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Stability and Performance Attainment with Fixed Order Controller Using Frequency Response

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ABSTRACT

Recently, a new data driven controller synthesis is presented for calculating the family of stabilizing first, second and fixed order controllers using frequency respons. However, this method is applicable just for plants that can guarantee some smoothness at the boundary of the resulted high dimension LMI. This paper solve that issue and extends the approach to fixed order controllers guaranteeing some performance criteria which are applicable for the more general types of plants. It is shown that knowing the frequency response of plant is sufficient to calculate the stabilizing fixed order controllers from a set of convex linear inequalities. The H_∞ norm on sensitivity and complementary sensitivity functions are satisfied from some frequency domain inequalities (FDI) that could be examined from frequency response data. The usefulness of the proposed approach is illustrated by an academic example.

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1. INTRODUCTION

Fixed order controller is the subject of many studies in literature. Toward other analytical methods such as H_∞ theorem, calculating the low order controller is the main advantage of fixed order controller design approaches. The most of these procedures lead to first or second order controllers [1-4] that are more conventional controllers toward other high order controllers. In fact, 90 percent of industrial controllers are currently belong to the family of PID controllers (for example see [5]). The PID

A polynomial method is implemented to calculate a fixed order controller so that the closed loop poles reside within a given region of complex plane [1]. A parameter space approach is presented [2] by constrained variance and minimum variance PID controller design for LTI models. The technique is based on plant mathematical model and noises. Using a version of Hermite-Biehler Theorem extended to quasipolynomials for first order plants, the complete family of stabilizing PID controllers is determined in [3]. Also, a modified chaotic genetic algorithm has been used to optimize the coefficients of PID controller [4]. Some optimization based approaches to PI/PID controller design are reviewed in literatures [6, 7]. These approaches are faced to the problem of non-convexity of the design parameters. A global lower bound on the achievable PID performance, defined in terms of output variance, is presented [8] that leads to solving a series of convex programs using sum of square programming. Another convex optimization of fixed order stabilizing controllers for systems with polytopic uncertainty is presented [9] as a linear matrix inequality using Kalman-Yakubovich-Popov (KYP) Lemma. Based on decoupling at singular frequencies, the algorithm of calculating the family of stabilizing PIDs is presented [10] in which nonconvex stability regions are built up by convex polygonal slices. All of these approaches are model based and need to mathematical model or time domain data of plant.

Some of model free PID tuning approaches based on feedback tuning [11, 12], un-falsified control [13],

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iterative feedback tuning [14] and extremum seeking [15] are already presented in the literature. A good review of these approaches could be found in literature [15]. Adaptive control is another model free approach [16, 17] that similar to other mentioned model free techniques relies on time domain data of plant. The first frequency based controller synthesis was based on loop shaping of open loop transfer function using Nyquist, Bode and Nicholz diagrams. The most practical issue in the frequency domain approaches is the difficulty in measuring the frequency response of unknown plants which are composed of hard nonlinear dynamics that triger high order harmonics. Also, the measurements are usually faced to measring noises.

Recently a new model free controller synthesis based on frequency response of plant is proposed [18] that can achieve to the set of stabilizing fisrt and second order controllers that satisfies some performance considerations. That approach is extended to simultaneously stabilizing [19], robust PID synthesis for plants with single uncertainty [20], robust stability for unstructured uncertainties [21] and fixed order controllers that can satisfy a particular performance criteria [22]. However, this criteria has a general drawback. In fact, since the approach applies a sort of gridding on a parameter of the performance criteria, the scope of solution is limited to plants that can guarantee some smoothness at the boundary of the resulted high dimension LMI. All the reviewed approaches have one or some of these limitations: (1) The mathematical model of plant is needed; (2) Some plants could not be stabilized by first or second order controllers [23]; (3) Some optimization approach may lead to a non-convex problem; (4) Some approach is applicable for plants that can that can guarantee some smoothness at the boundary of the resulted LMI. In the proposed approach of this paper, it is illustrated that for stability achievement, it is sufficient to examine the feasibility of some linear inequalities in terms of controller parameters. Many performance specifications can be achieved by satisfying H_∞ norm of sensitivity and complementary sensitivity functions. But there is a problem in dealing with LMI in a model free manner. It isn't possible to synthesis controller with these performance considerations using frequency response data. The frequency domain inequalities are model based and there isn't any FDI that can be solved using spectral model of plant. The contribution of the paper is proposing a new approach to satisfy the performance specifications using FDIs. In fact the performance considerations are transformed to some linear inequalities in terms of controller parameters and frequency. The feasibility of this problem can be analyzed using new FDIs by MATLAB. Moreover, since the proposed approach does not rely on gridding, it is applicable to more general types of plants.

