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Fuzzy Approximation Model-based Robust Controller Design for Speed Control of **BLDC** Motor

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ABSTRACT

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This paper presents a new controller for speed control problem of the BLDC motors. The nonlinear model of the motor is approximated by implementation of fuzzy rules. The uncertainties are considered in the fuzzy system. Using this model and linear matrix inequality (LMI) optimization, a robust controller for purpose of speed control of the motor has been designed and applied to it. The effectiveness of the designed controls demonstrated through simulation results.

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NOMENCLATURE			
$I_{a}(t), I_{b}(t), I_{c}(t)$	Stator winding currents, A	$\theta_r(t)$	Rotor position, rad
$I_{q}(t), I_{d}(t)$	qd axis currents, A	L_q , L_d	qd axis inductances, H
$e_{a}(t), e_{b}(t), e_{c}(t)$	Stator back-emf, V	ψ_m	Mutual air gap flux linkages, V-s
$V_{a}(t), V_{b}(t), V_{c}(t)$	Stator voltages, V	Т	Electromagnetic torque, N.m
$v_{q}(t), v_{d}(t)$	qd axis voltages, V	Р	Number of poles
R	Stator resistance/phase, Ω	T_L	Load torque, N.m
L	Stator inductance, H	J	Load inertia, kg-m ²
$\omega_r(t)$	Rotor speed, rad/s	В	Viscous friction, N.m/(rad/s)

1. INTRODUCTION

With rapid developments in power electronics, power semiconductor technologies, modern control theory for motors and manufacturing technology for high performance magnetic materials, the brushless DC (BLDC) motors have been widely used in many fields [1], including industrial automation, automotive,

aerospace, instrumentation and appliances since 1970's [2]. The BLDC motor is a novel type of DC motor in which commutation is done electronically instead of using brushes. Therefore, it needs less maintenance, and due to the advancement of small size, good performance, simple structure, high reliability and large output torque have attracted increasing attention [1, 3]. There are two types of BLDC with respect to back-emf signal of motor; sinusoidal and trapezoidal. There are also two types of BLDC according to havingor not havingsensors for detecting rotor position [3]. Normally,

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Hall Effect sensors were used for low cost, low resolution requirements and optical encoder for high resolution requirements [4]. Sensor signals are used to adjust PWM sequence of 3-phase bridge inverter . In sensor-less control back-emf sensing, back-emf integration, flux linkage- based, freewheeling diode conduction and speed independent position function technique are used for electronic commutation [5]. Due to electronical commutation, BLDC has more complex control algorithm compared to other motor types [3, 5]. In practice, the design of the BLDCM drive involves a complex process such as modeling, control scheme selection, simulation and parameters tuning etc. Recently, various modern control solutions are proposed for the speed control design of BLDC motor [1-3]. However, Conventional PID controller algorithm is simple, stable, easy adjustment and high reliability. Conventional speed control system used in conventional PID controller. But, in fact, most industrial processes have been with different degrees of nonlinear, parameter variability and uncertainty of mathematical model of the system. Tuning PID control parameters is very difficult due to its poor robustness; therefore, it is difficult to achieve the optimal state under field conditions in the actual production [6]. So far, there have been many different design methods and control schemes to overcome the uncertain nonlinear control problems such that neural network control system has a strong ability to solve the structure uncertainty but it requires more computing capacity and data storage space. For genetic algorithms, ant-colony algorithms, techniques can help improving performance but they also need longer computation time and larger storage capacity [7-11].

