

# **International Journal of Engineering**

Journal Homepage: www.ije.ir

# Distribution Design of Two Rival Decenteralized Supply Chains: a Two-person Nonzero Sum Game Theory Approach

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#### PAPER INFO

Paper history: Received 02 June2013 Received in revised form27 November2013 Accepted in 12December 2013

Keywords: Supply Chain Network Distribution Network Design Tactical and Strategic Decisions Nash Equilibrium Non-Zero Sum Game.

# 1. INTRODUCTION

A supply chain (SC) consists of collaborative agreements and contacts among parties which are integrated as a collaborative network [1]. Hence, a SC can be considered as a decentralized decision-making system in which each party takes decision to achieve its own goals, separately. Nowadays, the competition between firms is evolving to the competition among SCs [2]. For instance, SCs of Toyota and Honda compete with each other by establishing production facilities and distributing their products in several markets [1]. They try to be more responsive to markets and attract customers.

An efficient SCM needs appropriate decisions relating to the flow of product, information and fund. These decisions can be divided into three main categories of strategic, tactical and operational decisions [1]. SC decisions such as distribution network configuration, supplying sources for components, location and capacity of warehouses are required to be planned at the strategic levels [3]. However, less

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A B S T R A C T

We consider competition between two decentralized supply chain networks under demand uncertainty. Each chain consists of one risk-averse manufacturer and a group of risk-averse retailers. These two chains present substitutable products to the geographical dispensed markets. The markets' demand is a function of prices, service levels and advertising efforts of two chains. We formulate the distribution design problem of two rival chains as a two-person nonzero sum game theoretical model. Since strategic decisions of distribution network design offen have priority over tactical ones, we first calculate the equilibrium of tactical decisions for each pair of distribution design scenarios and according to the presented methodology, we find Nash equilibrium solutions of distribution network scenarios for two rival chains. Eventually, to illustrate the real applications of the methodology, a numerical example is presented and analyzed.

doi: 10.5829/idosi.ije.2014.27.08b.09

frequent decisions within shorter intervals are tactical decisions.

We consider each rival SC comprises one risk-averse manufacturer and a group of risk-averse retailers. Each rival manufacturer has the authority to take strategic decisions about his distribution network configuration. That is, the manufacturer can draw up contract with the appropriate subset of distributors from a potential distributors set and then the contractor distributors of two SCs distribute the products of manufacturer to different markets. At the tactical level, distribution network configurations of two SCs are considered fixed. Therefore, tactical decisions of two SCs such as pricing, service level, and advertisingshould be made regarding the distribution network configurations of the SCs.

In this article, we extend the research of Hafezalkotob et al. [4] and introduce two competitor SCs in which the manufacturers can choose the optimal scenario among a set of possible scenarios in order to maximize utility. The proposed methodology is able to capture some of the bewildering variety of the distribution design problems relating to competitive distribution networks that can be observed in the real world. For instance, in retail industry the locations of

Please cite this article as:A. Hafezalkotob, M. S. Babaei, A. Rasulibaghban, M. Noori-daryan, Distribution Design of Two Rival Decenteralized Supply Chains: a Twoperson Nonzero Sum Game Theory Approach. International Journal of Engineering (IJE) Transactions B: Applications Vol. 27, No. 8, (2014): 1233-1242 retailers depend upon those of rival retailers. The highly populated urban areas often have a considerable demand for products and services. If rival retailers were located in this area, they would satisfy high proportion of generated demand. Another case is in banking sector. When a well-known bank intends to open a new branch in an urban area, it should carefully consider the locations of rival banks' branches as well as distribution of population (demand) in that area.

In Section 2, the related literature is reviewed. Section 3 includes the problem description and related notations. A methodology for the competition between two SCs is presented in Section 4. The main model formulation for tactical and strategic decisions of SCs is given in Section 5. Section 6 contains some computational results of numerical example. Finally, research concludes are provided in Section 7.

# 2. LITERATURE REVIEW

In each SC, the central aims of the manufacturer are producing products and sending them to the markets. How to make the product available to the customers via intermediates is often a challenging problem. The appropriate selection of distribution channels makes an effective and profitable SC. A stream of multi-channel distribution literature deals with several retailers and each retailer covers specific markets. Some researchers also took multiple independent retailers into account [5, 6]. In this investigation, we focus on distribution network design of two rival SCs which each chain contains a manufacturer and multiple independent retailers. Xiao et al. [2] investigated impacts of service level and price on demand under uncertainty. In addition to price and service level, the advertisement efforts of competitive firms exert significant impact on customers' behavior [4]. Supply chain network design (SCND) is a strategic decision that affects SC's efficiency. Farahani et al. [7] provided a comprehensive review of SCND literature and highlighted the effects of competitive environment on SCND. Hammami et al. [8] proposed a mathematical model for the design of SCs in the delocalization context. Zamarripa et al. [9] extended the Mixed Integer Linear Programming model for decision making in cooperative or competitive scenarios of SCs. Badri et al. [10] presented a new mathematical model for multiple echelon, multiple commodity SCND and considered different time resolutions for tactical and strategic decisions. When customers' behavior is uncertain, they impose the risk on partners of SC. Different attitudes of the partners towards risk influence SC interactions and members' decision significantly. Hafezalkotob et al. [11] analyzed risk of losing a customer in a two-echelon SC facing on an integrated competitor. In this paper, we introduce two competing SCs under demand uncertainty. As our main

contribution, we assume that SC's partners make decision in the decentralized manner. That is, partners are independent firms with their own conflicting objectives. The coding for the corresponding models is demonstrated in Table 1. We surveyed the related recent researches which the features of them are presented in Table 2. It is obvious that several researches have performed in the area of SC under demand uncertainty. However, none of them studied the competition between SCs. We analyze two competing decentralized SCs. Hence, our research is related to [2], [4]. However, in this research, two manufacturers in both SCs are considered that have the ability to select distribution network from a set of feasible scenarios. To the best of authors' knowledge, no research was found that analyze the interaction of location problem of two rival SCs by game theory approach. There are four main contributions in this research. Firstly, we assume that the manufacturers encounter a set of possible scenarios for their distribution network. Instead of assigning demand markets by the manufacturer, he only determines which of the retailers are suitable for distribution network in geographical dispersed markets. Secondly, we formulate the strategic decisions of distribution design of both SCs as a nonzero two-person game theory model.

**TABLE 1.** Coding of the available problem

Characteristics	Coding
Objective function	Min cost (C)
	Max profit (P)
Game type	Cooperative (Co)
	Non-cooperative (Nco)
Demand type	Deterministic (D)
	Stochastic (S)
Type of equilibrium point	Nash (N)
	Stackelberg (St)
	Wide optimal (Wo)
Product	Multi production (Mp)
	Single production (Sp)
Location	(L)
Solution method	Optimization (O)
	Special algorithm (A)
	Integer linear programming (IIp)

Ref. No.	Author(s)	Articles code (Objective function, Game type, Demand, Type of equilibrium point, Product, Location, Solution method)
[2]	Xiao and Yang	P,_,S,Wo,Sp,_,O
[11]	Hafezalkotob and Makui	P,Nco,S,N,Sp,_,O
[12]	Meng et al.	P,_,_,Wo,Sp,L,A
[13]	Rezapour and Zanjirani Farahani	P,Nco,D,Wo,Sp,L,A
[14]	Hafezalkotob and Makui	P,Nco,D,N,Sp,_,O



Thirdly, we suggest a simultaneous game in the second step of decision making regarding the short-term scope of tactical decisions and symmetric power of players. The payoff matrix of the game (which is used in the strategic decision of competitive network design) is calculated based on the expected profit and risk criteria. Fourthly, the main aim of the present research is to suggest a new decision-making methodology for evaluating the distribution design problem of two rival chains.

