International Journal of Engineering

Journal Homepage: www.ije.ir

Global Stability for Thermal Convection in a Couple Stress Fluid Saturating a Porous Medium with Temperature-pressure Dependent Viscosity: Galerkin Method

Sunil [*], R. Devi

Department of Mathematics, National Institute of Technology, Hamirpur, (H.P.)–177005, India

A R T I C L E I N F O

Article history:
Received 24 August 2011
Received in revised form 5 February 2012
Accepted 19 April 2012

Keywords:
Couple-stress Fluid
Porous Medium
Global Stability
Temperature-pressure Dependent Viscosity
Different Conducting Boundaries (Free-free, Rigid-free, Rigid-rigid)

A B S T R A C T

A global nonlinear stability analysis is performed for a couple-stress fluid layer heated from below saturating a porous medium with temperature-pressure dependent viscosity for different conducting boundary systems. Here, the global nonlinear stability threshold for convection is exactly the same as the linear instability boundary. This optimal result is important because it shows that linearized instability theory has captured completely the physics of the onset of convection. The eigenvalue problems for different conducting boundary systems are solved by using Galerkin method. The effects of couple-stress parameter F, Darcy-Brinkman number $\tilde{D}a$ and variable viscosity parameter Γ on the onset of convection are also analyzed. The use of Darcy-Brinkman model makes the system thermally more stable than the Darcy model for all the different conducting boundary systems, couple-stress parameter and medium permeability promotes stabilization, and the variable viscosity destabilizes the system.

doi: *10.5829/idosi.ije.2012.25.03a.04*

1. INTRODUCTION

Conventional hydrodynamic stability theory is mainly concerned with the determination of critical values of Rayleigh number. This theory demarcates a region of stability from that of instability. The potentials of linear theory of stability and of the energy method are complementary to each other. The linear theory gives conditions under which hydrodynamic systems are definitely unstable. It tells nothing about the stability of the system. On the other hand, energy theory gives conditions under which hydrodynamic systems are definitely stable. It cannot with certainty conclude instability. Suffering from its basic assumptions, the validity of the linearized stability theory becomes questionable. Hence, the non-linear approach becomes inevitable to investigate the effect of finite disturbances. The formulation and derivation of the basic equation of a layer of fluid heated from below in porous medium using Boussinesq approximation has been given in a treatise by Joseph [1]. When a fluid flows in an isotropic and homogenous porous medium, the gross effect is represented by the Darcy's Law. The study of a layer of fluid heated from below in porous media is motivated by both theoretically and its practical application in engineering.

Among the applications in engineering disciplines one can find the application of the present study in food process industry, chemical process industry, solidification and centrifugal casting of metals. The oldest method of nonlinear stability analysis which can deal with finite disturbances is the energy method, originated by Orr [2] and its recent revival has been inspired by the work of Serrin [3] and Joseph [4,5]. Rapid improvements of the classical energy theory have been made in recent years (Galdi and Padula [6]). The motivation of the present study is due to the application of energy method, pioneered and developed in its modern use way by Straughan [7, 8]. Straughan [9] developed a sharp non-linear energy stability analysis for the Darcy's equations of thermal convection in a fluid saturated porous medium and the results obtained are the best possible showing that subcritical instabilities are not possible. By selecting the optimally, it has been possible to sharpen the stability bound in many physical problems [8]. A nonlinear stability

*Corresponding Author Email: *sunilnitham@gmail.com* (Sunil)

analysis of fluids by using generalized energy stability theory has been considered by many authors [10-15]. Recently, Sunil and Mahajan [16-21] studied the nonlinear stability analysis for thermal convection in a magnetized ferrofluid heated from below saturating a porous medium. They found that the non-linear critical stability magnetic thermal Rayleigh number does not coincide with that of the linear instability analysis, and thus indicates that the subcritical instabilities are possible. However, it is noted that, in the case of non-ferrofluid, global nonlinear stability Rayleigh number is exactly the same as that for linear instability.

There are a lot of analyses of performance and experiment in the couple-stress lubricant. Stokes [22] proposed the simplest theory called the Stokes microcontinum theory and which could be used for the simulation of couple-stress fluid. This kind of couple-stress model is intended to take account of the particle-size effects, and it is also very useful in the scientific and engineering application. One of the applications of couple-stress fluid is its use to the study of the μ_0, ρ_0, T_U, δ, γ mechanism of lubrication of synovial joints, which has become the object of scientific research. A human joint is a dynamically loaded bearing that has auricular cartilage as the bearing and synovial fluid as the lubricant. When a fluid film is generated, squeeze-film action is capable of providing considerable protection to the cartilage surface. Ramanaiah [23] applied the couple-stress fluid model to analyze the long slider bearing. Shehawey and Mekheimer [24] applied the couple-stress model to analyze the peristalsis problem for its relative mathematical simplicity. Das [25] proposed the analysis of elastohydrodynamic theory of line contacts. Das [26] studied the slider bearing lubricated with couple-stress, magnetic parameters and the shape of bearings. Gupta and Sharma [27] also used the couple-stress fluid model to carry out a hydrostatic truest bearing. Sharma, et al [28] and Sharma and Thakur [29] have studied the problems of couple-stress fluid heated from below in porous medium in hydromagnetic and rotation separately. Malashetty et al. [30] have studied the onset of convection in the couple-stress fluid saturating a porous layer by using the thermal non-equilibrium model. Abdallah and Lotfi [31] proposed an efficient numerical scheme to solve the direct lubrication problem for journal bearing lubricated with couple-stress fluids, which consists of the modified Reynolds equation, the film thickness equation, and the boundary for the pressure field. Hsu et al. [32] studied the combined effects of couple-stress and surface roughness using journal bearings lubricated with the non-Newtonian fluid. It was found that the combined effects of couple-stress and surface roughness can improve the load carrying capacity and decrease the attitude angle and friction parameter. Lahmar [33] also found that the lubricants with couple-stress fluid would increase the load carrying capacity and stability, and decrease the friction factor and the attitude angle. The above researches are about the application of couple-stress fluids, and all the results of their studies emphasized that the couple-stress fluids are more stable than the traditional Newtonian fluids. More recently, Sunil et al. [34] studied the global stability analysis for thermal convection in a couple-stress fluid.

