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Inverse Boundary Design Problem of Combined Radiation-convection Heat Transfer in Laminar Recess Flow

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ABSTRACT

In the present work, an inverse analysis of combined radiation and laminar forced convection heat transfer in a two-dimensional channel with variable cross sections is performed. The conjugate gradient method is used to find the temperature distribution over the heater surface to satisfy the prescribed temperature and heat flux distributions over the design surface. The fluid is considered to be a gray participating medium with absorption, emission and isotropic scattering. The discrete ordinate method is used to solve the radiative transfer equation. The effect of radiation-conduction parameter is studied on the amount of heat transfer from the heater surface.

NOMENCLATURE

CR	Contraction ratio	**Greek Symbols**		
d	Direction of descent	α	thermal diffusivity (m^2/s)	
E	Error	β	extinction coefficient (m^{-1})	
ER	Expansion ratio	δ	Dirac delta function	
G	Objective function	ε	wall emissivity	
h_1	Upstream height of the channel	ζ	derivation of temperature with respect to the heater element	
h_2	Downstream height of the channel	ϕ	scattering phase function, inclination angle	
H	Height of the channel, m	γ	conjugate coefficient	
I	Radiation intensity (W/m^2)	ν	kinematic viscosity (m^2/s)	
J	Sensitivity matrix	Θ, θ	dimensionless temperature	
I^*	Dimensionless radiation intensity	ρ	Density (kg/m^3)	
k	Thermal conductivity (W/m.K)	σ	Stefan Boltzmann's constant (5.67×10^{-8} W/m^2.K^4)	
L	Length of the channel, m	σ_a	absorption coefficient (m^{-1})	
p	Pressure (Pa)	σ_s	scattering coefficient(m^{-1})	
P	Dimensionless pressure	τ	optical thickness	
Pe	Peclet number	ω	scattering albedo	
Pr	Prandtl number	ψ	search step size	
q	Heat flux (W/m^2)	$\Im$	derivation of radiation intensity with respect to the heater element	
Q	Dimensionless heat flux			
$\vec{r}$	Radiation intensity position	Subscripts and Superscripts		
RC	Radiation-conduction parameter	C	Convective heat flux	
Re	Reynolds number	d, D	Design surface, desired heat flux	
$\vec{s}$	radiation intensity direction	h, H	Heater surface	
T	Temperature (K)	in	Inlet	

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U_0	average velocity at the inlet section, m/s	K	Iteration number
u, v	velocity components in x and y directions, respectively (m/s)	m, n	Node number on heater and design surfaces
		r	Radiative heat flux
U, V	Dimensionless velocity components in x and y directions, respectively (m/s)	rel	relative
x, y	Dimensional coordinates (m)	t	Total heatflux
X, Y	Dimensionless coordinates	w	Wall

1. INTRODUCTION

Forced convection flows inside channels with sudden expansion and contraction have many industrial applications, such as gas turbine blades, combustion chambers, heat exchangers, cooling of electronic devices and so on. Despite the simplicity of such geometries, fluid flow and heat transfer analysis is not that simple due to existence of separating and reattaching regions. One of the geometries in which separating and reattaching regions can be well seen is the recess, composed of a backward and a forward facing step. In the foregoing devices, simultaneous forced convection and radiation heat transfer may co-exist, especially when the temperature is high enough. Many industrial processes such as thermal processing of materials and air humidification systems require two thermal conditions to be satisfied in some parts of the thermal systems as a quality control sense. For instance, in order to produce materials with uniform quality all over them (heating process in furnaces), desired controlled heating can be achieved by simultaneously providing uniform temperature and uniform heat flux over heated objects. Inverse boundary design analysis can be employed in such cases to provide desired conditions.

Several studies have been carried out by many researchers in the field of convective heat transfer inside channels with backward or forward facing steps [1-4]. Heidary et al. [5] studied magnetic field effect on nanofluid forced convection in a channel whereas Sheikholeslami et al. [6] studied the effect of non-uniform magnetic field on forced convection heat transfer of nanofluid inside an enclosure. The combined effects of nanoparticle and magnetic field on the nonlinear Jeffery-Hamel flow were analyzed by Alama and Khanb [7]. In recent years, combined convective and radiative heat transfer inside channels with variable cross sections has been received more attention. Atashafrooz and Gandjalikhan Nassab [8, 9] investigated the effects of radiation-conduction parameter, optical thickness and albedo coefficient on the heat transfer behaviors of 2-D and 3-D laminar recess flows in a participating medium.

