

International Journal of Engineering

RESEARCH NOTE

Journal Homepage: www.ije.ir

Vibration Suppression of Fuel Sloshing using Subband Adaptive Filtering

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PAPER INFO

ABSTRACT

Paper history: Received 26 March 2014 Received in revised form 17 August 2015 Accepted 03 September 2015

Keywords: Fuel Sloshing Vibration Control Nonlinear Dynamics Lyapunov Theorem Subband Adaptive Filter One of the main vibration problems of aerospace vehicles with liquid fuel propulsion system is fuel sloshing. This phenomenon is a low frequency vibrational challenge which can affect the motion of the vehicle and degrade the stability of the main control system. In this regards, the motion of the liquid will be very critical when the frequency of the sloshing is very close to the frequencies of the main dynamic system. In addition, dominant frequencies of the sloshing are varied due to the flight time in the aerospace vehicles. In this paper, an aerospace launch vehicle with fuel sloshing is considered as a multi body dynamic system and in order to reduce the undesired effects of the fuel vibration, a new subband adaptive filter based on the lyapunov theorem and the discrete fourier transform (DFT) is designed. In this way, the new control system is implemented on the attitude control of the vehicle and the simulation of the nonlinear model is carried out. Numerical results of the simulation show that the effects of the fuel sloshing on the vehicle are effectively omitted by means of the new subband adaptive controller.

doi: 10.5829/idosi.ije.2015.28.10a.15

1. INTRODUCTION

One of the main vibration problems of aerospace vehicles with liquid fuel propulsion system is fuel sloshing. This phenomenon is a low frequency vibration problem which can affect the motion of the vehicle and degrade the stability of the main control system. In this regards, the motion of the liquid will be very critical when the frequency of the sloshing is very close to the frequencies of the main dynamic system or its subsystems. Moreover, in many aerospace vehicles, dominant frequencies of the sloshing are changed due to the flight time and this leads to variety of dynamic system behaviors.

Dynamic modeling of the fuel sloshing phenomenon has been widely investigated in many activities [1-3]. In this regards, equal mechanical models are very practical and useful tools to extract the dynamic model of the sloshing [4, 5]. The mechanical modeling of the sloshing phenomenon is basically different from the fluid structural modeling in which the sloshing fluid parameters are extracted. In fact, the fluid structural modeling is employed to analyze the behavior of the phenomenon in the point of view of fluid theories. In this paper, a nonlinear dynamic model of a slosh mass as a vibrational pendulum coupled with a launch vehicle dynamic in 6 degree-of-freedom (DOF) is investigated. The process of the dynamic modeling of a 7 DOF dynamic system is completely given elsewhere [6] and the equations of motion will be demonstrated in the appendix.

Design of passive and active vibration control system with the concept of reduction of the sloshing effects has been considered in several works in the point of view of a mechanical vibrational system [7-10]. Reyhanoglu et al. have proposed a Lyapunov-based nonlinear feedback controller for a multi-mass-spring model of sloshing [11]. Furthermore, Krishnaswamy and Bugajski have proposed an output feedback procedure for control of a multi body vehicle with the fuel sloshing [6] and Dong et al. have designed a controller to reduce the effects of the propellant sloshing in a spacecraft [11-14].

Please cite this article as: A. M. Khoshnood and O. Kavianipour, Vibration Suppression of Fuel Sloshing using Subband Adaptive Filtering, International Journal of Engineering (IJE), TRANSACTIONS A: Basics Vol. 28, No. 10 (October 2015) 1507-1514

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Although demonstrated works as well as optimal, robust and adaptive methods can reduce the effects of the fuel sloshing, the main problem refers to the uncertainty of the fuel sloshing parameters due to the flight time. Additionally, the frequency of the sloshing is often close to the frequency of the main dynamic system. Considerably, these problems require a high performance and precise estimator. Hence, one of the aims of this paper is to propose an approach for accurate estimating of the vibrational parameters of the fuel sloshing. In this regards, Jafari et al. have proposed an adaptive filter to reduce the destructive effects of the sloshing [14]. Their activities devoted to the fullband analysis and this may make some violations in the process of the estimation. However, in this paper, it is tried to extend the method of the adaptive filtering for the same model in the case of subband analysis and multi rate processing.

