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A New Technique for Image Zooming Based on the Moving Least Squares

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In this paper, a new method for gray-scale image and color zooming algorithm based on their local information is offered. In the proposed method, the unknown values of the new pixels on the image are computed by Moving Least Square (MLS) approximation based on both the quadratic spline and Gaussian-type weight functions. The numerical results showed that this method is more preferable to bilinear interpolation in term of quality (for the quadratic spline weight function), and is comparable to bicubic interpolation (for the Gaussian-type weight function).

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1. INTRODUCTION

Image interpolation, or zooming, is a basic problem in image processing. Nowadays, digital image processing that enlarges an image has become more important than ever. There are some applications that need methods of image zooming, such as electronic publishing, visible wireless telephone, digital camera, image-processing and, etc. The image enlargement is obtained by interpolating the discrete original image and is conventionally processed by methods such as pixel replication [1], bilinear interpolation [2, 3], and bicubic interpolation [4, 5] methods. Pixel replication method is a technique of nearest neighbor interpolation, which is simple to implement here for replicating the original pixels. This method is usually susceptible to the unfavorable defect of blocking effects. Generally, there are three kinds of blocking effects in JPEG decompressed images. One is the staircase noise along the image edges, another is the grid noise in the monotone area, and the other is the corner outlier in the corner point of the 8×8 DCT block [6]. Bilinear and bicubic interpolations use first-order spline and second-order spline model, respectively. Usually in these prevalent used interpolation techniques, there exists a problem where the low order interpolation method degrades the zoomed image quality, notwithstanding that lower order interpolation technique requires less computation unlike higher order interpolation technique, which yields better results but requires more computations. Other methods such as polynomial spline algorithm are described elsewhere [7]. For image zooming using interpolation in medical image processing, the reader is referred to the literature [8]. Battiato et al. [9] offered a locally adaptive zooming technique to resolve the problems mentioned above on these conventional interpolation methods. In this technique, the zooming algorithm is gradient-controlled, weighted non-linear interpolation by a multiple scan of the zoomed image. The technique, which relies on heuristic thresholds to decide the edge types for interpolation, is intricate for implementation.

Well-known and prevalent methods such as the bilinear, bicubic, and cubic spline interpolation, calculate the interpolated value as a weighted sum of the neighboring samples. However, these prevalent interpolation approaches do not consider local features, and thus may blur local structures or cause undesirable artifacts in reproduced high-resolution images. To improve the quality of the interpolated images, we present the moving least squares interpolation technique that adapts the interpolation weights according to splines.

The remainder of the paper is organized as follows: section 2 briefly introduces the moving least-squares method, and then spline weighted. Section 3 presents the examples of image magnification based on their local information by using moving least-squares based on two types of weight functions. Finally, section 4 concludes a brief conclusion.

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2. THE MOVING LEAST-SQUARE APPROXIMATION SCHEME

Moving Least Squares (MLS), originated by mathematicians for data fitting and surface construction, can be categorized as a method of finite series representation of functions. The MLS method is now a widely used alternative for constructing meshless shape functions for approximation. The MLS approximation has two major features that make it popular: (1) the approximated field function is continuous and smooth in the entire problem domain; and (2) it is capable of producing an approximation with the desired order of consistency. The MLS approximation is detailed in this part.

Consider a support domain Ω_x, which is located within the problem domain Ω (Figure 2) and has a number of randomly located nodes x_I ($I = 1, \dots, N$). The moving least squares approximate $\theta^h(x)$ of $\theta(x)$ which is defined as follows:

$$\theta^h(x) = \sum_{i=1}^{m} p_i(x) a_i(x) = p^T(x) a(x) \tag{1}$$

where $p^T(x)=[p_1(x),p_2(x), \dots ,p_m(x)]$ is a complete monomial basis, m is the number of terms in the basis, and $a(x)=[a_1(x),a_2(x), \dots ,a_m(x)]$ is the corresponding coefficient. For example, for a 2D problem, the basis can be chosen as:

$$\begin{aligned} &Linear\ \ basis\ (m=3):\ \boldsymbol{p}^{\boldsymbol{T}}(x) = [1, \mathrm{x}, \mathrm{y}] \\ &Quadratic\ \ basis\ (m=6):\ \boldsymbol{p}^{\boldsymbol{T}}(x) = [1, \mathrm{x}, \mathrm{y}, \mathrm{x}^2, \mathrm{xy}, \mathrm{y}^2] \end{aligned} \tag{2}$$

The coefficient vector $a(x)$ is determined by minimizing the difference between the local approximation and the function, and is defined as:

$$J(\boldsymbol{a}(x)) = \sum_{I=1}^{N} w_I(x)\left[\boldsymbol{P}^T(x_I)\boldsymbol{a}(x) - \hat{\theta}^I\right]^2 = \left[\boldsymbol{Pa}(x) - \hat{\theta}\right]^T \boldsymbol{W}\left[\boldsymbol{Pa}(x) - \hat{\theta}\right] \tag{3}$$

where x_I denotes the position vector of node I, $w_I(x)$ is the weight function associated with the node I, N is the number of node in Ω_x for which the weight functions $w_I(x) > 0$ are searched, the matrix P and the diagonal matrix W are defined as follows:

$$\boldsymbol{P} = \begin{bmatrix} p_1(x_1) & p_2(x_1) & \dots & p_m(x_1) \\ p_1(x_2) & p_2(x_2) & \dots & p_m(x_1) \\ & . & & \\ & . & & \\ & . & & \\ p_1(x_N) & p_2(x_N) & \dots & p_m(x_N) \end{bmatrix}_{N\times m} \tag{4}$$

$$\boldsymbol{W} = \begin{bmatrix} w_1(x) & . & . & . & 0 \\ . & . & & & . \\ . & & . & & . \\ . & & & . & . \\ 0 & . & . & . & w_N(x) \end{bmatrix}_{N\times N} \tag{5}$$

and

$$\hat{\theta}^T = \left[\hat{\theta}^1, \hat{\theta}^2, \dots, \hat{\theta}^N\right]_{1\times N} \tag{6}$$

