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# Train Scheduling Problem with Consideration of Praying Constraint as an Application of Job Shop Scheduling Problem 

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#### Abstract

$A B S T R A C T$

The present paper extends the idea of job shop scheduling problem with resting constraints to the train scheduling problem with the Muslim praying considerations. For this purpose, after proposing the new mathematical model, a heuristic algorithm based on the Electromagnetism-Like algorithm (EM) which is well adjusted to scheduling problems is employed to solve the large-size practical cases. The effectiveness of the proposed algorithm is then validated by comparing with optimum solution using small-size instances and simulated annealing algorithm, and Particle swarm Optimization (PSO) using medium and large-size instances. At the end, a practical case from Iranian railway network is studied and the results are reported. The results indicate that in the case of considering the Muslim praying constraint, the ratios of total tardiness of trains, and the total praying times are $14.5 \%$, and $3.5 \%$, respectively, while in the case of relaxing this constraint; the first ratio reduces to $12.3 \%$. This result demonstrates that the proposed algorithm is able to schedule the praying times so that in many cases the trains with different directions meet each other during the praying times.


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## 1. INTRODUCTION

Train scheduling problem has gained a lot of attentions in recent years. The problem can be defined in variety of types considering different objective functions and related constraints.

1. 2. Literature Review Amongst the solving methods, Branch and Bound as well as heuristic algorithms are more common ones. Burdett and Kozan [1] developed a simulated annealing and local search meta-heuristic algorithms to solve a train timetabling problem in a railway network modeled through a hybrid job shop scheduling problem. Cacchiani et al. [2] presented some exact and heuristic algorithms for solving the train timetabling problem using the solution of the LP relaxation of an integer linear programming formulation. Caprara et al. [3] addressed a double track railway line and introduced a heuristic algorithm based on the lagrange relaxation method. Jamili and Kianfar

[^0][4] Ghoseiri and Morshedsoluk [5] and Jamili et al., [6] applied simulated annealing, ant colony and hybrid particle swarm optimization and simulated annealing meta-heuristic algorithms for scheduling trains in single track railway lines, respectively.

Burdett and Kozan [7] exhibited a new train timetabling model based on a hybrid job shop scheduling problem and presented into a disjunctive graph. Sepehri and Pourseyed-Aghaee [8], presented a modified $\mathrm{B} \& \mathrm{~B}$ algorithm including 4 methods to achieve the optimum solutions for a single-track railway line. Zhou and Zhong [9] also introduced a modified B\&B algorithm which includes 3 methods to reduce the solution space. $\mathrm{B} \& \mathrm{~B}$ algorithms have also been experienced by other researchers as well as Walker, et al., [10]; Zhou and Zhong, [11] and D'Ariano et al., [12]. Shafia, et al., [13] proposed a new robust B\&B algorithm for train timetabeling problem.

Li, et al., [14] proposed a multi-objective train scheduling model by minimizing the energy and carbon emission cost as well as the total passenger-time, and named it as green train scheduling model. Sparing and

Goverde [15] proposed an optimization model for periodic timetable with dynamic frequencies, and used minimum cycle time as the objective function. Safarzadeh, et al., [16] proposed a new mathematical model with consideration of praying constraints. Their proposed model contains a large number of new variables, and constrains. The current proposed mathematical model and heuristic algorithm are computationaly tractable and are able to find the optimum solutions in a convenient time duration.

## 1. 2. Similarities of JSSP and Train Scheduling

 Problem A classical train scheduling problem is considered as scheduling $n$ trains on $m$ block sections, where all the safety requirements are satisfied. The objective function is to minimize the total weighted tardiness. Consider the railway network illustrated in Figure 1, which consists of 6 block sections embedded amongst 7 stations and 2 trains. Train $\mathrm{A}, \mathrm{T}_{\mathrm{A}}$, must pass block sections $1 \rightarrow 2 \rightarrow 3 \rightarrow 5 \rightarrow 6$ and train $\mathrm{B}, \mathrm{T}_{\mathrm{B}}$, passes block sections $4 \rightarrow 3 \rightarrow 2 \rightarrow 1$, as an example.It is easy to find that the above definition of train scheduling is similar to that of JSSP. To that end, by redefining the trains as jobs and block sections as machines, the JSSP mathematical model is also applicable to the train scheduling problem. Fakhrzad [17] Proposed a hybrid genetic algorithm for JSSP.