# **3. PROBLEM DESCRIPTION**

We consider two SCNs which each of them contains a manufacturer and a group of retailers. The retailers compete for the markets and the markets' demand are supposed to be random. The manufacturers and retailers in the both chains are risk-averse. The products of the chains are different and partially substitutable. The manufacturer in each chain sells his product in each market through the determined retailers for that market according to the pre-specified distribution design. Figure 1 presents the problem structure related to two rival SCs.

#### 3. 1. Specifications of Facilities in SCNs

- v Excluding the markets' demand, all parameters have been determined and defined in advance.
- v Each manufacturer faces with several possible scenarios to distribute his products to the markets. The locations of candidate retailers and markets covered by each retailer are determined by each scenario. Manufacturers and the retailers' capacities are sufficiently large.
- v The markets have been geographically scattered such that they can be assumed independent.
- v SCs' retailers collect the corresponding markets' demand and place order to the related manufacturer.
- v Retailers present various levels of service to the markets. On the other hand, the manufacturers are able to invest in advertisements to increase all markets' demand. The manufacturer and retailers determine the wholesale price and retail price, respectively. According to the network design

structure, the transportation costs are added to the retail price.

**3. 2. Specifications of Markets Demand** Markets' demand consists of two deterministic and random sections. The random section has a probability distribution function that is known by the SC decision-makers. Deterministic section of each market's demand depends on products prices, service levels, and market costs of two rival SCs.

**3. 3. Expense Parameters In Designing SCs Networks** Products' purchase price in a market is composed of manufacturer's wholesale price, retailers' profit margin, transportation costs between manufacturer and retailers, and between retailers and markets.

- v Each manufacturer has well defined production cost.
- v Retailers' costs to provide the same level of service are different, due to essence of the efficiency of their various service levels.

**3. 4. Sequence of Decision in The SCs** According to the time schedule of strategic and tactical decisions in a SC, we consider the following two phases in the competition game structure:

**Phase 1.** Each manufacturer in both SCs evaluates any possible scenario of designing distribution network. They select the scenario with the most utility. In each scenario, the active retailer and a group of markets covered by each retailer is determined.

**Phase 2.** The members of the both rival SCs make their tactical decisions in a decentralized decision-making process. That is, manufacturers and retailers determine the prices of the products, service levels, and advertisement costs in a non-cooperative manner.

Scenarios of a SC distribution network are defined according to the possible distribution network structure with one individual manufacturer and several retailers. The retailers of a SC do not interact with each other because each retailer has his defined pre-determined markets. Each member's goal is to maximize the average profit and minimize the variance of profit, simultaneously. The relative significance of these goals is determined by the risk sensitivity parameter.

# 4. MATHEMATICAL MODEL

SCs have been marked by indexes one and two, respectively. The following symbols, indexes, parameters, and decision variables have been utilized in the research.

# 4.1. The Set and Indexes

N : A set of market's demand,  $N = \{1, 2, ..., |N|\}, n \in N$ ;

- $J: \qquad \text{A set of candidate retailers in the first SC,} \\ J = \{1, 2, ..., |J|\}, j \in J ;$
- *I*: A set of candidate retailers in the second SC,  $I = \{1, 2, ..., |I|\}, i \in I;$
- $N_1^{D_1}$ : The partition of set N that shows which markets' demand is supplied by each contractor retailer under distribution scenario  $D_1$ , i.e.  $N_1^{D_1} = \left\{N_{11}^{D_1}, N_{12}^{D_1}, \dots, N_{|I_1|}^{D_1}\right\}$ ;
- $N_2^{D_2}$ : The partition of set N that shows which market's demand is supplied by each contractor retailer under distribution scenario  $D_2$ , i.e.  $N_2^{D_2} = \{N_{2l}^{D_1}, N_{22}^{D_2}, ..., N_{2|l|}^{D_2}\}$
- $N_{lj}^{D}, N_{2i}^{D}$  In turn, a subset from the set of markets' demand N supplied byj<sup>th</sup> retailer in the first SC under scenario  $D_1$  and a subset of the set of markets' demand N supplied by i<sup>th</sup> retailer in the second SC under scenario  $D_2$ .

#### 4.2. The Parameters

- $c_1, c_2$ : The production costs of each product unit related to the manufacturers in the first and second SCs, respectively;
- $\mathcal{A}_{In}$ : Random section of the  $n^{th}$  market's demand related to type one of product with mean of  $\overline{a}_{In} > 0$  and

variance of  $s_{ln}^2$ ;

- $\mathscr{H}_{2n}$ : Random section of the  $n^{th}$  market's demand related to type two of product with mean of  $\bar{a}_{2n} > 0$  and variance of  $s_{2n}^2$ ;
- d: Products substitutability coefficient of two SCs, 0 < d < 1;
- $b_n$ : Demand sensitivity of a retailer to his own service level in the  $n^{th}$  market,  $b_n > 0$ ;
- $g_n$ : Demand sensitivity of a retailer to the rival service level in  $n^{th}$  market which is called cross-service level coefficient,  $b_n > g_n > 0$ ;
- $r_n$ : Demand sensitivity of a retailer to his manufacturer's advertisement cost in the  $n^{th}$  market,  $r_n > 0$ ;
- $n_n$ : Demand sensitivity of a retailer to the rival manufacturer's advertisement cost in the  $n^{th}$  market which is called cross-marketing cost coefficient;
- $h_{lj}, h_{2i}$  The service investment efficiency coefficient of retailers in the first and second SCs  $(h_{lj}, h_{2i} > 0)$ . The larger value of  $h_{lj}(h_{2i})$ , indicates the high investment efficiency of ratailer i(i):
  - investment efficiency of retailer j(i);
- $TC_{1j},TC_{2i}$ : In turn, the transportation cost of a unit product between the manufacturer and retailer  $j^{th}$  in the first SC, and between the manufacturer and retailer $i^{th}$  in the second SC,  $TC_{1j},TC_{2i} > 0$ ;
- ${}^{TC_{1jn},TC_{2in}}$ : In turn, the transportation cost of a unit product between retailer  $j^{th}$  and market's demand  $n^{th}$  in the first SC and between  $i^{th}$  retailer and market's demand n in the second SC,  ${}^{TC_{1jn},TC_{2in}} > 0$ ;
- $I_{R_{1i}}, I_{R_{2i}}$  In turn, risk sensitivity coefficient or constant

absolute risk aversion (CARA) related to retailer j and*i*, which are defined as Arrow-Pratt,

$$I_{R_{1j}}, I_{R_{2i}} \ge 0$$

 $I_{M_1}, I_{M_2}$ : Risk sensitivity coefficient or constant absolute risk aversion (CARA) related to manufacturers in the first and second SCs, respectively,  $I_{M_1}, I_{M_2} \ge 0$ .

