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Size Effect on Free Transverse Vibration of Cracked Nano-beams using Couple Stress Theory

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ABSTRACT

In this paper, the transverse vibration of cracked nano-beam has been studied based on modified couple stress theory. Crack is modeled by a rotational spring that creates a discontinuity. First, transverse vibration equation of cracked nano-beam is derived along with its general response. Then, the frequency parameters are calculated for different crack positions, different lengths of the beam, different length scale parameter, different crack properties, and some typical boundary conditions. Results indicate that the effects of the crack parameter and crack location on transverse frequency of the cracked nano-beam are quite significant. In addition, scale effect parameter is one of the important parameters in nanoscale that must be taken into account. Finally, the results of special cases with simple support boundary condition for classic and nonlocal theories are compared with those available in the literature.

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1. INTRODUCTION

Crack often occurs in nanostructures and has different causes, for example, crack occurrence in ZnO nano-rods is due to thermal fabrication process [1, 2]. The crack presence results in changing the safety and reliability of nano devices and specially the system natural frequency that reduces their operation life [3]. It should be noted that structural elements like beams, shells, and membranes are the most widely-used components in nano/micro electromechanical systems (NEMS/MEMS) [4]. Drawing upon the above discussion, the presence of crack in nano structural elements is inevitable. Since the beam is one of the important components in NEMS/MEMS, studying the effects of cracks on nano-beams will be very important. Nowadays, experimental studies on nanomaterials are still a challenge because of difficulties confronted in the nanoscale. So, atomistic simulations and continuum mechanics theories have been used for nanostructure analysis [3]. Besides, atomistic simulations are costly and take much time. In addition, continuum theory is a convenient way to investigate the cracks effects on nano system performance. Scale-effect parameter (size effect) is an essential factor that the classic theory has failed to consider when the size reduces from macro to nano. Thus, using higher-order theories like modified couple stress theory [5] and modified strain gradient [6] are quite warranted.

Yuen [7] used finite element method to obtain the relation between damage location and corresponding change in Eringen value of a cantilever beam. Tada et al. [8] calculated the strain energy of a cracked classic beam, derived the compliance matrix, inversed it, and used it to establish the frequency equation of cracked beam. Freund et al. [9] found that a crack on a structure makes it more flexible. Adams et al. [10] attributed this flexibility to the existence of a spring on the beam. Loya et al. [11] modified the cracked classic beam equation and established the cracked nano-beam equation based on Eringen nonlocal theory. Hasheminejad et al. [12] studied free transverse vibration of cracked nano rod focusing on surface effect. Sherafatnia et al. [13] studied free vibration and buckling analysis of functionally-graded cracked beam via an analytic approach.

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The classical couple stress theory established by Mindlin et al. [14] is one of the other higher-order continuum theories that comprises two additional length scale parameters in addition to the classical constants for an elastic material. Recently, Yang et al. [5] proposed modified couple stress theory which contains only one additional length scale parameter. Many researchers have used this theory to study the function of beams in NEMS/MEMS [15-20]. In this paper, the transverse vibration of Euler-Bernoulli cracked nano-beam with some typical boundary conditions has been studied based on modified-couple stress theory. The authors modeled the cracked beam as two separated beams connected by a rotational spring. The equation of each beam's motion is obtained and continuity condition is satisfied at the crack location. The frequency parameters and the vibration modes of cracked nano-beams with some typical boundary conditions are calculated for different crack positions, crack severities, and values of the size-effect parameter. Finally, the results were compared with some data available in the literature. It is found that the impact of crack parameter, crack location and size effect parameter have significant effects on transverse frequency of the cracked nano-beams.

2. PRELIMINARIES

According to the coupled stress theory, the stored strain energy U_1 with infinitesimal deformation is written as:

$$U_1 = \frac{1}{2}\int_0^L \int_A \left(\sigma_{ij}\varepsilon_{ij} + m_{ij}^s \chi_{ij}^s\right) dxdA \tag{1}$$

where,

$$\varepsilon_{ij} = \frac{1}{2}\left(u_{i,j} + u_{j,i}\right) \tag{2}$$

$$\chi_{ij}^s = \frac{1}{2}(\theta_{i,j} + \theta_{j,i}) \tag{3}$$

$$\theta_i = \frac{1}{2}\left(curl(u)\right)_i \tag{4}$$

In the above equations, $\boldsymbol{u}$ and $\boldsymbol{\theta}$ are displacement and infinitesimal rotation vectors. In addition, $\boldsymbol{\varepsilon}$ and $\boldsymbol{\chi}$ are strain tensor and symmetric part of the rotation gradient tensor. $\boldsymbol{\sigma}$ is the Cauchy stress and $\boldsymbol{m}^s$ is usually called the higher-order stress. According to constitutive equation for a linear isotropic elastic material, the components of the stresses are written as follows:

$$\sigma_{ij} = \lambda tr(\varepsilon)\delta_{ij} + 2\mu\varepsilon_{ij} \tag{5}$$

$$m_{ij}^s = 2\mu l^2 \chi_{ij}^s \tag{6}$$

Parameters μ and λ in the constitutive equation of elastic material are called the Lame constants. Also, l is the additional independent material length scale parameter which appears in constitutive equations of higher-order stresses. For an Euler-Bernoulli beam, the displacement field can be expressed as:

$$u_1 = -z\frac{\partial w(x,t)}{\partial x}, \quad u_2 = 0, \quad u_3 = w(x,t) \tag{7}$$

where, u is axial displacement of the centroid of cross sections, and w denotes the lateral deflection of the beam and parameter z represents the distance of a point on the cross section with respect to the axis parallel to y-direction passing through the centroid.