The effects of thermal buoyancy and the radiative transfer on the distributions of the bulk fluid temperature, friction factor and Nusselt number in an inclined rectangular duct was studied by Chiu and Yan [10]. In that study, the integro-differential radiative transfer equation was solved by discrete ordinates method. Ansari and Gandjalikhan Nassab [11] studied the effects of radiation-conduction parameter, inclination angle, bleeding coefficient and optical thickness in laminar forced convection flow of a radiating gas over an inclined backward facing step of a horizontal duct under bleeding condition. In the foregoing studies, boundary conditions and fluid flow properties are specified as input data to find medium temperature and wall heat flux distributions. This trend is called direct procedure. Recently, researchers have shown much interest using inverse analysis of radiation combined with other modes of heat transfer to estimate boundary conditions, heat source distribution, temperature field and so on [12-17]. The optimization techniques based on different methods such as conjugate gradient and Levenberg-Marquardt is used by many investigators to find unknown strength of heater surface of enclosures with free convection and radiation [18-22]. Franca et al. [23] applied regularization methods in an inverse boundary design problem with laminar convection and radiation heat transfer in a participating medium between parallel plates. The fluid flow was assumed to have a fully developed velocity profile. Mossi et al. [24] used the same method to solve an inverse design problem with combined turbulent convection and radiation heat transfer. In that study, the researcher used a fan to generate turbulent flow inside an enclosure. An inverse boundary design problem of turbulent convection between parallel plates with surface radiation exchange was solved by Shokouhi [25] through optimization method. They used a fully developed turbulent velocity profile to model the fluid flow. The effects of Reynolds number and wall emissivity were investigated on the heat flux distribution over a heater surface. To the best of author's knowledge, the inverse boundary design technique has not been employed in combined convection-radiation heat transfer in separated fluid flows. That is, a few studies which were done by other investigators are limited to convective flow with simple geometries, i.e., square cavity flow and rectangular duct flow.

Thereby, this study considers an inverse analysis of combined laminar forced convection and radiation heat

transfer in a two-dimensional channel with two inclined backward and forward facing steps, performing a recess. The aim is to find the unknown temperature profile over the heater surface to satisfy both uniform temperature and heat flux distributions over the design surface. In most of the previous works, the fluid flow is considered to have a fully developed velocity profile. However, this assumption cannot be applied in this study due to existence of separating and reattaching regions. The fluid is considered to be a gray absorbing-emitting-scattering medium. All physical properties are assumed to be uniform. The channel walls are treated as gray-diffuse absorbers and emitters. An optimization technique based on conjugate gradient method is applied to minimize an objective function which is defined as sum of square errors between the exact and estimated heat fluxes over the design surface. The discretized form of the momentum equation is solved by finite volume method (FVM) through simple algorithm, while the finite difference method (FDM) is used to the solve energy equation. The discrete ordinate method (DOM) is applied to solve radiative transfer equation, which is discretized by the FVM. The block-off method is used in all of the above discretized equations including sensitivity problem.

2. PROBLEM DESCRIPTION

Figure 1 shows a schematic view of the two-dimensional rectangular channel with two inclined backward and forward facing steps. A steady state, laminar absorbing-emitting-scattering gas flows between the walls of the channel. The upstream height (h_1), the downstream height (h_2) and the height of channel in the recess region (H) are positioned in a manner that the expansion (ER=H/h_1) and the contraction (CR=h_2/H) ratios to be 2 and 0.5, respectively.
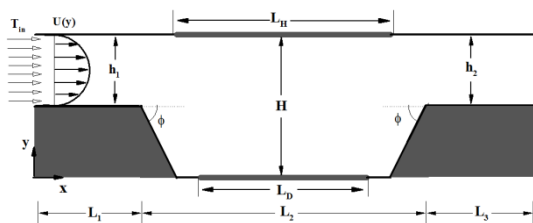
Figure 1. Schematic of the two-dimensional channel performing a recess

The upstream and downstream lengths of the channel are considered to be L_1=L_3=3H and the recess length is equal to L_2=8H. The heater and the design surfaces which have the length of L_H=6H and L_D=4H, are located at the center of the top and recess walls,

respectively. Each step is considered to have an inclination angle ϕ, which is measured from a horizontal sense.

The boundary conditions contain no slip condition at the solid walls and constant temperature of Tw at all boundary surfaces except the heater surface, which its temperature distribution is of our interest. There is also a uniform heat flux all over the design surface. At the inlet channel section, the flow is fully developed with uniform temperature of Tin which is considered to be less than Tw. At the outlet section, zero axial gradients for velocity components and gas temperature are employed. All walls are diffuse-gray with a constant emissivity. The inlet and outlet sections are treated as black walls at the inlet and outlet sections temperature, respectively. The inverse boundary design technique tries to find the unknown temperature distribution over the heater surface to recover uniform temperature and heat flux distribution over the design surface.