#### 4. 3. Decision Variables

- $D_1, D_2$ : The scenario of designing a possible distribution network related to the first and second SCs. Each scenario consists of a set of candidate retailers and a set of markets that can be covered and supplied by each candidate retailer;
- $w_1, w_2$ : The wholesale prices of each product unit related to the manufacturers in the first and second SCs, respectively;
- $m_{1j}, m_{2i}$ : The retailer j's profit margin in the first SC and retailer i's profit margin in the second SC, respectively;
- $p_{ljn}$ : The first SC's product price presented by retailer in  $n^{th}$  market,  $p_{ljn} = w_l + TC_{lj} + m_{lj} + TC_{ljn}$ ;
- $p_{2in}$ : The second SC's product price presented by retailer *i* in  $n^{th}$  market,  $p_{2in} = w_2 + TC_{2i} + m_{2i} + TC_{2in}$ ;
- $s_{lj}, s_{2i}$ : In turn, retailer j's service level in the first SC and retailer i's service level in the second SC;
- $a_1, a_2$ : The manufacturers' advertisement cost in the first and the second SCs, respectively.

In the both SCs, manufacturer determines a wholesale price for all of the retailers and each retailer specifies a profit margin for all of the allocated markets. As a result, the product's retailing price in each market is the sum of wholesale price, the related retailer's profit margin, and the transportation costs between manufacturer and retailer and between retailer and market. Transportation costs are affected by facilities geographical positions, transport methods, accessible vehicles, roads, and distances between the facilities. In our research, manufacturers in the both SCs will try to design their distribution network in order to deliver their products to the markets with the least possible retailing price. Therefore, they enjoy the most competitive advantages in the markets.

**4. 4. Demand Function in the Markets** We consider the joint impacts of price, service level, and advertisement costs on the demand. Therefore, we perceived the demand which is sensitive to price and service level (Xiao and Yang, [2]) and is a function of price and promotion expenses (Hafezalkotob et al. [4]). Under the conditions of  $D_1$  and  $D_2$ , we assume the demand of the  $n^{th}$  market for the products presented by

retailer j is

$$\partial_{l_{jn}}^{P_1,D_2} = \partial_{l_{jn}} - p_{l_{jn}} + dp_{2in} + b_n s_{l_j} - g_n s_{2i} + r_n \left( a_l \right)^{1/2} - n_n \left( a_2 \right)^{1/2}.$$
(1)

Index *i* refers to the rival retailer who covers the  $n^{th}$ 

market, which is determined by  $D_2$  (we show that with  $i \rightarrow D_2$ ). In addition, the demand of  $n^{th}$  market for the products presented by retailer i, under  $D_2$  condition, is as follows:

 $\mathcal{Q}_{j_{in}}^{D_1,D_2} = \mathcal{Q}_{2n} - p_{2in} + dp_{lin} + b_n s_{2i} - g_n s_{li} + r_n (a_2)^{1/2} - n_n (a_l)^{1/2},$ (2)where, retailer j who covers the  $n^{th}$  market is specified by  $D_1$  (i.e.  $i \to D_1$ ). Since the structures of the both competitive SCs impact on markets' demand, we characterized the demands by  $D_1$  and  $D_2$  in Equations (1) and (2). The demands' mean and variance differ in various markets because they depend on the customers' behavior and their understanding from the quality, brand, credit, position, and many others. Each market's demand related to each retailer is an ascending function of its rival retailing price, its service level, and its manufacturer's advertising cost. However, each market's demand related to each retailer is a descending function of its retailing price, its rival service level, and its rival manufacturer's advertising cost.

# 4. 5. Profit Functions and Partners Utility in SCs

The order made by retailer *j* to its manufacturer is equal to the sum of markets' demand that the retailer covers under  $D_1$  and  $D_2$  conditions. Thus, the order quantity of the retailer will be:

$$\mathcal{Q}_{lj}^{\mathcal{D}_{1},D_{2}} = \sum_{\substack{n \in N_{l,j} \\ \forall n, j \to D_{1} \\ \forall n, i \to D_{2}}} \mathcal{Q}_{l,j}^{D_{1},D_{2}} = \sum_{\substack{n \in N_{l,j} \\ \forall n, j \to D_{1} \\ \forall n, i \to D_{2}}} \left( \mathcal{A}_{n}^{\mathcal{D}_{1}} - p_{ljn} + dp_{2in} + b_{n}s_{lj} \\ -g_{n}s_{2i} + r_{n}\left(a_{l}\right)^{l/2} + n_{n}\left(a_{2}\right)^{l/2} \right) \tag{3}$$

Similarly, the order quantity of the  $i^{th}$  retailer to its manufacturer is equal to:

$$\mathcal{Q}_{2i}^{\mathcal{Q}_{1},D_{2}} = \sum_{\substack{n \in N_{2i}^{D_{2}} \\ \forall n_{i}, \rightarrow D_{2} \\ \forall n_{i}, \rightarrow D_{i}}} \mathcal{Q}_{2in}^{p_{1},D_{2}} = \sum_{\substack{n \in N_{2i}^{D_{2}} \\ \forall n_{i}, \rightarrow D_{i}}} \left( \frac{\mathcal{A}_{2n} - p_{2in} + dp_{ijn} + b_{n}s_{2i}}{-g_{n}s_{1j} + r_{n} \left(a_{2}\right)^{l/2} + n_{n} \left(a_{1}\right)^{l/2}} \right)$$
(4)

In Equations (3) and (4), retailers j and i related to any market are determined by  $D_1$  and  $D_2$  scenarios, respectively. Indexes  $D_1$  and  $D_2$  in  $\mathcal{O}_{1j}^{(D_1,D_2)}$  and  $\mathcal{O}_{2i}^{(D_1,D_2)}$ demonstrate that total order of retailers are dependent on the structures of both SCs. Like [2] and [4], we assume that the cost functions of service level costs of retailers j and i are equal to  $\frac{1}{2}h_{ij}s_{1j}^2, \frac{1}{2}h_{2i}s_{2i}^2$ , respectively. Considering the order quantities (3) and (4) the random

Considering the order quantities (3) and (4), the random profits of retailer j and i are:

$$\boldsymbol{p}_{R_{lj}}^{\mathcal{D}_{1},D_{2}} = m_{lj} \sum_{\substack{n \in N_{lj} \\ \forall n, i \to D_{2} \\ \forall n, i \to D_{2}}} \left( \frac{\mathscr{A}_{Pn} - p_{ljn} + dp_{2in} + b_{n}s_{lj}}{\left( -g_{n}s_{2i} + r_{n}\left(a_{l}\right)^{l/2} + n_{n}\left(a_{2}\right)^{l/2}} \right) - \frac{1}{2}h_{lj}s_{lj}^{2}, \ \forall j \in J.$$
(5)

$$\boldsymbol{p}_{R_{2i}}^{\boldsymbol{p}_{1,D_{2}}^{n,D_{2}}} = m_{2i} \sum_{\substack{n \in N_{2i}^{D_{2}} \\ \forall n, i \to D_{2} \\ \forall n, i \to D_{j}}} \left( \frac{\boldsymbol{a}_{2n}^{n} - p_{2in} + dp_{ljn} + \boldsymbol{b}_{n} \boldsymbol{s}_{2i}}{-\boldsymbol{g}_{n} \boldsymbol{s}_{lj} + \boldsymbol{r}_{n} \left(\boldsymbol{a}_{2}\right)^{1/2} + \boldsymbol{n}_{n} \left(\boldsymbol{a}_{1}\right)^{1/2}} \right) - \frac{1}{2} \boldsymbol{h}_{2i} \boldsymbol{s}_{2i}^{2}, \quad \forall i \in I,$$
(6)

where  $p_{1in} = w_1 + TC_{1i} + m_{1i} + TC_{1in}$  and  $p_{2in} = w_2 + TC_{2i} + m_{2i} + TC_{2in}$ .

