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State Estimation of MEMs Capacitor Using Taylor Expansion

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ABSTRACT

This paper deals with state estimation of micro tunable capacitor subjected to nonlinear electrostatic force. To this end, a nonlinear observer has been designed for state estimation of the structure. Necessary and sufficient conditions for construction of the observer are presented. Stability of the observer is checked using Lyapunov theorem. Observer design is based on converting differential equation of dynamic error from heterogeneous to homogenous. Thereby, non-linear electrostatic term is presented as coefficient of error which is done using decomposition of Taylor expansion of non-linear term. By stabilizing of homogenous differential equation, gains of observer can be obtained. Ability of the observer in state estimation of micro tunable capacitor is checked and related results are presented.

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1. INTRODUCTION

Currently, micro-electro mechanical systems (MEMS) have shown remarkable popularity in the engineering industry because of their several advantages such as order of magnitude, smaller size, better performance, possibilities for batch fabrication, cost effective integration with electronic systems, and low power consumption [1]. Electrostatically actuated MEMs devices such as, micro actuators [2, 3], mems capacitive microphone [4, 5], sensors [6, 7], capacitive micro-plate [8, 9], micro tunable capacitor [10-12], and micro-mirrors [13, 14] are broadly designed, fabricated, used and analyzed. With the fast growth of micro scale technology, necessity for state estimation of these devices will be taken into consideration for controlling, fault detecting and identifying structures.

Observer-based approach is a suitable tool for state estimation. This attention is mainly due to the associated advantages of this method such as quick detection and possibility of on-line implementation; it also does not require excitation signal. Moreover, control engineers are more familiar with the concepts of observer design [15].

The problem of designing observers for linear systems first introduced by Luenberger [16], then extended for nonlinear systems by Thau [17].

Overall, there are two procedures for design of observer for nonlinear systems. The first one is with respect to nonlinear state transformation in which dynamic error of state is rendered to linear and observer is designed using linear techniques [18-22]. The second method does not need the transformation [23-26].
Although various methods have been presented for design of non-linear observer, but use of linearized term or first order Taylor expansion of nonlinear function for observer design is admissible [27-29]. It may be due to simplicity of mentioned observers. However, if the operating region is too wide, the linearized model will deviate largely from the nonlinear model, particularly, if the system is operating away from the linearizing point [15]. For example, it is possible to point that filter extended Kalman filter is designed for minimization and error variance. Due to the fact that this filter takes advantage of using linearized form of non-linear term, in some cases this filter is not capable of correct state

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estimation and estimated state is deviated from actual state of non-linear system [30].

Furthermore using this method is not always possible. For this reason in this paper a method is presented that obviates above-mentioned shortages.

Presented method is based on using Taylor expansion of non-linear term in order to turn differential equation of dynamic error from heterogeneous to homogenous. Observer gains may be determined using stabilizing homogenous differential equation. In addition to having simplicity of linearized observers, this method has high accuracy due to the use of higher order terms of Taylor expansion.

2. CONSTRUCTION OF TAYLOR OBSERVER FOR NONLINEAR SYSTEM WITH LINEAR OUTPUT:

Consider the nonlinear system with form of:

$$\begin{aligned} \dot{x} &= Ax + uf(x) \\ y &= Cx \end{aligned} \tag{1}$$

where $x \in R^n$, $u \in R^r$ and $y \in R^m$ are state, input and output vectors, respectively, A and C are known system matrices, $f(x)$ represents the nonlinear function.

For implementation of the Taylor observer the following conditions must be satisfied:

1. Matrices A and C are observable.
2. The nonlinear function $f(x)$ is continuously differentiable. It satisfies also the Lipschitz condition at least locally with constant γ, i.e.

$$\| f(x) - f(\hat{x}) \| \le \gamma \| x - \hat{x} \| \tag{2}$$

$\|.\|$ denotes norm symbol.