At normal operating conditions, the viscosity of an incompressible fluid is assumed to be independent of the pressure. However, it is well known that the viscosity of a fluid can change with pressure, and if the pressure range is significantly large, the viscosity can change by several orders of magnitude. Thus one could consider such liquids as incompressible fluids with pressure dependent viscosities. In his celebrated paper on the response of fluids, Stokes [35] notes that the viscosity of a fluid could depend upon its pressure. However, based on the experiments of Du Buat on the flow of water in canals and normal operating conditions, Stokes suggested that the viscosity could be considered a constant for flows. Stokes is however very careful to delineate the class of flows wherein viscosity might be considered a constant and he also remarks that such an assumption would be invalid under other flow conditions. More recently, Laun [36] has modeled the viscosity of polymer melts through:

$$\mu(p,T) = \mu_0 \exp\left[\delta(p-p_0) - \gamma(T-T_U)\right]$$

where μ_0 is the viscosity at pressure p_0 and temperature T_U, and δ and γ are non-negative constant. There have been numerous other experiments by Bair and co-workers that show that the dependence of the viscosity on the pressure is exponential [37]. Mention must also be made of the work of Martin-Alfonso and co-workers [38, 39] wherein an intricate relationship between the temperature, viscosity and pressure are provided for bitumen. Rajagopal et al. [40] extended the approximation due to Oberbeck and Boussinesq to the case of a fluid whose viscosity depends on both the temperature and pressure. They showed that the principal of exchange of stabilities holds and that the critical Rayleigh numbers for the linear and nonlinear stability coincide.

The objective of the present article is to study the nonlinear stability analysis of couple-stress fluid heated from below saturating a porous medium of high permeability with temperature-pressure dependent viscosity via generalized energy method using a Brinkman model [41] for different conducting boundary systems by Galerkin method. It is believed that for the flow of a high porosity porous medium the Brinkman equation removes some of the deficiencies and gives preferable result. In the work of Qin and Kaloni [42], it was remarked that for high porosity materials and when boundary layer effects need to be taken into account, Brinkman model is superior to that of Darcy. Here, we establish the optimal result, that is the linear instability and nonlinear stability Rayleigh numbers coincides with each other, couple-stress parameter F and medium permeability in Darcy-Brinkman model stabilizes the system whereas variable-viscosity parameter destabilizes the system. This problem, to the best of our knowledge, has not been investigated yet.

2. MATHEMATICAL FORMULATION OF THE PBOBLEM

Consider an infinite, horizontal layer of thickness 'd' of incompressible thin couple-stress fluid with temperature-pressure dependent viscosity heated from below saturating an isotropic homogeneous porous medium of porosity ε and medium permeability K_1. The fluid is assumed to occupy the layer $z \in (0,\ \mathrm{d})$. The temperature T at the bottom and top surfaces $z = 0, d$ are T_L and T_U, respectively, and a uniform temperature gradient $\beta \left(= \left| \frac{dT}{dz} \right| \right)$ is maintained. The gravity field $\mathbf{g} = (0, 0, -g)$ pervades the system in the negative z-direction (Figure 1).

The equations governing the flow of an incompressible couple-stress fluid (utilizing the Boussinesq approximation), are given by the following [22, 43]:

$$\nabla p + \rho_0 g \hat{\mathbf{k}} = 0 \tag{1}$$

$$-\nabla P + \rho_0 g \alpha (T - T_U) \hat{\mathbf{k}} - \frac{1}{K_1}\left[\mu(p,T) - \mu' \nabla^2 \right] \mathbf{q}$$

$$+ \tilde{\mu} \nabla^2 \mathbf{q} = \mathbf{0} \tag{2}$$

$$\nabla \cdot \mathbf{q} = 0 \tag{3}$$

$$C^m \frac{\partial T}{\partial t} + \beta \rho_0 C_0 \mathbf{q} \cdot \nabla T = \kappa \nabla^2 T \tag{4}$$

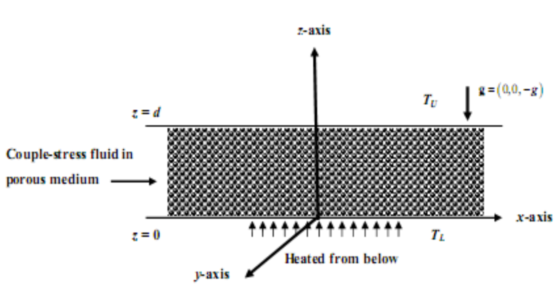

Figure 1. Geometrical configuration of the problem.

where ρ_0, $\mathbf{q}$, $\mathbf{g}$, t, p, P, μ, μ', $\tilde{\mu}$, κ, K_1, C^m, C_0 and α are the reference density of the fluid, filter velocity, acceleration due to gravity, time, pressure field due to gravity, pressure due to thermal expansion of the fluid, coefficient of viscosity, coefficient of visco-elasticity, effective viscosity, thermal conductivity, permeability of porous medium, overall heat capacity per unit volume, specific heat at constant pressure and coefficient of thermal expansion, respectively and assume that T_L and T_U are the constant temperatures of the lower and upper surfaces of the layer.

The appropriate boundary conditions to append to Equations (1-4) :

$$\mathrm{T}(\mathrm{x},\mathrm{y},0,\mathrm{t}) = \mathrm{T_L},\ \mathrm{T}(\mathrm{x},\mathrm{y},\mathrm{d},\mathrm{t}) = \mathrm{T_U},$$

$$\mathrm{p}(\mathrm{x},\mathrm{y},\mathrm{d},\mathrm{t}) = \mathrm{p_0} \tag{5}$$

where $\mathrm{p_0}$ is the reference pressure.