This paper is organized as follows. In section 2, the idea of fixed order controller design presented in [22] is reviewed. Section 3 illustrates how to achieve the performance specifications using frequency response data. In section 4, the effectiveness of the proposed approach is illustrated by an academic example. Some concluding remarks are mentioned in section 5.

2. FIXED ORDER CONTROLLER SYNTHESIS

In this section, we propose an algorithm to calculate the stabilizing fixed order controllers in the structure of Figure 1. The control objective is to synthesis a lowest order stabilizing controller that satisfies H_∞ norm of sensitivity and complementary sensitivity functions.
First, some mathematical preliminaries will be presented. Consider a real rational function

$$P(s)=\frac{A(s)}{B(s)} \tag{1}$$

where, $A(s)$ and $B(s)$ are polynomials with real coefficients and degrees m and n, respectively. We assume that $A(s)$ and $B(s)$ have no zero on $j\omega$ axis. Let z^+ and p^+ (z^-, p^-) determine the number of open right half plane (RHP) (open left half plane (LHP)) zeros and poles of $P(s)$. Also let $\Delta\angle P(j\omega)$ denotes the net change in phase as ω runs from 0 to $+\infty$. Then, we have

$$\Delta\angle P(j\omega)=\frac{\pi}{2}((z^- - z^+)-(p^- - p^+)) \tag{2}$$

The (Hurwitz) signature of $P(s)$ is defined as

$$\sigma(P)=z^- - z^+ - \left(p^- - p^+\right)=\frac{2}{\pi}\Delta\angle P(j\omega) \tag{3}$$

since $P(s)$ has no pole and zero on $j\omega$ axis, we have

$$\sigma(P)=-(n-m)-2(z^+ - p^+) \tag{4}$$

or

$$\sigma(P)=-r_p-2(z^+ - p^+) \tag{5}$$

where, r_p is the relative degree of plant $P(s)$ and can be obtained from high frequency slope of bode magnitude. The only available data is the frequency response of unknown plant $P(s)$ for $\omega\ge 0$. The frequency response of plants is as

$$c=\left|P(j\omega)\right|e^{j\phi(\omega)}=P_r(\omega)+jP_i(\omega) \tag{6}$$

where, $P_r(\omega)$ and $P_i(\omega)$ denote the real and imaginary parts of $P_i(\omega)$, respectively. Assume that the real, distinct, finite zeros of $P\ (j\omega)=0$ denote as $\omega_0,\omega_1,\ldots,\omega_{l-1}$ such that

$$0=\omega_0<\omega_1<\ldots<\omega_{l-1}<\omega_l=\infty. \tag{7}$$

Lemma 2.1: for $n-m$ is even

$$\begin{aligned}\sigma(P)&=(sgn(P_r(\omega_0))2\Sigma(-1)^j sgn(P_r(\omega_j))\\&+(-1)^l sgn(P_r(\omega_l)))(-1)^{(l-1)} sgn(P_i(\infty))\end{aligned} \tag{8}$$

and for $n-m$ odd

$$\begin{aligned}\sigma(P)&=(sgn(P_r(\omega_0))\\&+2\Sigma(-1)^j sgn(P_r(\omega_j)))(-1)^{(l-1)} sgn(P_i(\infty)).\end{aligned} \tag{9}$$