To design the BLDC motor drive system, it is necessary to have motor model give precise value of torque which is related to current and back-EMF [12]. Different models have been presented to analyze performance of BLDC motor [1-5], [11, 12]. However, the model of the motor is highly nonlinear. In this paper, the nonlinear model of the motor has been approximated by using Takagi-Sugeno (TS) fuzzy approximation model. Recently, the design of fuzzy $H\infty$ for a class of nonlinear systems which can be represented by a Takagi-Sugeno (TS) fuzzy model has been considered by many researchers[13-17]. In this TS Fuzzy model, local dynamics in different state space regions are represented by local linear systems. The overall model of the system is obtained by 'blending' these linear models through nonlinear membership functions. Hence, the aim of this paper is to design a robust $H\infty$ based on linear matrix inequality (LMI) approach for speed control problem of the BLDC motors. This controller is designed such that the f_2 gain of the mapping from the exogenous input noise to the regulated output is less than a prescribed value.



Figure 1. Overall diagram of BLDC motor with proposed controller



2. MATHEMATICAL MODEL

The overall system model is shown in Figure 1. As shown, it contains a 3-phase full bridge voltage source inverter supplying the BLDC motor. The inverter, in turn, is controlled through the signals of the robust controller. This controller generates the control signals using multiple feedback loops of currents and rotor position and has been designed in comingsections.

3. DYNAMIC MODEL OF BLDC MOTOR

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The coupled circuit equations of the 3-phase BLDC motor are [18]:

$$\begin{vmatrix} v_a(t) \\ v_b(t) \\ v_c(t) \end{vmatrix} = \begin{vmatrix} R & 0 & 0 & i_a(t) \\ 0 & R & 0 & i_b(t) \\ 0 & 0 & R & i_c(t) \end{vmatrix} + \begin{vmatrix} L & 0 & 0 & 0 \\ 0 & L & 0 & d \\ 0 & 0 & L & 0 \\ 0 & 0 & L & d \\ i_c(t) \end{vmatrix} \begin{vmatrix} i_a(t) \\ i_b(t) \\ i_c(t) \end{vmatrix} + \begin{vmatrix} e_a(t) \\ e_b(t) \\ e_c(t) \end{vmatrix}$$
(1)

Also, back-emf is depend to position of rotor because the permanent magnet is rotating with speed of rotor. So, the relationship between back-emfs and the function of rotor speed can be written as follows [19]:

$$\begin{bmatrix} e_{a}(t)\\ e_{b}(t)\\ e_{c}(t) \end{bmatrix} = \omega_{r}(t)\psi_{m} \begin{vmatrix} \sin(\theta_{r})\\ \sin(\theta_{r} - \frac{2\pi}{3})\\ \sin(\theta_{r} + \frac{2\pi}{3}) \end{vmatrix}$$
(2)

The *abc* variable via the Park's transform are applied such that the rotor frame qdvariable is obtained. A set of voltage is obtained as [20]:

$$v_q(t) = Ri_q(t) + \frac{d}{dt}\psi_q(t) - \omega_r(t)\psi_d(t)$$
(3)

$$v_d(t) = Ri_d(t) + \frac{d}{dt}\psi_d(t) - \omega_r(t)\psi_q(t)$$
(4)

$$\psi_{q}(t) = L_{q}(t)i_{q}(t)$$
(5)

$$\psi_{d}(t) = L_{d}(t)i_{d}(t) + \psi_{m}$$
(6)

And the electromagnetic torque of the motor is [20]:

$$T_{(i_q i_d)}(t) = \frac{3}{2} \left(P \left(\psi_m i_q(t) + (L_q - L_d) i_q(t) i_d(t) \right) \right)$$
(7)

Combining with equation of motion, the system equations in terms of the qd variables can be written as follows:

$$\omega_{r}(t) = -\frac{B}{J}\omega_{r}(t) + \frac{3}{2}\frac{P}{J}\psi_{m}i_{q}(t) - \frac{P}{J}T_{L}(t)$$

$$i_{q}(t) = -\frac{\psi_{m}}{L_{q}}\omega_{r}(t) - \frac{R}{L_{q}}i_{q}(t) - \omega_{r}(t)i_{d}(t)\frac{L_{d}}{L_{q}} + \frac{1}{L_{q}}v_{q}(t)$$

$$i_{d}(t) = \omega_{r}(t)i_{q}(t)\frac{L_{q}}{L_{d}} - \frac{R}{L_{d}}i_{d}(t) + \frac{1}{L_{d}}v_{d}(t)$$
(8)

