.08	0.07	0.08	0.10
KN	0.67	1.10	0.54	0.39	0.54	0.84	0.46	0.49
Т	0.62	1.17	1.83	2.38	0.20	0.52	1.66	1.95
	Base moment (kN.m)							
Tank	А	В	С	D	А	В	С	D
IF	16.8	30.0	47.3	27.8	14	19.6	38.9	24.9
IN	60.5	171	131	166	52	125	129	148
KF	25.4	40.5	43	48.8	26.4	39	42	48
KN	117	169	166	199	116	169	165	199
Т	45.9	114	235	315	19.8	51	225	292
			Base s	hear (k	N)			
Tank	А	В	С	D	А	В	С	D
IF	7.88	13	17.7	13.1	6.7	8.55	14.3	10.8
IN	25	55.8	54.3	60.4	24.2	47.5	52.5	56.5
KF	14.9	22.9	24.5	25.2	13.9	19.3	21.5	23.1
KN	65	93.0	93.9	103	63.8	87.6	90	99.4
Т	11.8	34.6	85.2	119	8.58	17.8	81	113

**TABLE 4.** Percentage of reduction in the response parameters due to adoption of air cushion.

Wave elevation (m)								
Tank	А	В	С	D				
IF	46	38	11	14				
IN	43	36	14	6				
KF	42	24	12	2				
KN	19	23	14	0				
Т	66	55	9	18				
Mean	43	35	12	8				
Base moment (kN.m)								
Tank	А	В	С	D				
IF	17	33	18	10				
IN	14	27	1	11				
KF	4	3	1	1				
KN	0	2	1	0				
Т	58	55	4	7				
Mean	18	24	5	6				
	Base shear (kN)							
Tank	А	В	С	D				
IF	16	34	19	17				
IN	3	15	3	7				
KF	6	16	12	8				
KN	2	6	4	4				
Т	27	48	5	5				
Mean	11	24	9	8				

The response for near field records in comparison with far field ones is remarkably larger. Smaller ordinate of acceleration spectrum of far field records in high period ranges could explain this difference in response magnitudes. This also justifies the large response amplitude for Tokachi-Oki record.

#### 7. TRANSFER FUNCTION

To assess the change in the efficiency of the air cushion for change in the tanks aspect ratio, the system transfer function is derived. For this purpose, the response of the system is evaluated for a harmonic excitation in the form of  $e^{i\omega t}$ . Assuming that the modal amplitudes have a harmonic response, we have:

$$\begin{aligned} x_g &= e^{i\omega t} \to \ddot{x}_g = (i\omega)^3 e^{i\omega t} = -i\omega^3 e^{i\omega t} \\ A_l &= \alpha_l e^{i\omega t} \to \dot{A}_l = \alpha_l i\omega e^{i\omega t} \to \ddot{A}_l = -\alpha_l \omega^2 e^{i\omega t} \\ B_m &= \beta_m e^{i\omega t} \to \dot{B}_m = \beta_m i\omega e^{i\omega t} \to \ddot{B}_m = -\beta_m \omega^2 e^{i\omega t} \\ C_n &= \gamma_n e^{i\omega t} \to \dot{C}_n = \gamma_n i\omega e^{i\omega t} \to \ddot{C}_n = -\gamma_n \omega^2 e^{i\omega t} \end{aligned}$$
(21)

Considering Equation (5), the modal amplitude *A* takes the following form:

$$\left(\Omega_l^2 - \omega^2\right)\alpha_l e^{i\omega t} = -a_l i\omega^3 e^{i\omega t} \to \alpha_l = \frac{a_l i\omega^3}{\omega^2 - \Omega_l^2}$$
(22)

and for the modal amplitudes B and C, we will have:

$$i\omega\beta_{m} = -b_{m} \left[ \frac{k_{ac}}{i\omega} \beta_{m} + c_{ac} \beta_{m} \right] +$$

$$\sum_{k=1}^{\infty} i\omega c_{mk} \left[ \alpha_{k} + \gamma_{k} \right]$$

$$-\omega^{2} \gamma_{n} = -\Omega_{n}^{2} \gamma_{n} - \sum_{j=1}^{\infty} \omega^{2} d_{nj} \beta_{j}$$
(23)

Rearranging this equation, we have:

$$\left(i\omega + c_{ac}b_m + \frac{k_{ac}b_m}{i\omega}\right)\beta_m - \sum_{k=1}^{\infty} i\omega c_{mk}\gamma_k +$$
$$= \sum_{k=1}^{\infty} i\omega c_{mk}\alpha_k$$
(24)

$$\sum_{j=1}^{\infty} \omega^2 d_{nj} \beta_j + \left[\Omega_n^2 - \omega^2\right] \gamma_n = 0$$