Now, it is convenient to non-dimensionalize Equations (1-5) by introducing the following set of non-dimensional quantities and parameters

$$\left.\begin{aligned}
& z^* = \frac{z}{d},\ \mathbf{q}^* = \frac{\mathbf{q}}{S},\ t^* = N\frac{S}{d}t,\ p^* = \frac{p - p_0}{\rho_0 g d}, \\
& \mu^* = \frac{\mu}{\mu_0},\ P^* = \frac{K_1 P}{\mu_0 S d},\ T^* = \frac{T}{\tilde{T}},\ F = \frac{1}{\nu}\frac{\mu'}{\rho_0 d^2}, \\
& S = \frac{\kappa}{\beta \rho_0 C_0 d},\ \tilde{T} = S\sqrt{\frac{\beta \mu_0 (T_L - T_U) d C_0}{g \alpha K_1 \kappa}}, \\
& N = \frac{\beta \rho_0 C_0}{C^m}, Da = \frac{K_1}{d^2},\ \tilde{D}a = \frac{\tilde{\mu} K_1}{\mu_0 d^2} = VrDa, \\
& Vr = \frac{\tilde{\mu}}{\mu_0},\ R = \rho_0 \sqrt{\frac{\beta g \alpha K_1 (T_L - T_U) d C_0}{\mu_0 \kappa}},
\end{aligned}\right\} \tag{6}$$

where, $\mu_0 = \mu(p_0, T_U)$ is the viscosity at the reference state (p_0, T_U), $\Re = R^2$ is the thermal Darcy-Rayleigh number, $\tilde{D}a$ is the Darcy-Brinkman number, Da is the Darcy number and F is the couple-stress parameter. With this scaling the non-dimensional form of Equations (1-5) becomes (omitting the asterisks):

$$\nabla p + \hat{\mathbf{k}} = \mathbf{0} \tag{7}$$

$$-\nabla P - \mu(p,T)\mathbf{q} + \left(F + \tilde{D}a\right)\nabla^2 \mathbf{q} + R\left(T - \frac{T_U}{\tilde{T}}\right)\hat{\mathbf{k}} = 0 \tag{8}$$

$$\nabla \cdot \mathbf{q} = 0 \tag{9}$$

$$\frac{\partial T}{\partial t} + \mathbf{q} \cdot \nabla T = \nabla^2 T \tag{10}$$

$$p = 0 \quad \text{at} \quad z = 1 \tag{11}$$

$$\mathbf{q}=\mathbf{0} \quad \text{at} \quad z=0,\ 1 \tag{12}$$

$$T=\frac{T_L}{\tilde{T}} \quad \text{at} \quad z=0 \tag{13}$$

$$T=\frac{T_L}{\tilde{T}}-R \quad \text{at} \quad z=1 \tag{14}$$

Our aim is to study the steady static conduction solution to Equations (7-14), given by the following system of :

$$\overline{p}=-(z-1) \tag{15}$$

$$\overline{\mathbf{q}}=\mathbf{0} \tag{16}$$

$$\overline{T}=-R(z-1)+\frac{T_U}{\tilde{T}} \tag{17}$$

$$\overline{P}=-\Re z\left(\frac{z}{2}-1\right)+P_0 \tag{18}$$

In order to study the stability of the conduction solution, we introduce the perturbations:

$\mathbf{q}'=u\hat{\mathbf{i}}+v\hat{\mathbf{j}}+w\hat{\mathbf{k}},\ \theta,\ p'$ and P' to $\overline{\mathbf{q}},\ \overline{T},\ \overline{p},\ \overline{P}$

respectively, i.e.:

$$\mathbf{q}=\overline{\mathbf{q}}+\mathbf{q}',\ \ T=\overline{T}+\theta,\ \ p=\overline{p}+p',\ \ P=\overline{P}+P' \tag{19}$$

From Equations (7-10), we find that the perturbations satisfy:

$$-\nabla P'-\mu\left(\overline{p},\overline{T}+\theta\right)\mathbf{q}'+\left(F+\tilde{D}a\right)\nabla^2\mathbf{q}'+R\theta\hat{\mathbf{k}}=\mathbf{0} \tag{20}$$

$$\nabla\cdot\mathbf{q}'=0 \tag{21}$$

$$\frac{\partial\theta}{\partial t}-Rw+\mathbf{q}'\cdot\nabla\theta=\nabla^2\theta \tag{22}$$

in $\mathbb{R}^2\times(0,d)\times(0,\infty)$ with the initial condition:

$$\theta(x,0)=\theta_0(x) \qquad \forall\ x\in\Omega \tag{23}$$

and the boundary conditions:

$$p'(x,y,d,t)=0 \text{ at } z=0,\ \ \theta=u=v=w=0 \text{ at } z=0,\ 1 \tag{24}$$

$\mathbf{q}$, θ and P must subject to the boundary conditions and we assume that $\mathbf{q}$, θ and P are periodic in x, y with periods $2\pi/a_x$ and $2\pi/a_y$ in the x and y directions. Let us denote by V the period cell

$$V=\left[0,\frac{2\pi}{a_x}\right]\times\left[0,\frac{2\pi}{a_y}\right]\times[0,1]$$

and let $a=\left(a_x^2+a_y^2\right)^{1/2}$ be the wave number.

