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Fitting Second-order Models to Mixed Two-level and Four-level Factorial Designs: Is There an Easier Procedure?

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ABSTRACT

Fitting response surface models is usually carried out using statistical packages to solve complicated equations in order to produce the estimates of the model coefficients. This paper proposes a new procedure for fitting response surface models to mixed two-level and four-level factorial designs. New and easier formulae are suggested to calculate the linear, quadratic and the interaction coefficients for mixed two-level and four-level factorial designs regardless of the number of factors included in the experiment. The results of the proposed procedure are in agreement with the results of least squares method. This paper could motivate researchers to study the possibility of applying a fixed formula to all factorial designs.

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1. INTRODUCTION

The concept of factorial designs was first introduced by Fisher in the mid- 1920's [1]. Researchers furthered the development of factorial design by introducing new methods, designs or applications. The concept of factorial designs became clear in the 1937 when Yates introduced a new method for analyzing two-level designs [2, 3]. Davies applied the idea of two-level designs to three-level designs, fitting response surface models to the latter [2]. Interest in factorial designs and their contribution to various fields increased in the 1960's. Margolin [4] combined Yates' two-level designs and Davies' three-level designs to develop mixed factorial designs, a procedure for analyzing and fitting response surface to mixed designs. Draper and Stoneman [5] studied the number of experiments (runs) required to fit response surface models to mixed two-level and three-level factorial designs and mixed two-level and four-level designs. Factorial designs increasingly contributed to experiment design and analysis and became the focus of many researchers, thereby necessitating a review of its impact. Herzberg and Cox [6] reviewed roughly 800 works that had been published since 1957 and provided a detailed bibliography for researchers. Addelman [1] reviewed factorial design application in works published from 1965 to 1972 and found that most of the studies utilized fractional factorial designs. Edmondson [7] fit a second-order model to a four-level design using pseudo-factors to represent the four-levels. Bisgaard [8] presented a method for accommodating four-level factors in two-level designs. This method converts two or more columns to accommodate multi-level factors. Abbas et al. [9] proposed new formulae for analyzing two-level designs and fitting a first-order response surface model to this type of experiment. Abbas [10] also fit a second-order model to three-level designs. Abbas and Low [11] presented a new procedure for analyzing mixed two-level and three-level designs. Wasin et al. [12] fit response surface to a four-level design using the coefficients of an orthogonal polynomial.

Statistical packages such as SAS, R, SPSS and others are used for analyzing data. However, these packages are costly or in the case of SAS and R, need

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special knowledge to carry out statistical analysis, creating an additional challenge to the researchers.

Design and analysis of experiments have been widely used by researchers from different fields for instance, different engineering areas, environmental technology, food science and other disciplines to carry out different experiments such as four-level design which was used by kovo [13] and response surface methodology which was used by Hosseinpour et al. [14], Sayyar Kavardi et al. [15] and Yahyaei et al. [16] for optimization different experiments.

The objectives of this research are 1) to investigate the possibility of using the linear coefficients of a polynomial contrast for fitting response surface models to mixed two-level and four-level factorial designs, thereby avoiding use of the least squares method, which is especially cumbersome when the number of independent variables is greater than two, and 2) to investigate the possibility of calculating each coefficient of the model individually. This procedure will streamline the method for fitting response surface models to mixed two-level and four-level factorial designs.

2. MIXED TWO-LEVEL AND FOUR-LEVEL FACTORIAL DESIGNS

The two-level and four-level factorial experiment is a factorial design with mixed levels, p factors each at two levels and q factors each at four levels and is denoted by $2^p 4^q$. The simplest design for two-level and four-level factorial design has two factors, one at two levels and one at four levels. The total number of runs required for $2^1 4^1$ is 8 runs for one replicate [2, 17].

3. PROPOSED PROCEDURE

The proposed procedure for fitting response surface models to a $2^p 4^q$ experiment splits the process into two experiments, type 2^p and type 4^q, then analyzes each experiment separately to find the linear, quadratic and interaction coefficients between the factors with equivalent levels.

The proposed procedure for fitting a response surface model to a mixed two-level and four-level factorial design is based on combining the two procedures for fitting experiments of type 2^p [9] and for experiments of type 4^q [12]. The data should be normally distributed.