In this study and in the case of condition in which the frequency of sloshing is close to the frequency of the main dynamic system, a controller for vibration control of the fuel sloshing is designed by means of Lyapunov based subband adaptive filter. In the first part of the control system, a combined form of the gradient and the Lyapunov methods in each subband is used. This idea not only holds the high performance of the gradient method but also guarantees the stability of the system considering the Lyapunov theorem. The second part of the control system includes several fullband filters in which the effects of the signals with undesired frequencies are eliminated.

This paper is organized as following: In the second section, the details of the new subband adaptive controller are given. Simulation results and conclusions of the work are expressed in the third and the final sections, respectively. Moreover, the complete equations of motion of the vehicle with the sloshing mass are given in the appendix.

2. DYNAMIC MODELING STRATEGY

2. 1. Dynamic Modeling of the LV with Fuel Sloshing Dynamic model of the launch vehicle with 6 DOF and one added mass-spring constructs a multi body system. For the fuel sloshing as illustrated in Figure 1 one can express the position vectors as:

$$\begin{cases} E^s = R + r \\ r = r_o + e^s \end{cases}$$
(1)

where E^s is position of the slosh mass in the inertial frame, R is position of the vehicle in the inertial frame, r_0 is rigid position in the body frame, and e^s is the displacement of the slosh mass in the body frame.

In the current dynamic modeling, it is better to use quasi-coordinate method in the form of Lagrange approach [13]. Considering Equation (1), the vehicle total kinetic energy in vector form is expressed as:

$$T = \frac{1}{2} \int \frac{dE}{dt_{I}}^{T} \frac{dE}{dt_{I}} dm + \frac{1}{2} \int \frac{dE^{s}}{dt_{I}}^{T} \frac{dE^{s}}{dt_{I}} dm^{s}$$

$$= \frac{1}{2} m^{s} V^{T} V + \frac{1}{2} (\overline{\omega}^{T} S \overline{\omega}) m^{s} + \frac{1}{2} \frac{d\overline{e}^{s}}{dt}^{T} \frac{d\overline{e}^{s}}{dt} m^{s}$$

$$+ \frac{1}{2} \overline{\omega}^{T} G \overline{\omega} + \overline{\omega}^{T} \widetilde{V}^{T} \int \overline{r}_{0s}^{s} dm^{s} + \overline{\omega}^{T} V^{T} \int \overline{e}^{s} dm^{s}$$

$$+ V^{T} \frac{d\overline{e}^{s}}{dt} m^{s} + \overline{\omega}^{T} \left(\frac{d\overline{e}^{s}}{dt_{c}} \right)^{T} \overline{r}_{0s}^{s} m^{s} + \overline{\omega}^{T} \frac{d\overline{e}^{s}}{dt}^{T} \overline{e}^{s} m^{s}$$

$$+ \overline{\omega}^{T} D \overline{\omega} + \frac{1}{2} m V^{T} V + \frac{1}{2} (\overline{\omega}^{T} I \overline{\omega})$$

$$(2)$$

where *E* is position of the main body in the inertial frame, *T* is kinetic energy, *V* is velocity vector of the vehicle, $\overline{\omega}$ is angular velocity vector, *I* is inertia matrix of the vehicle and m^s and *m* are slosh-mass and mass of the vehicle, respectively. Also, *S*, *G*, and *D* are defined in appendix. In addition, potential and dissipative energy of the system are given as:

$$U^{s} = \frac{1}{2} \left[k_{z} (e_{z}^{s})^{2} + k_{y} (e_{y}^{s})^{2} \right]$$
(3)

$$D^{s} = m^{s} \left[\mu_{z}^{s} \omega_{z}^{s} (\dot{e}_{z}^{s})^{2} + \pi_{y}^{s} \omega_{y}^{s} (\dot{e}_{y}^{s})^{2} \right]$$
(4)

where k_z and k_y are stiffness magnitude of the fuel sloshing in z and y directions, μ_z^s, μ_y^s are damping parameters, and ω_z^s, ω_y^s are frequencies of the sloshing in the same directions. The fuel sloshing has no component in x direction. Finally, the nonlinear equations of motion of the coupled rigid body and the fuel sloshing are derived by quasi-coordinate Lagrange method as follows:

$$\begin{cases} \frac{d}{dt} \left(\frac{\partial L}{\partial V} \right) + \tilde{\omega} \frac{\partial L}{\partial V} = F \\ \frac{d}{dt} \left(\frac{\partial L}{\partial \omega} \right) + \tilde{V} \frac{\partial L}{\partial V} + \tilde{\omega} \frac{\partial L}{\partial \omega} = M \\ \frac{d}{dt} \left(\frac{\partial L}{\partial e^{\epsilon}} \right) - \frac{\partial L}{\partial e^{\epsilon}} = Q_{e} \end{cases}$$
(5)

where L is Lagrangian, F is all the external force applied on the vehicle, M is all the external moments applied on the vehicle, and Q_e is generalized force. The complete equations of motion are given in appendix.

3. VIBRATION CONTROL STRATEGY

In many closed loop control systems, one of the possible problems refers to the interaction between the control system modes and the sub-systems vibration modes.

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Figure 1. Schematic of the approach for modeling the fuel sloshing



Figure 2. Block diagram of the excitation reducing strategy

In this situation, the excitation is significantly increased and can lead to undesired performance and in some cases instability. Therefore, excitation reducing is a practical and useful strategy to protect the main system from these undesired oscillations. In this approach, by adding an analyzer unit to the feedback of the control system, one can manage the feedback signal to reduce or even omit the excitation.

In this study, the analyzer unit is designed by the use of a new delayless subband adaptive filter, as explained in the next section. The block diagram of the proposed excitation reducing strategy is shown in Figure 2.

3. 1. Subband Adaptive Filter Design A11 researches related to the estimation of the sloshing frequency in the aerospace devices have been limited to the fullband analysis. In spite of this, one of the significant challenges in the estimation in the fullband analysis refers to convergence and persistency excitation. These problems currently lead to wide use of subband analysis and algorithms, especially in the signal processing. However, using the subband investigations instead of fullband can often solve the issues of signals correlation, persistency excitation, convergence, etc. [14, 15]. In the subband adaptive estimation, the input signal is partitioned to some alternative frequency bands. Consequently, each frequency band can be investigated separately in the case of frequency estimation. From this view, one of the practical subband adaptive methods known as delayless subband adaptive filtering is made from DFT filter bank [16]. This filter bank is used in the current study to analyze the output of the inertial navigation system (INS).

A delayless subband adaptive filter as illustrated in Figure 3 consists of two parts: analysis filters and estimation part. The analysis part includes some lowpass filters (h(z)) which is multiplied by an exponential function to generate complex signal as a Fourier transform. In this structure, each subband only consists of a limited band of frequency and this is very useful for frequency estimation. In this regards, partitioning the signal results in increase of the volume of computations. That is why the idea of multi rate filtering is employed in the subband filtering. On the other hand, before entering the signal in to the estimation algorithm, as demonstrated in Figure 4, an operator decimates the signal with decimation factor $(\downarrow L)$.

3. 2. Real Valued DFT Filter Bank The output signal of the analysis part of the DFT filter bank is commonly a complex valued signal. Considering the simplicity of the use of real valued signal in the frequency estimation, one can modify the DFT filter bank to produce a real valued signal [17] as shown in Figure 4. Although the use of real valued signal is simpler, the volume of computation in subband for both the real and the complex valued signal in multi rate systems is approximately equal.

In this paper, an un-uniform two band oversampled filter bank is proposed for estimating the slosh frequency. Since the total sampling rate of the system is selected 0.02 (with regard to the Nyquist sampling theorem), the subband system is made from 10 bands DFT filter bank. The first channel of the filter bank is separated from the others to construct an un-uniform two band filter bank. On the other hand, because the frequency band of the fuel sloshing only appears in the first channel, one can state:

$$y(t) = \sum_{i=1}^{M} x_i(Lt) \Longrightarrow y(t) = \underbrace{x_1(Lt)}_{Ch(1)} + \underbrace{\sum_{i=2}^{M} x_i(Lt)}_{Ch(2)}$$
(6)



Figure 3. Structure of two bands delayless subband adaptive filter



Figure 4. One channel of a real valued DFT filter bank

where y(t) and $x_i(Lt)$ are the output of the filter bank and decimated output of ith channel, respectively.