Choosing the state variables as;

 $x_1(t) = \omega_r(t), x_2(t) = i_q(t), x_3(t) = i_d(t)$, the system from (8) can be described by the following state equations:

$$\begin{aligned} & \cdot_{X_{1}(t)} = -\frac{B}{J} x_{1}(t) + \frac{3}{2} \frac{P}{J} \psi_{m} x_{2}(t) - \frac{P}{J} w_{1}(t) \\ & \cdot \\ & \cdot_{X_{2}}(t) = -\frac{\psi_{m}}{L_{q}} x_{1}(t) - \frac{R}{L_{q}} x_{2}(t) - x_{1}(t) x_{3}(t) \frac{L_{d}}{L_{q}} + 0.1 w_{2}(t) \\ & \cdot \\ & \cdot_{X_{3}}(t) = x_{1}(t) x_{2}(t) \frac{L_{q}}{L_{d}} - \frac{R}{L_{d}} x_{3}(t) + 0.1 w_{1}(t) \\ & z(t) = x_{1}(t) \end{aligned}$$
(9)

where $w_1(t)$ and $w_2(t)$ are the process noise, $w_3(t)$ the disturbance factor from torque load and z(t) the controlled output.

4. FUZZY APPROXIMATION AND NONLINEAR FUZZY MODE

Here, attention is particularly given to generalize the TS fuzzy system to represent a TS fuzzy system with parametric uncertainties. The TS fuzzy system with parametric uncertainties considered is as follows:

$$\mathbf{x}(t) = \sum_{i=1}^{l} \mu_{i}(\mathbf{v}(t)) [[A_{i} + \Delta A_{i}]\mathbf{x}(t) + [B_{i} + \Delta B_{i}]\mathbf{w}(t) + [B_{2i} + \Delta B_{2i}]\mathbf{u}(t)]$$

$$\mathbf{x}(t) = \sum_{i=1}^{l} \mu_{i}(\mathbf{v}(t)) [[C_{1i} + \Delta C_{1i}]\mathbf{x}(t) + [D_{12i} + \Delta D_{21i}]\mathbf{u}(t)]$$

$$\mathbf{y}(t) = \sum_{i=1}^{l} \mu_{i}(\mathbf{v}(t)) [[C_{2i} + \Delta C_{2i}]\mathbf{x}(t) + [D_{21i} + \Delta D_{21i}]\mathbf{u}(t)]$$
(10)

where $v(t) = [v_i(t)...v_g(t)]$ is the premise variable vector that may depend on states in many cases, $\mu_i(v(t))$ denotes the normalized time-varying fuzzy weighting functions for each rule (i.e., $\mu_i(v(t)) \ge 0$ and $\sum_{i=1}^r \mu_i(v(t))=1$),9 the number of fuzzy sets, $x(t) \in \Re^n$ the state vector, $u(t) \in \Re^m$ the input, $w(t) \in \Re^p$ the disturbance which belongs to L2 $[0,\infty]$, $y(t) \in \Re^\ell$ the measurement, and $z(t) \in \Re^s$ the controlled output. The matrices $A_i, B_{1i}, C_{1i}, C_{2i}, D_{12i}$ and D_{21i} are of appropriate dimension, and *r* is the number of IF-THEN rules. The matrices; $\Delta A_i, \Delta B_{1i}, \Delta C_{1i}, \Delta C_{2i}, \Delta D_{12i}$ and ΔD_{21i} represent the uncertainties in the system and satisfy the following assumptions:

$$\Delta A_{i} = F(x(t), t)H_{1i}$$

$$\Delta B_{1i} = F(x(t), t)H_{2i}, \Delta B_{21i} = F(x(t), t)H_{3i}$$

$$\Delta C_{1i} = F(x(t), t)H_{4i}, \Delta C_{2i} = F(x(t), t)H_{5i}$$

$$\Delta D_{12i} = F(x(t), t)H_{6i}, \Delta D_{21i} = F(x(t), t)H_{7i}$$

(11)

where $H_{ij} = 1, 2..., 7$ are known matrix functions which characterize the structure of the uncertainties. Furthermore, the following inequality holds:

$$\left\|F\left(\mathbf{x}(t),t\right)\right\| \le \rho \tag{12}$$

for any known positive constant ρ . If γ is a given positive number, then system is said to have L_2 -gain less than or equal to γ if:

$$\int_{0}^{T_{f}} z^{T}(t) z(t) dt \leq \gamma^{2} \left[\int_{0}^{T_{f}} w^{T}(t) w(t) dt \right], x(0) = 0$$
(13)

for all $T_r \ge 0$ and $w(t) \in L_2[0, T_r]$. Note that for the symmetric block matrices, we use (#) as an ellipsis for terms that are induced by symmetry. It is found that currents and the speed in dynamic model of BLDC motor from (9) are highly nonlinear. Thus, the nonlinearity and various uncertainties including external disturbances have to be taken into account. The nonlinear system plant can be approximated by TS fuzzy sets as Figure 2, the membership function can be written as:

$$M_1(x_1(t)) = \frac{-x_1(t) + N_2}{N_2 - N_1}; \qquad M_2(x_1(t)) = \frac{x_1(t) + N_2}{N_2 - N_1}$$
(14)

Using this membership functions, we can approximate the plant model as follows:Plant Rule I:

IF $x_1(t) \in \begin{bmatrix} -N_1 & N_1 \end{bmatrix}$ is $M_1(x_1(t))$ THEN

This can be written as:

Plant Rule II:

IF $x_1(t) \in \begin{bmatrix} -N_2 & N_2 \end{bmatrix}$ is $M_2(x_1(t))$ THEN

$$\mathbf{\dot{X}}(t) = \begin{bmatrix} -\frac{B}{J} & \frac{3}{2} \frac{P}{J} \psi_{m} & 0\\ -\frac{\psi_{m}}{L_{q}} & -\frac{R}{L_{q}} & -N_{2}\\ 0 & N_{2} & -\frac{R}{L_{d}} \end{bmatrix} \begin{bmatrix} x_{1}(t)\\ x_{2}(t)\\ x_{3}(t) \end{bmatrix} + \begin{bmatrix} 0 & 0\\ \frac{1}{L_{q}} & 0\\ 0 & \frac{1}{L_{d}} \end{bmatrix} \begin{bmatrix} u_{1}(t)\\ u_{2}(t) \end{bmatrix} + \begin{bmatrix} 0 & 0\\ 0 & \frac{1}{L_{d}} \end{bmatrix} \begin{bmatrix} u_{1}(t)\\ u_{2}(t) \end{bmatrix} + \begin{bmatrix} 0 & 0\\ 0 & \frac{1}{L_{d}} \end{bmatrix} \begin{bmatrix} u_{1}(t)\\ u_{2}(t) \end{bmatrix} + \begin{bmatrix} 0 & 0\\ 0 & \frac{1}{L_{d}} \end{bmatrix} \begin{bmatrix} u_{1}(t)\\ u_{2}(t) \end{bmatrix} + \begin{bmatrix} 0 & 0\\ 0 & \frac{1}{L_{d}} \end{bmatrix} \begin{bmatrix} u_{1}(t)\\ u_{2}(t) \end{bmatrix} + \begin{bmatrix} 0 & 0\\ 0 & \frac{1}{L_{d}} \end{bmatrix} \begin{bmatrix} u_{1}(t)\\ u_{2}(t) \end{bmatrix} + \begin{bmatrix} 0 & 0\\ 0 & \frac{1}{L_{d}} \end{bmatrix} \begin{bmatrix} u_{1}(t)\\ u_{2}(t) \end{bmatrix} + \begin{bmatrix} 0 & 0\\ 0 & \frac{1}{L_{d}} \end{bmatrix} \begin{bmatrix} u_{1}(t)\\ u_{2}(t) \end{bmatrix} + \begin{bmatrix} 0 & 0\\ 0 & \frac{1}{L_{d}} \end{bmatrix} \begin{bmatrix} u_{1}(t)\\ u_{2}(t) \end{bmatrix} + \begin{bmatrix} 0 & 0\\ 0 & \frac{1}{L_{d}} \end{bmatrix} \begin{bmatrix} u_{1}(t)\\ u_{2}(t) \end{bmatrix} + \begin{bmatrix} 0 & 0\\ 0 & \frac{1}{L_{d}} \end{bmatrix} \begin{bmatrix} u_{1}(t)\\ u_{2}(t) \end{bmatrix} + \begin{bmatrix} 0 & 0\\ 0 & \frac{1}{L_{d}} \end{bmatrix} \begin{bmatrix} u_{1}(t)\\ u_{2}(t) \end{bmatrix} + \begin{bmatrix} 0 & 0\\ 0 & \frac{1}{L_{d}} \end{bmatrix} \begin{bmatrix} u_{1}(t)\\ u_{2}(t) \end{bmatrix} + \begin{bmatrix} 0 & 0\\ 0 & \frac{1}{L_{d}} \end{bmatrix} \begin{bmatrix} u_{1}(t)\\ u_{2}(t) \end{bmatrix} + \begin{bmatrix} 0 & 0\\ 0 & \frac{1}{L_{d}} \end{bmatrix} \begin{bmatrix} u_{1}(t)\\ u_{2}(t) \end{bmatrix} + \begin{bmatrix} 0 & 0\\ 0 & \frac{1}{L_{d}} \end{bmatrix} \begin{bmatrix} u_{1}(t)\\ u_{2}(t) \end{bmatrix} + \begin{bmatrix} 0 & 0\\ 0 & \frac{1}{L_{d}} \end{bmatrix} \begin{bmatrix} u_{1}(t)\\ u_{2}(t) \end{bmatrix} + \begin{bmatrix} 0 & 0\\ 0 & \frac{1}{L_{d}} \end{bmatrix} \begin{bmatrix} u_{1}(t)\\ u_{2}(t) \end{bmatrix} + \begin{bmatrix} 0 & 0\\ 0 & \frac{1}{L_{d}} \end{bmatrix} \begin{bmatrix} u_{1}(t)\\ u_{2}(t) \end{bmatrix} + \begin{bmatrix} 0 & 0\\ 0 & \frac{1}{L_{d}} \end{bmatrix} \begin{bmatrix} u_{1}(t)\\ u_{2}(t) \end{bmatrix} + \begin{bmatrix} 0 & 0\\ 0 & \frac{1}{L_{d}} \end{bmatrix} \begin{bmatrix} u_{1}(t)\\ u_{2}(t) \end{bmatrix} + \begin{bmatrix} 0 & 0\\ 0 & \frac{1}{L_{d}} \end{bmatrix} \begin{bmatrix} u_{1}(t)\\ u_{2}(t) \end{bmatrix} + \begin{bmatrix} 0 & 0\\ 0 & \frac{1}{L_{d}} \end{bmatrix} \begin{bmatrix} u_{1}(t)\\ u_{2}(t) \end{bmatrix} + \begin{bmatrix} 0 & 0\\ 0 & \frac{1}{L_{d}} \end{bmatrix} \begin{bmatrix} u_{1}(t)\\ u_{2}(t) \end{bmatrix} + \begin{bmatrix} 0 & 0\\ 0 & \frac{1}{L_{d}} \end{bmatrix} \begin{bmatrix} u_{1}(t)\\ u_{2}(t) \end{bmatrix} + \begin{bmatrix} 0 & 0\\ 0 & \frac{1}{L_{d}} \end{bmatrix} \begin{bmatrix} u_{1}(t)\\ u_{2}(t) \end{bmatrix} + \begin{bmatrix} 0 & 0\\ 0 & \frac{1}{L_{d}} \end{bmatrix} \end{bmatrix} \begin{bmatrix} 0 & 0\\ 0 & \frac{1}{L_{d}} \end{bmatrix} \end{bmatrix} \begin{bmatrix} 0 & 0\\ 0 & \frac{1}{L_{d}} \end{bmatrix} + \begin{bmatrix} 0 & 0\\ 0 & \frac{1}{L_{d}} \end{bmatrix} \begin{bmatrix} 0 & 0\\ 0 & \frac{1}{L_{d}} \end{bmatrix} \end{bmatrix} + \begin{bmatrix} 0 & 0\\ 0 & \frac{1}{L_{d}} \end{bmatrix} \end{bmatrix} \begin{bmatrix} 0 & 0\\ 0 & \frac{1}{L_{d}} \end{bmatrix} \end{bmatrix} \end{bmatrix} \begin{bmatrix} 0 & 0\\ 0 & \frac{1}{L_{d}} \end{bmatrix} \end{bmatrix} \begin{bmatrix} 0 & 0\\ 0 & 0\\ 0 & \frac{1}{L_{d}} \end{bmatrix} \end{bmatrix} \begin{bmatrix} 0 & 0\\$$