The amount of production in each SC is equal to the sum of orders of corresponding retailers. Each manufacturer's total profit is equal to his profit margin multiplied by the total amount of the sold products to all retailers minus of the promotion costs. Thus, the random profits of the manufacturers are

$$p_{M_{j}}^{\mathcal{O}_{1},D_{2}} = (w_{1} - c_{1}) \sum_{j \in J} \sum_{\substack{n \in N_{j} \\ \forall n, j \to D_{1} \\ n, i \to D_{2}}} \left( \frac{\mathcal{A}_{n} - p_{ijn} + dp_{2in} + b_{n}s_{1j}}{-g_{n}s_{2i} + r_{n} \left(a_{1}\right)^{1/2} - n_{n} \left(a_{2}\right)^{1/2}} \right)^{-a_{i}},$$

$$(7)$$

$$\mathbf{p}_{M_{2}}^{D_{1},D_{2}} = (w_{2} - c_{2}) \sum_{i \in I} \sum_{\substack{n \in N_{2}^{D_{2}} \\ \forall n, i \to D_{2} \\ \forall n, j \to D_{1}}} \begin{pmatrix} \mathcal{A}_{2n} - p_{2in} + dp_{1jn} + \mathbf{b}_{n} s_{2i} \\ -g_{n} s_{1j} + \mathbf{r}_{n} (a_{2})^{1/2} + \mathbf{n}_{n} (a_{l})^{1/2} \end{pmatrix} - a_{2}.$$
(8)

Random markets demands result in uncertainty in all above profit functions. Manufacturers and retailers may show different sensitivities to the risk arising from these uncertainties. Risk-neutral retailers (manufacturers) are entirely indifferent towards the profit fluctuations. However, risk-averse retailers (manufacturers) define their strategies in a way to reduce the profit uncertainty. Bar-Shira and Finkelshtain [15] expressed that using utility function, which increases the mean and decreases the variance, is more appropriate, in regard to the approaches which are just based on the expected utility (mean). As a result, we assume that each player has a utility function of  $\{E(\beta) - IVar(\beta)\}$ . That is, each player's utility is an ascending function of his expected profit. However, it is a descending function of profit uncertainty and his sensitivity towards risk [2,4]. Adopting mean-variance concept for random functions (5)-(8), the following utilities for random profit for the retailers and manufacturers are obtained as follows:

$$u_{R_{ij}}(\boldsymbol{\mathcal{P}}_{R_{ij}}^{p_1, D_2}) = m_{Ij} \sum_{\substack{n \in N_{ij}^{(2)} \\ \forall n, i \to D_2}} \left( \frac{a_{ln} - p_{ljn} + dp_{2\,in}}{b_{R} s_{Ij} - g_{R} s_{2\,i}} + r_{n}\left(a_{j}\right)^{l/2} + n_{n}\left(a_{2}\right)^{l/2} \right)^{-\frac{1}{2}h_{Ij}s_{Ij}^{2} - I_{R_{ij}}m_{Ij}^{2} \sum_{n \in N_{Ij}} s_{In}^{2}, \forall j \in J,$$
(9)

$$u_{R_{2i}}(\beta_{R_{2i}}^{D_1,D_2}) = m_{2i} \sum_{\substack{n \in N_{2i}^{D_2} \\ \forall n_i, J = D_2 \\ \forall n_i, J = D_2 \\ \forall n_i, J = D_2}} \begin{pmatrix} \bar{a}_{2n} - p_{2in} + dp_{1jn} \\ + b_n s_{2i} - g_n s_{1j} \\ + r_n (a_2)^{1/2} + n_n (a_i)^{1/2} \end{pmatrix} - \frac{1}{2} h_{2i} s_{2i}^{2i} - I_{R_{2i}} m_{2i}^{2i} \sum_{n \in N_{2i}^{D_2}} s_{2n}^{2i}, \forall i \in I,$$
(10)

$$u_{M_{j}}(\boldsymbol{\beta}_{M_{j}}^{D_{j},D_{2}}) = (w_{j} - c_{j}) \sum_{\substack{j \in J_{n} \in M_{j}^{D_{j}} \\ \forall n_{i} \to D_{2}}} \left( \frac{a_{ln} - p_{ljn} + dp_{2in}}{b_{n_{j}} + d_{n_{j}} s_{2i}} \right) - q - l_{M_{j}} (w_{l} - c_{l})^{2} \sum_{n \in N} s_{ln}^{2}, \quad (11)$$

$$u_{M_{2}}(p_{M_{2}}^{p_{1},D_{2}}) = (w_{2} - c_{2}) \sum_{\substack{i \in I \ n \in N_{2}^{D_{1}} \\ \forall n, j \rightarrow D_{2} \\ \end{pmatrix}} \begin{pmatrix} \mathcal{B}_{2n} - p_{2n} + dp_{1jn} \\ + b_{n}s_{2i} - g_{n}s_{ij} \\ + r_{n}(a_{2})^{V_{2}} + n_{n}(a_{j})^{V_{2}} \end{pmatrix} - a_{2} - I_{M_{2}}(w_{2} - c_{2})^{2} \sum_{n \in N} s_{2n}^{2}, \quad (12)$$

where  $p_{1jn} = w_1 + TC_{1j} + m_{1j} + TC_{1jn}$  and  $p_{2in} = w_2 + TC_{2i} + m_{2i} + TC_{2in}$ . In utility functions (9)-(12), parameters  $I_{R_{1j}}$ ,  $I_{R_{2i}}$ ,  $I_{M_1}$ , and  $I_{M_2}$  are the coefficients of CARA which define the sensitivity of the retailers and manufacturers towards the risk in regard to uncertainty. The zero amount for these coefficients means that the firms are risk-neutral; on the contrary,  $I_{R_{1j}}$ ,  $I_{R_{2i}}$ ,  $I_{M_1}$ ,  $I_{M_2} > 0$  demonstrates that the firms are risk-averse and the larger the amount of CARA, the more conservative the firms are.