3. DIRECT PROBLEM

To express principal equations and related boundary conditions, we define the following non-dimensional parameters as:

$$(X,Y) = (\frac{x}{H},\frac{y}{H}) \quad (U,V) = (\frac{u}{U_0},\frac{v}{U_0}) \quad P = \frac{p}{\rho U_0^2}$$

$$\Theta = \frac{T-T_{in}}{T_w-T_{in}} \quad \theta_1 = \frac{T_{in}}{T_w-T_{in}} \quad \theta_2 = \frac{T_w}{T_{in}} \quad I^* = \frac{I}{\sigma T_w^4}$$

$$\tau = \beta.H \quad (1-\omega) = \frac{\sigma_a}{\beta} \quad \omega = \frac{\sigma_s}{\beta} \quad \text{Pr} = \frac{\nu}{\alpha} \tag{1}$$

$$Pe = \text{Re.Pr} \quad RC = \frac{\sigma T_w^3 H}{k} \quad Q = \frac{q}{\sigma T_w^4} \quad \text{Re} = \frac{\rho U_0 H}{\mu}$$

For incompressible, steady state, two-dimensional flow with uniform properties, continuity, momentum and energy equations can be expressed in a non-dimensional form as below:

$$\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0 \tag{2}$$

$$U\frac{\partial U}{\partial X} + V\frac{\partial U}{\partial Y} = -\frac{\partial P}{\partial X} + \frac{1}{\text{Re}}\left(\frac{\partial^2 U}{\partial X^2} + \frac{\partial^2 U}{\partial Y^2}\right) \tag{3}$$

$$U\frac{\partial V}{\partial X} + V\frac{\partial V}{\partial Y} = -\frac{\partial P}{\partial Y} + \frac{1}{\text{Re}}\left(\frac{\partial^2 V}{\partial X^2} + \frac{\partial^2 V}{\partial Y^2}\right) \tag{4}$$

$$U\frac{\partial \Theta}{\partial X} + V\frac{\partial \Theta}{\partial Y} = \frac{1}{Pe}\left(\frac{\partial^2 \Theta}{\partial X^2} + \frac{\partial^2 \Theta}{\partial Y^2}\right) - \frac{1}{Pe}\nabla.Q_r \tag{5}$$

3. 1. Gas Radiation Modeling   The governing equations for gas radiation model can be represented by

[26]. The non-dimensional radiative term in the energy equation can be calculated as:

$$\nabla . Q_r = \tau(1-\omega)RC\theta_1\theta_2 \left[\frac{4}{\theta_2^{\,4}}\left(\frac{\Theta}{\theta_1}+1\right)^4 - \int_{4\pi} I^*(\vec{r},\vec{s})\,d\Omega \right] \qquad (6)$$

where $I^*(\vec{r},\vec{s})$ is non-dimensional radiation intensity at the position $\vec{r}$ in the $\vec{s}$ direction which can be calculated from the radiation transfer equation (RTE). For an absorbing, emitting and scattering gray medium, dimensionless form of RTE is as follows:

$$(\vec{s}.\nabla)I^*(\vec{r},\vec{s}) = -I^*(\vec{r},\vec{s}) + \frac{(1-\omega)}{\pi\theta_2^{\,4}}\left(\frac{\Theta}{\theta_1}+1\right)^4$$
$$+\frac{\omega}{4\pi}\int_{4\pi} I^*(\vec{r},\vec{s})\phi(\vec{s},\vec{s}')\,d\Omega' \qquad (7)$$

In the above equation, $\phi(\vec{s},\vec{s}')$ is the scattering phase function, which its value is equal to unity in an isotropic scattering medium, as in this study. The RTE must be solved simultaneously with the energy equation due to Θ term which is unknown. In this study, the discrete ordinate method is used to solve the RTE. The radiative boundary condition for diffuse- gray wall is:

$$I^*(\vec{r_w},\vec{s}) = \frac{\varepsilon_w}{\pi\theta_2^{\,4}}\left(\frac{\Theta}{\theta_1}+1\right)^4 + \frac{(1-\varepsilon_w)}{\pi}$$
$$\int_{\vec{n_w}.\vec{s}'<0} I^*(\vec{r_w},\vec{s})\left|\vec{n_w}.\vec{s}'\right|\,d\Omega' \qquad (8)$$

The details of numerical solution of RTE with DOM are given in the previous work by the second author [11].