3. NONLINEAR STABILITY ANALYSIS

Since, we have assumed that the viscosity is an analytic function of the temperature and pressure, for sufficiently small disturbances, we can expand the viscosity in the following manner:

$$\mu\left(\overline{p},\overline{T}+\theta\right)\mathbf{q}'=\left[\sum_{n=0}^{\infty}\frac{1}{n!}\frac{\partial^n\mu}{\partial T^n}\left(\overline{p},\overline{T}\right)\theta^n\right]\approx\mu(z)\mathbf{q}' \tag{25}$$

where

$$\mu(z)=\mu\left(\overline{p},\overline{T}\right) \tag{26}$$

On multiplying Equation (20) by $\mathbf{q}'$, Equation (22) by θ, and then integrating over V, we get:

$$R\langle w\theta\rangle-\left(F+\tilde{D}a\right)\|\nabla\mathbf{q}'\|^2-\mu\left(\overline{p},\overline{T}+\theta\right)\|\mathbf{q}'\|^2=0 \tag{27}$$

$$\frac{1}{2}\frac{d}{dt}\|\theta\|^2=R\langle w\theta\rangle-\|\nabla\theta\|^2 \tag{28}$$

where $\langle\cdot\rangle$ denotes the integration over V and $\|\cdot\|$ denotes the $L^2(V)$ norm.

To study the nonlinear stability of the basic state, an L^2 energy, $E(t)$, is constructed by using Equations (27) and (28), and the evolution of $E(t)$ is given by:

$$\frac{dE}{dt}=I_0-D_0 \tag{29}$$

where

$$E=\frac{1}{2}\|\theta\|^2 \tag{30}$$

$$I_0=(1+\lambda)R\langle w\theta\rangle \tag{31}$$

$$D_0=\|\nabla\theta\|^2+\lambda\mu(z)\|\mathbf{q}'\|^2+\lambda\left(F+\tilde{D}a\right)\|\nabla\mathbf{q}'\|^2 \tag{32}$$

with λ as the positive coupling parameter. We now define,

$$m=\max_H\frac{I_0}{D_0} \tag{33}$$

where H is the space of admissible solutions. Then, we require $m<1$ so that:

$$\frac{dE}{dt}\le -a_0 D_0 \tag{34}$$

where $a_0=1-m(>0)$. From the Poincar inequality, we have:

$$D_0\ge\pi^2\|\theta\|^2\ge 2\pi^2 E \tag{35}$$

this gives:

$$\frac{dE}{dt}\le -2\pi^2 a_0 E \tag{36}$$

implying

$$E(t)\le e^{-2\pi^2 a_0 t}E(0) \tag{37}$$

Thus, E decays at least exponentially fast and nonlinear stability is assured for all values of *E(0)*. It is important to note that this result holds for all initial data.

3. 1. Variational Problem We now return to Equation (33) and use calculus of variation to find the maximum problem at the critical argument $m=1$. The associated Euler-Lagrange equations after taking transformations $\hat{\mathbf{q}}=\sqrt{\lambda}\mathbf{q}$ (dropping caps) are:

$$2\left(F+\tilde{D}a\right)\nabla^2\mathbf{q}-2\mu(z)\mathbf{q}+R(1+\lambda)\frac{1}{\sqrt{\lambda}}\theta\,\hat{\mathbf{k}}=2\nabla\eta \tag{38}$$

$$2\nabla^2\theta+R(1+\lambda)\frac{1}{\sqrt{\lambda}}w=0 \tag{39}$$

where η is a Lagrange's multiplier introduced, since $\mathbf{q}$ is solenoidal. On taking curl curl of Equation (38) and then taking third component of the resulting equation, we find:

$$2\mu'(z)\frac{\partial w}{\partial z}+2\mu(z)\nabla^2 w-2\left(F+\tilde{D}a\right)\nabla^4 w$$

$$-R(1+\lambda)\frac{1}{\sqrt{\lambda}}\nabla_1^2\theta=0 \tag{40}$$

Now, we assume a plane tiling form

$$(w,\theta)=[W(z),\Theta(z)]\,g(x,y) \tag{41}$$

where $\nabla_1^2 g+a^2 g=0$, a being the wave number [9, 44] .

The wave number is found a *posteriori* to be non-zero, so from Equations (40) and (39), we see that W, Θ satisfy

$$-2\left(F+\tilde{D}a\right)\left(D^2-a^2\right)^2W+2\mu(z)\left(D^2-a^2\right)W \tag{42}$$

$$+2\mu'(z)DW+Ra^2(1+\lambda)\frac{1}{\sqrt{\lambda}}\Theta=0$$

$$2\left(D^2-a^2\right)\Theta+R(1+\lambda)\frac{1}{\sqrt{\lambda}}W=0 \tag{43}$$

subject to the boundary conditions

$$W=0,\quad D^2W=0,\quad \Theta=0 \quad \text{at} \quad z=0,\ 1 \tag{44}$$
(Free-Free conducting boundaries),

$$\left.\begin{array}{llll} W=0, & DW=0, & \Theta=0 & \text{at } z=0 \\ W=0, & D^2W=0, & \Theta=0 & \text{at } z=1 \end{array}\right\} \tag{45}$$
(Rigid-Free conducting boundaries),

$$W=0,\quad DW=0,\quad \Theta=0 \quad \text{at} \quad z=0,\ 1 \tag{46}$$
(Rigid-Rigid conducting boundaries).

For linear instability analysis, we return to the perturbed Equations (20-22) and after neglecting the nonlinear terms, arrive at the linearized form

$$-\nabla P'-\mu\left(\overline{p},\overline{T}+\theta\right)\mathbf{q}'+\left(F+\tilde{D}a\right)\nabla^2\mathbf{q}'+R\theta\hat{\mathbf{k}}=0 \tag{47}$$

$$\nabla\cdot\mathbf{q}'=0 \tag{48}$$

$$\frac{\partial\theta}{\partial t}-Rw=\nabla^2\theta \tag{49}$$

Performing the standard normal mode analysis technique, we obtain the eigenvalue problem in the form with boundary conditions Equations (44)-(46).

$$-\left(F+\tilde{D}a\right)\left(D^2-a^2\right)^2W+\mu(z)\left(D^2-a^2\right)W \tag{50}$$

$$+\mu'(z)DW+Ra^2\Theta=0$$

$$\left(D^2-a^2\right)\Theta+RW=0 \tag{51}$$

This eigenvalue problem is exactly the same as that of the eigenvalue problem Equations (42-45) obtained by nonlinear stability analysis. Hence, in the presence of couple-stresses, the critical Rayleigh numbers for the linear and non-linear stability problems coincide.