The formulae for fitting two-level factorial design based on two factors [9] are:

$$b_l = \frac{Contrast\ \ for\ X_l}{4n} \qquad l = 1,\ 2,\ \ldots,\ p,$$

for linear coefficients and the interaction:

$$b_{lq=} \frac{Contrast\ \ for\ X_l X_q}{4n} \qquad l \neq q$$

and the formulae for four-level factorial design [4] are:

a. Linear coefficient

$$b_L = \frac{Linear\ \ contrast\ \ for\ X_L}{20 \times n} \quad L = 1,\ 2,\ \ldots,\ q$$

b. Quadratic coefficient

$$b_{LL} = \frac{Quadratic\ \ contrast\ \ for\ X_L}{16 \times n}$$

c. Interaction coefficient

$$b_{LQ} = \frac{Linear\ \ contrast\ \ for\ X_L X_Q}{400 \times n},\ L \neq Q$$

Below are the formulae for intercept and interaction between factors that have two levels and factors that have four levels will be derived in the next section.

4. PROPOSED FORMULAE FOR INTERCEPT AND INTERACTION

Consider a second-order response surface model with p factors each at two levels ($X_1, X_2, \ldots, X_p$) and q factors each at four levels ($Z_1, Z_2, \ldots, Z_q$) as given below:

$$\begin{aligned} Y_i &= b_0 + b_1 X_{1i} + b_2 X_{2i} + \ldots + b_p X_{pi} + b_{12} X_{1i} X_{2i} \\ &+ \ldots + b_{(p-1)p} X_{(p-1)i} X_{pi} + \gamma_1 Z_{1i} + \gamma_2 Z_{2i} + \ldots \\ &+ \gamma_q Z_{qi} + \gamma_{11} Z_{1i}^2 + \ldots + \gamma_{qq} Z_{qi}^2 + \gamma_{12} Z_{1i} Z_{2i} + \ldots \\ &+ \gamma_{(q-1)q} Z_{(q-1)i} Z_{qi} + \alpha_{11} X_{1i} Z_{1i} + \ldots + \alpha_{pq} X_{pi} Z_{qi} + \varepsilon_i \end{aligned} \tag{1}$$

$i = 1,\ 2,\ \ldots,\ k$

The levels of each factor represent the coded form, which is similar to the linear coefficients of the orthogonal contrast. The relationship between actual and coded variables for two levels is:

$$X = \frac{C - (High + Low)/2}{(Highe - Low)/2}$$

and for four levels is:

$$X = \frac{C - \text{Average of all levels } (a_1 + a_2 + a_3 + a_4)/4}{(\text{Range of any two consecutive levels})/2}$$

The model in Equation (1) obeys some constraints. The constraints are obtained from the linear coefficients of the orthogonal polynomial contrast for four levels (-3 -1 1 3) and the contrast for two levels (-1 1). The constraints are as follows:

a. Constraints for the factors at two levels:

1- $\sum_{i=1}^{k} X_i = 0$ 2- $\sum_{i<j}^{k} X_i X_j = 0$

3- $\sum_{i<j}^{k} X_i^2 X_j = 0$ 4- $\sum_{i<j}^{k} X_i X_j^2 = 0$

5- $\sum_{i<j<h}^{k} X_i X_j X_h = 0$ 6- $\sum_{i=1}^{k} X_i^2 = k$

7- $\sum_{i<j}^{k} (X_i X_j)^2 = k$

b. Constrains for the factors at four levels:

1- $\sum_{i=1}^{k} Z_i = 0$ 2- $\sum_{i<j}^{k} Z_i Z_j = 0$ 3- $\sum_{i=1}^{k} Z_i Z_i^2 = 0$

4- $\sum_{i<j}^{k} Z_i Z_j^2 = 0$ 5- $\sum_{i=1}^{k} Z_i^2 = 20\times 4^{q-1}$ 6- $\sum_{i=1}^{k} Z_i^4 = 164\times 4^{q-1}$

7- $\sum_{i<j}^{k} Z_i^2 Z_j^2 = 400\times 4^{q-2}$ 8- $\sum_{i<j<z}^{k} Z_i Z_j Z_z = 0$ 9- $\sum_{i<j}^{k} (Z_i Z_j)^2 = 400\times 4^{q-2}$

c. Constraints for the joint effects between factors at two levels and factors at four levels:

1- $\sum_{i=1}^{k} (Z_i X_i)^2 = 40\times(4^{q-1}\times 2^{p-1})$

2- $\sum_{i=1}^{k} X_i Z_i = \sum_{i=1}^{k} X_i^2 Z_i = \sum_{i=1}^{k} X_i Z_i^2 = 0$

3- $\sum_{i<j<l}^{k} X_i X_j Z_l = \sum_{i<j<l}^{k} Z_i Z_j X_l = 0$

The formula for the intercept b_o for mixed two-level and four-level factorial design can be derived by summing Equation (1) over i and applying the constraints that yield the formula in Equation (2).

$$b_0 = \bar{Y} - b_{11}\bar{Z}_1 - b_{22}\bar{Z}_2 - \ldots - b_{qq}\bar{Z}_q \tag{2}$$

where $\bar{Z}_i = \frac{\sum Z_i^2}{k}$, and k is the total number of observations.

The formula for the interaction coefficient between factors at two levels and four levels can be derived by multiplying Equation (1) by $X_{li}\ Z_{Li}$ and summing over i, which give the formula in Equation (3).

$$\sum X_{li} Z_{Li} Y_i = \alpha_{lL} \sum (X_{li} Z_{Li})^2$$

$$\alpha_{lL} = \frac{\sum X_{li} Z_{Li} Y_i}{\sum (X_{li} Z_{Li})^2} \quad l = 1,\ 2,\ \ldots, p, \qquad L = 1,\ 2,\ \ldots, q \tag{3}$$

The formula for the interaction coefficient between factors at two levels and four levels in Equation (3) can be written in terms of linear joint contrast. To illustrate the procedure, consider a $2^2 4^2$ experiment without losing information from the general formula.

TABLE 1. A design with four factors where X_1 and X_2 have two levels each, and Z_1 and Z_2 have four levels each

Z_1	Z_2	X_1	-1	-1	1	1
		X_2	-1	1	-1	1
-3	-3		Y_1	Y_{17}	Y_{33}	Y_{49}
-1	-3		Y_2	Y_{18}	Y_{34}	Y_{50}
1	-3		Y_3	Y_{19}	Y_{35}	Y_{51}
3	-3		Y_4	Y_{20}	Y_{36}	Y_{52}
-3	-1		Y_5	Y_{21}	Y_{37}	Y_{53}
-1	-1		Y_6	Y_{22}	Y_{38}	Y_{54}
1	-1		Y_7	Y_{23}	Y_{39}	Y_{55}
3	-1		Y_8	Y_{24}	Y_{40}	Y_{56}
-3	1		Y_9	Y_{25}	Y_{41}	Y_{57}
-1	1		Y_{10}	Y_{26}	Y_{42}	Y_{58}
1	1		Y_{11}	Y_{27}	Y_{43}	Y_{59}
3	1		Y_{12}	Y_{28}	Y_{44}	Y_{60}
-3	3		Y_{13}	Y_{29}	Y_{45}	Y_{61}
-1	3		Y_{14}	Y_{30}	Y_{46}	Y_{62}
1	3		Y_{15}	Y_{31}	Y_{47}	Y_{63}
3	3		Y_{16}	Y_{32}	Y_{48}	Y_{64}

Suppose there are four factors: X_1, X_2 at two levels and Z_1, Z_2 at four levels. The design for this experiment is given in Table 1. Consider the formula for α_{11},

$$\alpha_{11} = \frac{\sum X_{1i} Z_{1i} Y_i}{\sum (X_{1i} Z_{1i})^2} \tag{4}$$

The denominator of Equation (4) equals to: $\sum (X_{1i} Z_{1i})^2 = 40\ (4^{q-1} 2^{p-1}) = 40\ (4^1 2^1) = 320$. This is equal to 40×8, where 8 represents the number of replicates at each joint level. The numerator of Equation (4) is:

$$\begin{aligned}\sum_{i=1}^{64} X_{1i} Z_{1i} Y_i &= 3(Y_1 + Y_{17} + Y_5 + Y_{21} + Y_9 + Y_{25} + Y_{13} \\ &+ Y_{29} + Y_{36} + Y_{52} + Y_{40} + Y_{56} + Y_{44} + Y_{60} + Y_{48} + Y_{64}) \\ &+ (Y_2 + Y_{18} + Y_6 + Y_{22} + Y_{10} + Y_{26} + Y_{14} + Y_{30} + Y_{35} \\ &+ Y_{51} + Y_{39} + Y_{55} + Y_{43} + Y_{59} + Y_{47} + Y_{63}) - (Y_3 + Y_{19} \\ &+ Y_7 + Y_{23} + Y_{11} + Y_{27} + Y_{15} + Y_{31} + Y_{34} + Y_{50} + Y_{38} \\ &+ Y_{54} + Y_{42} + Y_{58} + Y_{46} + Y_{62}) - 3(Y_4 + Y_{20} + Y_8 + Y_{24} \\ &+ Y_{12} + Y_{28} + Y_{16} \quad + Y_{32} + Y_{33} + Y_{49} + Y_{37} + Y_{53} + Y_{41} \\ &+ Y_{57} + Y_{45} + Y_{61})\end{aligned} \tag{5}$$

The same result can be obtained if the coefficients of orthogonal contrast are used. To show that the numerator of Equation (4) is equal to the linear joint contrast between X_1 and $Z1$, a $2^1 \times 4^1$ experiment is utilized. In this case, one factor has two levels, and the other factor has four. Observations that have the same joint level before finding the joint contrast, as shown in

Table 2 for X_1 and Z_1, are summed in the cells. Similar tables can be constructed for X_1Z_2, X_2Z_1, and X_2Z_2.
From Table 2, the linear joint contrast for X_1Z_1 is:

$$\begin{aligned}&=(\ 3)(1)[Y_1+Y_5+Y_9+Y_{13}+Y_{17}+Y_{21}\\&+Y_{25}+Y_{29}]+(1)(1)[Y_2+Y_6+Y_{10}+Y_{14}\\&+Y_{18}+Y_{22}+Y_{26}+Y_{30}]+\ldots+(3)(1)[Y_{36}\\&+Y_{40}+Y_{44}+Y_{48}+Y_{52}+Y_{56}+Y_{60}+Y_{64}]\end{aligned} \tag{6}$$

TABLE 2. The results for factors X_1 (at two levels) and Z_1 (at four levels)

Z_1	X_1	Response
-3	-1	$Y_1+Y_5+Y_9+Y_{13}+Y_{17}+Y_{21}+Y_{25}+Y_{29}$
-1	-1	$Y_2+Y_6+Y_{10}+Y_{14}+Y_{18}+Y_{22}+Y_{26}+Y_{30}$
1	-1	$Y_3+Y_7+Y_{11}+Y_{15}+Y_{19}+Y_{23}+Y_{27}+Y_{31}$
3	-1	$Y_4+Y_8+Y_{12}+Y_{16}+Y_{20}+Y_{24}+Y_{28}+Y_{32}$
-3	1	$Y_{33}+Y_{37}+Y_{41}+Y_{45}+Y_{49}+Y_{53}+Y_{57}+Y_{61}$
-1	1	$Y_{34}+Y_{38}+Y_{42}+Y_{46}+Y_{50}+Y_{54}+Y_{58}+Y_{62}$
1	1	$Y_{35}+Y_{39}+Y_{43}+Y_{47}+Y_{51}+Y_{55}+Y_{59}+Y_{63}$
3	1	$Y_{36}+Y_{40}+Y_{44}+Y_{48}+Y_{52}+Y_{56}+Y_{60}+Y_{64}$

TABLE 3. The actual and coded form for the selected variables

Factor level	X_1	coded	X_2	Coded	X_3	coded
a1	5	-1	3	-3	15	-3
a2	9	1	4	-1	20	-1
a3			6	1	25	1
a4			8	3	30	3

The result in Equation (6) is the same as the result in Equation (5); therefore, the numerator of Equation (4) can be written as the linear joint contrast for X_1 and Z_1. Thus, Equation (4) can be written using linear contrast as given below:

$$\alpha_{11}=\frac{\sum X_{1i}Z_{1i}Y_i}{\sum (X_{1i}Z_{1i})^2}=\frac{Linear\ contrast\ \ for\ \ X_1Z_1}{40\times 8}$$

where 8 represents the number of replicates at the joint levels. Similarly, the same steps can be used to find the formula for other coefficients. In general, let *n* represent the number of replicates at the joint levels, then the formula becomes:

$$\alpha_{lL}=\frac{Linear\ contrast\ \ for\ \ X_lZ_L}{40\times n} \tag{7}$$

In summary, it can be said that the proposed procedure will eliminate the need to use the least squares method, which is particularly cumbersome when the number of independent variables is more than two, making it difficult to find the inverse matrix. Furthermore, the proposed procedure provides a simple way to check individual coefficients without affecting other coefficients in the model. Alternatively, the least squares method results in a different intercept for each variable and possibly different coefficients when all terms are included in the model.