The frequency response of the proposed delayless subband adaptive filter bank is demonstrated in Figure 5. In this figure, partitioning the input signal in two subbands is shown. The lowpass filter of the DFT filter bank must be designed through a prototype filter and with low aliasing and low phase distortion in the adjacent subbands. The order of the prototype filter in the current filter bank is selected 20 and the decimation factor is selected 10.

3.3. Frequency Estimation in Subband There are several studies associated with the frequency estimation in the vibrational systems. In one of these works, the rigid body model of the system has been proposed as a model reference for the flexible body [18]. In this approach, engaging the gradient method leads to instability of the algorithm in some cases. This makes a limitation for using the gradient algorithm. Hence, it is necessary to improve the algorithm of frequency estimation in order to solve the challenge of the stability of the rigid model reference approach.

In this paper, a new algorithm based on the Lyapunov theorem is proposed. In this way, a new Lyapunov candidate function based on the gradient method is extracted. On the other hand, the new algorithm is designed by combination of the previous method (gradient) and the Lyapunov theorem. In this regards, for extracting the error signal one can express:

$$e(n) = y(n) - y_m(n) \tag{7}$$

where y(n) is filtered output of the INS. Now, a Lyapunov candidate function in discrete form can be introduced as:

$$V = \frac{1}{2}e^2 + P^2$$
(8)

Thus,

$$\Delta V(n) = V(n+1) - V(n) = -B^2 + f(x,n)$$
(9)

where *P* and *B* are the hypothetic parameters.



Figure 5. Frequency response of two channel filter bank constructed from a 10 channel DFT filter bank

According to the Lyapunov theorem for time variant systems [19], the algorithm will be stable if *V* is a descending function and $\Delta V(n) < 0$. This condition is shown as follows:

If
$$f(x,n) \to 0 \Longrightarrow \Delta V(n) < 0$$
 (10)

In this situation, the estimation algorithm is asymptotically stable. In order to explain the process of derivation of the algorithm, it is necessary to describe the gradient method as:

$$\begin{cases} J(\theta) = \frac{1}{2}e^2 \\ \frac{d\theta}{dt} = -\gamma \frac{\partial J}{\partial \theta} \Rightarrow \frac{d\theta}{dt} = -\gamma e \frac{\partial e}{\partial \theta} \end{cases}$$
(11)

where $J(\theta)$ is the cost function, θ is an adjustable parameter (center frequency of the filter), and γ is the gain of the estimation. Consequently, Equation (11) in discrete form can be expressed as [15]:

$$\frac{\Delta\theta}{\Delta t} = -\gamma (y - y_m) (\frac{\Delta e}{\Delta \theta})$$
(12)

For estimating the frequency of the system, the output signal must be passed from a filter. For an FIR (Finite Impulse Response) filter one can state:

$$\begin{cases} H(z) = \frac{N(z)}{D(z)} \\ N(z) = 1 + (2\theta)Z^{-1} + Z^{-2} \end{cases}$$
(13)

and

$$x(n) = \frac{1}{D(z)}u(n) \tag{14}$$

where u(n) is the input of the filter (output of the INS). Consequently, the output of the filter is given as:

$$y(n) = x(n) + (2\theta)x(n-1) + x(n-2)$$
(15)

Using the above equations, the adaptive algorithm is extracted as follows [14]:

$$\theta(n) = \frac{y_m(n) - x(n) - x(n-2)}{2x(n-1)} \Longrightarrow \theta(n+1) = -\gamma e(n)x(n-1)$$
(16)

In spite of the high speed and performance of this algorithm, it encounters the challenge of stability. In another word, increasing the gain of the estimation leads to degradation of the adaptation algorithm stability in the gradient method. Hence, in this paper, it is tried to propose a combined method by means of the previous approaches considering the Lyapunov theorem. As an idea, one can introduce a new candidate function for the Lyapunov theorem as follows:

$$V = \sum \frac{1}{2}e^{2}(n) + \sum \frac{1}{2\delta}(\theta(n) - \frac{y_{m}(n) - x(n) - x(n-2)}{2x(n-1)})^{2}$$
(17)

where δ is a gain for enhancing the estimation performance. According to Equation (17), the variation of the Lyapunov function is derived as:

$$\Delta V = \sum \frac{1}{2} e^{2} (n+1) + \sum \frac{1}{2\gamma} (\theta(n+1)) - \frac{y_{m}(n+1) - x(n+1) - x(n-1)}{2 x(n)} + \sum \frac{1}{2} e^{2} (n) - \frac{1}{2\gamma} (\theta(n) - \frac{y_{m}(n) - x(n) - x(n-2)}{2 x(n-1)})^{2}$$
(18)