The following observer is proposed for state reconstruction of system (1).

$$\begin{aligned} \dot{\hat{x}} &= A\hat{x} + uf(\hat{x}) + L(x, \hat{x})(y - \hat{y}) \\ \hat{y} &= C\hat{x} \end{aligned} \tag{3}$$

where $\hat{x}$ and $\hat{y}$ represent estimated state and output and $L(x, \hat{x})$ is the unknown gain of observer that must be obtained. For the sake of simplicity, in the rest of paper observer gain is denoted by L. Defining the observer error as $e = x - \hat{x}$, we have:

$$\dot{e} = (A - LC)e + uf(x) - uf(\hat{x}) \tag{4}$$

where $\dot{e}$ is the dynamic error. We want to find an observer gain, L, such that the observer error dynamics is asymptotically stable. Equation (4) is a heterogeneous differential equation. In this paper we attempt to turn aforementioned equation from heterogeneous condition to homogeneous.

If we can find continuous function $F(x, \hat{x})$ in which $f(x) - f(\hat{x}) \simeq (x - \hat{x})F(x, \hat{x}) = eF(x, \hat{x})$.

Then, Equation (4) can be presented as:

$$\dot{e} = (A - LC + uF(x, \hat{x}))e \tag{5}$$

This is a homogeneous equation. For asymptotic stability of dynamic error, coefficient matrix must be Hurwitz, or in another word matrix $A - LC + uF(x, \hat{x})$ must be negative definite. Considering this point observer gains can be found.

Function $F(x, \hat{x})$ can be obtained using Taylor expansion of $f(x) - f(\hat{x})$ about $x = \hat{x} = x_0$. Taylor expansion of $f(x)$ and $f(\hat{x})$ are presented in the following equations:

$$\begin{aligned} f(x) &= f(x_0) + \frac{df(x_0)}{dx}(x - x_0) + \frac{d^2 f(x_0)}{dx^2}\frac{(x - x_0)^2}{2!} \\ &+ \frac{d^3 f(x_0)}{dx^3}\frac{(x - x_0)^3}{3!} + ... \\ f(x) &= f(x_0) + \sum_{n=1}^{\infty} \frac{d^n f(x_0)}{dx^n}\frac{(x - x_0)^n}{n!} \end{aligned} \tag{6}$$

$$\begin{aligned} f(\hat{x}) &= f(x_0) + \frac{df(x_0)}{d\hat{x}}(\hat{x} - x_0) + \frac{d^2 f(x_0)}{d\hat{x}^2}\frac{(\hat{x} - x_0)^2}{2!} \\ &+ \frac{d^3 f(x_0)}{d\hat{x}^3}\frac{(\hat{x} - x_0)^3}{3!} + ... \\ f(\hat{x}) &= f(x_0) + \sum_{n=1}^{\infty} \frac{d^n f(x_0)}{d\hat{x}^n}\frac{(\hat{x} - x_0)^n}{n!} \end{aligned} \tag{7}$$

$\frac{d^n f(x_0)}{dx^n} = \frac{d^n f(x_0)}{d\hat{x}^n}$ So $f(x) - f(\hat{x})$ can be presented as:

$$\begin{aligned} f(x) - f(\hat{x}) &= \sum_{n=1}^{\infty} \frac{d^n f(x_0)}{dx^n}\frac{(x - x_0)^n}{n!} - \sum_{n=1}^{\infty} \frac{d^n f(x_0)}{d\hat{x}^n}\frac{(\hat{x} - x_0)^n}{n!} \\ f(x) - f(\hat{x}) &= \sum_{n=1}^{\infty} \frac{d^n f(x_0)}{n!dx^n}\left[(x - x_0)^n - (\hat{x} - x_0)^n\right] \end{aligned} \tag{8}$$

Following equality can be proved using mathematic induction:

$$(x - x_0)^n - (\hat{x} - x_0)^n = (x - \hat{x})\sum_{k=o}^{n-1}(x - x_0)^{n-1-k}(x - \hat{x})^k \tag{9}$$