4. INVERSE PROBLEM

In this study, the Conjugate Gradient Method is used to find the unknown temperature distribution, $\Theta_h(x)$ over the heater surface. The Conjugate Gradient Method, is a straightforward and powerful iterative technique for solving linear and nonlinear inverse problems of parameter estimation. In the iterative procedure of the Conjugate Gradient Method, at each iteration, a suitable step size is taken along a direction of descent to minimize the objective function. The direction of descent is obtained as a linear combination of the negative gradient direction at the current iteration with the direction of descent of the previous iteration. The linear combination is such that the resulting angle between the direction of descent and the negative gradient direction is less than 90' and the minimization of the objective function is assured [27]. The heat flux distribution over the design surface is additional information for estimating the heater temperature distribution, which can be measured by some sensors installed on the design surface. The conjugate gradient method is based on minimization of an objective function, G, which is defined as the sum of square errors between desired and estimated heat fluxes over the design surface as below[26]:

$$G = \sum_{n=1}^{N}(Q_{d,n} - Q_{e,n})^2 \qquad (9)$$

In the above equation, Q_d, Q_e are the desired and estimated dimensionless heat fluxes over the design surface, respectively. N and subscript n are number of nodes and the node number on design surface, respectively. The iterative procedure of the conjugate gradient method is explained below. The heater surface temperature at iteration k+1 is advanced by:

$$\Theta_{h,m}^{k+1} = \Theta_{h,m}^{k} - \psi^k d_m^k \qquad (10)$$

where superscript k is the iteration number, ψ^k the search step size and d^k direction of descent given by:

$$d_m^{\,k} = \nabla G_m^k + \gamma^k d_m^{\,k-1} \qquad (11)$$

which is a conjugation of gradient direction vector ∇G and the direction of descent in the previous iteration. In Equation (13) γ is the conjugate coefficient which can be expressed as the Polak-Ribiere expression given in the form:

$$\gamma^k = \frac{\sum_{m=1}^{M}\left[(\nabla G_m^k)(\nabla G_m^k - \nabla G_m^{k-1})\right]}{\sum_{m=1}^{M}(\nabla G_m^{k-1})^2} \qquad (12)$$

with $\gamma^0 = 0$ when k=0.
To obtain ∇G, differentiation from Equation (11) with respect to unknown parameter $\Theta_{h,m}$ should be performed. Thus:

$$\nabla G_m^k = -2\sum_{n=1}^{N} J_{nm}^k(Q_{d,n} - Q_{e,n}^k) \qquad (13)$$

Here, $J_{nm}^k = \dfrac{\partial Q_{e,n}^k}{\partial \Theta_{h,m}^k}$ is sensitivity coefficient which is an element of the sensitivity matrix. The sensitivity coefficients are calculated in sensitivity problem. Finally, by obtaining sensitivity coefficients and direction of descent, the search step size can be defined as:

$$\psi^k = \frac{\sum_{m=1}^{M}\sum_{n=1}^{N}(J_{nm}^k d_m^k)(Q_{e,n}^k - Q_{d,n})}{\sum_{m=1}^{M}\sum_{n=1}^{N}(J_{nm}^k d_m^k)(J_{nm}^k d_m^k)} \qquad (14)$$

4. 1. Sensitivity Problem   The sensitivity coefficient in Equation (13) is a measure of sensitivity of the estimated heat flux $Q_{e,n}$ over the design surface with respect to changes in temperature of node m, over the heater surface ($\Theta_{h,m}$). The sensitivity coefficients are obtained by differentiating the governing equations and boundary conditions with respect to unknown nodal temperatures over the heater surface. The differentiated equations can be written as follows:

$$U\frac{\partial \zeta^m}{\partial X} + V\frac{\partial \zeta^m}{\partial Y} = \frac{1}{Pe}\left(\frac{\partial^2 \zeta^m}{\partial X^2} + \frac{\partial^2 \zeta^m}{\partial Y^2}\right) -$$

$$\frac{\tau(1-\omega)RC\theta_1\theta_2}{Pe}\left[\frac{16}{\theta_1\theta_2^4}\left(\frac{\Theta}{\theta_1}+1\right)^3\zeta^m - \sum_{i=1}^{n}\Im_i^m w_i\right] \tag{15}$$