Now, if the following part of Equation (18) is tuned to zero, the stability of the algorithm is satisfactorily guaranteed.

$$\sum \frac{1}{2}e^{2}(n+1) + \sum \frac{1}{2\gamma}(\theta(n+1)) - \frac{y_{m}(n+1) - x(n+1) - x(n-1)}{2x(n)})^{2} = 0$$
(19)

It is noticeable that Equation (19) proposes an adaptive algorithm including the unknown parameter of the filter. For this reason, expanding this equation leads to the final algorithm of frequency estimation:

$$\theta(n-1)^{2} + \theta(n-1)A(n) + B(n) = 0$$
(20)

where A(n) and B(n) are defined from Equation (21) and $\theta(n)$ is the adjustable parameter of the filter.

$$\begin{cases} A(n) = \left[-\frac{y_m(n-1)}{x(n-2)} + \frac{x(n-1)}{x(n-2)} + \frac{x(n-3)}{x(n-2)}\right] \\ B(n) = \left[\gamma e^2(n-1) + \frac{y_m^2(n-1)}{4x(n-2)^2} - \frac{y_m(n-1)x(n-1)}{2x(n-2)^2} - \frac{y_m(n-1)x(n-1)}{2x(n-2)^2} + \frac{x(n-1)x(n-3)}{2x(n-2)^2} + \frac{x(n-1)x(n-3)}{2x(n-2)^2} + \frac{x(n-1)x(n-3)}{2x(n-2)^2} + \frac{x(n-3)^2}{4x(n-2)^2}\right] \end{cases}$$
(21)

The process of the frequency estimation is implemented in the subband channel and in the decimated sample time. For comparing the fullband and the subband analyses, in the final part of the paper the proposed algorithm in the both frequency bands is examined.

4. SIMULATION RESULTS

In order to demonstrate the performance of the new subband adaptive controller, this control system is implemented on the LV modeled in section 2. On the other hand, the LV with its attitude control system is modified with the new augmented vibration control system as demonstrated in Figure 2. In this model, the sloshing frequency is increased from 0.88 to 3.5 (Hz) due to the flight time. The trend of the frequency estimation in one condition of the flight time is shown in Figure 6. Moreover, the stability of this algorithm in spite of the increase of the gain of the estimation is illustrated in this figure.

The expected frequency in this figure is approximately extracted from the calculation of the parameters of the mechanical model of the fuel sloshing. In Figure 7, sloshing frequency estimation in the nonlinear model in the subband is shown and compared with the expected one. In this figure, the trend of the estimation is improved at the final step of the flight time. This is arisen from the increasing the sloshing effects at this part of the flight time. In addition, in order to show the effectiveness of the subband analysis instead of fullband, the frequency estimation from both views (subband and fullband [6]) is compared as shown in Figure 8. This comparison considerably shows the advantage of the subband analysis. The error of the frequency estimation is shown in Figure 9. This range of the error is appropriate for the current vibration control strategy.

Final figures show the main aim of the current study related to the active vibration control. In Figure 10, the mass displacement of the fuel sloshing is illustrated. This result appropriately demonstrates the excitation reduction of the slosh-mass. In fact, the new control system protects the slosh-mass against the increase of the excitation. As demonstrated in Figure 11, the effect of the fuel sloshing on the angular velocity of the LV is considerably reduced using the new control system. In another word, without using the new control system, the vibration arisen from the fuel sloshing can significantly affect the angular velocity of the LV.

One of the latest control systems used for vibration control is L1 adaptive controller [20]. In order to investigate the preference of the subband adaptive control system with the L1 adaptive controller, the angular velocity of the system is controlled using both methods. In spite of the advantages of the L1 adaptive controller in contrast with the other control systems, according to Figure 12 for sloshing vibration control the subband adaptive controller shows better performance. This is a result of low frequency of the sloshing phenomenon.