Jamili proposed a new mathematical model for a new variant JSSP, where for each job and/or a machine an operator is assigned. No operation can be initiated unless the operator is present. It was assumed that the operators must have some resting times during the daily working hours. Therefore, it is required to schedule the operations considering the required resting times.

Further to the mentioned similarities of classical JSSP and the train scheduling problem, the resting time constraints proposed by Jamili (2013) could be redefined as the praying constraints which are considered for Iranian railway planning as a Muslim society. Based on religion rules, Moslems are obliged to pray the God five times a day in specific durations. However, in the Shiat's case which is subsection of Islam, it accumulates to three times, including before sunrise, middle of the day and after sunset. The durations are not fixed and alter as the sunset and sunrise times change. To illustrate the conversion of JSSP to train scheduling problem, one can consider example 1.

Example 1: Consider tabular representation of a JSSP shown in Tables 1 and 2. All release times and due dates of jobs are supposed to be equal to 0 . The weights are all equal. Considering the TWT as the objective function, the optimum solution which is yielded through coding the proposed mathematical model in Lingo, will be found as Figure 2. Surprisingly, this example can be redefined as a complicated train scheduling problem which is shown in Figure 3.


Figure 1. A simple railway network


Figure 2. Optimal solution for Ex. 1


Figure 3. Rail Network, Ex. 1

TABLE 1. Processing Times (Ex. 1)

| Jobs |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $\mathbf{1}$ | $\mathbf{2}$ | $\mathbf{3}$ | $\mathbf{4}$ | $\mathbf{5}$ | $\mathbf{6}$ |
| $\mathbf{1}$ | 20 | 15 | 35 | 15 | 25 | 20 |
| $\mathbf{2}$ | 20 | 15 | 35 | 15 | 17 | 19 |
| $\mathbf{3}$ | 19 | 18 | 15 | 15 | 27 | -- |
| $\mathbf{4}$ | 19 | 18 | 15 | 15 | 17 | 19 |
| $\mathbf{5}$ | 19 | 18 | 15 | 21 | 15 | 27 |
| $\mathbf{6}$ | 14 | 16 | 20 | 21 | 15 | 27 |
| $\mathbf{7}$ | 20 | 14 | 15 | 25 | 20 | --- |
| $\mathbf{8}$ | 14 | 16 | 20 | 15 | 25 | 20 |

TABLE 2. Routing (Ex. 1)

| Jobs | Operation |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $\mathbf{1}$ | $\mathbf{2}$ | $\mathbf{3}$ | $\mathbf{4}$ | $\mathbf{5}$ | $\mathbf{6}$ |
| $\mathbf{1}$ | 1 | 2 | 3 | 4 | 5 | 6 |
| $\mathbf{2}$ | 1 | 2 | 3 | 7 | 8 | 9 |
| $\mathbf{3}$ | 6 | 5 | 10 | 2 | 1 | --- |
| $\mathbf{4}$ | 6 | 5 | 11 | 7 | 8 | 9 |
| $\mathbf{5}$ | 6 | 5 | 4 | 3 | 2 | 1 |
| $\mathbf{6}$ | 9 | 8 | 7 | 3 | 2 | 1 |
| $\mathbf{7}$ | 1 | 2 | 10 | 5 | 6 | --- |
| $\mathbf{8}$ | 9 | 8 | 7 | 11 | 5 | 6 |