4. METHOD OF SOLUTIONS

Equations 42 and 43 with given boundary conditions are solved by using Galerkin-type method. Accordingly, we set the solutions satisfying the boundary conditions as:

$$W=\sum_{i=1}^{n}A_iW_i \tag{52}$$

$$\Theta=\sum_{i=1}^{n}B_i\Theta_i \tag{53}$$

in which A_i, B_i are constant and the basis functions W_i, Θ_i will be represented by the power series satisfying the boundary conditions. When the series Equations (52) and (53) are substituted back into Equations (42) and (43), and the Galerkin procedure of demanding that the residues be normal to the basis functions is applied, we obtain a system of homogeneous linear algebraic equations

$$C_{ji}A_i+D_{ji}B_i=0 \tag{54}$$

$$E_{ji}A_i+F_{ji}B_i=0 \tag{55}$$

The coefficients C_{ji} to F_{ji} involve the inner products of the basis functions and are given by:

$$C_{ji}=2\left(F+\tilde{D}a\right)\left\langle D^2W_jD^2W_i\right\rangle+\left[4a^2\left(F+\tilde{D}a\right)+2\mu(z)\right]$$

$$\left\langle DW_jDW_i\right\rangle+\left[2a^4\left(F+\tilde{D}a\right)+2a^2\mu(z)\right]\left\langle W_jW_i\right\rangle$$

$$-2\mu'(z)\left\langle W_jDW_i\right\rangle$$

$$D_{ji}=-R\frac{a^2(1+\lambda)}{\sqrt{\lambda}}\left\langle W_j\Theta_i\right\rangle$$

$$E_{ji}=-R\frac{(1+\lambda)}{\sqrt{\lambda}}\left\langle\Theta_jW_i\right\rangle$$

$$F_{ji}=2\left\langle D\Theta_jD\Theta_i\right\rangle+2a^2\left\langle\Theta_j\Theta_i\right\rangle$$

where the inner product is defined as $\langle\cdots\rangle=\int_0^1(\cdots)\,dz$.

The polynomial types of basis functions which satisfy the boundary conditions are used to solve the eigenvalue for different types of boundaries.

4. 1. Free-free Conducting Boundaries In this case, the boundary conditions are:

$W_i = D^2W_i = 0 = \Theta_i$ at $z = 0, 1$

The basis functions satisfying the boundary conditions are:

$$W_i = z^i - 2z^{i+2} + z^{i+3} \quad \text{and} \quad \Theta_i = z^i - z^{i+1}$$

4. 2. Rigid-free Conducting Boundaries In this case, the boundary conditions are:

$W_i = DW_i = 0 = \Theta_i$ at $z = 0$,

$W_i = D^2W_i = 0 = \Theta_i$ at $z = 1$

The basis functions satisfying the boundary conditions are:

$$W_i = 3z^{i+1} - 5z^{i+2} + 2z^{i+3} \quad \text{and} \quad \Theta_i = z^i - z^{i+1}$$

4. 3. Rigid-rigid Conducting Boundaries In this case, the boundary conditions are:

$W_i = DW_i = 0 = \Theta_i$ at $z = 0, 1$

The basis functions satisfying the boundary conditions are:

$$W_i = z^{i+1} - 2z^{i+2} + z^{i+3} \quad \text{and} \quad \Theta_i = z^i - z^{i+1}$$

In order to have a nontrivial solution for Equations (54) and (55), the determinant of the coefficient matrix should be zero. That is,

$$\begin{vmatrix} C_{ji} & D_{ji} \\ E_{ji} & F_{ji} \end{vmatrix} = 0 \qquad (56)$$

From Equation (56), we get thermal Darcy-Rayleigh number $\Re$ in terms of coupling parameter λ, wave number a, couple-stress parameter F, Darcy-Brinkman number $\tilde{D}a$ and variable viscosity parameter Γ. Then, optimum value of λ is determined by the condition $\frac{\partial \Re}{\partial \lambda} = 0$, and is found to be $\lambda = 1$. After fixing the values of couple-stress parameter F, Darcy-Brinkman number, $\tilde{D}a$ and variable viscosity parameter, Γ, we find approximations for critical wave number, a_c and critical thermal Darcy-Rayleigh number, $\Re_c$.

5. DISCUSSION OF RESULTS AND CONCLUSIONS

The effects of couple stress-fluid in porous medium with temperature-pressure dependent viscosity on the onset of convection are investigated for free-free, rigid-free and rigid-rigid conducting boundary systems. The critical thermal Darcy-Rayleigh number $\Re_c$ and critical wave numbers a_c, obtained by Galerkin method, are shown in Tables 1-3 for different values of various parameters.

Table 1 shows the variation of critical wave number a_c and critical thermal Darcy-Rayleigh number $\Re_c$ with couple-stress parameter F for different values of variable viscosity parameter Γ and $\tilde{D}a = 0$ (Darcy model). This table shows that in the absence of couple-stress parameter F and variable viscosity parameter Γ, when Darcy model is used, the critical thermal Darcy-Rayleigh number for different conducting boundary systems (free-free, rigid-free, rigid- rigid) are 39.79, 44.80 and 45.54, respectively. But, when F goes on increasing, the critical thermal Darcy-Rayleigh number also keeps increasing showing the stabilizing effect of couple-stress parameter F for all the conducting boundary systems which we have considered here. Furthermore, it is also clear from this table that variable viscosity parameter Γ has the destabilizing effect on the onset of convection as with the increase in Γ, $\Re_c$ goes on decreasing for all the three types of conducting boundary systems.