In SCM, the goal of tactical decisions is to optimize the profit in the planning horizon with considering the strategic decisions taken in the strategic planning horizon [16]. Hence, for distribution scenarios  $D_1$  and  $D_2$ , now we compute the optimal solution for the tactical decisions. Retailers determine the profit margin and service level considering the SCs' distribution design scenarios (i.e.  $N_1^{D_1}$  and  $N_2^{D_2}$ ). Moreover, the tactical decisions of the manufacturers (i.e. wholesale prices and advertising costs) are determined with regard to the strategic decisions  $N_1^{D_1}$  and  $N_2^{D_2}$ , as well. Respectively, Hessian matrixes  $u_{R_i}(p_{R_{2i}}^{D_1,D_2})$  and  $u_{R_{2i}}(p_{R_{2i}}^{D_1,D_2})$  with respect to the profit margin and service level decisions are equal to:

$$H_{R_{ij}}^{D_{1}} = \begin{bmatrix} -2\left| \left| N_{ij}^{D_{1}} \right| + I_{R_{ij}} \sum_{n \in N_{ij}^{D_{1}}} s_{1n}^{2} \right| & b_{n} \left| N_{ij}^{D_{1}} \right| \\ B_{R_{ij}} + B_{R_{ij}} = \begin{bmatrix} -2\left| \left| N_{2i}^{D_{2}} \right| + I_{R_{ij}} \sum_{n \in N_{2i}^{D_{2}}} s_{2n}^{2} \right| & b_{n} \left| N_{2i}^{D_{1}} \right| \\ B_{n} \left| N_{ij}^{D_{1}} \right| & -b_{ij} \end{bmatrix} \\ B_{n} \left| N_{2i}^{D_{1}} \right| & -b_{ij} \end{bmatrix}$$

Besides, Hessian matrixes  $u_{M_1}(\mathbf{p}_{M_1}^{p_1,D_2})$  and  $u_{M_2}(\mathbf{p}_{M_2}^{p_1,D_2})$  concerning the wholesale price and advertising expenses decisions are:

$$H_{M_{k}} = \begin{bmatrix} -2\left(|N| + I_{M_{k}} \sum_{n \in N} S_{n}^{2}\right) & \frac{a_{k}^{-1/2} \sum_{n \in N} r_{n}}{2} \\ \frac{a_{k}^{-1/2} \sum_{n \in N} r_{n}}{2} & \frac{-(w_{k} - c_{k}) x_{k}^{-3/2} \sum_{n \in N} r_{n}}{4} \end{bmatrix}, \forall k \in \{1, 2\}$$

|N|,  $|N_{1j}^{D_1}|$ , and  $|N_{2i}^{D_2}|$  are cardinalities of N,  $N_{lj}^{D_1}$ , and  $N_{2i}^{D_2}$  which indicate the number of elements in these sets, respectively. If and only if Hessian matrixes  $H_{R_{1j}}$ ,  $H_{R_{2i}}$ ,  $H_{M_1}$ , and  $H_{M_2}$  are negative definite, retailers and manufacturers' utilities are concave functions on the corresponding tactical decisions. Now, let us introduce the following parameters:

$$B_{lj}^{D_{1}} = \left| N_{lj}^{D_{1}} \right| + I_{R_{lj}} \sum_{n \in N_{lj}^{D_{1}}} s_{ln}^{2} - \left( b_{n} \left| N_{lj}^{D_{1}} \right| \right)^{2} / 2h_{lj} , \qquad (13)$$

$$B_{2i}^{D_2} = \left| N_{2i}^{D_2} \right| + I_{R_{2i}} \sum_{n \in N_{2i}^{D_2}} S_{2n}^2 - \left( b_n \left| N_{2i}^{D_2} \right| \right)^2 / 2h_{2i} , \qquad (14)$$

$$A_{M_{k}} = |N| + I_{M_{k}} \sum_{n \in N} s_{kn}^{2} - \left(\sum_{n \in N} r_{n}\right)^{2} / 4, k = 1, 2.$$
(15)

**4. 6. Optimal Tactical Decisions** Assume the distribution design scenarios for the first and second SCs are  $D_1$  and  $D_2$ , respectively. Yet, the question is how the optimal tactical decisions of SCs are

determined for these scenarios under the competitive situation. In Proposition 1, we calculate the optimal decisions concerning retailing prices, wholesale prices, advertisement costs, and service levels for both SCs. Afterwards, Proposition 2 gives the utility of each manufacturer under scenarios  $D_1$  and  $D_2$ .

**Proposition 1.** If  $B_{1j} > 0, \forall j \in J, B_{2i}^{D_2} > 0, \forall i \in I$ , and  $A_{M_1}, A_{M_2} > 0$ , then the optimal profits margins of all retailers  $j \in J$  and  $i \in I$  satisfy the following linear system of equations:

$$\begin{split} &\sum_{n \in N_{ij}} \left[ \left( \frac{r_n}{2} \sum_{n \in N} r_n - 1 \right)_{j \in J} \mathbf{q}_{jj}^{D_j} m_{ij}^* + \left( \frac{b_n}{h_{l\,j}} \sum_{n \in N_{ij}} b_n - 1 \right) m_{ij}^* + \left( \frac{n_n}{2} \sum_{n \in N} r_n + d \right)_{j \in J} \mathbf{q}_{2i}^{D_2} m_{2i}^* \\ &- \left( \frac{g_n}{h_{2i}} \sum_{n \in N_{2i}^{D_2}} b_n - d \right) m_{2i}^* + \overline{a}_{ln} - c_l - TC_{l\,j} - TC_{ljn} + d \left( c_2 + TC_{2i} + TC_{2in} \right) \right] \\ &- \left[ \left| N_{lj}^{D_j} \right| + 2I_{R_{ij}} \sum_{n \in N_{ij}} S_{ln}^2 \right] m_{lj}^* = 0, \ \forall j \in J , \\ &\sum_{n \in N_{2i}^{D_2}} \left[ \left( \frac{r_n}{2} \sum_{n \in N} r_n - 1 \right)_{i \in I} \mathbf{q}_{2i}^{D_2} m_{2i}^* + \left( \frac{b_n}{h_{2i}} \sum_{n \in N_{2i}^{D_2}} b_n - 1 \right) m_{2i}^* + \left( \frac{n_n}{2} \sum_{n \in N} r_n + d \right) \sum_{j \in J} \mathbf{q}_{jj}^{D_i} m_{ij}^* \\ &- \left( \frac{g_n}{h_{ij}} \sum_{n \in N_{ij}} b_n - d \right) m_{lj}^* + \overline{a}_{2n} - c_2 - TC_{2i} - TC_{2in} + d \left( c_l + TC_{lj} + TC_{ljn} \right) \right] - \\ &\left[ \left| N_{2i}^{D_2} \right| + 2I_{R_{2i}} \sum_{n \in N_{2i}^{D_2}} S_{2n}^2 \right] m_{2i}^* = 0, \ \forall i \in I. \end{split}$$

Afterwards, other tactical decisions (i.e. wholesale prices, service levels, and advertising costs) are determined as follows:

$$\begin{split} w_{I}^{*} &= \sum_{j \in J} q_{lj} m_{Ij}^{*} + c_{I}, \ w_{2}^{*} = \sum_{i \in I} q_{2i}^{D_{2}} m_{2i}^{*} + c_{2}, \\ a_{I}^{*} &= \left( \left( \frac{I}{2} \sum_{n \in N} r_{n} \right) \sum_{j \in J} q_{lj} m_{Ij}^{*} \right)^{2}, \ a_{2}^{*} &= \left( \left( \frac{I}{2} \sum_{n \in N} r_{n} \right) \sum_{j \in J} q_{2i}^{D_{2}} m_{2i}^{*} \right)^{2}, \\ a_{2}^{*} &= \left( \left( \frac{I}{2} \sum_{n \in N} r_{n} \right) \sum_{j \in J} q_{2i}^{D_{2}} m_{2i}^{*} \right)^{2}, \ s_{Ij}^{*} &= \left( \frac{I}{h_{Ij}} \sum_{n \in N_{Ij}} b_{n} \right) m_{Ij}^{*}, \forall j \in J, \\ s_{2i}^{*} &= \left( \frac{I}{h_{2i}} \sum_{n \in N_{2i}^{D_{2}}} b_{n} \right) m_{2i}^{*}, i \in I, \\ \text{where} \\ q_{Ij} &= \left( \left| N_{Ij} \right| + 2I_{R_{Ij}} \sum_{n \in N_{In}} s_{In}^{2} \right) / \left( |N| + 2I_{M_{I}} \sum_{n \in N} s_{In}^{2} \right) \end{split}$$

and

$$q_{2i}^{D_2} = \left( \left| N_{2i}^{D_2} \right| + 2I_{R_{2i}} \sum_{n \in N_{2i}^{D_2}} S_{2n}^2 \right) \right) / \left( \left| N \right| + 2I_{M_2} \sum_{n \in N} S_{2n}^2 \right)^2$$