Let the controller in the structure of Figure 1 be as

$$C(s)=\begin{cases}\dfrac{\rho_1+\rho_2 s+\sum_{i=0}^{\frac{N}{2}-1}\rho_{(i+3)}s^{2(i+1)}}{s(1+sT_1)(1+sT_2)\ldots(1+sT_{N-1})} & : N\ even\\[2ex] \dfrac{\rho_1+\rho_2 s+\sum_{i=0}^{\frac{N-3}{2}}\rho_{(i+3)}s^{2(i+1)}}{s(1+sT_1)(1+sT_2)\ldots(1+sT_{N-1})} & : N\ odd\end{cases} \tag{10}$$

where, $\rho_1,\rho_2,\ldots,\rho_{i+3}$, $T_1,T_2,\ldots,T_{N-1}$ and N are design parameters, arbitrary constants and the order of controller, respectively. Now consider N be even. Let

$$\begin{aligned}F(s)&=s(1+sT_1)(1+sT_2)\ldots(1+sT_{N-1})\rho_1\\&+\rho_2 s+\sum_{i=0}^{(N-1)/2}\rho_{(i+3)}s^{2(i+1)}P(s).\end{aligned} \tag{11}$$

Lemma 2.2: The closed loop stability is equivalent to the below equality

$$\sigma(F(s))=n-m+2z^{+}+N. \tag{12}$$

Let $\overline{F}(s)=F(s)P(-s)$ and write

$$\overline{F}(\omega)=\overline{F}_r(\omega,\rho_1,\rho_3,\rho_4,\ldots,\rho_{i+3})+j\omega\overline{F}_i(\omega,\rho_2). \tag{13}$$

Consider $\overline{F}_i(\omega,\rho_2)=0$ and define

$$\rho_2:=g(\omega)=\frac{aP_i(j\omega)-bP_r(j\omega)}{\omega|P(j\omega)|^2} \tag{14}$$

in which, a and b can be obtained from the denominator of controller as

$$j\omega(1+j\omega T_1)\ldots(1+j\omega T_N)=a+jb \tag{15}$$

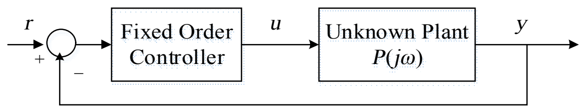

Figure 1. The feedback structure of plants with fixed order controller.

And define $J=sgn[\overline{F}_i(\infty^{-},\rho_2)]$ where $\rho_2^{min}<\rho_2^{*}<\rho_2^{max}$.

Theorem 2.1: Let $\omega_1<\omega_2<\ldots<\omega_{l-1}$ denote the distinct frequencies of odd multiplicities which are solutions to $\overline{F}_i(\omega,\rho_2)=0$. Determine string of integers $I=[i_0,i_1,i_2,\ldots,i_l]$ where $i_t\in\{-1,1\}$ such that for $n-m$ even:

$$[i_0-i_1+\ldots+(-1)^{(l-1)}2i_{(l-1)}(-1)^l i_l](-1)^{(l-1)}J=n-m+2z^{+}+N \tag{16}$$

and for $n-m$ odd:

$$\left[i_0-i_1+\ldots+(-1)^{l-1}2i_{l-1}\right](-1)^{l-1}J=n-m+2z^{+}+N \tag{17}$$

and $i_l=sgn(\overline{F}_r(\infty_{-},\rho_1,\rho_3,\rho_4,\ldots,\rho_{i+3}))$. Then for $\rho_2=\rho_2^{*}$, the $(\rho_1,\rho_3,\rho_4,\ldots,\rho_{i+3})$ values corresponding to closed loop stability can be found by solving the problem of feasibility of

$$\begin{pmatrix}\overline{f}_{r1} & 0 & 0 & 0\\ 0 & \overline{f}_{r1} & 0 & 0\\ 0 & 0 & \ddots & 0\\ \vdots & \vdots & \ddots & \vdots\\ 0 & 0 & 0 & \overline{f}_{rt}\end{pmatrix}\prec 0 \tag{18}$$