As it is shown in Figure 3, this network consists of 9 stations. The distance between each one called blocksection is shown by the colored lines on the figure. The block sections are playing similar role as the machines in JSSP. The path between stations C and D is a doubletrack line, while the other routes are the single-track. Besides, station D is a passenger exchange one in which some trains must stop in tracks 4 or 11, depending on its destinations, for mounting and dismounting purposes. In this example, 8 trains, jobs in JSSP mode, travel form their origins to their destinations. The relevant data of trains and travelling time for each block are specified in Tables 3 and 4. Obviously, the data presented in these tables are completely adjusted to Tables 1 and 2. In other words, Tables 3 and 4 present the same data shown in Tables 1 and 2 but in a different form. For example, consider Job 1, similar to train 1, in Table 1. This table shows the processing times, similar to traveling times as shown in Table 4. Considering the routing shown in Table 2, similar to Table 3, the data presented in Table 1 matches the Table 4. The other jobs (trains) can be checked in a same way.

TABLE 3. Trains data

|  | OABLE 3. Trains data |  |  |
| :--- | :---: | :---: | :---: |
|  | Origin | Destination | Block- sections sequence |
| Train 1 | A | F | $1-2-3-4-5-6$ |
| Train 2 | A | I | $1-2-3-7-8-9$ |
| Train 3 | F | A | $6-5-10-2-1$ |
| Train 4 | F | I | $6-5-11-7-8-9$ |
| Train 5 | F | A | $6-5-4-3-2-1$ |
| Train 6 | I | A | $9-8-7-3-2-1$ |
| Train 7 | A | F | $1-2-10-5-6$ |
| Train 8 | I | F | $9-8-7-11-5-6$ |

TABLE 4. Required times to pass block-sections

| Train | Block Section |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $\mathbf{1}$ | $\mathbf{2}$ | $\mathbf{3}$ | $\mathbf{4}$ | $\mathbf{5}$ | $\mathbf{6}$ |
| $\mathbf{1}$ | 20 | 14 | 33 | 15 | 25 | 20 |
| $\mathbf{2}$ | 20 | 14 | 33 | 0 | 0 | 0 |
| $\mathbf{3}$ | 27 | 15 | 0 | 0 | 18 | 19 |
| $\mathbf{4}$ | 0 | 0 | 0 | 0 | 18 | 19 |
| $\mathbf{5}$ | 27 | 15 | 21 | 15 | 18 | 19 |
| $\mathbf{6}$ | 27 | 15 | 21 | 0 | 0 | 0 |
| $\mathbf{7}$ | 20 | 14 | 0 | 0 | 25 | 20 |
| $\mathbf{8}$ | 0 | 0 | 0 | 0 | 25 | 20 |
|  | $\mathbf{7}$ | $\mathbf{8}$ | $\mathbf{9}$ | $\mathbf{1 0}$ | $\mathbf{1 1}$ |  |
| $\mathbf{1}$ | 0 | 0 | 0 | 0 | 0 |  |
| $\mathbf{2}$ | 15 | 17 | 19 | 0 | 0 |  |
| $\mathbf{3}$ | 0 | 0 | 0 | 15 | 0 |  |
| $\mathbf{4}$ | 15 | 17 | 19 | 0 | 15 |  |
| $\mathbf{5}$ | 0 | 0 | 0 | 0 | 0 |  |
| $\mathbf{6}$ | 20 | 16 | 14 | 0 | 0 |  |
| $\mathbf{7}$ | 0 | 0 | 0 | 15 | 0 |  |
| $\mathbf{8}$ | 20 | 16 | 14 | 0 | 15 |  |

The train schedule is generally shown by a special graph which is somehow different from Figure 2, but it presents similar information. Therefore, as a result it is not repeated here.

1. 3. Contribution of the Paper In this paper, a new formulation for the train timetabling problem with consideration of Muslim praying constraints is presented. The proposed model is based on the JSSP. As the exact algorithms are able to solve only small-size instances in a reasonable amount of time, we present Electromagnetism-Like Mechanism algorithm to find some good solutions. These solutions are not guaranteed to be optimal, i.e., near-optimal solutions in a limited amount of time.
1. 4. Outline The current paper is organized as follows: In section 2, the problem formulation is introduced. Section 3 addresses the proposed heuristic algorithm to solve the problem. In section 4, the proposed algorithm is evaluated comparing with exact solutions as well as the simulated annealing and PSO algorithms. Finally, the concluding remarks are given at the end to summarize the contribution of this paper.