Considering that $f(x) - f(\hat{x}) \simeq (x - \hat{x})F(x, \hat{x}) = eF(x, \hat{x})$ and regarding Equation (9), $F(x, \hat{x})$ can be presented in the following form:

$$F(x, \hat{x}) \simeq \sum_{n=0}^{N} \frac{d^n f(x_0)}{n!\,dx^n}\left(\sum_{k=0}^{n-1}(x - x_0)^{n-1-k}(\hat{x} - x_0)^k\right) \tag{10}$$

where N is finite integer. Theorem 1. Consider the nonlinear system (1), the nonlinear observer (3) and the dynamic error of (4). The dynamic error (4) is asymptotically stable. If there exists a $n \times m$ matrix L and a positive definite, symmetric $n \times n$ matrix P such that:

$$P[A - LC + uF(x, \hat{x})] < 0 \tag{11}$$

i.e. it is uniformly negative-definite for all magnitude of x and $\hat{x}$. Proof: consider the following Lyopunov function candidate:

$$v = e^T P e \tag{12}$$

Then, time derivative of v for system (4) is:

$$\begin{aligned} \dot{v} &= 2 e^T P \dot{e} \\ \dot{v} &= 2\left\{e^T P[(A - LC)e + uf(x) - uf(\hat{x})]\right\} \\ \dot{v} &= 2\left\{e^T P[A - LC + uF(x, \hat{x})]e\right\} \end{aligned} \tag{13}$$

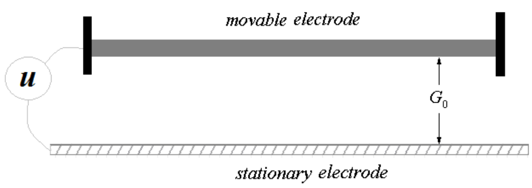
Since form Equation (11) $P[A - LC + uF(x, \hat{x})] < 0$ so $\dot{v} < 0$. Therefore v is a Lyapunov function for system (4). Theorem 2. The dynamic error of the observer is (globally) asymptotically stable if eigenvalues of $[A - LC + uF(x, \hat{x})]$ have negative real part.

Proof: theorem 2 can be proved easily, if matrix P set equal with identity matrix.

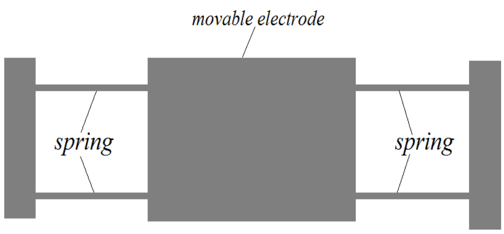
3. NUMERICAL EXAMPLE

In this section, ability of the Taylor observer is examined with implementing of it to state estimation of micro tunable capacitor subjected to nonlinear electrostatic force. To this end geometrical and mathematical model of the capacitor are presented.

3.1. Model Description Figure 1a, shows schematic view of a classic parallel plate micro capacitor. This device consists of a movable electrode suspended over a stationary conductor plate. As shown in this figure, the initial gap between the movable electrode and substrate is G_o. Attractive electrostatic force due to applied bias voltage u pulls movable electrode down towards the stationary plate. Figure 1b shows top view of the movable electrode. This electrode is suspended by four supporting beams (two at each side). The area and thickness of movable electrode are S and h, respectively.



a. Front view of capacitor



b. Top view of capacitor

Figure 1. Parallel plate tunable capacitor

All supporting beams are identical and width, thickness, and length of each beam are b, h, and l, respectively. The equivalent stiffness of each beam is $k = 12EI/l^3$, where E and I are Young's modulus and cross section moment of inertia, respectively. The movable electrode is considered isotropic with density ρ.