**Proof:** See reference [4] for the detailed proof. **Proposition 2.** If  $B_{1j} > 0$ ,  $\forall j \in J$ ,  $B_{2i}^{D_2} > 0$ ,  $\forall i \in I$ , and  $A_{M_1}$ ,  $A_{M_2} > 0$ , then the optimal expected demand and optimal utility of SCs' partners are:

$$\begin{split} & \mathcal{Q}_{lj}^{D_2^*} = \left( \left| N_{lj} \right| + 2I_{R_{lj}} \sum_{n \in N_{lj}} S_{ln}^2 \right) m_{lj}^*, \ \forall j \in J, \\ & \mathcal{Q}_{2i}^{D_2^*} = \left( \left| N_{2i}^{D_2} \right| + 2I_{R_{lj}} \sum_{n \in N_{2i}^{D_2}} S_{2n}^2 \right) m_{2i}^*, \ \forall i \in I, \end{split}$$

$$\begin{split} &\sum_{j \in J} \mathcal{Q}_{lj}^{D_2*} = \left( \left| N \right| + 2I_{M_1} \sum_{n \in N} s_{ln}^2 \right) (w_1^* - c_l), \\ &\sum_{i \in I} \mathcal{Q}_{2i}^{D_2*} = \left( \left| N \right| + 2I_{M_2} \sum_{n \in N} s_{2n}^2 \right) (w_2^* - c_2), \\ &u_{R_{lj}} \left( \mathcal{P}_{R_{lj}}^{D_2*} \right) = m_{lj}^{*2} B_{lj} , \forall j \in J, \\ &u_{R_{lj}} \left( \mathcal{P}_{R_{lj}}^{D_2*} \right) = m_{lj}^{*2} B_{lj}^{D_j} , \forall j \in J, \\ &u_{M_1} \left( \mathcal{P}_{M_1}^{D_2*} \right) = U_1^{D_1,D_2} = (w_1^* - c_l)^2 A_{M_1}, \\ &u_{M_2} \left( \mathcal{P}_{M_2}^{D_1,D_2} \right) = U_2^{D_1,D_2} = (w_2^* - c_2)^2 A_{M_2}. \end{split}$$

Therefore, the optimal utility function of each manufacturer is obtained under distribution scenarios  $D_1$  and  $D_2$ . In the next section, we obtain the best distribution scenarios for the manufacturers in a competitive situation.

4. 7. Optimal Strategic Decisions Retailers in each SC are different with respect to geographical locations, transportation costs, service level efficiencies, and also their sensitivity towards risk. Picking out a suitable retailer to supply the markets concerning these factors, results in improvement of product's competitive advantage in the markets and increase of the manufacturer's profit. Distribution network design is a strategic decision which includes the long-term contracts with the retailers. We assume that the SC designs and the markets that each candidate retailer can supply are given as a set of possible scenarios for the manufacturer. Considering the optimal tactical decisions for service level, price, promotion, and marketing cost, each manufacturer should determine his distribution network design. That is, he should decide which retailers should be selected from the candidate retailers in order to maximize his distribution network's utility. For instance, assume that the manufacturer in the first SC considers three independent candidate retailers in order to distribute his products in five markets. He evaluates four different distribution network design scenarios. In the first scenario, as it is shown in Figure 2, the second and the third retailers are selected and markets 1 to 3 are covered by the second retailer while the other markets are covered by the third retailer. Consequently, the first scenario can be demonstrated as  $N_1^1 = \{N_{11}^1, N_{12}^1, N_{13}^1\} = \{\{\}, \{1, 2, 3\}, \{4, 5\}\}.$  Similarly, the other three scenarios can be represented as  $N_1^2 = \{\{1, 2, 3\}, \{\}, \{4, 5\}\}, N_1^3 = \{\{1, 2\}, \{\}, \{3, 4, 5\}\},\$ and  $N_1^4 = \{\{1,2\},\{3,4\},\{5\}\}$ . Moreover, the distribution network design scenarios of the second SC scenario can shown he as  $N_{2}^{1} = \{N_{21}^{1}, N_{22}^{1}\} =$  $\{\{1,2\},\{3,4,5\}\},\$  $N_2^2 = \{\{1,2,3,4,5\}, \{\}\}, \text{ and } N_2^3 = \{\{\}, \{1,2,3,4,5\}\}.$  Contemplating the tactical decisions in Propositions 1 and 2, the rival manufacturers are able to survey the utility of any possible combinations of the distribution design scenarios.

**TABLE 3.** Bimatrix of nonzero sum game of distribution design of two SCs

$\overline{}$	$U_{2}^{D_{1},D_{2}}$	Manufacturer 2 $(D_2)$				
$U_1^{D_1,D_2}$	-	Scenario 1	•••	Scenario n		
1	Scenario 1	$(U_1^{1,1},\!U_2^{1,1})$	•••	$(U_1^{1,n},\!U_2^{1,n})$		
urer	•	•	•			
$(D_i)$	•					
lanu	•		•			
2	Scenario m	$(U_1^{m,1},\!U_2^{m,1})$	•••	$(U_1^{m,n},U_2^{m,n})$		



The combination of possible design scenarios can be evaluated by a payoff matrix. This matrix indicated in Table 3 is a two-person non-zero-sum bimatrix game. Each element of this matrix represents the utilities obtained by the first and second manufacturers with regard to the corresponding distribution scenarios.

In each row of the payoff matrix, according to the different scenarios that the first manufacturer selects, there are different utilities for the second manufacturer. On the other hand, in each column, according to the different scenarios that the second manufacturer selects, there are different utilities for the first manufacturer.

To obtain the equilibrium point of the payoff matrix, we use Nash strategy [17]. First of all, in each column of the matrix, the maximum utility of the first manufacturer for each strategy of the second manufacture should be indicated by sign +. Afterwards, in each row of the matrix, the maximum utility of the second manufacturer for each strategy of the first manufacturer should be pointed out by sign \*. For instance, see Table 7.

Nash equilibrium point is where the utilities of both manufacturers are marked, simultaneously. The following three cases may occur:

v There is a unique Nash equilibrium point.

v There are two or more Nash equilibrium points.

v There is no Nash equilibrium point.