3. GOVERNING EQUATIONS OF MOTION AND CORRESPONDING BOUNDARY CONDITIONS

In this section, the governing equation and corresponding classical and non-classical boundary conditions of nano-beam are derived based on the couple stress theory. By assuming small slopes in the beam after deformation, the non-zero component of strain (axial strain) can be expressed as:

$$\varepsilon_{11} = -z\frac{\partial^2 w}{\partial x^2} \tag{8}$$

Substitution of Equation (8) for Equations (3) and (4), yields the non-zero component of χ_{ij}^s as:

$$\chi_{12}^s = \chi_{21}^s = -\frac{1}{2}\frac{\partial^2 w}{\partial x^2} \tag{9}$$

Consequently, by substituting Equations (8) and (9) for Equations (5) and (6), one can get the nonzero components of the Cauchy and higher-order stresses as follows:

$$\sigma_{11} = E\varepsilon_{11} = -Ez\frac{\partial^2 w}{\partial x^2} \tag{10}$$

$$m_{12}^s = m_{21}^s = -\mu l^2 \frac{\partial^2 w}{\partial x^2} \tag{11}$$

It ought to be noted that in Equation (10), due to assumption of thin beam and small amount of Poisson's ratio, the Poisson's effect has been ignored. By substituting Equations (8)-(11) for Equation (1) the yield is as follows:

$$U_1 = \frac{1}{2}\int_0^L S\left(\frac{\partial^2 w}{\partial x^2}\right) dx \tag{12}$$

where,

$$S = EI + \mu A l^2 \tag{13}$$

In Equation (13), E is Young's modulus. The strain energy U_2, due to the existence of the initial axial force

(N) and the kinetic energy T of the beam, is written as follows:

$$U_2 = \frac{1}{2}\int_0^L N\left(\frac{\partial w}{\partial x}\right)^2 dx \tag{14}$$

$$T = \frac{1}{2}\int_0^L \int_A \left\{\left(-z\frac{\partial^2 w}{\partial x \partial t}\right)^2 + \left(\frac{\partial w}{\partial t}\right)^2\right\} dxdA \tag{15}$$

The work done by the external loads acting on the beam is also expressed as:

$$\begin{aligned}\delta W = &\int_0^L F(x,t)\delta w dx + \int_0^L G(x,t)\delta u dx + \left(\hat{N}\delta u\right)\Big|_{x=0}^{x=L} \\ &+ \left(\hat{V}\delta w\right)\Big|_{x=0}^{x=L} + \left(\hat{M}\delta\left(\frac{\partial w}{\partial x}\right)\right)\Big|_{x=0}^{x=L}\end{aligned} \tag{16}$$

where, $F(x,t)$ and $G(x,t)$ are the external transverse and longitudinal forces, respectively. $\hat{N}$ and $\hat{V}$ represent the resultant axial and transverse forces in a cross section caused by the classical stress components acting on the plane; the parameter $\hat{M}$ is the resultant moment in a cross section caused by the classical and higher-order stress components.
Now, the Hamilton principle is considered as:

$$\int_{t_1}^{t_2}\left(\delta U_1 + \delta U_2 - \delta T + \delta W\right)dt = 0 \tag{17}$$

By substituting aforementioned expressions for T, U_1, U_2 and δW in Equation (17) and some mathematical operations, the final transverse vibration equation of the cracked nano-beam can be written as:

$$S\frac{\partial^4 w}{\partial x^4} - N\frac{\partial^2 w}{\partial x^2} + \rho A\frac{\partial^2 w}{\partial t^2} + \rho I\frac{\partial^4 w}{\partial x^2 \partial t^2} + F(x,t) = 0 \tag{18}$$

$$\left(-S\frac{\partial^3 w}{\partial x^3} + N\frac{\partial w}{\partial x} + \rho I\frac{\partial^3 w}{\partial t \partial x^2} - \hat{V}\right)\Bigg|_{x=0,L} = 0 \quad or \quad \delta w\big|_{x=0,L} = 0 \tag{19}$$

$$\left(S\frac{\partial^2 w}{\partial x^2} - \hat{M}\right)\Bigg|_{x=0,L} = 0 \qquad or \qquad \delta\left(\frac{\partial w}{\partial x}\right)\Bigg|_{x=0,L} = 0 \tag{20}$$

where, $I=bh^3/12$ is the cross section moment of inertia, b and h are the width and the thickness of the beam, respectively. Also, ρ and A are density and cross section area, respectively.