Figure 6. The expected and estimated frequencies of the fuel sloshing obtained from the Lyapunov approach



Figure 7. The estimation of the sloshing frequency in the nonlinear system via the subband adaptive estimator



Figure 8. Comparison of the frequency estimation via the subband and fullband analyses [6]



Figure 9. The frequency estimation error of the fuel sloshing due to the flight time



Figure 10. The mass displacement of the fuel sloshing with and without subband adaptive filtering



Figure 11. The angular velocity in the presence of the sloshing with and without subband adaptive filtering



Figure 12. comparison of the subband adaptive control performance and L1 adaptive controller

5. CONCLUSIONS

Fuel sloshing currently is an important vibration problem in the family of liquid fuel engine for aerospace vehicles. The undesired effects of this phenomenon in the worst case can degrade the stability of the control system and also lead to violated performance. However, one of the main challenges due to this problem refers to low value of the sloshing frequency and its variation due to the flight time. For this reason, estimation of the sloshing frequency and reducing the resulted effects in the aerospace devices are very important.

In this paper, a nonlinear dynamic model of a LV with fuel sloshing as a multi body dynamic system is completely proposed. In continue, a subband adaptive filter based on the Lyapunov theorem which is generated from DFT filter bank is derived and proposed for vibration suppression of the fuel sloshing. The new proposed subband adaptive filter works stably and precisely in contrast with the previous approaches in which the estimation is implemented in the fullband analysis. Moreover, the stability of the estimation algorithm is currently guaranteed by the use of the Lyapunov theorem. Consequently, by means of the properties of the subband analysis and the Lyapunov theorem, vibration suppression strategy is improved in the view of performance and also stability.

Simulation results show the considerable improvement in the vibration reduction and the stability of the new subband adaptive algorithm in contrast with the fullband analysis. Furthermore, the precision of the estimated frequency is acceptable in spite of the fact that the value of the sloshing frequency in a part of flight time is very close to the frequency of the main control system.

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APPENDIX

Definitions

$$S = \tilde{r}_0^s \tilde{r}_0^{s^t} \tag{1}$$

$$G = \tilde{e}^s \tilde{e}^{s^T} \tag{2}$$

$$D = \int \begin{bmatrix} \overline{z}^s \overline{e}_z^s + \overline{y}^s \overline{e}_y^s & -\overline{x}^s \overline{e}_y^s & -\overline{x}^s \overline{e}_z^s \\ 0 & \overline{z}^s \overline{e}_z^s & -\overline{y}^s \overline{e}_z^s \\ 0 & -\overline{z}^s \overline{e}_y^s & \overline{y}^s \overline{e}_y^s \end{bmatrix} dm$$
(3)

Nonlinear Equations of Motion

$$\begin{aligned} & \vec{e}^{s} m^{s} + \dot{V}m^{s} + (\tilde{\vec{r}}_{0}^{s^{T}} \vec{\varpi})m^{s} + \vec{\vec{e}}^{s^{T}} \overline{\varpi}m^{s} + \vec{\vec{e}}^{s^{T}} \vec{\varpi}m^{s} \\ & - [(\tilde{\varpi}^{T} \tilde{\varpi})\bar{e}^{s}]m^{s} - \tilde{\varpi}^{T} Vm^{s} \\ & - \frac{d\tilde{\vec{e}}^{s}}{dt} \overline{\varpi}m^{s} - \overline{\varpi}^{T} \tilde{\vec{r}}_{0}^{s^{T}} \widetilde{\varpi}m^{s} + \Omega \dot{e}^{s} + Ke^{s} = 0 \end{aligned}$$

$$(4)$$

$$\dot{V}m^{s} + \dot{V}m + \tilde{\tilde{r}}_{0c}^{sT}\bar{\omega}m^{s} + \tilde{\overline{\omega}}e^{s}m^{s} + \tilde{\overline{\omega}}e^{s}m^{s} + \tilde{\overline{e}}^{s}m^{s} + \tilde{\overline{\omega}}e^{s}m^{s} + \tilde{\overline{\omega}}e^{s}m^{s$$