This can be written as:

$$\begin{aligned} \mathbf{X}(t) &= A_2 \, \mathbf{x}(t) + B_2 \, u(t) + B_w \, w(t) \\ \mathbf{z}(t) &= C \mathbf{x}(t) \end{aligned} \tag{18}$$

Using the values of parameters in Table 1 we have:

$$A = \begin{bmatrix} -80 & 1109154 & 0 \\ -4.976 & -33.175 & -N_1 \\ 0 & N_1 & -33.175 \end{bmatrix}, A_2 = \begin{bmatrix} -80 & 1109154 & 0 \\ -4.976 & -33.175 & -N_2 \\ 0 & N_2 & -33.175 \end{bmatrix}$$

$$B_i = \begin{bmatrix} 0 & 0 \\ 47.393 & 0 \\ 0 & 47.393 \end{bmatrix}, B_w = \begin{bmatrix} 0 & 0 & -7042253 \\ 0 & 0.1 & 0 \\ 0.1 & 0 & 0 \end{bmatrix}, C = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}$$
(19)

5. ROBUST CONTROLLER DESIGN

Here, we are going to find a robust $H\infty$ controller of the form:

$$u(t) = \sum_{j=1}^{r} \mu_{j}(v(t)) K_{j} x(t)$$
(20)

$$\mathbf{X}^{(t)} = \sum_{i=1}^{t} \sum_{j=1}^{t} \mu_{i} \mu_{j} \begin{bmatrix} [(A_{i} + B_{2i}K_{j}) + (\Delta A_{i} + B_{2i}K_{j})]\mathbf{x}(t) \\ + [B_{1i} + \Delta B_{1i}]\mathbf{w}(t) \end{bmatrix}$$
(21)

where x(0) = 0.