In free vibrations, the transverse displacement is assumed to be in the form of $w(x,t) = w(x)e^{i\omega t}$ where ω is the transverse frequency of the beam. Furthermore, the transverse inertia, as usual, is neglected [21]. Consequently, after introducing the following dimensionless variables:

$$\bar{x} = \frac{x}{L}, \quad \bar{w} = \frac{w}{L}, \quad \Lambda^4 = \frac{\rho A L^4 \omega^2}{S}, \quad \Pi = \frac{NL^2}{S} \tag{21}$$

Transverse dimensionless equation of motion of beam in Equation (18) is written as:

$$\bar{w}^{(4)} - \Pi\bar{w}'' - \Lambda^4\bar{w} = 0 \tag{22}$$

The general solution for the above differential equation is as follows:

$$\bar{w}(\bar{x}) = c_1 e^{(R_1\bar{x})} + c_2 e^{(R_2\bar{x})} + c_3 e^{(R_3\bar{x})} + c_4 e^{(R_4\bar{x})} \tag{23}$$

where, $R_1 - R_4$ are the roots of Eigen values problem of the Equation (22) and can be calculated through the following equation:

$$R^4 - \Pi R^2 - \Lambda^4 = 0 \tag{24}$$

4. VIBRATION OF CRACKED NANO-BEAM

In this study, the cracked beam is modeled as two beam segments connected by a rotational spring. Consider a nano-beam with an edge crack located at the distance M from the left end with corresponding dimensionless variable $l_c=M/L$ (see Figure 1). In addition, let $\Delta\theta$ and Δu, respectively, be the rotated angle by the rotational spring and the relative horizontal displacement at $x=M$, described by Loya et al. [11]:

$$\Delta\theta = K_{11}\frac{\partial^2 w}{\partial x^2} + K_{12}\frac{\partial u}{\partial x}, \quad \Delta u = K_{22}\frac{\partial u}{\partial x} + K_{21}\frac{\partial^2 w}{\partial x^2} \tag{25}$$

where, K_{11}, K_{22}, K_{21} and K_{12} are the flexibility constants. There is no longitudinal displacement in free transverse vibration ($u(x,t)=0$), and the flexibility K_{22}, K_{21} and K_{12} are generally considered to be small [11]. Thus, the Equation (25) is written in the dimensionless form:

$$\Delta\theta = \frac{K_{11}}{L}\frac{\partial^2 w}{\partial x^2} = K_t\frac{\partial^2 \bar{w}}{\partial \bar{x}^2}\,(at\ \bar{x} = l_c) \quad \& \quad K_t = \frac{K_{11}}{L} \tag{26}$$

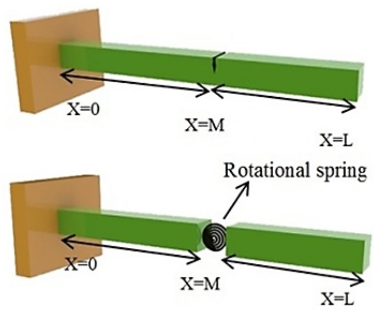

Figure 1. a) Cracked beam b) Cracked beam model

TABLE 1. Material and geometrical properties of nano beam

Young's modulus	E=1.44GPa
density	ρ=1600kg/m^3
Length scale parameter	l=17.6 µm
Shear modulus	μ=G=521.7MPa
Length of beam	L= 1µm
Thickness of cross section	h= 100nm
Width of cross section	b = 300 nm

It should be noted that we extended the method initially proposed by [10, 11, 22] in order to take into account the effects of the crack properties. Therefor, the cracked-beam has been considered to be two beams connected by a rotational spring at the cracked section as depicted in Figure 1. According to Equation (26), stiffness coefficient of torsional spring (K_t) is obtained from the increment of strain energy (ΔU_c) corresponding to the crack. In the case of local cracked beams, the additional strain energy due to the crack ΔU_c can be calculated from the fracture mechanics theory in macro scale [23]. As for nanobeams, ΔU_c must be obtained from either *ab initio* studies or molecular dynamics calculations in the classic theory [11]. However, considering size-dependency mechanical properties in nanoscale of structure or length scale parameters they must be modeled by proper continuum theory such as modified couple stress theory. Finally, increment of strain energy (ΔU_c) in a nanobeam by taking a size effect is a new phenomenon which can be dealt with in future researches. By drawing on the dimensionless transverse motion equation of the beam, free vibration equation of the cracked beam is described as:

$$\begin{aligned} &\bar{w}_1^{(4)} - \Pi \bar{w}_1'' - \Lambda^4 \bar{w}_1 = 0 \qquad 0 \le \zeta \le l_c \\ &\bar{w}_2^{(4)} - \Pi \bar{w}_2'' - \Lambda^4 \bar{w}_2 = 0 \qquad l_c \le \zeta \le 1 \end{aligned} \tag{27}$$

In Equation (27), Λ is the frequency parameter of the cracked nano-beam, which is related to its natural frequency. The general solutions of above equations are:

$$\begin{aligned} &\bar{w}_1(\bar{x}) = c_1 e^{(R_1 \bar{x})} + c_2 e^{(R_2 \bar{x})} + c_3 e^{(R_3 \bar{x})} + c_4 e^{(R_4 \bar{x})}, \quad 0 \le \zeta \le l_c \\ &\bar{w}_2(\bar{x}) = c_5 e^{(R_1 \bar{x})} + c_6 e^{(R_2 \bar{x})} + c_7 e^{(R_3 \bar{x})} + c_8 e^{(R_4 \bar{x})}, \quad l_c \le \zeta \le 1 \end{aligned} \tag{28}$$