Over heater surface:

$$\zeta_h^m(X) = \delta(X - X_m) \tag{16.a}$$

Over other surfaces:

$$\zeta^m(X) = 0 \tag{16.b}$$

$$(\vec{s}.\nabla)\Im^m(\vec{r},\vec{s}) = -\Im^m(\vec{r},\vec{s}) + \frac{4(1-\omega)}{\pi\theta_1\theta_2^4}\left(\frac{\Theta}{\theta_1}+1\right)^3\zeta^m$$

$$+\frac{\omega}{4\pi}\int_{4\pi}\Im^m(\vec{r},\vec{s})\phi(\vec{s},\vec{s'})\,d\Omega' \tag{17}$$

with boundary condition:

$$\Im^m(\vec{r_w},\vec{s}) = \frac{4\varepsilon_w}{\pi\theta_1\theta_2^4}\left(\frac{\Theta}{\theta_1}+1\right)^3\zeta^m$$

$$+\frac{(1-\varepsilon_w)}{\pi}\int_{\vec{n_w}.\vec{s'}<0}\Im^m(\vec{r_w},\vec{s})\left|\vec{n_w}.\vec{s'}\right|d\Omega' \tag{18}$$

In the above equations, $\zeta^m(X) = \dfrac{\partial \Theta}{\partial \Theta_{h,m}}$ and

$\Im^m(X) = \dfrac{\partial I^*}{\partial \Theta_{h,m}}$. These equations are solved in a similar manner to the one used in direct problem. By solving these equations, sensitivity coefficients are calculated as:

$$J_{nm} = \frac{\partial Q_{e,n}}{\partial \Theta_{h,m}} \qquad n = 1, 2, \ldots, N \tag{19}$$

which are updated at each iteration. In order to complete the sensitivity matrix, sensitivity problem must be solved *M* times.

5. COMPUTATIONAL PROCEDEURE

The solution algorithm of the above problem with conjugate gradient method is summarized as follows.

Start with an initial guess of the unknown temperature distribution over the heater surface, $\Theta_h^0(X)$, set k=0, and then perform the steps below:

Step 1. Solve the direct problem to obtain temperature distribution inside the medium.

Step 2. Calculate the objective function G by Equation (9) and check if it is smaller than a pre-specified value. This small value is chosen in a manner to ensure that the answer has enough accuracy. If it is satisfied, terminate procedure, else go to step 3.

Step 3. By knowing the temperature distribution inside the medium from step 1, solve the sensitivity problem.

Step 4. Calculate the gradient direction, ∇G by Equation (13).

Step 5. Compute the conjugation coefficient by Equation (12).

Step 6. Calculate direction of descent, d_m^k and the search step size, ψ^k, by Equation (11) and Equation (14), respectively.

Step 7. Calculate the new estimation for $\Theta_h^{k+1}(X)$, replace k by k+1 and go to step 1.

6. VERIFICATION OF SOLUTION

6. 1. Verification of Direct Problem   In order to verify the accuracy of the direct problem, the convective and radiative Nusselt numbers along the bottom wall of a recess are compared with those obtained by Atashafrooz and Gandjalikhan Nassab [8]. In that study, a gray participating medium flows in a laminar regime, over a recess. All surfaces are at constant temperature of T_w. The comparison of results is presented in Figure 2. As seen, the results of the present work with those reported by [8] have a very good agreement.
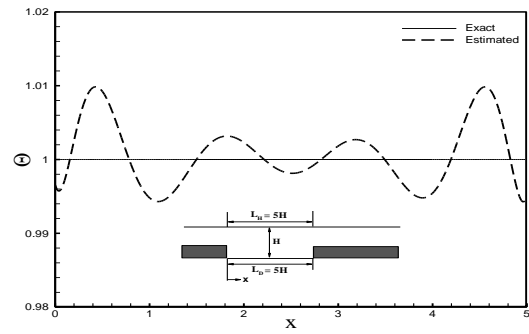
6. 2. Verification of Inverse Problem   To verify the accuracy of the inverse problem, we use the results of previous study by [8]. After insuring verification of the direct problem, total heat flux distribution over bottom wall of the recess is calculated as additional information for an inverse problem. The entire length of recess bottom wall and its corresponding equal length over the top wall are considered to be the design and heater surface, respectively. Now, the problem is to find the unknown temperature distribution over the heater surface located on the top wall, to obtain both uniform temperature and prescribed heat flux distributions over the design surface. All boundary conditions and physical properties are those used in [8]. As we expect, the heater must have a uniform temperature all over its surface. Figure 3 shows the estimated and the exact temperature distribution over the heater surface. As

seen, the estimated temperature has an acceptable deviation with the exact one.