Table 2 shows the variation of critical wave number a_c and critical thermal Darcy-Rayleigh number $\Re_c$ with couple stress parameter F for different values of Γ and $\tilde{D}a = 0.05$ (Darcy-Brinkman model). This table shows that in the absence of couple-stress parameter F and variable viscosity parameter Γ, when Darcy-Brinkman model is used, the critical thermal Darcy-Rayleigh numbers for free-free, rigid-free, rigid-rigid conducting boundaries are 75.62, 102.92 and 133.16, respectively. This table also shows the stabilizing effect of couple-stress parameter F and destabilizing effect of variable viscosity parameter Γ on the onset of convection as with the increase in F, $\Re_c$ goes on increasing and with the increase in Γ, $\Re_c$ goes on decreasing.

Furthermore, from both of these tables, we conclude that for both the models (Darcy model and Darcy-Brinkman model), couple-stress parameter has the stabilizing effect on the onset of convection whereas variable viscosity parameter Γ has the destabilizing effect on the onset of convection. But, among these two models, the use of Darcy-Brinkman model makes the system thermally more stable for all the conducting boundary systems which we have considered than the Darcy model.

TABLE 1. Variation of the Critical Thermal Darcy-Rayleigh number $\Re_c$ with couple-stress parameter F for various values of Γ, $\tilde{D}a = 0$.

Γ	F	Free-free		Rigid-free		Rigid-rigid	
		a_c	$\Re_c$	a_c	$\Re_c$	a_c	$\Re_c$
0	0	3.152	39.72	3.265	44.80	3.310	45.54
	0.2	2.377	176.71	2.736	213.93	3.132	395.68
	0.4	2.310	310.04	2.705	378.77	3.125	745.68
0.5	0	3.152	31.83	3.265	35.40	3.310	36.43
	0.2	2.352	168.08	2.734	261.43	3.129	386.55
	0.4	2.295	301.28	2.708	485.25	3.123	736.54
1	0	3.152	26.52	3.265	29.65	3.310	30.36
	0.2	2.334	162.29	2.733	252.73	3.127	380.45
	0.4	2.284	295.41	2.710	473.78	3.122	730.45
1.5	0	3.152	22.74	3.265	25.47	3.310	26.02
	0.2	2.321	158.14	2.731	246.40	3.126	376.10
	0.4	2.277	291.21	2.712	465.45	3.121	726.10

TABLE 2. Variation of the Critical Thermal Darcy-Rayleigh number $\Re_c$ with couple-stress parameter *f* for various values of Γ, $\tilde{D}a = 0.05$.

Γ	F	Free-free		Rigid-free		Rigid-rigid	
		a_c	$\Re_c$	a_c	$\Re_c$	a_c	$\Re_c$
0	0	2.612	75.62	2.865	102.92	3.166	133.16
	0.2	2.352	210.10	2.724	331.12	3.129	483.18
	0.4	2.302	343.32	2.702	558.94	3.123	833.18
0.5	0	2.566	67.35	2.848	93.323	3.158	124.03
	0.2	2.330	201.41	2.724	317.39	3.127	474.05
	0.4	2.288	334.54	2.705	541.19	3.122	824.18
1	0	2.530	61.80	2.834	86.75	3.153	117.94
	0.2	2.315	195.60	2.724	308.00	3.125	476.95
	0.4	2.279	328.66	2.708	529.04	3.121	817.95
1.5	0	2.502	57.82	2.823	81.97	3.348	113.58
	0.2	2.304	191.43	2.724	301.17	3.124	463.60
	0.4	2.272	324.46	2.710	520.21	3.121	813.60

The variation of critical wave number a_c and thermal Darcy-Rayleigh number $\Re_c$ with variation in Darcy-Brinkman number $\tilde{D}a$ for different values of Γ and $F = 0.3$ is shown by Table 3. It is depicted from this table that medium permeability in Brinkman model delays the onset of convection because, as Darcy-Brinkman number increases, the value of thermal Darcy-Rayleigh number increases. Thus, medium permeability has the stabilizing effect on the onset of convection. Whereas variable viscosity parameter Γ causes for the early onset of convection as with the increase in Γ for the corresponding values of $\tilde{D}a$, the thermal Darcy-Rayleigh number goes on decreasing for all the conducting boundary systems.

The principal conclusions from the above analysis are as under:

- The result we establish is that both the linear instability and nonlinear stability Rayleigh numbers coincide.
- The use of Darcy-Brinkman model makes the system thermally more stable for all the different conducting boundary systems which we have considered than the Darcy model.
- The couple-stresses and medium permeability both have the tendency to slow down the motion of the fluid in the boundary layer; thus reducing the heat transfer from bottom to top. The decrease in heat transfer is responsible for delaying the onset of convection. Thus, both the couple-stress parameter F and medium permeability promote stabilization.
- The variable viscosity has the tendency to grow up the motion of the fluid in the boundary layer; thus, increasing the heat transfer from bottom to top. The increase in heat transfer is responsible for early onset of convection. Thus, variable viscosity destabilizes the system.

TABLE 3. Variation of the Critical Thermal Darcy-Rayleigh number $\Re_c$ with darcy-brinkman number $\tilde{D}a$ for various values of Γ, $F = 0.3$.

Γ	$\tilde{D}a$	Free-free		Rigid-free		Rigid-rigid	
		a_c	$\Re_c$	a_c	$\Re_c$	a_c	$\Re_c$
0	0.01	2.331	250.10	2.715	339.48	3.127	588.18
	0.03	2.325	263.43	2.712	422.26	3.126	623.18
	0.05	2.320	276.75	2.710	445.04	3.126	658.18
	0.07	2.316	290.07	2.708	467.82	3.125	693.18
	0.09	2.312	303.39	2.706	490.61	3.125	728.18
0.5	0.01	2.312	241.38	2.716	384.54	3.125	579.05
	0.03	2.380	254.69	2.714	406.92	3.125	614.05
	0.05	2.304	268.01	2.712	429.30	3.124	649.05
	0.07	2.300	281.31	2.710	451.68	3.124	684.04
	0.09	2.297	294.62	2.709	474.06	3.123	719.04
1	0.01	2.300	235.54	2.717	374.32	3.123	572.95
	0.03	2.296	248.85	2.715	396.42	3.123	607.95
	0.05	2.292	262.15	2.714	418.53	3.123	642.95
	0.07	2.289	275.46	2.712	440.63	3.122	677.95
	0.09	2.286	288.76	2.711	462.73	3.122	712.95
1.5	0.01	2.290	231.35	2.718	366.88	3.122	568.60
	0.03	2.287	244.66	2.716	388.79	3.122	603.60
	0.05	2.283	257.96	2.715	410.69	3.122	638.60
	0.07	2.281	271.26	2.713	432.59	3.121	673.60
	0.09	2.278	284.56	2.712	454.50	3.121	708.60