This presents a set of linear coupled equations, which by solving it, the modal amplitudes could be obtained. After obtaining the modal amplitudes, the transfer function for pressure on the tank wall could be evaluated as:

$$\rho \frac{\partial \varphi}{\partial t} = \rho \left( \frac{\partial \varphi_1}{\partial t} + \frac{\partial \varphi_2}{\partial t} + \frac{\partial \varphi_3}{\partial t} \right) = \begin{bmatrix} r\ddot{x}_g + \sum_{k=1}^{\infty} \dot{A}_k(t) J_1(\lambda_k r) \cosh(\lambda_k z) + \\ \sum_{j=1}^{\infty} \dot{B}_j(t) I_1(\mu_j r) \cos(\mu_j z) + \\ \sum_{k=1}^{\infty} \dot{C}_k(t) J_1(\lambda_k r) \cosh(\lambda_k z) \end{bmatrix} \rho \cos\theta$$
(25)

This could be rewritten as:

$$\rho \frac{\partial \varphi}{\partial t} = \begin{bmatrix} -\omega^{2}r + \\ \sum_{k=1}^{\infty} i\omega\alpha_{k}J_{1}(\lambda_{k}r)\cosh(\lambda_{k}z) + \\ \sum_{j=1}^{\infty} i\omega\beta_{j}I_{1}(\mu_{j}r)\cos(\mu_{j}z) + \\ \sum_{k=1}^{\infty} i\omega\gamma_{k}J_{1}(\lambda_{k}r)\cosh(\lambda_{k}z) \end{bmatrix} \rho e^{i\omega t}\cos\theta$$
(26)

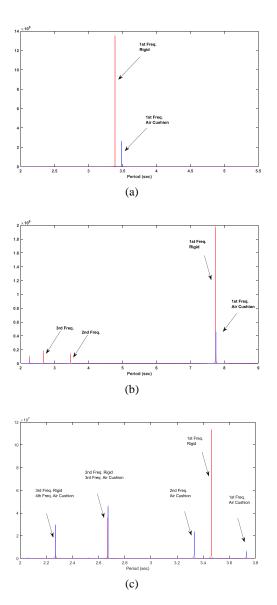
Figure 7 presents the pressure transfer function for  $k_{ac}$  and  $c_{ac}$  of 10 kN/m and 100 N.sec/m, respectively. Small damping is considered in this example, only to be able to evaluate transfer function near the tanks natural periods.

Design codes (e.g. ACI 350.3 [16]) usually adopting Housner [17] frequency domain solution for rigid tanks, decomposes response to impulsive and convective components. In this treatment, the impulsive response is associated with a relatively high frequency (a frequency in constant acceleration zone of response spectrum) and convective term is associated with the fundamental period of the tank ( $T_1$  in Table 1). By this way, codes treatment ignores contribution of higher modes in the response.

For the tank B, as could be seen, there is only significant frequency shift for the fundamental period (convective mode in code treatment) of the rigid tank (from 3.391 sec for rigid tank to 3.489 sec for tank with air cushion), and the frequency shift for other natural frequencies are smaller (including impulsive mode). Note that the transfer function does not cover the phase of different frequencies.

Figure 7b explains why the efficiency of the proposed system decreases for the tanks with larger aspect ratios. This figure compares the transfer function for the fundamental period (convective mode) of the tank D with and without air cushion. As could be inferred from this figure, for increasing aspect ratio (the tank D), the frequency shift is very small (from 7.745 sec for rigid tank to 7.753 sec for tank with air cushion). This explains why there is little, if any reduction of pressure in Table 3 for this tank.

The small shift in the fundamental period is due to the use of the optimized air cushion stiffness derived for the tank B. Figure 7c depicts the frequency shift for tank D for air cushion stiffness of 1 kN/m. As could be seen, in this case there is good frequency shift between fundamental periods of rigid tank and tank with air cushion.



**Figure 7.** Evolution of pressure transfer function, a) Tank B:  $k_{ac}$ =10 kN, b) Tank D:  $k_{ac}$ =10 kN, c) Tank D:  $k_{ac}$ =5 kN.

#### 8. CONCLUSIONS

A novel retrofit technique using air cushion is proposed to reduce seismic demand in cylindrical water storage tanks. Using potential function, analytical solution for the tank with air cushion under lateral excitation is derived. Efficiency of the proposed retrofit technique is investigated for tanks of different aspect ratios under different seismic excitation including near source and far field ground records. It is shown that the proposed retrofit technique works well for tanks of small aspect ratio, but efficiency decreases for tanks of large aspect ratio. Finally, this change in efficiency is attributed to the pressure transfer function and resulting frequency shift due to adoption of air cushion.