In the first case, there is only one combination of distribution scenarios of the two manufacturers where both utilities are marked. Therefore, the manufacturers select the corresponding scenarios. In the second case, more than one combination of utilities is marked. However, in the third case, none of the combination of the utilities is marked in the payoff matrix. Nash [18] discussed bargaining problem for understanding how players should cooperate when non-cooperation leads to Pareto-inefficient results. Several games have multiple equilibriums with different payoff for each player, compelling the players to negotiate on which equilibrium to target. Nash [18] purposed a solution for nonzero two-person game based on four assumptions including invariant to affine transformations, Pareto optimality, independence of irrelevant alternatives, and symmetry. Nash bargaining theory is widely employed in the real cases when unique Nash equilibrium does not exist. The Nash bargaining problem for our competitive distribution network design can be developed as

$$\max \left( U_{1}^{D_{1},D_{2}} - d_{1} \right) \left( U_{2}^{D_{1},D_{2}} - d_{2} \right),$$
  
st:  

$$d_{1} \leq U_{1}^{D_{1},D_{2}} \leq \overline{U_{1}}, \ \forall D_{1},D_{2},$$
  

$$d_{2} \leq U_{2}^{D_{1},D_{2}} \leq \overline{U_{2}}, \ \forall D_{1},D_{2}.$$
(16)

where,  $U_1^{D_1,D_2}$  and  $U_1^{D_1,D_2}$  are, in turn, the utilities of manufacturers one and two in combination of distribution scenarios  $(D_1,D_2).\overline{U_1}$  and  $\overline{U_2}$  are the maximum uilities of manufacturers one and two, respectively. Moreover,  $d_1$  and  $d_1$  are the minimum utilities of the first and second manufacturers, respectively.

We say that a pair of distribution scenarios  $(D_1^*, D_2^*)$  is a Nash bargaining solution if it solves problem (16). That is, the two rival players seek to maximize the product of the excess utilities. We suggest Nash bargaining problem (16) for cases two and three. Accordingly, a pair of distribution scenarios of Table 3 which optimizes problem (16) is a solution for the competitive distribution network design.

#### 5. METHODOLOGY

follows:

Here, we summarize the main stages of our proposed methodology for choosing the best distribution network scenarios for the two rival decenteralized SCs.

**Stage1.***Generating Feasible Scenarios*: First of all, each manufacturer determines a set of potential distribution network scenarios by choosing different subsets of candidate retailers. Then, he should gain social and economic information about them (such as transportation cost, retailer's demand sensitivity to its service level and advertisement cost).

**Stage2.** *Evaluation of Feasible Scenarios:* Then, both manufacturers calculate the utility of each distribution network scenario according to Proposition 2. Then, they form bimatrix game of Table 3 using the obtained utilities for scenarios.

**Stage3.** Choosing the Optimal Scenarios: If the bimatrix game in Table 3 has a unique Nash equilibrium, the corresponding scenarios are optimal competitive distribution scenario. Otherwise, if the manufacturers behave according to the basic assumptions of Nash bargaining problem, the pair of distribution network scenarios which maximize problem (16) are the optimal competitive distribution scenarios.

#### 6. NUMERICAL RESULTS AND DISCUSSION

In this section, we use the proposed methodology for a numerical example to illustrate the corresponding results. Our numerical examples comprise two competitive networks in which the manufacturer has several potential retailers. Two SCs compete for five different markets as shown in Figure 1.

**6. 1. The First Numerical Example** This numerical example comprises two competitive networks; in the first network, the manufacturer has three potential retailers and in the second one, the manufacturer has two potential retailers. We assume the default values of parameters are  $c_1 = c_2 = 10$ ,  $I_{M1} = 0.2$ , and  $I_{M2} = 0.2$ . The markets' data of the first and second SCs are listed in Tables 4, 5, and 6, respectively. Assume that the manufacturer of the first SC faces four scenarios of distribution deign that can be represented by  $\{1,1,3,3,3\}$ ,  $\{1,1,1,3,3\}$ ,  $\{2,2,2,3,3\}$ , and  $\{1,1,2,2,3\}$ .

**TABLE 4.** Markets data in the first numerical example

п	$\boldsymbol{b}_n$	$g_n$	$r_n$	$u_n$	$\overline{a_{ln}}$	$\boldsymbol{s}_{ln}$	$\overline{a_{2n}}$	$s_{2n}$
1	1	0.6	0.2	-0.5	10	2	20	4
2	1.5	0.8	0.5	-0.8	15	4	15	2
3	0.9	0.7	0.7	-0.1	20	4	10	2
4	1.1	0.5	0.4	-0.3	15	2	15	4
5	1.2	0.4	0.6	-0.4	10	2	20	4

**TABLE 5.** Data of the first SC in the first numerical example

j	$h_{lj}$	$I_{lj}$	$TC_{lj}$	$TC_{ljl}$	$TC_{lj2}$	$TC_{Ij3}$	$TC_{1j4}$	$TC_{1j5}$
1	5	0.1	1	1	1	2	3	4
2	5	0.2	1.5	3	2	1	2	3
3	5	0.1	2	4	3	2	1	1

**TABLE 6.** Data of the second SC in the first numerical example

i	$h_{2i}$	$I_{2i}$	$TC_{2i}$	$TC_{2i1}$	$TC_{2i2}$	$TC_{2i3}$	$TC_{2i4}$	$TC_{2i5}$
1	5	0.1	1	1	1	1	4	4
2	5	0.1	1	4	4	1	1	1

**TABLE 7.** Bimatrix of nonzero sum game of distribution design of two SCs in the first numerical example

Manufacturer 1 (D1)	Manufacturer 2 (D2) $U_2^{D_1,D_2}$					
$U_1^{D_1,D_2}$	{1,1,2,2,2}2	{1,1,1,1,1}}	{2,2,2,2,2}}			
{1,1,3,3,3}	(104.42,131.65)	(99.85,165.75*)	(99.76,165.17)			
{1,1,1,3,3}	(105.41+,130.57)	(99.27+,166.98*)	(101.16+,163.3)			
{2,2,1,3,3}	(99.52,131.9)	(93.04,170.07*)	(95.16,165.45)			
{1,1,2,2,3}	(100.27,125.69)	(95.15,158.74*)	(96.33,157.73)			

That is, in the first design, the first and second markets are supplied by the first retailers and the other markets are supplied by the third retailer. In the second design, the third retailer supplies the fourth and fifth markets and the first retailer supplies the other markets. Other distribution designs can be interpreted similarly. The assignment of markets to retailers can also be represented by partitions of market set N as follows:  $N_1^1 = \{N_{11}^1, N_{12}^1, N_{13}^1\} = \{\{1, 2\}, \{\}, \{3, 4, 5\}\}, N_1^2 = \{\{1, 2, 3\}, \{\}, \{4, 5\}\},\$  $N_1^3 = \{\{\}, \{1, 2, 3\}, \{4, 5\}\}, \text{ and } N_1^4 = \{\{1, 2\}, \{3, 4\}, \{5\}\}.$  Moreover, suppose that the manufacturer of the second SC faces three scenarios of distribution designs that can be represented by  $\{1,1,2,2,2\}$ ,  $\{1,1,1,1,1\}$ ,  $\{2,2,2,2,2\}$ . Similar to the first SC, we can show the assignment of markets to retailers by partitions of market set N as follows:  $N_2^1 = \{\{1,2\},\{3,4,5\}\}, N_2^2 = \{\{1,2,3,4,5\},\{\}\}, \text{ and }$  $N_2^3 = \{\{\}, \{1, 2, 3, 4, 5\}\}$ . We use Proposition 2 to calculate the optimal utility of each combination of distribution network scenarios for the manufacturers. The results (payoff matrixes) have been shown in Table 7. For obtaining the Nash equilibrium point form the table, we mark the best response strategy of manufacturer 1 against the rival manufacturer, by (+). Moreover, we mark the best response strategy of manufacturer 2 against the rival manufacturer by (\*). Therefore,  $D_1 = \{1,1,1,3,3\}$  and  $D_2 = \{1,1,1,1,1\}$  (which both utilities are marked) are equilibrium distribution networks for the rival manufacturers. In this situation, the first manufacturer selects retailer one to cover markets 1, 2, and 3 and retailer three to cover markets 4 and 5. However, the second manufacturer only chooses retailer 1 to supply all markets.