6. REFERENCES

1. Joseph, D.D., "Stability of Fluid Motions", Vols. I-II, Berlin, Springer, New York, (1976).
2. Orr, W.McF, "Stability or Instability of the Steady Motions of a Perfect Liquid", *Proceeding of the Royal Irish Academy Section A*, Vol. 27, (1907), 69-138.
3. Serrin, J., "On the Stability of Viscous Fluid Motions", *Archive of Rational Mechanics Analysis*, Vol. 3, (1959), 1-13.
4. Joseph, D.D., "On the Stability of the Boussinesq Equations", *Archive of Rational Mechanics Analysis*, Vol. 20, (1965), 59-71.
5. Joseph, D.D., "Nonlinear Stability of the Boussinesq Equations by the Method of Energy", *Archive of Rational Mechanics Analysis*, Vol. 22, (1966), 163-184.
6. Galdi, G.P. and Padula, M., "A New Approach to Energy Theory in the Stability of Fluid Motion", *Archive of Rational Mechanics Analysis*, Vol. 110, (1990), 187-286.
7. Straughan, B., "Explosive Instabilities in Mechanics", Berlin, Springer, New York, (1998).
8. Straughan, B., "The Energy Method, Stability, and Nonlinear Convection", Verlag, Springer, New York, (2004).
9. Straughan, B., "A Sharp Nonlinear Stability Threshold in Rotating Porous Convection", *Proceeding of the Royal Society of London Series A*, Vol. 457, (2001), 87-93.
10. Kaloni, P.N. and Qiao, Z., "Non-Linear Stability of Convection in a Porous Medium With Inclined Temperature Gradient", *International Journal of Heat and Mass Transfer*, Vol. 40, (1997), 1611-1615.
11. Kaloni, P.N. and Qiao, Z., "Non-Linear Convection With Inclined Temperature Gradient and Horizontal Mass Flow", *International Journal of Engineering Science*, Vol. 35, (1997), 299-309.
12. Kaloni, P.N. and Qiao, Z., "Non-Linear Convection in a Porous Medium With Inclined Temperature Gradient and Variable Gravity Effects", *International Journal of Heat and Mass Transfer*, Vol. 44, (2001), 585-1591.
13. Guo, J. and Kaloni, P.N., "Nonlinear Stability Problem of a Rotating Doubly Diffusive Porous Layer", *Journal of Mathematical Analysis and Application,*, Vol. 190, (1995), 373-390.
14. Guo, J., Qin, Y. and Kaloni, P.N., "Nonlinear Stability Problem of a Rotating Doubly Diffusive Fluid Layer", *International Journal of Engineering Science*, Vol. 32, (1994), 1207-1219.
15. Payne, L.E. and Straughan, B., "Unconditional Nonlinear Stability in Temperature-Dependent Viscosity Flow in a Porous Medium", *Studies in Applied Mathematics*, Vol. 105, (2000), 59-81.
16. Sunil and Mahajan, A., "A Nonlinear Stability Analysis for Magnetized Ferrofluid Heated From Below", *Proceeding of the Royal Society of London Series A*, Vol. 464, (2008), 83-98.
17. Sunil and Mahajan, A., "A Nonlinear Stability Analysis for Rotating Magnetized Ferrofluid Heated From Below", *Applied Mathematics and Computation*, Vol. 204, (2008), 299-310.
18. Sunil and Mahajan, A., "A Nonlinear Stability Analysis of a Doubly-Diffusive Magnetized Ferrofluid", *Zeitschrift für Naturforschung*, Vol. 63a, (2008), 797-807.
19. Sunil and Mahajan, A., "A Nonlinear Stability Analysis in a Doubly-Diffusive Magnetized Ferrofluid Layer Saturating a Porous Medium", *Journal of Geophysics and Engineering*, Vol. 5, (2008), 311-322.
20. Sunil, Mahajan, A., "A Nonlinear Stability Analysis for Thermoconvective Magnetized Ferrofluid Saturating a Porous Medium", *Transport in Porous Media*, Vol. 76, (2009), 327-343.
21. Sunil and Mahajan, A., "A Nonlinear Stability Analysis for Rotating Magnetized Ferrofluid Heated From Below Saturating a Porous Medium", *The Journal of Applied Mathematics and Physics*, Vol. 60, (2009), 344-362.
22. Stokes, V.K., "Couple Stresses in Fluids", *Physics of Fluids*, Vol. 9, (1966.), 1709-1715.
23. Ramanaiah, G., "Slider Bearings Lubricated by Fluids With Couple-Stress", *WEAR*, Vol. 52, (1979), 27-36.
24. Shehawey, E.F.EI. and Mekheimer, K.S., "Couple-Stresses in Peristaltic Transport of Fluids", *Journal of Physics D: Applied Physics*, Vol. 27, (1994), 1163-1170.
25. Das, N.C., "Elastohydrodynamic Lubrication Theory of Line Contacts: Couple Stress Fluid Model", *Society of Tribologists and Lubrication Engineers (STLE), Tribology Transactions*, Vol. 40, (1997), 353-359.
26. Das, N.C., "A Study of Optimum Load-Bearing Capacity for Slider Bearing Lubricated With Couple Stress Fluids in Magnetic Field", *Tribology International*, Vol. 31, (1998), 393-400.
27. Gupta, R.S. and Sharma, L.G., "Analysis of Couple-Stress Lubricant in Hydrostatic Thrust Bearings", *WEAR*, Vol. 125, (1998), 257-269.
28. Sharma, R.C., Sunil and Pal, M., "On Couple-Stress Fluid Heated From Below in Porous Medium in Presence of Rotation", *Applied Mechanics and Engineering*, Vol. 5, (2000), 883-896.
29. Sharma, R.C. and Thakur, K.D., "On Couple-Stress Fluid Heated From Below in a Porous Medium in Hydromagnetic", *Czechoslovak Journal of Physics*, Vol. 50, (2000), 753-758.
30. Malashetty, M.S., Shivakumara, I.S., Sridhar Kulkarni, "The onset of convection in a couple-stress fluid saturated porous layer using a thermal non-equilibrium model", *Physics Letters A*, Vol. 373, (2009), 781-790.
31. Abdallah, A.E. and Lotfi, H.G., "An Inverse Solution for Finite Journal Bearing Lubricated With Couple-Stress Fluids", *Tribology International*, Vol. 34, (2001), 107-118.
32. Hsu, C.H., Lin, J.R. and Chiang, H.L., "Combined Effects of Couple-Stresses and Surface Roughness on the Lubrication of Short Journal Bearings", *Indus Tribal*, Vol. 55, (2003), 233-243.
33. Lahmar, M., "Elastohydrodynamic Analysis of Double-Layered Journal Bearings Lubricated With Couple-Stress Fluids", *Proceedings of the Institution of Mechanical Engineers,* Part *J*: *Journal of Engineering Tribology,* Vol. 219, (2005), 145-165.
34. Sunil, Devi, R. and Mahajan, A., "Global Stability for Thermal Convection in a Couple-Stress Fluid", *International Communication in Heat and Mass Transfer*, Vol. 38, (2011), 938-942.
35. Stokes, G.G., "On the theories of the internal friction of fluids in motions, and of elastic solids", *Transactions* of the *Cambridge Philosophical Society*, Vol. 8, (1845), 287-305.
36. Laun, H.M., "Pressure dependent viscosity and dissipative heating in capillary rheometry of polymer melts", *The journal Rheologica Acta*, Vol. 42, (2005), 295-308.
37. Bair, S. and Kottke, P., "Pressure-viscosity relationship for elastohydrodynamics", *Tribology Transactions*, Vol. 46, (2003), 353-359.
38. Martin-Alfonso, M.J., Martinez-Boza, F.J., Partal, P. and Gallegos, C., " Influence of pressure and temperature on the