where, $\overline{f}_{r1}<0$, $i=1,2,\ldots,t$ are corresponding to inequalities

$$\overline{F}_r(\omega_t,\rho_1,\rho_3,\rho_4,\ldots,\rho_{i+3})i_t>0 \tag{19}$$

and i_t's are taken from strings satisfy Equations (16) or (17) and ω_t's are taken from the Equation (14). The proof for Lemmas 2.1 and 2.2 and Theorem 2.1 can be found in [18]. This theorem can be proved using the stability criteria of Equation (18) and applying Lemma 2.1 to compute the signature of $\overline{F}(s)$.

Theorem 2.2: The necessary condition for PID stabilizing is that there exists ρ_2 such that Equation (14) has at least R distinct roots of odd multiplicities such that

$$\begin{cases}R\geq\dfrac{n-m+2z^{+}+N}{2}-1 & : if\, n-m\ \ even\\[2ex] R\geq\dfrac{n-m+2z^{+}+N+1}{2}-1 & : if\, n-m\ \ odd.\end{cases} \tag{20}$$

Using Theorem 2.2, we can calculate the range of ρ_2 and then by sweeping on this range, the range of admissible $(\rho_1,\rho_3,\rho_4,\ldots,\rho_{i+3})$ can be obtained for an unknown plant. If the plant be unstable then we can't directly obtain the frequency response. In this case, it should be exist a stabilizing known controller $C(s)$ with any order. Then, the frequency response of unstable plant can be obtained from

$$P(j\omega)=\frac{H(j\omega)}{C(j\omega)(1-C(j\omega))} \tag{21}$$

The next theorem illustrates how to calculate the number of right half plan (RHP) zeros and poles of plant $P(s)$. As it will be shown, there is no need to plant mathematical model to calculate RHP poles and zeros. *Theorem 2.3:* The number of RHP zeros and poles can be obtained from

$$z^{+}=\frac{1}{2}[-r_p-r_C-2z_C^{+}-\sigma(H)] \tag{22}$$

$$p^{+}=\frac{1}{2}\left[\sigma(P)-\sigma(H)-r_C\right]-2z_C^{+} \tag{23}$$

where, z_C^{+} and r_C are the number of RHP zeros and the relative degree of $C(s)$, respectively.

The proof for Theorems 2.2 and 2.3 can be found in research [22]. The procedure of calculating the lowest order controllers with frequency response data is summarized in the following algorithm.

Algorithm 2.1: Calculating the lowest order controllers with frequency response.

1- Determine the relative degrees of plant from high frequency slope of Bode magnitude diagram.
2- Determine z^{+} for every plant from (4).
3- Set $N=1$.
4- Determine the range of ρ_2 from Theorem 2.2.
5- If there is not any ρ_2 that satisfies the conditions of Theorem 2.2 then set $N=N+1$ and go to Step 4, else go to the next step.
6- For $\rho_2=\rho_2^{*}$ solve (14) and obtain roots with odd multiplicity as $\omega_1<\omega_2<\ldots<\omega_{l-1}$.
7- Let $\omega_1=0$ and $\omega_l=\infty$ and define $J=sgn[\overline{F}_i(\infty^{-},\rho_2)]$. Determine $i_0,i_1,\ldots,i_{l-1}$ from Equations (16) or (17).
8- For $\rho_2=\rho_2^{*}$, determine the $(\rho_1,\rho_3,\rho_4,\ldots,\rho_{i+3})$ values from Equation (18).
9- Change the value of ρ_2 and go to Step 4 to obtain another stabilizing PID controller.