6. 3. Grid Study   Since computation of sensitivity matrix is a time-consuming process when the number of elements to be estimated is large, working with the optimum grid is necessary to obtain grid-independent solutions and also to reduce the time of calculations. Thus, maximum value of the total heat flux and its corresponding location on the design surface was used in a direct problem for grid study. The optimized mesh size is chosen 700×60 (X×Y) based on grid study results. Figure 4 shows the discretized computational domain for numerical calculations. As seen, the grid is non-uniform and concentrated near solid walls.

7. RESULTS AND DISCUSSION

The 2-D channel shown in Figure 1 consists of two inclined backward and forward facing steps which has an step inclination angle of $\phi = 45°$. The expansion and contraction ratios are $ER = 2$ $CR = 0.5$, respectively. The heater and the design surface which have a length of $L_H = 6H$ and $L_D = 4H$, are located at the center of the top and recess walls, respectively.

(a)

(b)

Figure 2. Comparison of (a) convective Nusselt number and (b) radiative Nusselt number along the bottom wall.
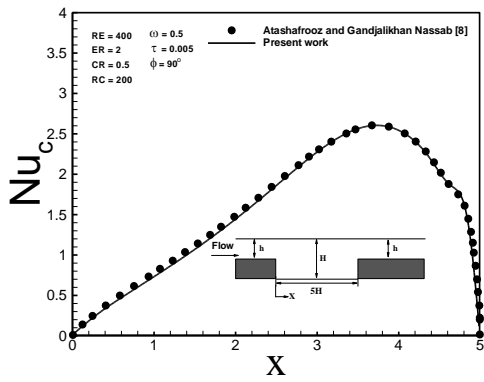

Figure 3. Comparison of estimated and exact temperature distribution over the heater surface
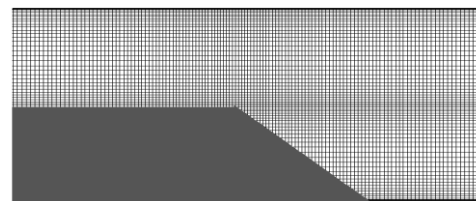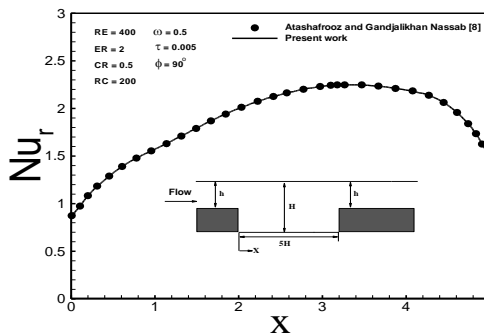

Figure 4. Discretization of the computational domain
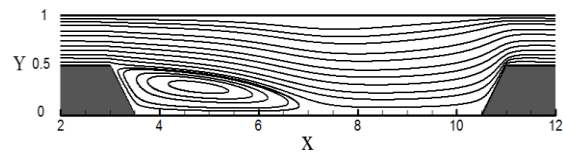
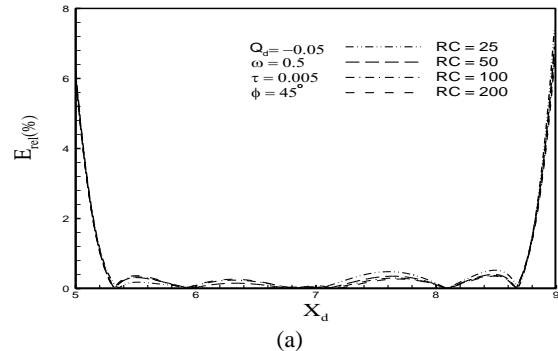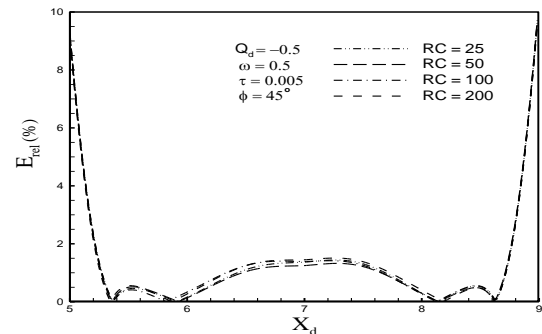**Figure 5**. Flow streamlines inside recess

(a)

(b)

Figure 6. Relative error distribution over the design surface for different values of RC (a) $Q_d = -0.05$ (b) $Q_d = -0.5$