## 2. PROBLEM FORMULATION

To formulate the introduced problem, the following notations are used in the mathematical model.

## 2. 1. Sets and Parameters

A Set of block-sections
$J \quad$ Set of trains
$t_{i j} \quad$ Traveling time of train $i$ in block-section $j$
$w_{i} \quad$ Importance weight of train $i$
$d_{i} \quad$ The last block-section which is met by train $i$
$o_{i} \quad$ The first block-section which is met by train $i$
$r t^{k} \quad$ The assigned time for the $k$-th praying time
$N R \quad$ Number of praying times
$\underline{\tau}^{k} \quad$ Lower bound (LB) of the $k$-th praying time
$\bar{\tau}^{k} \quad$ Upper bound of the $k$-th praying time
$r_{i} \quad$ Release time of train $I$ at its origin
A parameter which equals 1 if the release time of train $i$ is more than $\underline{\tau}^{k}$ and 0 , otherwise

## 2. 2. Variables

$\operatorname{arr}_{i j} \quad$ The arrival time of train $i$ on block-section $j$
$y_{i p j} \quad$ A binary variable which equals 1 if train $i$ precedes train $p$ on block-section $j$ and 0 , otherwise
$p_{i j}^{k} \quad$ A binary variable which equals 1 if the $k$-th praying time is taken part just after passing block-section $j$ by train $i$ and 0 , otherwise.
2. 3. Mathematical Model The mathematical model for the train scheduling problem with consideration of Muslim praying constrains is designed based on the JSSP with resting constraints proposed by Jamili and proposed as follows:
Model P1:
$\min \sum_{i \in J} w_{i} \times \operatorname{arr}_{i d_{i}}$
S.t.
$\operatorname{arr}_{p j}-\operatorname{arr}_{i j}+M\left(1-y_{i p j}\right) \geq t_{i j}, \quad \forall i, p \in J, j \in A$
$\operatorname{arr}_{i j}-\operatorname{arr}_{p j}+M \times y_{i p j} \geq t_{p j}, \quad \forall i, p \in J, j \in A$
$\operatorname{arr}_{i j} \geq \operatorname{arr}_{i j^{j}}+t_{i j}+\sum_{k=1}^{N R}\left(r t^{k} \times p_{i j}^{k}\right), \quad \forall i \in J, j \in A$
$\sum_{j \in A} p_{i j}^{k} \leq 1, \quad \forall i \in J, k \in[1, N R]$
$\operatorname{ar}_{r_{i j}} \geq t_{i j}+p_{i j}^{k} \times\left(\underline{\tau}^{k}+r t^{k}\right), \quad \forall i \in J, j \in A, k \in[1, N R]$
$\operatorname{arr}_{i j} \leq \underline{\tau}^{k}+M \times \sum_{j \in A^{\prime}} p_{i j}^{k}+M \times z_{i}^{k}$,
$\forall i \in J, j \in A, k \in[1, N R]$
$\operatorname{arr}_{i j} \leq \bar{\tau}^{k}-r t^{k}+M \times\left(1-p_{i j}^{k}\right)+M \times z_{i}^{k}$
$\forall i \in J, j \in A, k \in[1, N R]$
$\operatorname{arr}_{i_{i o_{i}}} \geq r_{i}+t_{i i_{i}}$,
$\forall i \in J$
$y_{i p k}, p_{i j}^{\prime}, \ldots, p_{i j}^{(n)} \in\{0,1\}$
The objective function is to minimize the total arrival times to destinations. Constraints 2 and 3 eliminate any possible train crashing in block-sections. Constraint 4 indicates that the arrival time of train $i$ on block-section $j$ is bigger than the arrival time of this train on the previous block-section, $j^{\prime}$, plus the traveling time and possible resting time. Constraints 5, 6,7 and 8 are used to adjust variable $p_{i j}^{k}$. Constraint 9 indicates that train $i$ allows to depart its origin only after its release time.