3. 2. Mathematical Modeling The governing equation of motion in a micro capacitor such as the one in Figure 1a can be described as:

$$m\frac{d^2 z}{dt^2} + c\frac{dz}{dt} + k_{eq} z = q_{elec} \tag{14}$$

where z, m, c, and k_{eq}, are the deflection, mass, damping coefficient, and equivalent stiffness ($k_{eq}=4k$) of the movable electrode, respectively. Also, q_{elec} represents electrostatic force. When the actuating voltage u is applied between the movable and stationary electrodes, the electrostatic force is computed using a standard parallel capacitance model, which yields [1]:

$$q_{elec} = \frac{\varepsilon_0 S u^2}{2(G_0 - z)^2} \tag{15}$$

where $\varepsilon_0 = 8.854 \times 10^{-12} C^2 N^{-1} m^{-2}$ is the permittivity of the vacuum within the gap. For convenience, Equation (14) can be rewritten in a non-dimensional form by defining the following parameters:

$$w = \frac{z}{G_0}, \quad \tau = \frac{t}{t^*} \tag{16}$$

where τ is the dimensionless time, and $t^* = \sqrt{m/k_{eq}}$. Therefore, Equation (14) may be written as:

$$\frac{d^2 w}{d\tau^2} + c'\frac{dw}{d\tau} + w = \frac{\alpha u^2}{(1-w)^2} \tag{17}$$

where c' and α are dimensionless damping and electrostatic coefficients, respectively, defined as:

$$c' := \frac{c}{t^* k_{eq}}, \quad \alpha := \frac{\varepsilon_0 S}{2k_{eq} G_0^3} \tag{18}$$

3. 2. 1. Mathematical Model in State Space Form Consider $x_1 = w$ and $x_2 = \frac{dw}{d\tau}$; so Equation (17) is rewritten in the state space form as:

$$\begin{cases} \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -1 & -c' \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ \dfrac{\alpha u^2}{(1-x_1)^2} \end{bmatrix} \\ y = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \end{cases} \tag{19}$$

Matrix $\begin{bmatrix} C \\ CA \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ is full rank therefore system is observable. As it is stated in the literature [1], $0 \le x \le 0.66$ so term $\frac{\alpha u^2}{(1-x)^2}$ is locally Lipschitz and terms 1 and 2 for observer design are fulfilled.

In this paper, output is considered as the non-dimensional deflection x_1. Following the same procedure for construction of Taylor observer for nonlinear system (section 2), the structure of the direct observer may be rewritten as:

$$\begin{bmatrix} \dot{\hat{x}}_1 \\ \dot{\hat{x}}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -1 & -c' \end{bmatrix} \begin{bmatrix} \hat{x}_1 \\ \hat{x}_2 \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{\alpha u^2}{(1-\hat{x}_1)^2} \end{bmatrix} + \begin{bmatrix} L_1 \\ L_2 \end{bmatrix} (y - \hat{y}) \tag{20}$$

Regarding Equations (19) and (20), dynamic error can be obtained as:

$$\begin{bmatrix} \dot{e}_1 \\ \dot{e}_2 \end{bmatrix} = \left(\begin{bmatrix} 0 & 1 \\ -1 & -c' \end{bmatrix} - \begin{bmatrix} L_1 \\ L_2 \end{bmatrix} \begin{bmatrix} 1 & 0 \end{bmatrix} \right) \begin{bmatrix} e_1 \\ e_2 \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{\alpha u^2}{(1-x_1)^2} - \frac{\alpha u^2}{(1-\hat{x}_1)^2} \end{bmatrix} \tag{21}$$

Using the same procedure of section 2, above equation can be rewritten as:

$$\begin{bmatrix} \dot{e}_1 \\ \dot{e}_2 \end{bmatrix} = \left(\begin{bmatrix} -L_1 & 1 \\ -1-L_2 & -c \end{bmatrix} \right) \begin{bmatrix} e_1 \\ e_2 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ \alpha u^2 F(x_1, \hat{x}_1) & 0 \end{bmatrix} \begin{bmatrix} e_1 \\ e_2 \end{bmatrix}$$

or

$$\begin{bmatrix} \dot{e}_1 \\ \dot{e}_2 \end{bmatrix} = \left(\begin{bmatrix} -L_1 & 1 \\ \alpha u^2 F(x_1, \hat{x}_1) - 1 - L_2 & -c \end{bmatrix} \right) \begin{bmatrix} e_1 \\ e_2 \end{bmatrix} \tag{22}$$

where

$$F(x_1, \hat{x}_1) = \left[2 + 3(x_1 + \hat{x}_1) + 4(x_1^2 + x_1\hat{x}_1 + \hat{x}_1^2) + 5(x_1^3 + x_1^2\hat{x} + x_1\hat{x}_1^2 + \hat{x}_1^3) + \ldots \right]$$