The above two equations have eight unknown constants that must get satisfied by four boundary condition at two beam ends and four following compatibility conditions at the cracked section as:

Continuity of the vertical displacement

$$\bar{w}_1(l_c) = \bar{w}_2(l_c) \tag{29}$$

Change in bending slope

$$\Delta\theta = \bar{w}_2'(l_c) - w_1'(l_c) = K_t \bar{w}_1''(l_c) \tag{30}$$

Continuity of classic and higher-order bending moment

$$S\bar{w}_1''(l_c) = S\bar{w}_2''(l_c) \tag{31}$$

Continuity of the shear force

$$\frac{S}{L^2}\bar{w}_1''' + N\bar{w}_1' = \frac{S}{L^2}\bar{w}_2''' + N\bar{w}_2' \tag{32}$$

Also, the following four sets of boundary conditions are discussed:

Simple support (SS) beam

$$\bar{w}_1\big|_{\bar{x}=0} = S\bar{w}_1''\big|_{\bar{x}=0} = \bar{w}_2\big|_{\bar{x}=1} = S\bar{w}_2''\big|_{\bar{x}=1} = 0 \tag{33}$$

Clamped-clamped (CC) beam

$$\bar{w}_1\big|_{\bar{x}=0} = \bar{w}_1'\big|_{\bar{x}=0} = \bar{w}_2\big|_{\bar{x}=1} = \bar{w}_2'\big|_{\bar{x}=1} = 0 \tag{34}$$

Cantilevers (CF) beam

$$\bar{w}_1\big|_{\bar{x}=0} = \bar{w}_1'\big|_{\bar{x}=0} = S\bar{w}_2''\big|_{\bar{x}=1} = \frac{S}{L^2}\bar{w}_2''' + N\bar{w}_2'\Big|_{\bar{x}=1} = 0 \tag{35}$$

Simple-clamped (SC) beam

$$\bar{w}_1\big|_{\bar{x}=0} = S\bar{w}_1''\Big|_{\bar{x}=0} = \bar{w}_2\big|_{\bar{x}=1} = \bar{w}_2'\Big|_{\bar{x}=1} = 0 \tag{36}$$

For each of the above boundary conditions for nano-beam, the free vibration equation becomes an eight-variable linear system that is described as the following non-homogeneous system of linear equations:

$$A_{ij} c_j = 0 \quad (i, j = 1, \ldots, 8) \tag{37}$$

To avoid the trivial solution, it is necessary to impose the determinant of the coefficients matrix equal to zero:

$$\det(A_{ij}) = 0 \tag{38}$$

With the expansion of the Equation (38), the vibration equation and frequency parameter of each boundary condition set could be determined.

5. NUMERICAL EXAMPLE

For the following results, the beam is taken to be made of epoxy with the material properties in Table 1.

Due to the fact that experimental results concerning the vibration frequency of cracked nano beam are not available in the literature and also the fact that a new size-dependent formulation based on couple stress theory has been presented in this article, the results were evaluated in two stages.

In the first stage, with regard to the fact that the existing formulation in a specific case is reduced to the continuum classic theory, the classical theory results in this formulation have been compared with the results of the reference [11] shown in Figure 1. For comparison and verification of the present work, the frequency parameter of the supported cracked beam (with central crack) was calculated as a function of crack severity by ignoring initial axial force in the mathematical code.

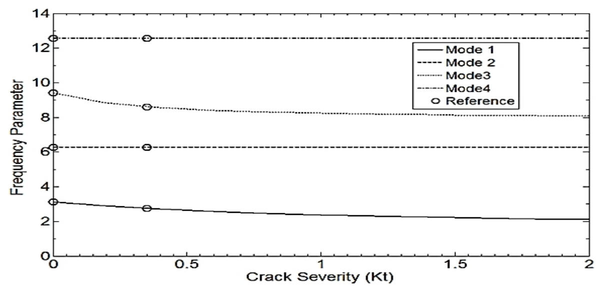

Figure 2. Frequency parameter of a cracked nano-beam as a function of crack severity without considering length scale parameter and initial axial force.

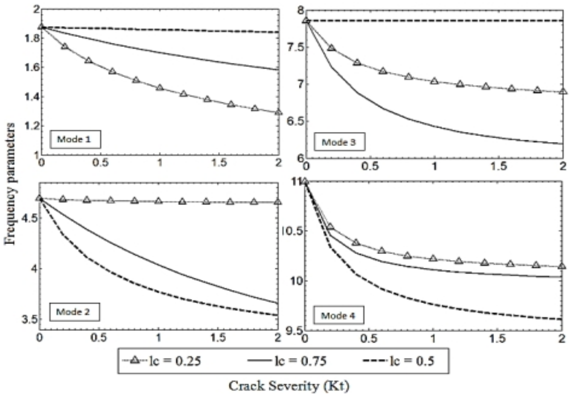

Figure 3. First four frequency parameters for a cantilever beam as a function of crack severity