To validate the proposed mathematical model, the example specified in section 1.2., is solved by Lingo software package, and the optimum solution is presented in Figure 2. It can be easily found that the trains (jobs) passes block-sections (machines) in a way that the delays are minimized and the praying constraint is well performed in the colored box. The optimum solution is presented if Table 5.

TABLE 5. Optimum Solution for Ex. 1

| Train No. | Block-section No. | Departure | Arrival |
| :---: | :---: | :---: | :---: |
| 1 | 1 | 49 | 69 |
| 1 | 2 | 69 | 83 |
| 1 | 3 | 120 | 153 |
| 1 | 4 | 153 | 168 |
| 1 | 5 | 168 | 193 |
| 1 | 6 | 193 | 213 |
| 2 | 1 | 0 | 20 |
| 2 | 2 | 20 | 34 |
| 2 | 3 | 34 | 67 |
| 2 | 7 | 70 | 85 |
| 2 | 8 | 85 | 102 |
| 2 | 9 | 122 | 141 |
| 3 | 1 | 69 | 96 |
| 3 | 2 | 54 | 69 |
| 3 | 5 | 19 | 37 |
| 3 | 6 | 0 | 19 |
| 3 | 10 | 39 | 54 |
| 4 | 5 | 38 | 56 |
| 4 | 6 | 19 | 38 |
| 4 | 7 | 85 | 100 |
| 4 | 8 | 104 | 121 |
| 4 | 9 | 141 | 160 |
| 4 | 11 | 70 | 85 |
| 5 | 1 | 150 | 177 |
| 5 | 2 | 135 | 150 |
| 5 | 3 | 90 | 111 |
| 5 | 4 | 75 | 90 |
| 5 | 5 | 57 | 75 |
| 5 | 6 | 38 | 57 |
| 6 | 1 | 123 | 150 |
| 6 | 2 | 88 | 103 |
| 6 | 3 | 67 | 88 |
| 6 | 7 | 30 | 50 |
| 6 | 8 | 14 | 30 |
| 6 | 9 | 0 | 14 |
| 7 | 1 | 20 | 40 |
| 7 | 2 | 40 | 54 |
| 7 | 5 | 75 | 100 |
| 7 | 6 | 100 | 120 |
| 7 | 10 | 54 | 69 |
| 8 | 5 | 100 | 125 |
| 8 | 6 | 145 | 165 |
| 8 | 7 | 50 | 70 |
| 8 | 8 | 34 | 50 |
| 8 | 9 | 14 | 28 |
| 8 | 11 | 85 | 100 |

The JSSP as well as train scheduling problem are known to be NP-Hard. Therefore, the optimum solutions for real-world problems could not be found in a reasonable amount of time. Moreover, real case studies are always large in scale, and as a result the exact methods are not practical. Jamili shows that in this particular problem, meta heuristic methods outperforms the well known heuristic beam search method in large
scale examples. Thus, in the next section, a heuristic algorithm based on The Electromagnetism-like Mechanism (EM) is introduced.

## 3. A HEURISTIC ALGORITHM BASED ON ELECTROMAGNETISM-LIKE MECHANISM

Electromagnetism-like mechanism (EM) is a population based meta-heuristic which simulates the attractionrepulsion mechanism of electromagnetism theory that is based on Coulomb's law. EM embodies some attractive characteristics: (1) comparing with other evolutionary algorithms (e.g., GA), it possesses memory; whereas in GA, the previous characteristics of the problem are lost once the population alters. (2) EM has constructive cooperation amongst the particles. From the other point of view, similar to evolutionary algorithms, EM often easy to be premature convergence so that exploration should be enhanced and well balanced to achieve better performance. Thus, the authors of this paper employ a huristic method into EM in order to achieve the results with higher quality.