Following conditions guarantee the asymptotic stability of dynamic error:

$$\begin{aligned} & L_1 > -c \\ & L_2 > -1 - L_1 c + \alpha u^2 F(x_1, \hat{x}_1) \\ & \text{or} \\ & L_1 = -c + \varepsilon_1 \\ & L_2 = -1 - L_1 c + \alpha u^2 F(x_1, \hat{x}_1) + \varepsilon_2 \end{aligned} \tag{23}$$

where ε_1 and ε_2 are positive value. Regarding Equation (23), it is concluded that L_2 is a variable gain and depends on actual and estimated states of system.

4. SIMULATION RESULTS

In this section, state estimation of the micro parallel plate capacitor subjected to nonlinear electrostatic force has been developed using Taylor and linearized observer and related results are compared to each other. Spatial properties of the capacitor are presented in Table 1. The initial conditions of system and observer are $[x_1 \;\; x_2] = [0 \;\; 0]$ and $[\hat{x}_1 \;\; \hat{x}_2] = [0.1 \;\; 0.1]$, respectively.

Figures 2 and 3 show state estimation results for applied step DC voltage *2V*. Dynamic pull-in voltage for the capacitor is about *4.08 V*. Since there is a considerable difference between applied and pull-in voltage, the effects of nonlinearity may be neglected. In this condition, linearized observer has acceptable capability in state estimation of the capacitor. With comparing Figures 2 and 3, it may be concluded that, there is no significant difference between results of linearized and Taylor observer for applied voltage *2V*.

Figures 4 and 5 illustrate estimation results for applied voltage *4V*. As this voltage is close to dynamic pull-in voltage, the effects of nonlinear term increased and we can't use linearized observer for state estimation of capacitor. As it is shown in Figure 4, linear observer can't estimate the state and difference between actual state and estimated state is not decreased with respect to time. But as it is shown in Figure 5, Taylor observers guarantee the stability of dynamic error and estimate state of the capacitor with good accuracy even in the vicinity of dynamic pull-in voltage.

In the following, sensitivity of observer to noise is discussed. It is assumed that applied voltage is contaminated by $\pm 10\%$ noise. Figure 6 indicates state estimation of micro tunable capacitor for the applied voltage *2V*. In this condition, applied voltage varies in the range $1.8 \le u \le 2.2$. As it is clear by this figure, observer has good ability for state estimation of micro structure. Presented result indicates observer is robust against noise.

TABLE 1. Spatial properties of the micro parallel plate capacitor

Properties	**Value**
Area of movable electrode (S)	$400\mu m \times 400\mu m$
Thickness of movable electrode	$2\mu m$
Thickness of beam	$2\mu m$
Length of beams	$200\mu m$
Width of beams	$5\mu m$
Young's modulus of beams	$169 GPa$
G_0	$3\mu m$
Density	$2300\, Kg/m^3$

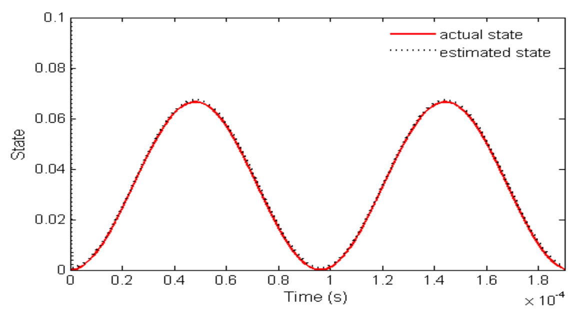


Figure 2a. Actual and estimated values of the state for applied voltage 2 using linearized observer