3. PERFORMANCE CONSIDERATION WITH FIXED ORDER CONTROLLER

Many performance attainment problems for plant $P(s)$ can be cast as the problem of achieving an H_∞ norm specification on the sensitivity and complementary sensitivity functions [18]. In Section 2, the stabilizing controllers obtained from solving the feasibility problem of the linear matrix inequality [18]. Here, the main contribution of the paper is presented. To obtain the lowest order controllers that achieves to stability and performance, another constraints are needed to add to Equation (18), but there isn't any linear performance condition that can be examined by frequency response data. In the other hand, every FDI needs to state space or transfer function of the plant. In the next theorem and the following corollaries, we transform the problem of performance attainment to the feasibility problem of some linear inequalities that can be examined by new FDIs.

Theorem 3.1: Consider the structure of Figure 1 with controller of Equation (10). Then:

(1) The closed loop system achieves to the H_∞ norm of sensitivity function, i.e. $\left|\frac{1}{1+c(s)P(s)}\right|<\gamma$, if the controller parameters satisfy one of the below sets of inequalities for $\omega\geq 0$:

$$\begin{cases}-Re(1+CP)+Im(1+CP)-\frac{1}{\gamma}:=h_1<0\\ Re(1+CP)\,Im(1+CP)>0\\ Re(1+CP)-Im(1+CP)>0\end{cases} \tag{24}$$

$$\begin{cases}-Re(1+CP)+Im(1+CP)-\frac{1}{\gamma}:=h_2<0\\ Re(1+CP)\,Im(1+CP)>0\\ Re(1+CP)-Im(1+CP)<0\end{cases} \tag{25}$$

$$\begin{cases}-Re(1+CP)+Im(1+CP)-\frac{1}{\gamma}:=h_3<0\\ Re(1+CP)\,Im(1+CP)<0\\ Re(1+CP)-Im(1+CP)>0\end{cases} \tag{26}$$

$$\begin{cases}-Re(1+CP)+Im(1+CP)-\frac{1}{\gamma}:=h_4<0\\ Re(1+CP)\,Im(1+CP)<0\\ Re(1+CP)-Im(1+CP)<0\end{cases} \tag{27}$$

(2) The closed loop system achieves to the H_∞ norm of complementary sensitivity function, i.e. $\left|\frac{C(s)P(s)}{1+C(s)P(s)}\right|<\gamma'$, if the controller parameters satisfy one of the below sets of inequalities for $\omega\geq 0$:

$$\begin{cases}-Re(CP)+Im(CP)-\frac{\gamma}{\gamma'}:=J_1<0\\ Re(CP)\,Im(CP)<0\\ Re(CP)+Im(CP)>0\end{cases} \tag{28}$$

$$\begin{cases}Re(CP)-Im(CP)+\frac{\gamma}{\gamma'}:=J_2<0\\ Re(CP)\,Im(CP)>0\\ Re(CP)+Im(CP)<0\end{cases} \tag{29}$$

$$\begin{cases} -Re(CP) - Im(CP) - \gamma' / \gamma := J_3 < 0 \\ Re(CP)\, Im(CP) < 0 \\ Re(CP) - Im(CP) > 0 \end{cases} \tag{30}$$

$$\begin{cases} Re(CP) + Im(CP) + \gamma' / \gamma := J_4 < 0 \\ Re(CP)\, Im(CP) < 0 \\ Re(CP) + Im(CP) < 0 \end{cases} \tag{31}$$

Proof: for part (1) we have the performance constraint as

$$\begin{aligned} &\left|1 + C(s)P(s)\right| > \frac{1}{\gamma} \\ &=> \sqrt{Re^2(1 + CP) + Im^2(1 + CP)} > \frac{1}{\gamma} := d \end{aligned} \tag{32}$$

Let $a = Re(1 + C(j\omega)P(j\omega))$ and $b = Im(1 + C(j\omega)P(j\omega))$. Then from $a^2 + b^2 > d^2$ we have two following cases:

1) If $ab > 0$, then $a^2 + b^2 > (a - b)^2$ and we have:

- If $(a - b) > 0$, then the performance condition transforms to the constraint $(a - b) > d$ that is equivalent to Equation (24).
- If $(a - b) < 0$, then the performance condition transforms to the constraint $(a - b) < -d$ that is equivalent to Equation (25).