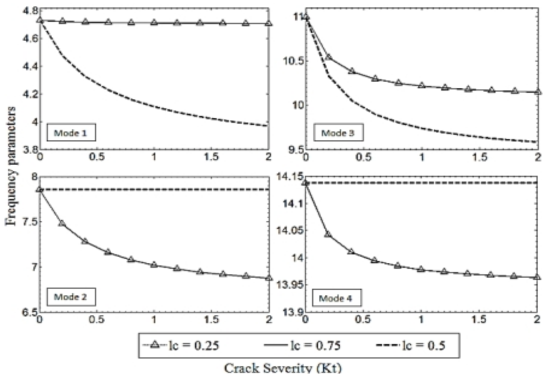

Figure 4. First four frequency parameters for a clamped-clamped beam as a function of crack severity

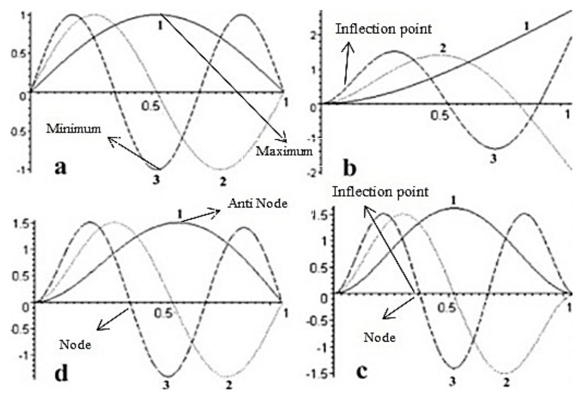

Figure 5. First three frequency mode shapes of a) simply support, b) cantilever, c) clamped-clamped and d) simply-clamped beams

The results illustrated in Figure 1 indicate their consistency with those of Loya et al. [11]. In the second stage, the results of the new size- dependent formulation have been compared with nonlocal theory in Tables 2 and 3 for supported cracked beam (with central crack). It should be mentioned that the effects of the type of high-order theory on the vibrational frequency are mixed. For example, the effect of nonlocal theory on the nanobeam rigidity and structure is softening and by increasing the size parameter, the natural frequency decreases. However, using couple stress theory the nanobeam rigidity and structure are hardened. Therefore, increasing the size parameter can raise natural frequency. Moreover, comparing results in Tables 2 and 3 reveal that increasing the length scale parameter will raise natural frequency of nanobeam in couple stress theory, whereas it is reduced in nonlocal theory.

Table 4 shows the frequency changes for an edge cracked beam (located at its middle point) with four types of boundary conditions as a function of different length scale parameter and crack severity. It is obvious that by considering the length scale effect in couple stress theory, the beam stiffness increases and so does the frequency. On the other hand, the physics of the problem shows that any increase in crack stiffness results in a fall in the frequency which is contrary to the effect of length scale parameter.

Figures 3 and 4 show the first four frequency parameters of cantilever and clamed-clamped beam, for three different crack locations, respectively (l_c=0.25, 0.5, 0.75). As it can be seen in Figure 4, the first and third modes for the beam with central crack have a lower frequency than that with lateral crack; but this finding is not valid for cantilever beam in Figure 3.

It could be seen in the second and fourth modes of Figure 4 that the frequency parameter of the beam with central crack is invariant with respect to crack severity. The reason is that if a crack stands on vibration nodes, the vibration of the beam will get insensitive to crack severity. To examine this result, Figure 5 is provided. Figure 5 shows the beam mode shapes with different boundary conditions. In this figure, the location of the horizontal axis which intersects the mode shapes curve is called vibration nodes and the maximum and minimum value of the mode shapes curve called vibration anti nodes.

According to the second mode shapes of Figure 5-c, the second frequency mode parameter of the clamped-clamped beam must be insensitive to crack severity. This state can be seen in the second mode at Figure 4. Also, in the third mode shapes of Figure 5-b, there is a node in the middle of the beam. In other words, the third frequency mode parameter of cantilever must be insensitive to crack severity. This can be seen in the third mode of Figure 3. Based on the above discussion, a suitable comparison could be made among Figures 3, 4 and 5; and the phenomenon of the insensitivity of vibration could be gained with respect to the crack severity at vibration node. Furthermore, for all cases, an increase in crack severity results in a reduction in frequency parameters, except where the crack stands on vibration node.

Figures 6 and 7 show the first four frequency parameters of different boundary conditions as a function of cracked location for selected beam crack severity (K_t=1). As it is seen, the crack location effect is significant on its frequency. In addition, the simply supported, clamed-clamped and simply clamped beams have higher frequency parameters value than cantilever beams. A comparison of Figures 5, 6 and 7, suggests that there is also a relation between mode shapes and variation of frequency parameters. For every boundary condition, if a crack stand on maximum, minimum or inflection point of the shape mode, the frequency parameter will have severe variation and reach to its minimum or maximum value. For example, in the third mode shape of Figure 5-b at point l_c=0.1, there is an inflection point. In other words, the frequency parameter of cantilever beam in this mode has a local maximum or minimum value. In the third mode of Figure 7, it can be observed that the frequency parameter reaches the maximum at point l_c=0.1. In the first mode shape of Figure 5-c, there is an anti-node at the center of the beam. From the first mode of Figure 6, it could be concluded that the frequency parameter has a minimum value at the center of the beam. It should be noted that nodes are inflection points and anti-nodes are minimum or maximum points of mode shapes. Mode shapes may have an inflection point but it is not the case for node or antinodes like l_c=0.1 at the third mode shape of the cantilever beam. In all mode shapes of a simply supported beam, all of the inflection points are nodes; while in the clamped-clamped and simply-clamped beams, in each corresponded mode shapes, there are respectively two and one inflection point more than the simply supported beam, as shown in Figure 6. The clamed-clamped and simply clamped beams have respectively, two and one local extremum point more than the simply supported beam.