Each particle represents a solution and the charge of each particle relates to its objective function value. The better OFV of the particle, the higher charge the particle has. To compute the force between two points, a charge, such as value $q_{i}$, is assigned to each point. The charge of the point is calculated according to the relative efficiency of the objective function values in the current population, as given in Equation (10).
$q_{i}=\exp \left(-n \times \frac{f\left(x_{i}\right)-f\left(x_{\text {best }}\right)}{\sum_{j=1}^{p o p s i z e}\left(f\left(x_{j}\right)-f\left(x_{\text {best }}\right)\right)}\right)$,
$\forall i=1,2, \ldots$, popsize
where, $q_{i}$ is the charge of particle $i$. In addition, $f\left(x_{i}\right)$, $f\left(x_{\text {best }}\right)$, and $f\left(x_{j}\right)$ denote the objective values of particle $i$, the best solution, and particle $j$, respectively. Finally, popsize is the population size. In this way, the points that have better objective function values possess higher charges. Note that, unlike electrical charges, no signs are attached to the charge of an individual point in the Equation (10). Instead, the direction of a particular force between two points is determined after comparing their objective function values. The total force, $F_{i}$, exerted on candidate solution $i$ is also calculated by:
$F_{i}=\left\{\begin{array}{c}\sum_{j \neq i}^{\text {popsize }}\left(x_{j}-x_{i}\right) \frac{q_{j} \times q_{i}}{\left\|x_{j}-x_{i}\right\|^{2}} ; \\ f\left(x_{j}\right)<f\left(x_{i}\right) \\ \sum_{j \neq i}^{\text {popsize }}\left(x_{i}-x_{j}\right) \frac{q_{j} \times q_{i}}{\left\|x_{j}-x_{i}\right\|^{2}} ; \\ f\left(x_{j}\right) \geq f\left(x_{i}\right)\end{array}\right.$
The next procedure is to move the candidate solutions based on the total force calculated by Equation
(11). All the candidate solutions are moved with the exception of the current best solution. The move for each candidate solution is in direction of total force exerted on it by a random step length. This length is generated from uniform distribution between $(0,1)$, see Equation (12). We can guarantee that candidate solutions have a nonzero probability to move to the unvisited solution along this direction by selecting random length.
$\left\{\begin{array}{c}x_{i k}=x_{i k}+\operatorname{Random}(0,1) \times F_{i k}\left(1-x_{i k}\right) ; \\ F_{i k}>0 \\ x_{i k}=x_{i k}+\operatorname{Random}(0,1) \times F_{i k}\left(x_{i k}\right) ; \\ F_{i k} \leq 0\end{array}\right.$
The random keys ( RKs ) are selected as the encoding scheme which is well experienced and is easy to adjust to the EM algorithm. Using the idea of Rks is well exploited in the literature. For example, Jamili, et al. [18] studied the periodic JSSP, and presented an EM hybridized with a simulated annealing algorithm and used random key representation to encode a solution. Tavakkoli-Moghaddam, et. al, [19] proposed a hybridization of simulated annealing and electromagnetic-like mechanism for a variant type of JSSP, and used the RKs to represent a schedule. In order to represent a solution by Rks, a random number between 0 and 1 is assigned to each operation. These RKs are then sorted to find the relative order of operations. The operations are then scheduled based on the order of assigned random keys. In the remaining part of this section, the proposed algorithm based on the EM is introduced. At first, the proposed method to find a candidate solution, i.e. a train time schedule, based on the random keys are as follows:
Step 1. $\mathrm{k} \leftarrow 0$
Step 2. Based on the assigned randome key find the k-th train-block section.
Step 3. Consiedring the pre-specified praying time thresholds if the selected train should be stopped for praying purpose, conisder this dwell time in the time schedule.
Step 4. Compute the departure and arrival times for the selected train based on the arriavl time of selected train to the end of the previous block section and the occupation times of the selected block section.
Step 5. If all train-blocksections are scheduled, terminate the algorithm, otherwise $\mathrm{k} \leftarrow \mathrm{k}+1$, and go to Step 2.
Note. In train scheduling problem, the act of passing a block section by a train shown by train-blocksection in the above algorithm has the similar role as the operation in JSSP. Moreover, a local search procedure is used to perform a quick exploration around a solution. The procedure is to explore the possibility of finding a solution which may improve the objective function. This method is based on finding the most delayed trains in the ttimetable and increasing the assigned random
keys to these trains. The comparative outcomes demonstrated that the procedure is very effective for the proposed problem. The introduced local search is simply applied to each candidate solution in each population of EM algorithm. To that end, for each candidate solution, the following steps are employed:
Step 1: Select one of the RKs randomly
Note that each random key is associated to one distinct operation.
Step 2: Register the random key, i.e. a random number between 0 and 1 is assigned to it.
Step 3: Find the new schedule considering the new changes on the order of operations.
Step 4: Go to Step 1, if the objective function is not improved or the termination criterion is not fulfilled.
The proposed EM algorithm for the proposed train scheduling problem is as follows:
Step 1: Generate popsize initial schedules: $s_{1}^{r}$, $r=1, \ldots$, popsize. Find the objective function value (OFV) for each particle. Set $P_{l}^{r} \leftarrow s_{1}^{r}$.
Step 2: Calculate $q_{r}$ and $F_{r}$ for each schedule. Update the schedules based on Equation (12).
Step 3: Run the Local Search Algorithm. If the OFV of each of the schedules improved let $s_{1}^{r} \leftarrow P_{l}^{r}$. Update $P_{g}$.
Step 4: If the termination criterion is not met, go to Step 2; otherwise, return $P_{g}$ as the best found schedule.
where, $P_{l}^{r}$ is the best local solution that the $r$-th particle has achieved. $P_{g}$ is the best solution obtained in the whole population.