For the system (10), a given prescribed $H\infty$ performance $\gamma > 0$ and positive constant δ , if there is a matrix $P = P^T$ and matrices Y_i , j = 1, 2, ..., r, satisfying the following linear matrix inequalities (LMI):

$$P > 0
\Omega_{ij} < 0
\Omega_{ij} < 0
\Omega_{ij} + \Omega_{ji} < 0
Where: (22)$$

$$\Omega_{ij} = \begin{pmatrix} (A_i P + PA_i^T + B_{2i} Y_j + Y_j^T B_{2i}^T & (\#)^T & (\#)^T \\ B_{1i}^T & -\gamma I & (\#)^T \\ C_{1i} P + D_{12i} Y_j & 0 & -\gamma I \end{pmatrix}$$
(23)

with

$$B_{1i} = \begin{bmatrix} \delta I & I & \delta I & B_{1i} \end{bmatrix}$$

$$C_{1i} = \begin{bmatrix} \frac{\gamma \rho}{\delta} H_{1i}^{T} & 0 & 2\lambda\rho H_{4i}^{T} & 2\lambda\rho C_{12i}^{T} \end{bmatrix}^{T}$$

$$D_{12i} = \begin{bmatrix} 0 & \frac{\gamma \rho}{\delta} H_{3i}^{T} & 2\lambda\rho H_{6i}^{T} & 2\lambda\rho D_{12i}^{T} \end{bmatrix}^{T}$$

$$\lambda = \left(1 + \rho^{-2} \sum_{i=1}^{r} \sum_{j=1}^{r} [\|H_{2i}^{T}\|] \right)^{1/2}$$
(24)

Then, inequality (13) holds. Also, a suitable choice of the controller is:

$$u(t) = \sum_{j=1}^{r} \mu_{j} (v(t)) K_{j} x(t)$$
(25)

where:

$$K_{j} = Y_{j} P^{-1}$$
(26)

Using the LMI optimization algorithm and following the Equation (13) with set as $\gamma = 1$, we obtain:

$$P = \begin{bmatrix} 1.247 & 0.5671 & 0\\ 0.0034 & 2.6578 & 0\\ 0 & 0 & 2.6578 \end{bmatrix}$$

$$Y_{1} = \begin{bmatrix} 0.541 & 0.045 & -0.2347\\ 0 & 0.015 & 0.0054 \end{bmatrix}, Y_{2} = \begin{bmatrix} 0.016 & 12.2413 & 0.0053\\ 0.8795 & 0.0025 & 0.76318 \end{bmatrix}$$

$$K_{1} = \begin{bmatrix} 1.543 & 0.0071 & -1.86\\ -0.0024 & 0.4583 & 0.0098 \end{bmatrix}, K_{2} = \begin{bmatrix} -1.0822 & 15.7548 & 0.0875\\ 0.9577 & 0.0011 & 0.989 \end{bmatrix}$$
(27)

6. SIMULATION RESULTS

In this section, two case studies were considered to evaluate the performance of proposed robust controller. In the first case, an optimal PI controller was designed and applied to the BLDC motor. In the second case, the performance of the proposed robust controller was extensively compared to that of [6].

Case I. Proportional-integral (PI) type controllers are commonly used in industrial applications. Thus, it is worth comparing the performance of these controllers as opposed to common robust controller mentioned here. In this study, for the purpose of comparison, a PI controller is considered for speed control of BLC motor and its parameters have been optimized by using genetic algorithm (GA) [21]. In the GA optimization, the optimization objective function is considered as the integral time absolute error (ITAE) .This well-known index could be used for optimization and controller tuning and is defined as:

$$ITAE = \int_{0}^{t} t \left| \Delta \omega \right| dt$$
 (28)

where, $\Delta \omega = \omega^* \cdot \omega$ is the error between the reference speed ω^* and the motor speed ω . In order to calculate PI controller's parameters, a 0.1 step change in references input (ω^*) is assumed, and the performance index ITAE

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is minimized using GA. In order to acquire better performance, population size, number of chromosomes, mutation rate and mutating rate are chosen as 80, 4, 10% and 50%, respectively. The obtained parameters of the optimal PI are $K_p = 0.8$ and $K_I = 0.02$.