2) If $ab < 0$, then $a^2 + b^2 > (a + b)^2$ and we have:

- If $(a + b) > 0$, then the performance condition transforms to the constraint $(a + b) > d$ that is equivalent to Equation (26).
- If $(a + b) < 0$, then the performance condition transforms to the constraint $(a + b) < -d$ that is equivalent to Equation (27).

For part (2) of theorem, we have the performance constraint as

$$\frac{C(s)P(s)}{1 + C(s)P(s)} < \gamma'. \tag{33}$$

From part (1) of theorem we have

$$\frac{1}{1 + C(s)P(s)} < \gamma => \frac{C(s)P(s)}{1 + C(s)P(s)} < \gamma \left|C(s)P(s)\right| \tag{34}$$

It can be written

$$\sqrt{e^2 + f^2} < \frac{\gamma'}{\gamma} = h \;=>\; e^2 + f^2 < h^2 \tag{35}$$

1) If $ef > 0$, then $(e^2 + f^2) > (e + f)^2$ and we have:

- If $(e + f) > 0$, then the performance condition transforms to the constraint $(e + f) < h$ that is equivalent to Equation (28).
- If $(e + f) < 0$, then the performance condition transforms to the constraint $(e + f) > -h$ that is equivalent to Equation (29).

2) If $ef < 0$, then $(e^2 + f^2) > (e - f)^2$ and we have:

- If $(e - f) > 0$, then the performance condition transforms to the constraint $(e - f) < h$ that is equivalent to Equation (30).
- If $(e - f) < 0$, then the performance condition transforms to the constraint $(e - f) > -h$ that is equivalent to Equation (31). ■

The following two corollaries introduce two FDIs to obtain lowest order controllers that satisfy H_∞ norm on sensitivity and complementary sensitivity functions.

Corollary 3.1: To satisfy stability and H_∞ norm on sensitivity function, the feasibility of the following FDI must be satisfied:

$$\begin{pmatrix} \bar{f}_{r1} & 0 & 0 & 0 & 0 \\ 0 & \bar{f}_{r2} & 0 & 0 & 0 \\ 0 & 0 & \ddots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \bar{f}_{rt} & 0 \\ 0 & 0 & 0 & 0 & h_i \end{pmatrix} \prec 0 \tag{36}$$

where, h_i , $i = 1,\ldots,4$ are corresponding to Equations (24-27). For every h_i and every frequency ω a set of controller parameters will be obtained. Obviously, we select the controller that leads to best closed loop response.

Corollary 3.2: To satisfy stability and H_∞ norm on sensitivity and complementary sensitivity functions, the feasibility of the following FDIs must be satisfied:

$$\begin{pmatrix} \bar{f}_{r1} & 0 & 0 & 0 & 0 & 0 \\ 0 & \bar{f}_{r2} & 0 & 0 & 0 & 0 \\ 0 & 0 & \ddots & 0 & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \bar{f}_{rt} & 0 & 0 \\ 0 & 0 & 0 & 0 & h_i & 0 \\ 0 & 0 & 0 & 0 & 0 & J_i \end{pmatrix} \prec 0 \tag{37}$$

where, J_i , $i = 1,\ldots,4$ are corresponding to Equations (28)-(31). For every h_i, J_i and every frequency ω, a set of controller parameter will be obtained. Obviously, we select the controller that leads to best closed loop response.