Table 5 shows the effects of dimensionless initial axial force on frequency parameter of the cracked nano-beam for selected beam crack severity (K_t=1) and crack location (l_c=0.5) in four typical boundary conditions. The most important observations are as follow: by increasing the amount of this force, the frequency parameter will rise gradually. In addition, this force has the most impact on the frequency parameter of the simply supported beam and the least effect on that of cantilever beam. Another finding is that the initial axial force can increase the frequency parameter of the cracked nano-beam more than when it has no crack. For example, the frequency parameter of a simply supported beam with no crack is Λ=3.1416 while Table 5 shows that for the case of dimensionless initial axial force Π=10, the frequency parameter will be Λ=3.4336 for a cracked beam. Similarly, the non-cracked clamped-clamped beam has a frequency parameter Λ=4.77 while a cracked beam one with Π=40 has frequency parameter of Λ=4.7595. Thus, by changing the initial axial force, the frequency parameter of cracked beam could be controlled.

TABLE 2. Four frequency of the cracked nano-beam with mid span as a function of scale effect parameter and crack severity with nonlocal theory [11]

h=0 (μm)				
Frequency (THz)	**K_t=0**	**K_t=0.065**	**K_t =0.35**	**K_t =2**
1	0.0000326	.0000302	0.0000280	.0000231
2	0.0001723	0.0001723	0.0001723	0.0001723
3	0.000258	0.000251	0.000236	0.000221
4	0.000344	0.000344	0.000344	0.000344
h=0.6 (μm)				
Frequency (THz)	**K_t=0**	**K_t=0.065**	**K_t =0.35**	**K_t =2**
1	0.0000589	0.0000571	0.0000512	0.0000383
2	0.0000871	0.0000871	0.0000871	0.0000871
3	0.000108	0.000104	0.0000965	0.0000895
4	0.000125	0.000125	0.000125	0.000125

TABLE 3. Four frequency of the cracked nano-beam with mid span as a function of scale effect parameter and crack severity with present couple stress theory

l=0 (μm)				
Frequency (THz)	**K_t=0**	**K_t=0.065**	**K_t =0.35**	**K_t =2**
1	0.0000326	.0000302	0.0000280	.0000231
2	0.0001723	0.0001723	0.0001723	0.0001723
3	0.000258	0.000251	0.000236	0.000221
4	0.000344	0.000344	0.000344	0.000344
l=40 (μm)				
Frequency (THz)	**K_t=0**	**K_t=0.065**	**K_t =0.35**	**K_t =2**
1	0.1730	0.1611	0.1004	0.09543
2	0.2288	0.2288	0.2288	0.2288
3	0.3012	0.2944	0.2639	0.2402
4	0.3983	0.3983	0.3983	0.3983

TABLE 4. Frequency of the cracked nano-beam with mid span crack in four typical boundary conditions as a function of scale effect parameter and crack severity

	l=0 (μm)			l=10 (μm)		
Frequency (THz)	**K_t=0**	**K_t=1**	**K_t =2**	**K_t =0**	**K_t =1**	**K_t =2**
SS beam	0.00003	0.000027	0.000023	0.05636	0.03243	0.002508
CC beam	0.000025	0.000022	0.000019	0.1277	0.0963	0.09
SC beam	0.000028	0.000024	0.000023	0.088	0.06561	0.06038
CF beam	0.000027	0.000026	0.000026	0.02007	0.01651	0.01428
	l=20 (μm)			**l=30 (μm)**		
Frequency (THz)	**K_t =0**	**K_t =1**	**K_t =2**	**K_t =0**	**K_t =1**	**K_t =2**
SS beam	0.1127	0.06486	0.05017	0.1690	0.09729	0.07525
CC beam	0.2555	0.1927	0.18	0.3832	0.289	0.27
SC beam	0.176	0.1312	0.1207	0.2641	0.1968	0.1811
CF beam	0.04015	0.03303	0.02856	0.06023	0.04953	0.04284

TABLE 5. Effects of dimensionless initially axial force on first mode frequency parameter of the beams

	Π				
	10	**20**	**30**	**40**	**50**
SS beam	3.4336	3.6567	3.8416	4.0005	4.0398
CC beam	4.5230	4.6341	4.6957	4.7595	4.8244
SC beam	4.0113	4.1490	4.2708	4.3814	4.4833
CF beam	2.6770	2.7344	2.7867	2.8350	2.8798