 $\frac{1}{2}(\dot{S}+\dot{S}^{T})\overline{\omega}m^{s} + \frac{1}{2}(S+S^{T})\dot{\overline{\omega}}m^{s} + \dot{\overline{e}}^{s}\tilde{\overline{e}}^{sT}\overline{\omega}m^{s} \\
+ \tilde{\overline{e}}^{s}\dot{\overline{e}}^{sT}\overline{\omega}m^{s}\tilde{\overline{e}}^{s}\tilde{\overline{e}}^{sT}\dot{\overline{\omega}}m^{s} + \tilde{\overline{f}}_{0}^{s}\dot{V}m^{s} + \dot{\overline{e}}^{s}Vm^{s} + \tilde{\overline{e}}^{s}\dot{V}m^{s} \\
+ \tilde{\overline{f}}_{0}^{s}\ddot{\overline{e}}^{s}m^{s} + (\dot{\overline{e}}^{s})^{2}m^{s} + \tilde{\overline{e}}^{s}\dot{\overline{e}}^{s}m^{s} + (\dot{D}+\dot{D}^{T})\overline{\omega} + (D+D^{T})\dot{\overline{\omega}} \\
+ I\dot{\overline{\omega}} + \tilde{V}[Vm^{s} + Vm + \tilde{\overline{f}}_{0}^{sT}\overline{\omega}m^{s} + \tilde{\overline{\omega}}^{c}\overline{\overline{e}}^{s}m^{s} + \frac{d\overline{\overline{e}}^{s}}{dt}m^{s}]$ $(6) \\
+ \tilde{\omega}[\frac{1}{2}(S+S^{T})\overline{\omega}m^{s} + \tilde{\overline{e}}^{s}\tilde{\overline{e}}^{sT}\overline{\omega}m^{s} + \tilde{\overline{f}}_{0}^{s}Vm^{s} + \tilde{\overline{e}}^{s}Vm^{s} \\
+ \tilde{\overline{f}}_{0}^{s}\frac{d\overline{\overline{e}}^{s}}{dt}m^{s} + \tilde{\overline{e}}^{s}\frac{d\overline{\overline{e}}^{s}}{dt}m^{s} + (D+D^{T})\overline{\omega} + I\overline{\omega}] = M$

where *L* is the Lagrangian, *F* and *M* are the external forces and the momentums vector of the vehicle, Ω is the damping ratio matrix and *K* is the stiffness matrix associated with the fuel sloshing.

Vibration Suppression of Fuel Sloshing using Subband Adaptive RESEARCH Filtering NOTE

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PAPER INFO

Paper history: Received 26 March 2014 Received in revised form 17 August 2015 Accepted 03 September 2015

Keywords: Fuel Sloshing Vibration Control Nonlinear Dynamics Lyapunov Theorem Subband Adaptive Filter یکی از مسایل اصلی در ارتعاشات سیستمهای هوافضایی با سیستم پیشران مایع، تلاطم سیال مایع می باشد. این پدیده یک چالش ارتعاشی فرکانس پایین است که می تواند روی حرکت سیستم تاثیر گذاشته و پایداری سیستم اصلی کنترل را کاهش دهد. دراین خصوص، حرکت مایع زمانی خیلی بحرانی می شود که فرکانس ارتعاشات تلاطم به فرکانس کاری سیستم اصلی بسیار نزدیک باشد. علاوه بر این، محدوده فرکانسهای تلاطم در یک سیستم هوافضایی با تغییر زمان تغییر میکند. در این مقاله، یک سیستم هوافضایی با سوخت مایع با دیدگاه چند جسمی مورد بررسی قرار گرفته و برای کاهش تائیرات نامطلوب ارتعاش سوخت، یک فیلتر زیرباند تطبیقی جدید بر اساس تئوری لیاپانوف و تبدیل فوریه زمان گسسته طراحی شده است. در این زمینه، سیستم کنترل جدید روی کنترل وضعیت موجود سیستم هوافضایی پیاده سازی شده و شبیه سازی عددی استخراج شده است. نتایج عددی شبیه سازی نشان می دهند که تاثیرات تلاطم سوخت بر روی سیستم با استفاده از سیستم کنترل تطبیقی زیرباند به طور موثری کاهش یافته است.

چکیدہ

doi: 10.5829/idosi.ije.2015.28.10a.15