All walls are considered to be diffuse-gray with constant emissivity equal to $\varepsilon_w = \varepsilon_d = \varepsilon_h = 0.8$. The design surface and all other solid boundary surfaces except the heater surface has a dimensionless temperature of $\Theta_w = \Theta_d = 1$. A laminar air flow enters the duct with uniform dimensionless temperature of $\Theta_{in} = 0$. In this study, the values of the second and third non-dimensional temperatures are $\theta_1 = 4$ and $\theta_2 = 1.25$. The Reynolds number is $\text{Re} = 400$ and the Prandtl number is $\text{Pr} = 0.71$. The optical thickness and scattering albedo of the participating medium are $\tau = 0.005$ and $\omega = 0.5$, respectively. The propose of the inverse design problem is to study the effect of radiation-conduction parameter on the amount of heat transfer from the heater surface for two different values of non-dimensional uniform heat flux over the design surface.

In order to have a better comprehension of the effect of sudden expansion and contraction on the amount of convective heat transfer in the inverse analysis, the streamlines are shown in Figure 5 when the recess length is equal to $L_2 = 8H$. It is depicted in this figure, a recirculation region exists downstream the backward step adjacent the bottom wall. It should be noted that this circulation region is not the only one in this kind of geometry, such that more circulation regions can exist by increasing the Reynolds number or inclination angle.

One of the main parameters in the design of high temperature thermal systems is the radiation-conduction parameter (RC) which indicates the significance of the radiation heat transfer relative to the conduction counterpart. High values of RC shows the radiative heat transfer is the dominant mode in the thermal behavior of the system. In order to investigate the effects of RC parameter in the inverse design problem, two cases for total uniform heat flux over the design surface is considered. Those are, $Q_d = -0.05$ and $Q_d = -0.5$, which are named case 1 and case 2, respectively.

The distribution of relative error (E_{rel}) of the estimated heat flux over the design surface in both cases for different values of RC is presented in Figure 6. The relative error is defined as:

$$E_{rel} = \frac{\left| Q_{d,n} - Q_{e,n} \right|}{\left| Q_{d,n} \right|} \times 100 \qquad (20)$$

The non-dimensional temperature distribution over the heater surface (propose of the inverse problem) for different values of RC is presented in Figure 7. Figure 7(a) shows that by increasing the RC parameter, the temperature of the heater surface decreases, while Figure 7(b) reveals that the temperature distribution of

the heater surface is nearly the same for different values of RC.

In order to have a better understanding of this thermal behavior of the system, the convective, radiative and total heat flux distributions over the heater surface are presented in Figure 8. For small values of the non-dimensional heat flux over the design surface (case 1), Figure 8 shows that by increasing the RC parameter, convective, radiative and therefore total heat flux over the heater surface decreases. Comparison of Figures 8(a) and 8(b) reveals that for small values of RC, convective heat transfer has significant effect on the total heat flux distribution over the heater surface and cannot be neglected, while it is seen that for large values of RC, the radiative heat transfer is dominant. According to Figure 8(c), one can find that a designer prefers to have thermal systems with high values of the RC parameter in comparison to low value ones, since the heaters installed on the heater surface needs much less power. It is also seen that no parts of the heater surface have zero or negative heat flux (cooling on the heater surface), so the solution achieved by the inverse method for this case is a realistic one.

(a)

(b)
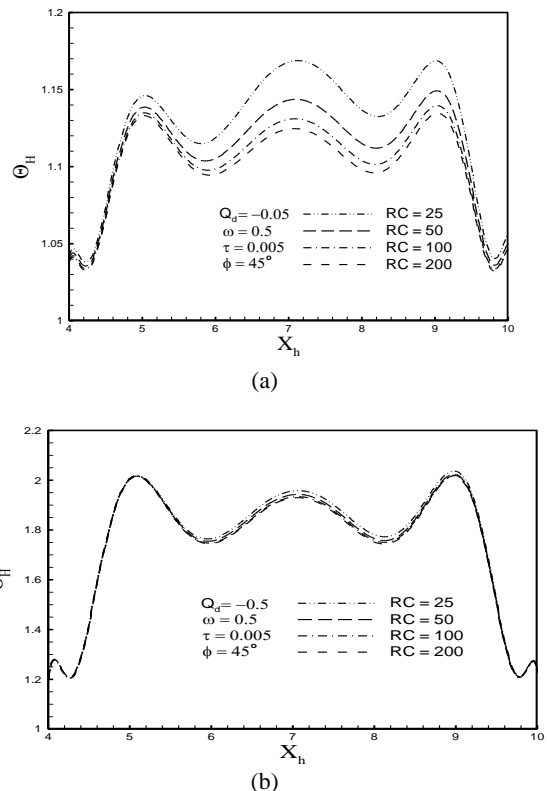
Figure 7. Temperature distribution over the heater surface for different values of RC (a) $Q_d = -0.05$ (b) $Q_d = -0.5$