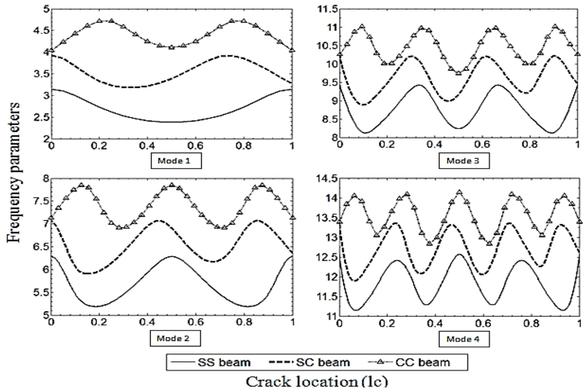

Figure 6. First four frequency parameters the for simply supported, clamped-clamped and simply-clamed beams

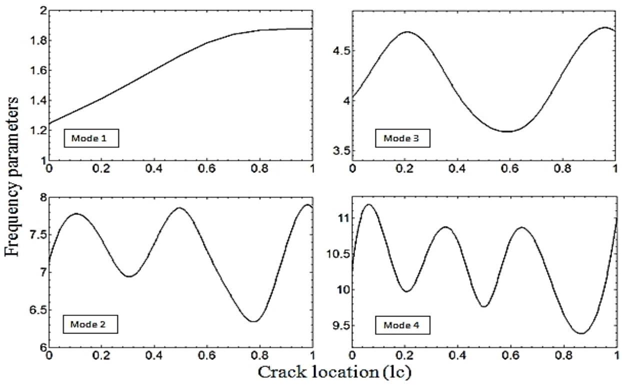

Figure 7. First four frequency parameters for cantilever

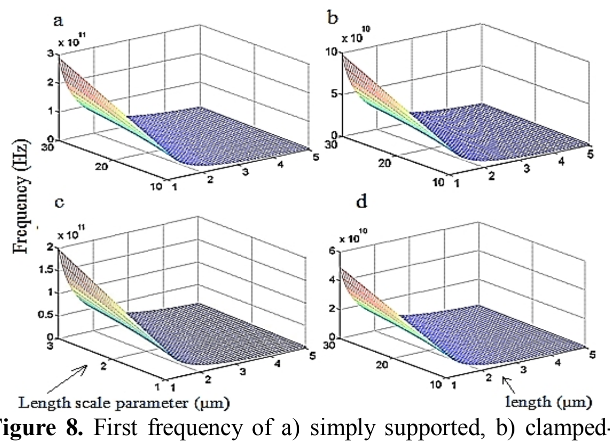

Figure 8. First frequency of a) simply supported, b) clamped-clamped, c) simply-clamped and d) cantilever beams

Figure 8 shows the transverse frequency of a cracked nano-beam with selected crack severity (K_t=1) and crack location ($l_c=0.5$) as a function of beam length and scale effect parameter, simultaneously. Some results of these diagrams are as follow: A shorter length beam has a higher frequency than a longer one. For a fixed length, as the scale effect parameter increases, the frequency gradually increases.

6. CONCLUSION

This paper examines the free transverse vibration of a cracked nano-beam. The modified couple stress theory is used to account for the scale effect parameter. The most salient observations are abbreviated as follow:

1. A clamped-clamped beam with central crack has a lower frequency parameter than a beam with lateral crack except when the crack stands on vibration node.
2. In a simple support (SS) or clamped-clamped (CC) beam, the crack may be put in the center of the beam (*l*c=0.5) to cause the decrease of frequency parameter more than once by the cracks put in the end of beam (*l*c= 0.25 or *l*c=0.75), except in cases where crack put on nodes or anti nodes vibration. However, in a cantilever beam, in general the above findings cannot concluded.
3. It is illustrated that the crack location has a significant effect on the frequency parameter of the cracked beam. It is also clear from mode shapes that if the crack locates on the inflection or the extremum points of mode shapes, the frequency will reach its extremum value.
4. It was observed that the presence of initial axial force makes the beam stiffer and increases its frequency parameter.
5. Based on the analysis, it is observed that the frequency goes up with a reduction in scale effect parameter value and decreases with an increase in beam length.

7. REFERENCES

1. Fang, T.-H., Chang, W.-J. and Liao, S.-C., "Simulated nanojet ejection process by spreading droplets on a solid surface", ***Journal of Physics: Condensed Matter***, Vol. 15, No. 49, (2003), 8263-8271.
2. Fang, T.-H. and Chang, W.-J., "Sensitivity analysis of scanning near-field optical microscope probe", ***Optics & Laser Technology***, Vol. 35, No. 4, (2003), 267-271.
3. Chen, D.-L., Chen, T.-C. and Lai, Y.-S., "New bending algorithm for field-driven molecular dynamics", ***Nanoscale Research Letters***, Vol. 5, No. 2, (2010), 315-322.