#### 9. REFERENCES

- Hashemi, S., Saadatpour, M. and Kianoush, M., "Dynamic behavior of flexible rectangular fluid containers", *Thin-Walled Structures*, Vol. 66, (2013), 23-38.
- Kianoush, M. and Ghaemmaghami, A., "The effect of earthquake frequency content on the seismic behavior of concrete rectangular liquid tanks using the finite element method incorporating soil–structure interaction", *Engineering Structures*, Vol. 33, No. 7, (2011), 2186-2200.
- Shekari, M., Khaji, N. and Ahmadi, M., "A coupled be–fe study for evaluation of seismically isolated cylindrical liquid storage tanks considering fluid–structure interaction", *Journal of Fluids* and Structures, Vol. 25, No. 3, (2009), 567-585.
- Shekari, M., Khaji, N. and Ahmadi, M., "On the seismic behavior of cylindrical base-isolated liquid storage tanks excited by long-period ground motions", *Soil Dynamics and Earthquake Engineering*, Vol. 30, No. 10, (2010), 968-980.
- Malhotra, P.K., "New method for seismic isolation of liquidstorage tanks", *Earthquake Engineering & Structural Dynamics*, Vol. 26, No. 8, (1997), 839-847.
- De Angelis, M., Giannini, R. and Paolacci, F., "Experimental investigation on the seismic response of a steel liquid storage tank equipped with floating roof by shaking table tests", *Earthquake engineering & Structural Dynamics*, Vol. 39, No. 4, (2010), 377-396.
- 7. Anderson, J.G., "Liquid sloshing in containers: Its utilisation and control", Victoria University of Technology, (2000).
- Guzel, B.U., Gradinscak, M., Semercigil, S.E. and Turan, O.F., "Control of liquid sloshing in flexible containers: Part 1. Added mass", in 15th Australasian Fluid Mechanics Conference, University of Sydney, Sydney, Australia. (2004).

- Guzel, B., Gradinscak, M., Semercigil, S. and Turan, Ö, ... "Tuning flexible containers for sloshing control", *IMAC XXIII*, (2004).
- Gradinscak, M., "Liquid sloshing in containers with flexibility", Victoria University, (2009).
- Mousavi, J. and Tariverdilo, S., "Employing internal flexible wall as mass absorber in tanks subjected to harmonic excitations", *International Journal of Engineering-Transactions A: Basics*, Vol. 27, No. 10, (2014), 1527-1536.
- Mahmoudi, R., Ashrafzadeh, F. and Tariverdilo, S., "Introducing padded wall to reduce sloshing induced wall pressure in water storage tanks", *International Journal of Engineering-Transactions C: Aspects*, Vol. 27, No. 12, (2014), 1823-1832.
- 13. Ibrahim, R.A., "Liquid sloshing dynamics: Theory and applications, Cambridge University Press, (2005).
- Hernandez-Barrios, H., Heredia-Zavoni, E. and Aldama-Rodriguez, A.A., "Nonlinear sloshing response of cylindrical tanks subjected to earthquake ground motion", *Engineering Structures*, Vol. 29, No. 12, (2007), 3364-3376.
- Council, B.S.S., "Nehrp recommended provisions for seismic regulations for new buildings and other structures (fema 450): Provisions, Building Seismic Safety Council, National Institute of Building Sciences, (2004).
- 16. American Concrete Institute, "Seismic design of liquid containing concrete structures", (2006).
- Housner, G., "Dynamic pressure on fluid containers", *TID-7027*, *Nuclear Reactors and Earthquakes*, Vol., No., (1963), 183-209.

چکيد

## Introduction of Air Cushion to Reduce Seismic Demand in Cylindrical Water Storage Tanks

#### F. Ashrafzadeh, S. Tariverdilo, M.R. Sheidaii

Department of Civil Eng., Faculty of Eng., Urmia University, Urmia, Iran

#### PAPER INFO

Paper history: Received 20 December 2015 Received in revised form 29 March 2016 Accepted 14 April 2016

Keywords: Cylindrical Water Storage Tanks Air Cushion Seismic Response Boundary Condition Wall Pressure مخازن ذخیره آب که برای بارهای لرزهای طراحی نشدهاند، اغلب نیازمند بهسازی هستند. روش معمول بهسازی لرزهای در برگیرنده تغییراتی در سیستم سازهای مخازن می باشد که هزینه بر بوده و توام با خروج از سرویس مخزن برای زمان طولانی است. مقاله حاضر روشی جدید مبتنی بر کاهش نیاز در دیواره های مخازن تحت بارگذاری لرزهای معرفی می نماید که نیازمند تغییر در سازه مخزن نیست. این امر با تعبیه بالشتک هوا در حد فاصل دیوار مخزن و سیال انجام می شود. در مخازن عادی (بدون بالشتک) شرایط مرزی دیواره مخزن سینماتیکی است و هیچ گونه کنترلی روی فشار سیال وارد بر دیواره مخزن وجود ندارد. با تعبیه بالشتک، شرایط مرزی در دیواره سینتیکی شده، امکان کنترل فشار سیال بر روی دیواره مخزن ایجاد می گردد. در این مقاله، پاسخ مخزن شامل فشار سیال بر دیواره، ارتفاع موج، برش و لنگر خمشی در مخازن استوانهای به ابعاد مختلف با و بدون بالشتک هوا برای زلزله های حوزه دور و نزدیک ارائه شده و کارایی سیستم پیشنهادی بررسی شده است. نتایج نشانگر آن است که روش ارائه شده یک روش مؤثر برای کنترل نیروی ناشی از مواج شدگی است. doi: 10.5829/idosije.2016.29.04.04