## 4. EVALUATION OF THE PROPOSED EM-BASED ALGORITHM

In this section, the performance of the proposed algorithm is compared with that of a simulated annealing (SA) algorithm, and PSO algorithm proposed by Jamili [13] using medium and large-size problems. The required data contains: (1) number of trains, (2) number of Block-sections, (3) travelling times, (4) routing of trains and (5) the praying times. The comparison results are shown in Table 6.

Note that RPD stands for relative percentage deviation which is used as a performance measure to compare the algorithms and is obtained by the following formula.
$R P D_{k}=\frac{O F V_{k}-\text { Min }_{k}}{\operatorname{Min}_{k}}$
The results are illustrating that the RPD avarage for SA, PSO, and EM algorithms equal $2.14 \%, 0.83 \%$, and $0.65 \%$ respectively. The computed avearge RPD demonstrates that the EM relatively outperformes the SA, and PSO. Further to the randomly generated instances, the authors are studied the Tehran-Isfahan Railway line as a real case study issue. This line consists of 19 stations which are connected by 18 block sections. It is aimed at scheduling 16 trains, 4 low-speed and 12 medium speed. The travelling times of mediumspeed trains are shown in Table 7. The travelling times of low-speed trains are $50 \%$ greater than that of medium-speed ones. Trains must stop 20 minutes in stations immediately after the commence of praying times. The lower and upper bounds of praying times are shown in Table 8. The authors have solved the problem by the proposed EM algorithm. Based on all above assumptions, the best found schedule is illustrated in Figure 4. The train stops are shown through colored circles..

TABLE 6. The comparison results between proposed EM algorithm with those obtained by SA and PSO algorithms

| Trains | Block sections | TWT SA | TWT PSO | TWT EM | RPD SA \% | RPD PSO\% | RPD EM\% |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 10 | 10 | 681 | 681 | 681 | 0.0 | 0.0 | 0.0 |
| 10 | 12 | 724 | 720 | 720 | 0.6 | 0.0 | 0.0 |
| 10 | 15 | 800 | 834 | 796 | 0.5 | 4.8 | 0.0 |
| 12 | 15 | 1175 | 1175 | 1175 | 0.0 | 0.0 | 0.0 |
| 15 | 12 | 1870 | 1852 | 1872 | 1.0 | 0.0 | 1.1 |
| 15 | 14 | 2340 | 2296 | 2311 | 1.9 | 0.0 | 0.7 |
| 15 | 15 | 3198 | 3073 | 3112 | 4.1 | 0.0 | 1.3 |
| 16 | 14 | 2965 | 2666 | 2665 | 11.3 | 0.0 | 0.0 |
| 18 | 14 | 2877 | 2875 | 2887 | 0.1 | 0.0 | 0.4 |
| 20 | 15 | 3434 | 3444 | 3427 | 0.2 | 0.5 | 0.0 |
| 25 | 15 | 8187 | 7858 | 7901 | 4.2 | 0.0 | 0.6 |
| 30 | 15 | 10354 | 11008 | 10656 | 0.0 | 6.3 | 2.9 |
| 35 | 15 | 17980 | 17621 | 17910 | 2.0 | 0.0 | 1.6 |
| 40 | 15 | 19665 | 18884 | 18999 | 4.1 | 0.0 | 0.6 |