4. SIMULATION

The Bode diagram for an academic example is shown in Figure 2. The corresponding function $g(\omega)$ is shown in Figure 3. Note that we examine a 2-order controller as

$$C(s) = \frac{\rho_1 + \rho_2 s + \rho_3 s^2}{s(1+sT)} \tag{38}$$

where, $T = 1$. From Equation (20) we have

$$R \geq \frac{2+2}{2} - 1 = 1. \tag{39}$$

For $\rho_2 = 10$, from Figure 3 $\omega_1 = 26.96$ and the corresponding inequalities for stability region are

$$\begin{cases} \rho_1 > 0 \\ \rho_1 - 723\rho_3 - 497 < 0 \end{cases} \tag{40}$$

Choosing $\rho_1 = 150$ and $\rho_3 = 10$ is led to step response of Figure 4-a. Obviously, these parameters are corresponding to only stabilizing region and there isn't any performance consideration. As you see in Figure 4, the overshoot is 50% and the rise and settling times are large. For the problem of satisfying H_∞ norm of sensitivity function, the feasibility of Equation (36) must be examined through checking Equations (24)-(27) for $\omega > 0$. This, in turn, will lead to a high dimension LMI in MATLAB. The simulation is accomplished and the parameters are obtained as $\rho_1 = 33.9978$ and $\rho_3 = -1$. So the corresponding controller is

$$C(s) = \frac{33.9978 + 10s - s^2}{s(1+s)} \tag{41}$$

And the corresponding step response is shown in Figure 4-b. There isn't overshoot in this case but still the rise and settling times are large. For the problem of satisfying H_∞ norm of sensitivity and complenetray sensitivity function, the feasibility of Equation (37) must be examined through checking Equations (28-31) for $\omega > 0$. This, in turn, will lead to a high dimension LMI in MATLAB. The simulation is accomplished and the parameters are obtained as $\rho_1 = 9.0228$ and $\rho_3 = -0.1$ and the corresponding controller is

$$C(s) = \frac{9.0228 + 10s - 0.1s^2}{s(1+s)}. \tag{42}$$

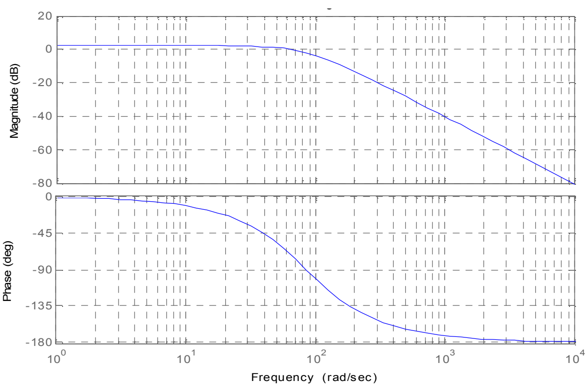

Figure 1. The Bode diagram of plant $P(s)$.

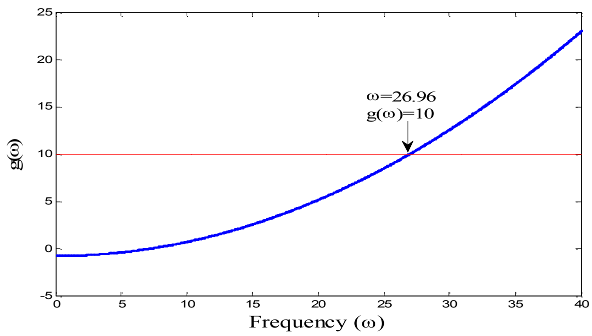

Figure 2. The plot of function $g(\omega)$

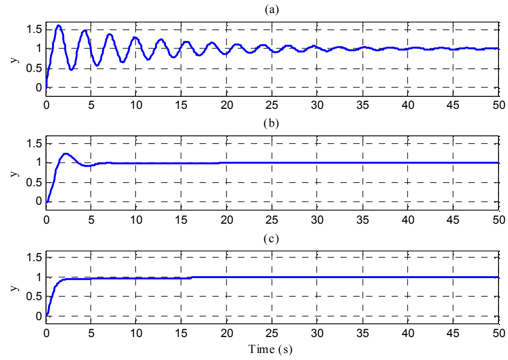

Figure 3. Step responses corresponding to (a), stabilizing controller without any performance consideration; (b), stabilizing controller of Equation (41) which satisfies H_∞ norm of sensitivity function; (c), stabilizing controller of Equation (42) which satisfies norm of sensitivity and complementary sensitivity functions.