5. A NUMERICAL EXAMPLE

The following example illustrates the steps of the proposed procedure and compares the results with the least squares method. The calculations for the new procedure were done by using normal calculator to show that the new procedure is simple whilst SPSS version 20 was used for the least squares method.
The production of lactic acid from mango peels using the bio-fermentation method was investigated. The effect of three factors on lactic acid yield was studied: initial medium pH (X_1) at two levels (5 and 9), fermentation time (X_2) at four levels (2, 4, 6 and 8 days) and temperature (X_3) at four levels (15, 20, 25, and 30°C). The design used to run this experiment is a 2^14^2 type. The total number of runs is 64-run with two replicates. The levels of each factor in actual and coded form are given in Table 3. The researcher wants to fit a second-order response surface model to this experiment. The model is given in Equation (6).

$$\begin{aligned}Y&=b_0+b_1X_1+\gamma_1Z_1+\gamma_2Z_2+\gamma_{11}Z_1^2\\&+\gamma_{22}Z_2^2+\alpha_{11}X_1Z_1+\alpha_{12}X_1Z_2+\gamma_{23}Z_1Z_2\end{aligned} \tag{6}$$

Both the least squares method and the proposed procedure will be used to fit the model in Equation (6).
Fitting a response surface model to this type of experiment using the proposed procedure requires separating this experiment into two shorter experiments, one of type 2^1 and another of type 4^2. The order in which the experiments are conducted is irrelevant, and the 2^1 experiment will be analyzed first. The two-level experiment can be implemented using the formula proposed by Abbas et al. [9].

The calculation of the contrast is based on two factors ($A_l=(a_l-1)(a_q+1)$). In this example, there is only one factor at two levels, thus the denominator of the formula should be modified to $2n$ as given below:

$$b_l=\frac{Contrast\ \ for\ X_l}{2n}$$

The linear contrast for X_1 is:

$$\begin{aligned}&\text{Contrast for } X_1=(-1)(139.17)+1(260.93)\\&=121.76\end{aligned}$$

The number of observations at each level is 32; therefore, the number of replicates is $n = 32$.

$$b_1 = \frac{Contrast\ \ for\ X_1}{2n} = \frac{121.76}{2(32)} = 1.902625$$

The second step in the analysis is to start with four-level experiment 4^2. There are two factors, each at four levels. Four-level design can be analyzed using the formulae introduced by Wasin et al. [12]. This requires treating each factor as an experiment of type 4^1 to find the linear and quadratic coefficients and then applying a 4^2 experiment to find the interaction.

The linear (*L*) and quadratic (*Q*) contrasts for Z_1 are:

$$L_{Z1} = 3(112.28) + 1(98.12) + (\ \ 1)(95.64) + (\ \ 3)(94.04) = \ \ 57.18$$

$$Q_{Z1} = 1(112.28) + (\ \ 1)(98.12) + (\ \ 1)(95.64) + 1(94.04) = 12.54$$

The linear and quadratic coefficients for Z_1 are:

$$\gamma_1 = \frac{Linear\ \ contrast\ \ for\ Z_1}{20 \times n} = \frac{57.18}{20(16)} = \ \ 0.17869$$

$$\gamma_{11} = \frac{Quadratic\ \ contrast\ \ for\ Z_1}{16 \times n} = \frac{12.54}{16(16)} = 0.048984$$

and the linear and quadratic coefficients for Z_2 are:

$$\gamma_2 = \frac{Linear\ \ contrast\ \ for\ Z_2}{20 \times n} = \frac{36.15}{20(16)} = \ \ 0.11297$$

$$\gamma_{22} = \frac{Quadratic\ \ contrast\ \ for\ Z_2}{16 \times n} = \frac{35.19}{16(16)} = \ \ 0.13746$$

The coefficient of the linear interaction between Z_1 and Z_2 each at four levels is:

$$\gamma_{12} = \frac{Linear\ \ contrast\ \ for\ Z_1 Z_2}{400 \times n}$$

The linear contrast between Z_1 and Z_2 is:

$$L_{Z1Z_2} = (\ \ 3)(\ \ 3)(18.118) + (\ \ 3)(\ \ 1)(21.95) + \ \ldots + (3)(1)(16.31) + (3)\ \ (3)\ \ (11.97) = \ \ 397.165$$

There are 4 observations at each joint level which means that the number of replicates is $n = 4$. Applying the formula for γ_{12} results in:

$$\gamma_{12} = \frac{397.165}{400 \times 4} = \ \ 0.24823$$

The last step in the analysis is to find the linear coefficients for the interaction between the factors at two levels (here, only X_1) and factors at four levels (here, Z_1 and Z_2). The linear coefficients for the linear interaction can be calculated using the formula given in Equation (7).