The specification of the examined BLDC motor considered in the simulation is given in Table 1 . All simulations have been evaluated extensively using commercially available software package, MATLAB¹. Each simulation result presented in this section consists of two different plots (PI controller and Robust controller). The results of the time-domain simulations are shown in Figure 3. These figures show simulation results for speed reference input of 1800 rpm with a step change in load torque. The disturbance input signals $w_1(t), w_2(t)$ and $w_3(t)$ which were used during the simulation are given in Figure 4. Figure 3 (a) shows overshoot in speed is zero for the case of using the robust controller and settling time is about 0.04 sec. whereas these values for the PI controller are about 33% and 0.28 sec. respectively. Figure 3 (b) shows the electromagnetic torque of the motor dramatically has fewer ripples by implementing of the robust controller than the PI controller. The current of phase "a" of the machine also is shown in Figure 3 (c). This figure shows that the current waveform is smoother by use of the robust controller compared to the case of PI controller.

Case II. Here, the performance of the proposed robust controller was compared to the adaptive fuzzy proportional-integral- derivative (PID) controller suggested by [6]. The simulation results are shown in Figure 5. As shown in Figure 5 (a), the settling time has been reduced by using the proposed robust controller. Figures 5 (b), (c) show that the ripple in the electromagnetic torque and in the phase current, with the proposed robust controller, is much lower than those with the fuzzy PID controller introduced in [6].



1The MATHWORKS, "MATLAB Software", , Version 7.14, Inc., 2012



(c). Phase a current of the machine

Figure 3. Simulation results, Case I: response of BLDC motor with the designed robust and PI controllers for a step change in load torque.



Figure 4. Disturbance inputs $w_1(t)$, $w_2(t)$ and $w_3(t)$ used in simulation.

TABLE 1.BLDC motor specification		
Parameter	Value	
Rated power	2 hP	
Number of poles	4	
Number of phases	3	
Connection	Star	
13658	2.4544	
V _{dc}	160 V	
R _S	0.7 Ω	
Ψ_m	0.105 Wb	
Self-inductance	2.72 mH	
Mutual inductance	1.5 mH	
Motor inertia (J)	0.000284 kg.m ²	
Damping constant (B)	0.02 N.m/rad/sec	







Figure 5. Simulation results, Case II: Response of BLDC motor with the designed robust and fuzzy controller of [6] for a change in load torque.

7. CONCLUDING REMARKS

Brushless DC (BLDC) motors are preferred as small horsepower control motors due to their high efficiency, silent operation, compact form, reliability, and low maintenance. However, the problems are encountered in these motor for variable speed operation over last decades continuing technology development in power semiconductors, microprocessors, adjustable speed drivers control schemes and permanent-magnet brushless electric motor production have been combined to enable reliable, cost-effective solution for a broad range of adjustable speed applications.

In this paper, the nonlinear model of brushless DC motor was approximated using implementation of fuzzy rules. Based on this model, linear matrix inequality algorithm, and $H\infty$ theorem, a new robust controller was designed for speed control of BLDC motor. Simulation results demonstrated the effectiveness of the proposed controller in reducing the ripple in torque and zero overshoot in speed response of the motor.

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Fuzzy Approximation Model-based Robust Controller Design for Speed Control of BLDC Motor

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Keywords: BLDC Motor Fuzzy Approximation Robust Controller در این مقاله یک کنترلکنندهی جدید برای مسالهی کنترل سرعت موتورهای جریان مستقیم بدون جاروبک ارائه شده است. مدل غیرخطی موتور با استفاده از قوانین منطق فازی تقریب زده شده و عدمقطعیتها در سیستم فازی لحاظ شدهاند. با استفاده از این مدل و نیز با استفاده از الگوریتم بهینهسازی LMI یک کنترلکنندهی مقاوم برای کنترل سرعت موتور طراحی شده و به آن اعمال شده است. موثر بودن این رویهی کنترلی در کنترل مناسب سرعت موتور با استفاده از شبیهسازی نشان داده شده است.

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چكيده