TABLE 7. The travelling times of trains in passing block sections

| Block sections | $\mathbf{1}$ | $\mathbf{2}$ | $\mathbf{3}$ | $\mathbf{4}$ | $\mathbf{5}$ | $\mathbf{6}$ | $\mathbf{7}$ | $\mathbf{8}$ | $\mathbf{9}$ |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Travelling times | 12 | 10 | 12 | 22 | 23 | 13 | 17 | 18 | 17 |
| Block sections | $\mathbf{1 0}$ | $\mathbf{1 1}$ | $\mathbf{1 2}$ | $\mathbf{1 3}$ | $\mathbf{1 4}$ | $\mathbf{1 5}$ | $\mathbf{1 6}$ | $\mathbf{1 7}$ | $\mathbf{1 8}$ |
| Travelling times | 17 | 32 | 14 | 19 | 24 | 29 | 15 | 16 | 20 |

TABLE 8. The lower and upper bounds of praying times

| Parameters | $\underline{\tau}^{1}$ | $\bar{\tau}^{1}$ | $\underline{\tau}^{2}$ | $\bar{\tau}^{2}$ | $\underline{\tau}^{3}$ | $\bar{\tau}^{3}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Time | $04: 00$ | $05: 00$ | $12: 00$ | $17: 00$ | $17: 00$ | $24: 00$ |



Figure 4. The Tehran-Isfahan Graph

Figure 4 shows that all trains are stopped for 20 minutes whenever they passed the time-thresholds specified for praying purpose. Note that in the studied case, each block section is considered as the distance between two adjacent stations. However, the idea can be extended where the distance between each pair of stations are decomposed into several block sections. In real-time scheduling problem, the same method should be considered to cover the praying conditions. For simplicity, in order to reach a new timetable in a limited amount of time, it is suggested to prioritize the trains based on the remained free slack times to satisfy the praying conditions. The new timetable is generated considering the computed priorities.

## 5. CONCLUDING REMARKS

Train scheduling problem with praying constraints was discussed as an application of Job shop scheduling problem with consideration of resting times. An
effective heuristic algorithm which is based on the EM enriched by a local search algorithm was presented. Finally the implementation of the proposed EM algorithm was demonstrated as a proof of the effectiveness of the new approach by comparing the outputs with those of PSO, and SA algorithm. Finally, Tehran-Esfahan railway line is studied and the best found time table is reported. The results demonstrated that the proposed algorithm is able to schedule the praying times so that in many cases the trains with different directions meet each other during the praying times. By this performance, the generated timetables were successful in reducing the unwilling delays. In the studied railway line, $33 \%$ of praying times had dual functions, and the other praying times were scheduled so that, they did not affect the other trains.

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# Train Scheduling Problem with Consideration of Praying Constraint as an Application of Job Shop Scheduling Problem 

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اين مقاله ايده زمانبندى كار كار گاهى با محدوديت در نظر گيرى زمان استراحت را به مساله زمانبندى حركت قطارها با


قرار گرفته است. در ادامه كارايى روش بيشنهادى با مقايسه آن با جوابهاى بهينه براى مثالهاى كوچکى و جوابهاى


در صورت در نظر گيرى محدوديت نماز، نرخ جمع كل تاخيرات قطارها و زمانهاى توقف به دليل نماز، به ترتيب برابر


قطارهاى با جهات مختلف يكديگر را در زمانهاى نماز ملاقات مى نمايند.


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