flow behavior of heavy fluid oils", *The journal Rheologica Acta*, Vol. 45, (2006), 357-365.

39. Martin-Alfonso, M.J., Martinez-Boza, F.J., Navarro, F.J., Fernandez, M. and Gallegos, C., "pressure-temperature-viscosity relationship for heavy petroleum fractions", *Fuel*, Vol. 86, (2007), 227-233.
40. Rajagopal, K.R., Saccomandi, G. and Vergori, L., "Stability analysis of the Rayleigh-Bènard convection for a fluid with temperature and pressure dependent viscosity", *The Journal of Applied Mathematics and Physics*, Vol. 60, (2009), 739-755.
41. Nield, D.A. and Bejan, A., "Convection in Porous Media", Springer, New York, (2006).
42. Qin Y. and Kaloni, P.N., "Nonlinear Stability Problem of a Rotating Porous Layer", *The Quarterly of Applied Mathematics*, Vol. 53, (1995), 129-142.
43. Rajagopal, K.R., Saccomandi, G. and Vergori, L., "Stability Analysis of Rayleigh-Bènard Convection in Porous Medium", *The Journal of Applied Mathematics and Physics*, Vol. 62, (2011), 149-160.
44. Chandrasekhar, S., "Hydrodynamic and Hydromagnetic Stability", Dover, New York, (1981)

Global Stability for Thermal Convection in a Couple Stress Fluid Saturating a Porous Medium with Temperature-pressure Dependent Viscosity: Galerkin Method

Sunil , R. Devi

Department of Mathematics, National Institute of Technology, Hamirpur, (H.P.) –177005, India

ARTICLE INFO

Article history:
Received 24 August 2011
Received in revised form 5 February 2012
Accepted 19 April 2012

Keywords:
Couple-stress Fluid
Porous Medium
Global Stability
Temperature-pressure Dependent Viscosity
Different Conducting Boundaries (Free-free, Rigid-free, Rigid-rigid)

چکیده

آنالیز پایدار غیر خطی جامع برای یک تنش مکمل در یک لایه سیال که تحت شرایط اشباع و محیط متخلخل گرم شده است و در آن گران‌روی به فشار و دما در سیستم‌های مرزی هدایتی متفاوت وابسته می‌باشد، انجام شده است. در این‌جا آستانه پایداری غیرخطی جامع برای هدایت، کاملاً شبیه لایه ناپایدار خطی است. نتیجه بهینه به‌دست امده، به این علت که نشان می‌دهد تئوری ناپایداری خطی، به‌طور کامل فیزیک شروع جابه‌جایی را کنترل کرده است، مهم می‌باشد. مقدار مشخصه مسائل برای سیستم‌های مرزی هدایت متفاوت به‌وسیله روش گالرکین حل شده است. تاثیرات پارامتر تنش دوتایی F، عدد دارسی برینکمن $\tilde{D}a$، پارامتر تغییرات گران‌روی Γ، در شروع جابه‌جایی نیز مورد آنالیز قرار گرفته است. استفاده از مدل دارسی برینکمن پایداری حرارتی سیستم را نسبت به مدل دارسی برای همه سیستم‌های مرزی هدایتی متفاوت، پارامتر تنش دوتایی و افزایش پایداری نفوذ واسطه افزایش می‌دهد که تغییرات گران‌روی موجب بی‌ثباتی سیستم می‌شود.

doi: *10.5829/idosi.ije.2012.25.03a.04*