4. Tadi Beni, Y., Koochi, A. and Abadyan, M., "Theoretical study of the effect of casimir force, elastic boundary conditions and size dependency on the pull-in instability of beam-type nems", ***Physica E: Low-dimensional Systems and Nanostructures***, Vol. 43, No. 4, (2011), 979-988.
5. Yang, F., Chong, A., Lam, D. and Tong, P., "Couple stress based strain gradient theory for elasticity", ***International Journal of Solids and Structures***, Vol. 39, No. 10, (2002), 2731-2743.
6. Lam, D., Yang, F., Chong, A., Wang, J. and Tong, P., "Experiments and theory in strain gradient elasticity", ***Journal of the Mechanics and Physics of Solids***, Vol. 51, No. 8, (2003), 1477-1508.
7. Yuen, M., "A numerical study of the eigenparameters of a damaged cantilever", ***Journal of Sound and Vibration***, Vol. 103, No. 3, (1985), 301-310.
8. Tada, H., Paris, P. and Irwin, G., "The analysis of cracks handbook, New York: ASME Press, (2000).
9. Freund, L., "A simple model of the double cantilever beam crack propagation specimen", ***Journal of the Mechanics and Physics of Solids***, Vol. 25, No. 1, (1977), 69-79.
10. Adams, R., Cawley, P., Pye, C. and Stone, B., "A vibration technique for non-destructively assessing the integrity of structures", ***Journal of Mechanical Engineering Science***, Vol. 20, No. 2, (1978), 93-100.
11. Loya, J., López-Puente, J., Zaera, R. and Fernandez-Saez, J., "Free transverse vibrations of cracked nanobeams using a nonlocal elasticity model", ***Journal of Applied Physics***, Vol. 105, No. 4, (2009), 443-449.
12. Hasheminejad, S.M., Gheshlaghi, B., Mirzaei, Y. and Abbasion, S., "Free transverse vibration of crack nano-beams with surface effects", ***Thin Solid Films***, Vol. 519, (2011), 2477-2482.
13. Sherafatnia, K., Farrahi, G. and Faghidian, S.A., "Analytic approach to free vibration and buckling analysis of functionally graded beams with edge cracks using four engineering beam theories", ***International Journal of Engineering-Transactions C: Aspects***, Vol. 27, No. 6, (2013), 979-990.
14. Mindlin, R. and Tiersten, H., "Effects of couple-stresses in linear elasticity", ***Archive for Rational Mechanics and Analysis***, Vol. 11, No. 1, (1962), 415-448.
15. Noghrehabadi, A., Eslami, M. and Ghalambaz, M., "Influence of size effect and elastic boundary condition on the pull-in instability of nano-scale cantilever beams immersed in liquid electrolytes", ***International Journal of Non-Linear Mechanics***, Vol. 52, (2013), 73-84.
16. Abdi, J., Koochi, A., Kazemi, A. and Abadyan, M., "Modeling the effects of size dependence and dispersion forces on the pull-in instability of electrostatic cantilever nems using modified couple stress theory", ***Smart Materials and Structures***, Vol. 20, No. 5, (2011), 55-66.
17. Akbari, A.R. and Abolghasemi, A., "A size-dependent bernoulli-euler beam formulation based on a new model of couple stress theory", (2014).
18. Noori, H.R. and Jomehzadeh, E., "Length scale effect on vibration analysis of functionally graded kirchhoff and mindlin micro-plates", ***International Journal of Engineering, Transactions C: Aspects***, Vol. 27, No. 3, (2014), 431-440.
19. Rezazadeh, G., Rashvand, K. and Madinei, H., "Effect of length-scale parameter on pill-in voltage and natural frequency of a micro-plate", ***International Journal of Engineering, Transactions C: Aspects***, Vol. 27, No. 3, (2014), 375-384.
20. Dashtaki, P.M. and Beni, Y.T., "Effects of casimir force and thermal stresses on the buckling of electrostatic nanobridges based on couple stress theory", ***Arabian Journal for Science and Engineering***, (2014), 1-11.
21. Nayfeh, A.H. and Mook, D.T., "Nonlinear oscillations, John Wiley & Sons, (2008).
22. Loya, J.A., Rubio, L. and Fernandez-Saez, J., "Natural frequencies for bending vibrations of timoshenko cracked beams", ***Journal of Sound and Vibration***, Vol. 290, No. 3, (2006), 640-653.
23. Irwin, G., "Structural mechanics", Pergamon Press, New York, (1960), 557-592.

Size Effect on Free Transverse Vibration of Cracked Nano-beams using Couple Stress Theory

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چکیده

در این مقاله ارتعاشات عرضی نانوتیر ترک‌دار با استفاده از تئوری تنش کوپل مورد مطالعه قرار گرفته‌است. در اینجا ترک توسط فنر پیچشی مدل شده است که یک ناپیوستگی در تیر ایجاد می‌کند. ابتدا معادله ارتعاش عرضی نانو تیر ترک دار بدست آمده و پاسخ عمومی آن استخراج شده است. سپس پارامتر برای محل‌های مختلف ترک، طول‌های مختلف تیر، پارامتر اثر اندازه مختلف، مشخصات متفاوت ترک و شرایط مرزی مختلف نانو تیر محاسبه گردیده است. با توجه به نتایج می‌توان گفت تاثیر پارامتر ترک و محل قرارگیری ترک بر مقادیر فرکانس‌ها کاملا قابل توجه است. به علاوه، پارامتر اثر اندازه یکی از پارامترهای مهم در مقیاس نانو می‌باشد که باید مورد توجه قرار گیرد. در نهایت نتایج بدست آمده در حالت خاص برای تیر با شرایط مرزی ساده–ساده برای تئوری کلاسیک و تئوری غیر موضعی با نتایج مشابه موجود در مراجع مقایسه شده است.

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