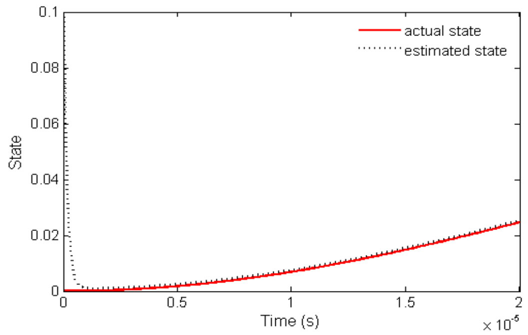


Figure 2b. A detailed closed view of Figure 2a

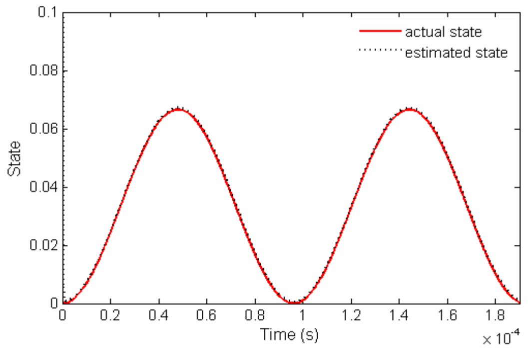


Figure 3a. Actual and estimated values of the state for applied voltage 2 using Taylor observer

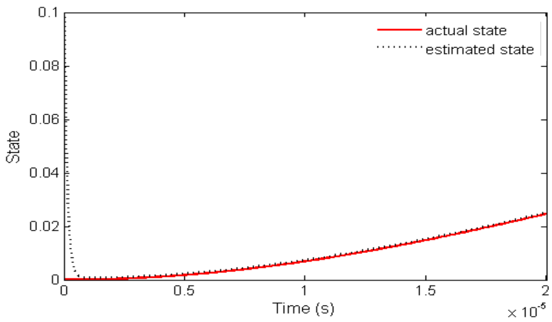


Figure 3b. A detailed closed view of Figure 3a

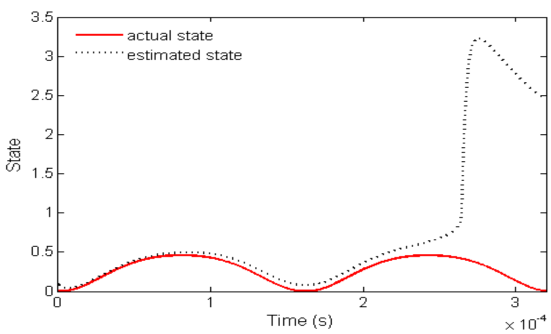


Figure 4a. Actual and estimated values of the state for applied voltage 4 using linearized observer

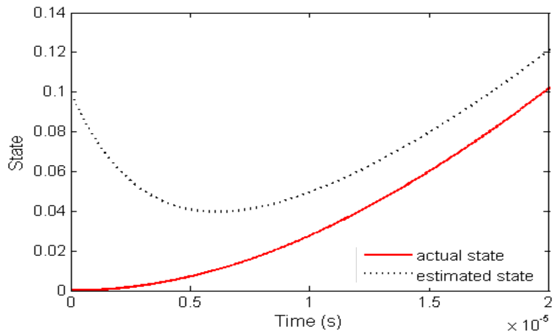


Figure 4b. A detailed closed view of Figure 4a

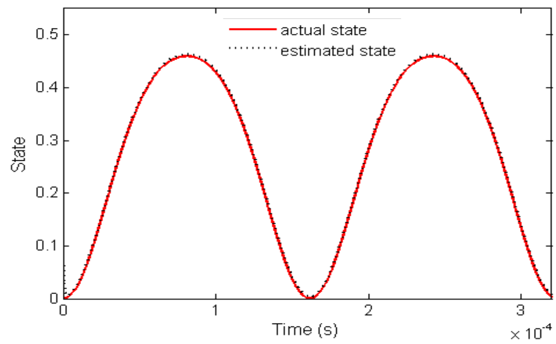


Figure 5a. Actual and estimated values of the state for applied voltage 4 using Taylor observer