(a)

(b)

(c)
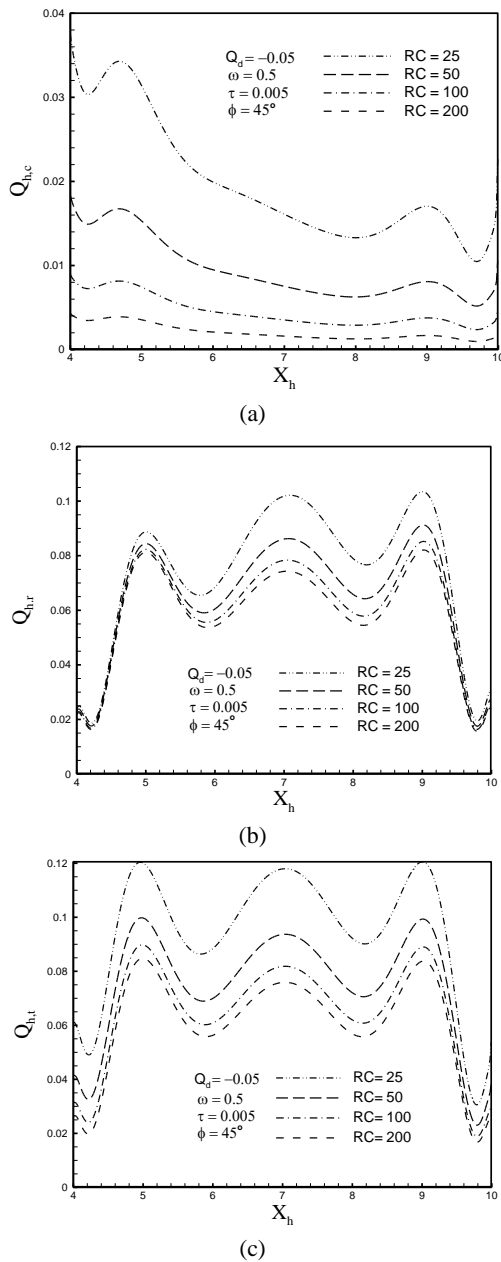
Figure 8. Comparison of heat flux distribution over the heater surface for $Q_d = -0.05$ (a) convective heat flux, (b) radiative heat flux, (c) total heat flux

8. CONCLUSIONS

The present work describes the inverse boundary design where the temperature distribution on the heater surface was determined to satisfy the specified temperature and heat flux distributions along the design surface. The theoretical formulation was explained by a set of high non-linear equations govern to a combined radiation-convection heat transfer in a recess flow. The DOM was

employed to solve the RTE, while the flow equations were numerically solved by finite volume technique using the SIMPLE algorithm for velocity-pressure coupling. Besides, the conjugate gradient method was used to find the unknown temperature distribution over the heater surface. The examples treated two cases where both the prescribed temperature and heat flux on the design surface were uniform with two different values. Numerical results showed that the conduction-radiation parameter has considerable effect on the result of such inverse problem when light heat flux is considered on the design surface.

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Inverse Boundary Design Problem of Combined Radiation-convection Heat Transfer in Laminar Recess Flow

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چکیده

در کار حاضر آنالیز معکوس انتقال حرارت ترکیبی جابجایی تابشی در جریان آرام سیال داخل کانال همراه با تغییر سطح مقطع انجام شده است. از روش گرادیان مزدوج جهت محاسبه تغییر دما روی سطح گرمکن به منظور ایجاد توزیع دما و شار حرارتی دلخواه روی سطح طراحی استفاده گردید. محیط مورد نظر تابشی با رفتار خاکستری در نظر گرفته شده که می‌تواند انرژی تشعشعی را جذب، صادر و پخش نماید. معادلات حاکم بر جریان به صورت عددی و با استفاده از تکنیک دینامیک سیالات محاسباتی حل شده و به منظور محاسبه جمله تابشی در معادله انرژی، معادله انتقال تابش به روش جهات مجزا به جواب رسیده است. در کار حاضر اثر پارامتر تابش-هدایت بر توزیع دمای گرمکن مورد ارزیابی قرار گرفته است.

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