The linear contrast between X_1 and Z_1 should be calculated first.

$$L_{X_1Z_1} = (\ \ 3)(\ \ 1)(42.95) + (\ \ 1)(\ \ 1)(35.38) + \ldots + (1)\ \ (1)\ \ (64.65) + (3)\ \ (1)\ \ (64.21) = 30.28$$

The number of observations at each joint level is 8, thus the number of replicates is $n = 8$. The linear coefficient for interaction α_{11} is:

$$\alpha_{11} = \frac{Linear\ \ contrast\ \ for\ X_1 Z_1}{40 \times n} = \frac{30.28}{40(8)} = 0.094625$$

and b_{12} is:

$$\alpha_{12} = \frac{Linear\ \ contrast\ \ for\ X_1 Z_2}{40 \times n} = \frac{130.72}{40(8)} = \ \ 0.4085$$

The intercept term can be calculated using the formula given in Equation (2).

$$b_0 = \overline{Y} - b_{22}\overline{Z}_2 - b_{33}\overline{Z}_3 = 6.251563 - 0.048984(320/64) - (-0.3746)(320/64) = 6.693943$$

The formula given in Equation (6) becomes:

$$Y = 6.693943 + 1.9025\ \ X_1 \text{ - } 0.17869\ \ Z_1 \text{ - } 0.11297\ \ Z_2 + 0.048984\ \ Z_1^2 \text{ - } 0.13746\ \ Z_2^2 + 0.094625\ \ X_1 Z_1 \text{ - } 0.4085\ \ X_1 Z_2 \text{ - } 0.24823\ \ Z_1 Z_2$$

TABLE 4. Comparison between the proposed procedure and the least squares method for fitting a second-order model to $2^1 4^2$ design

Parameter	Proposed procedure	Least squares method
b_0	-27.228	-27.228
b_1	2.254	2.254
b_2	1.054	1.054
b_3	1.051	1.051
b_{22}	0.087	0.087
b_{33}	-0.005	-0.005
b_{12}	0.063	0.063
b_{13}	0.041	0.041
b_{23}	0.066	0.066

To write this equation in actual variables, the relationship between the actual and coded form is used.

$$Y = 6.693943 + (1.9025)[\frac{x_1 \quad 7}{2}] - (0.17869)[\frac{z_1 \quad 5.25}{0.75}]$$
$$- (0.11297)[\frac{z_2 \quad 40}{5}] + (0.04898)[\frac{z_1 \quad 5.25}{0.75}]^2 - (0.1374)$$
$$[\frac{z_2 \quad 40}{5}]^2 + (0.094625)[\frac{x_1 \quad 7}{2}][\frac{z_1 \quad 5.25}{0.75}] - (0.4085)$$
$$[\frac{x_1 \quad 7}{2}][\frac{z_2 \quad 40}{5}] - (0.24823)[\frac{z_1 \quad 5.25}{0.75}][\frac{z_2 \quad 40}{5}]$$

$$Y = 27.228 + 2.254 \ x_1 + 1.054 \ z_1 + 1.051 \ z_2$$
$$+ 0.087 \ z_1^2 - 0.005 \ z_2^2 + 0.063 \ x_1 z_1 + 0.041 \ x_1 z_2$$
$$+ 0.066 \ z_1 z_2$$

The results of the least squares method and the proposed procedure are given in Table 4 and show that the least squares method and the proposed procedure have the same results. Furthermore, the proposed procedure calculated the coefficients individually, which cannot be performed with the least squares method.

6. CONOCLUSION

Based on the above results and discussion, the proposed procedure for fitting a second-order model to mixed two-level and four-level designs provides fixed formulae regardless of the number of factors, avoids the arduous least squares method, makes it possible to calculate each coefficient individually, and eliminates the need for costly and complicated statistical software.