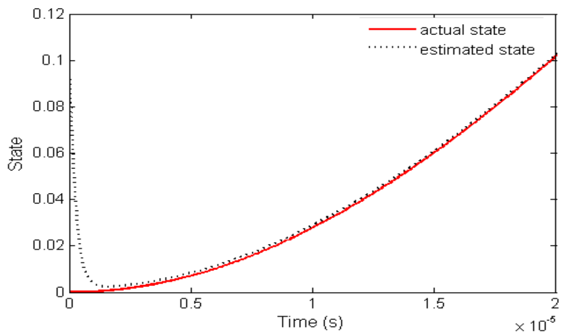


Figure 5b. A detailed closed view of Figure 5a

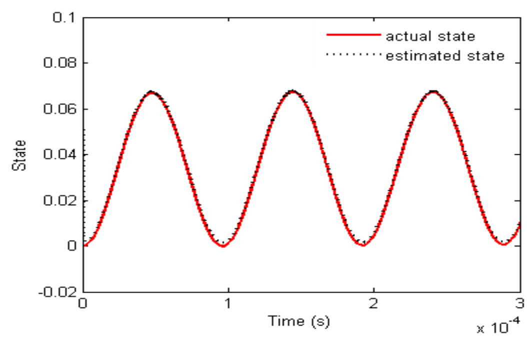


Figure 6a. Actual and estimated values of the state for applied voltage 2 contaminated by $\pm 10\%$ noise using Taylor observer.

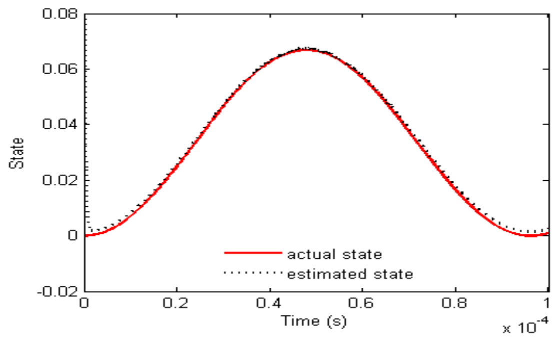


Figure 6b. A detailed closed view of Figure 6a

5. CONCLUDING REMARKS

In this paper, a new method is presented for observer design of Lipschitz non-linear system. Necessary and sufficient conditions for construction of the observer are presented. Stability of the observer is checked using Lyapunov theorem. The presented method is based on stabilizing of dynamic error. To this end, the non-linear term of dynamic error differential equation is presented as coefficient of error in order to change differential equation from heterogeneous to homogenous. It is done using decomposition of Taylor expansion of non-linear term to error term. With stabilization of homogenous equation, gains of observer are extracted. Simulating results for state estimation of micro capacitor are obtained and presented. Furthermore, the obtained results are compared to the results obtained by linearized observer. The results show that Taylor observer has good ability in the state estimation of micro capacitor, even in the vicinity of dynamic pull-in voltage.

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State Estimation of MEMs Capacitor Using Taylor Expansion

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چکیده

در این مقاله به تخمین حالت میکروخازن قابل‌تنظیم که تحت تاثیر نیروی غیرخطی الکترواستاتیک قرار دارد؛ پرداخته می-شود. بدین منظور مشاهده‌گر غیرخطیی برای تخمین حالت این ساختار طراحی می‌شود. شرایط لازم وکافی برای طراحی مشاهده‌گر ارائه شده است و پایداری مشاهده‌گر با استفاده از تئوری لیاپانوف بررسی می‌شود. طراحی مشاهده‌گر بر پایه‌ی تبدیل معادله دیفرانسیل خطای دینامیکی از حالت غیرهمگن به همگن می‌باشد. برای این تبدیل از بسط تیلور استفاده شده و ترم غیر خطی نیروی الکترواستاتیک بصورت ضریبی از خطا ارائه می‌شود. با استفاده از پایدارسازی معادله دیفرانسیل همگن، ضرایب بهره‌ی مشاهده‌گر را می‌توان تعیین کرد. در نهایت توانایی مشاهده‌گر در تخمین حالت میکروخازن سنجیده می‌شود و نتایج بدست آمده ارائه می‌شود.

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