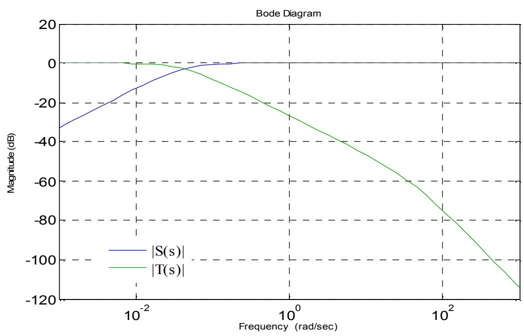

Figure 5. The magnitude of sensitivity and complementary sensitivity functions corresponding to Equation (42).

The corresponding step response is shown in Figure 4-c. The rise and settling time are improved and there isn't overshoot in the response. Finally, the magnitude plot of sensitivity and complementary sensitivity functions, i.e. $|S(s)|$ and $|T(s)|$, with controller as the form of Equation (42) are shown in Figure 5. It could be concluded that the proper shaping of these diagrams is resulted to better specifications in the step response of the plant with frequency response of Figure 2.

Other admissible parameters can be obtained by sweeping on the admissble range of ρ_2. All resulted controllers can guarantee the performance criteria used in this paper. However, solving Equations (36) and (37) may result in a non-minimum phase controller.

For the time being, there has not been found any general rule on the effect of sweeping on controller parameters. Since the controller parameters must be selected among a set of convex inequalities through Equations (24) to (31), the computed parameters by MATLAB might construct a conservative controller. This is a work in progress to find the optimal parameters among the admissible range. There has been provided lots of examples on this issue in literature [24].

5. CONCLUDING REMARKS

Through the paper, an algorithm for calculating the fixed order controller proposed using only frequency domain data is reviewed. The previous technique was applicable just for plants that can guarantee some smoothness moothness at the boundary of the resulted high dimension LMI. This paper solve that issue and extends the approach to fixed order controllers guaranteeing some performance criteria which are applicable for the more general types of plants. It is shown that the performance specifications such as H_∞ norm of sensitivity and complementary sensitivity functions can be examined by new linear inequalities in terms of controller parameters and frequency. This is the important feature of these constraints by which the problem of stabilizing controller synthesis with performance consideration can be transformed to the feasibility problem of new FDIs that could be analyzed using frequency response data.

Another important feature of the proposed approach is that there is no need to exact data on the whole range of frequency response. In fact, exact data are needed just at low frequency band and beyond which data might be rough or approximated.

Althought there are some useful techniques for measuring the frequency response such as attaining the input-output data using Fourier analysis, virtual sine sweeping, spectrum analyzer, network analyzer or using audio sine-wave generators and the sine function of function generators, but measuring the frequency response of plants with hard nonlinearity is still a challenging issue. This needs to more improvement to the measuring technologies and instruments.

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Stability and Performance Attainment with Fixed Order Controller Using Frequency Response

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چکید

اخیرا روشی برای محاسبه خانواده کنترل‌کننده‌های مرتبه اول و دوم پایدارساز با استفاده از داده‌های حوزه فرکانس ارائه شده است. در این مقاله، این روش به طراحی کنترل‌کننده مرتبه ثابت (از مرتبه N) تعمیم داده می‌شود که قادر به برآورده نمودن معیارهای عملکرد نیز می‌باشد. برای محاسبه کنترل‌کننده مرتبه ثابت پایدارساز به کمک مجموعه‌ای از ناتساویهای خطی محدب، کافیست پاسخ فرکانسی فرآیند دردسترس باشد. نرم H_∞ توابع حساسیت و مکمل حساسیت با استفاده از تعدادی ناتساوی خطی در حوزه فرکانس ارضا می‌شوند که این ناتساویها با استفاده از داده‌های پاسخ فرکانسی بررسی می‌شوند. کارآیی روش پیشنهادی با یک مثال اکادمیک نشان داده شده است.

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