## 7. CONCLUSION

A new stream of distribution networks literature has recently emerged that deals with competitive location and allocation problems. These problems have been raised in many sectors such as retail, banking, and postal industries. In this regard, we considered two SCs that compete for a set of common geographically dispersed markets. Each rival SC consists of one riskaverse manufacturer and a set of candidate risk-averse retailers. We used game theory approach for finding the equilibrium solution for distribution networks of both manufacturers. We proposed a novel methodology for the tactical and strategic decisions of two SCs. At the first step, optimal tactical decisions of SCs such as price, service level, and advertisement are calculated for each pair of distribution networks. Afterwards, the optimal utilities of distribution networks for manufacturers are formulated as a payoff matrix of a non-zero sum, non-cooperative, two-person game model. We found that Nash equilibrium solution can be achieved in closed form. Eventually, two numerical examples reveal two cases which either unique or multiple equlibrium solutions exist. We showed that Nash bargaining problem gives the equilibrium solution for the multiple equilibrium case. Although our model is restricted to duopoly of two SCs, one can easily generalize it to competition of more than two SCs. In this prospective condition, the model would be transform into a three-person or multiple-person game between the manufacturers. There are also other directions and suggestions for the future research. Firstly, we assumed that the decision-making structures of SCs are decentralized. However, the competition of centralized and decentralized SCs seems highly interesting. Secondly, in this paper, Nash equilibrium solution is presumed for tactical decisions, but contemplating Stackelberg equilibrium structure for leader and follower interaction between manufacturer and retailers could be particularly attractive. Thirdly, it is appealing but challenging to investigate how other well-known demand functions affect the market equilibrium. Eventually, one can develop our model by considering other criteria for customer purchasing decisions such as travel time, distance, service quality, and brand.

# 8. REFERENCES

- Kogan, K. and Tapiero, C.S., "Supply chain games: Operations management and risk valuation: Operations management and risk evaluation, Springer, Vol. 113, (2007).
- 2 Xiao, T. and Yang, D.," Price and service competition of supply chains with risk-averse retailers under Demand uncertainty", *International Journal of Production Economics*, Vol. 114, No. 1, (2008), 187–200.
- 3. Chandra, C. and Grabis, J., "Supply chain configuration: Concepts, solutions, and applications, Springer, (2007).
- Hafezalkotob, A., Makui, A. and Sadjadi, S.J., "Strategic and tactical design of competing decentralized supply chain networks with risk-averse participants for markets with uncertain demand", *Mathematical Problems in Engineering*, Vol. 2011, No., (2011).
- Ingene, C.A. and M. E. Parry, "Coordination and producer profit maximization: The multiple retailer channel", *Journal of Retailing*, Vol. 71, No. 2, (1995), 129–151.
- 6. Chen, F., Federgruen, A. and Zheng, Y.-S., "Coordination mechanisms for a distribution system with one supplier and

multiple retailers", *Management Science*, Vol. 47, No. 5, (2001), 693-708.

- Farahani, R.Z., Rezapour, S., Drezner, T. and Fallah, S., "Competitive supply chain network design: An overview of classifications, models, solution techniques and applications", *Omega*, Vol. 45, No., (2014), 92-118.
- Hammami, R., Frein, Y. and Hadj-Alouane, A.B., "A strategictactical model for the supply chain design in the delocalization context: Mathematical formulation and a case study", *International Journal of Production Economics*, Vol. 122, No. 1, (2009), 351-365.
- Zamarripa, M.A., Aguirre, A.M., Méndez, C.A. and Espuña, A., "Improving supply chain planning in a competitive environment", *Computers & Chemical Engineering*, Vol. 42, No., (2012), 178-188.
- Badri, H., Bashiri, M. and Hejazi, T.H., "Integrated strategic and tactical planning in a supply chain network design with a heuristic solution method", *Computers & Operations Research*, Vol. 40, No. 4, (2013), 1143-1154.
- Hafezalkotob, A. and Makui, A., "Modeling risk of losing a customer in a two-echelon supply chain facing an integrated competitor: A game theory approach", *International Journal of Engineering*, Vol. 25, No. 1, (2012), 11-34.

- Meng, Q., Huang, Y. and Cheu, R.L., "Competitive facility location on decentralized supply chains", *European Journal of Operational Research*, Vol. 196, No. 2, (2009), 487-499.
- Rezapour, S. and Farahani, R.Z., "Strategic design of competing centralized supply chain networks for markets with deterministic demands", *Advances in Engineering Software*, Vol. 41, No. 5, (2010), 810-822.
- Hafezalkotob, A. and Makui, A., "Supply chains competition under uncertainty concerning player's strategies and customer choice behavior: A generalized nash game approach", *Mathematical Problems in Engineering*, Vol. 2012, No., (2012).
- Bar-Shira, Z. and Finkelshtain, I., "Two-moments decision models and utility-representable preferences", *Journal of Economic Behavior & Organization*, Vol. 38, No. 2, (1999), 237-244.
- Chopra, S. and Meindl, P., "Supply chain management. Strategy, planning & operation, Springer, (2007).
- 17. Owen, G., Game theory. 1995, Academic Press.
- Nash Jr, J.F., "The bargaining problem", *Econometrica: Journal* of the Econometric Society, Vol., No., (1950), 155-162.

# Distribution Design of Two Rival Decenteralized Supply Chains: a Two-person Nonzero Sum Game Theory Approach

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PAPER INFO

Paper history: Received 02 June 2013 Received in revised form 27 November2013 Accepted in 12 December 2013

Keywords: Supply Chain Network Distribution Network Design Tactical and Strategic Decisions Nash Equilibrium Non-Zero Sum Game. در این تحقیق ما رقابت بین دو شبکه زنجیره تأمین غیرمتمرکز را تحت شرایط عدم قطعیت تقاضا در نظر گرفتهایم. هر یک از این زنجیره ها از یک تولیدکننده و همچنین مجموعهای از خرده فروشان ریسک گریز تشکیل شده است. این دو زنجیره، کالاهای قابل جایگزین را به بازارهایی ارائه میکنند که در جغرافیای مشخصی استقرار یافته اند. تقاضای بازارها تابعی از قیمت، سطح خدمت و میزان تبلیغات دو زنجیره است. ما مسأله طراحی شبکه توزیع دو زنجیره تأمین را در قالب یک بازی غیرمجموع صفر دو نفره مدلسازی کرده ایم. از آنجایی که تصمیمات استراتژیک طراحی شبکه توزیع موزیع دمینی به تصمیمات تاکتیکی اغلب دارای اولویت است، تعادل تصمیمات تاکتیکی را به ازای هر جفت سناریوهای توزیع محاسبه می نماییم و مطابق با روش ارائه شده، جواب تعادلش را برای سناریوهای شبکه توزیع دو زنجیره جستجو می نماییم. در پایان، برای تشریح کاربردهای عملی این روش، مثال عددی ارائه شده و مورد تحلیل قرار گرفته است.

#### doi:10.5829/idosi.ije.2014.27.08b.09

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