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8. REFERENCES

1. Addelman, S., "Recent developments in the design of factorial experiments", ***Journal of the American Statistical Association***, Vol. 67, No. 337, (1972), 103-111.
2. Davies, O.L., "The design and analysis of industrial experiments", ***The Design and Analysis of Industrial Experiments,*** (1954) 234-251.
3. Yates, F., "The design and analysis of factorial experiments", ***Imperial Bureau of Soil Science***, (1978) 113-125.
4. Margolin, B.H., "Systematic methods for analyzing $2^n 3^m$ factorial experiments with applications", ***Technometrics***, Vol. 9, No. 2, (1967), 245-259.
5. Draper, N.R. and Stoneman, D.M., "Response surface designs for factors at two and three levels and at two and four levels", ***Technometrics***, Vol. 10, No. 1, (1968), 177-192.
6. Herzberg, A.M. and Cox, D., "Recent work on the design of experiments: A bibliography and a review", ***Journal of the Royal Statistical Society. Series A (General)***, (1969), 29-67.
7. Edmondson, R., "Agricultural response surface experiments based on four-level factorial designs", ***Biometrics***, (1991), 1435-1448.
8. Bisgaard, S., "Accommodating four-level factors in two-level factorial designs", ***Quality Engineering***, Vol. 10, No. 1, (1997), 201-206.
9. Abbas, F.M.A., Low, H.C. and Quah, S.H., "A new method for analyzing experiments of type 2^p", Proceedings of the National Conference on Management Science/Operation Research, (2000), 123-130.
10. Abbas, F.M.A., "A new method for analyzing experiments of type 2p, 3m, and 2p3m", Ph.D thesis, Universiti Sains Malaysia, (2001).
11. Alkarkhi, A.F. and Low, H., "A proposed technique for analyzing experiments of type", ***Modern Applied Science***, Vol. 3, No. 1, (2008), 95-102.
12. Alqaraghuli, W.A., Alkarkhi, A.F. and Low, H., "A new procedure for fitting second-order model to four-level factorial designs", ***World Applied Sciences Journal***, Vol. 22, No. 8, (2013), 1116-1128.
13. Kovo, A., "Application of full 42 factorial design for the development and characterization of insecticidal soap from neem oil", ***Leonardo Electronic Journal of Practices and Technologies***, Vol. 8, No. 1, (2006), 29-40.
14. Hosseinpour, M.N., Najafpoura, G.D., Younesi, H., Khorrami, M. and Vaseghi, Z., "Lipase production in solid state fermentation using aspergillus niger: Response surface methodology", ***International Journal of Engineering***, Vol. 25, No. 3, (2012), 151-159.
15. Kavardi, S.S., Alemzadeh, I. and Kazemi, A., "Optimization of lipase immobilization", ***International Journal of Engineering***, ***Transactions A: Basics*** Vol. 25, No. 1, (2012), 1-9.
16. Yahyaei, M., Bashiri, M. and Garmeyi, Y., "Multicriteria logistic hub location by network segmentation under criteria weights uncertainty" ***International Journal of Engineering***, ***Transactions B: Applications*** Vol. 27, No. 8 (2014) 1205-1214
17. Roger G. P., "Design and analysis of experiment", New York, USA: Marcel Dekker, INC, (1985).

Fitting Second-order Models to Mixed Two-level and Four-level Factorial Designs: Is There an Easier Procedure?

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چکیده

برازش مدلهای سطح پاسخ معمولا با استفاده از بسته های آماری برای حل معادلات پیچیده به منظور تولید برآورد ضرایب مدل انجام می‌شود. در این مقاله یک روش جدید برای برازش مدل سطح پاسخ با مخلوط طرح فاکتوریل دو سطح و چهار سطح پیشنهاد می شود. فرمولهای جدید و آسان تر پیشنهاد می‌شود که برای محاسبه خطی، درجه دوم و ضرایب تعامل برای مخلوط طرح فاکتوریل دو سطح و چهار سطح صرف نظر از تعداد عوامل موجود در آزمایش است. نتایج حاصل از روش پیشنهادی در توافق با نتایج حاصل از روش حداقل مربعات می باشد. این مقاله می‌تواند محققان را برای مطالعه امکان استفاده از یک فرمول ثابت برای تمام طرحهای فاکتوریل ترغیب کند.

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