In Eq. (6) $\hat{\theta}^I$ is the fictitious nodal value. It is not the nodal value of trial functions denoted by $\theta^h(x)$. To find the coefficient $a(x)$, we obtain the extremum by:

$$\frac{\partial J(\boldsymbol{a}(x))}{\partial(\boldsymbol{a}(x))} = 2\sum_{I=1}^{N} w_I(x)\left[\sum_{i=1}^{m} p_i(x_I)a(x) - \hat{\theta}^I\right] p_i(x_I) = 0 \tag{7}$$

This leads to the following set of linear relations:

$$\begin{aligned} &\boldsymbol{A}(x)\boldsymbol{a}(x) = \boldsymbol{B}(x)\hat{\boldsymbol{\theta}} \\ &(m\times m)(m\times 1) = (m\times N)(N\times 1) \end{aligned} \tag{8}$$

where the matrixes $A(x)$ and $B(x)$ are defined by:

$$\boldsymbol{A}(x) = \boldsymbol{P}^T\boldsymbol{W}\boldsymbol{P} = \boldsymbol{B}(x)\boldsymbol{P} = \sum_{I=1}^{N} w_I(x)\boldsymbol{p}(x_I)\boldsymbol{p}^T(x_I) \tag{9}$$

$$\boldsymbol{B}(x) = \boldsymbol{P}^T\boldsymbol{W} = \left[w_1(x)\boldsymbol{p}(x_1), w_2(x)\boldsymbol{p}(x_2), \dots, w_N(x)\boldsymbol{p}(x_N)\right] \tag{10}$$

Solving $a(x)$ from Eq. (8), and substituting it into Eq. (1), we can obtain the final form of the MLS approximation as Eq. (11).

$$\begin{aligned} &\theta^h(x) = \boldsymbol{\Phi}^T(x)\hat{\boldsymbol{\theta}} = \sum_{I=1}^{N} \varphi^I(x)\hat{\theta}^I \\ &\theta^h(x_I) = \theta^I \neq \hat{\theta}^I, \quad x \in \Omega_x \end{aligned} \tag{11}$$

where $\Phi^T(x)=p^T(x)A^{-1}(x)B(x)$ is the shape function, and its partial derivative is:

$$\phi^I_{,K} = \sum_{j=1}^{m}\left[p_{j,k}\left(\boldsymbol{A}^{-1}\boldsymbol{B}\right)_{jI} + p_j\left(\boldsymbol{A}^{-1}\boldsymbol{B}_{,K} + \boldsymbol{A}^{-1}_{,K}\boldsymbol{B}\right)_{jI}\right] \tag{12}$$

In practical applications, the weight function $w_I(x)$ is generally nonzero over the small neighborhood of point x_I, and this neighborhood is called the domain of influence or the domain of definition (Fig. 1).

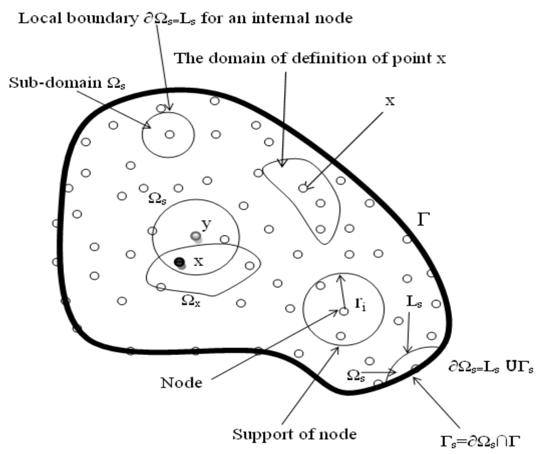

Fig. 1. Schematics of the MLS approximation

Typically, the shape of the domain in the two dimensional space can be circular, ellipse, rectangular or any other convenient regular closed lines and in the three dimensional space can be sphere, ellipsoid, cube or any other simple cubic volume. In the present analysis a circular domain has been selected. The choice of weight function $w_I(x)$ affects the resulting approximation $\theta^h(x)$; therefore, its selection is of essential importance. Numerical practices have shown that the quadratic spline and Gaussian-type weight functions work well [10, 11]. Hence in this article, both weight functions are used. Thus, the quadratic spline weight function could be written as:

$$w_I(x)=\begin{cases}1-6\left(\frac{d_I}{r_I}\right)^2+8\left(\frac{d_I}{r_I}\right)^3-3\left(\frac{d_I}{r_I}\right)^4 & 0\le d_I\le r_I\\ 0 & d_I\ge r_I\end{cases} \tag{13}$$

where d_I is the distance between points x and nod x_I and r_I is the size of support (Fig. 1) for the weight functions. It can be seen that the quadratic spline weight function is C^1 continuous over the entire domain. Furthermore, the Gaussian-type weight function corresponding to node I may be written as:

$$w_I(x)=\begin{cases}\dfrac{\exp[-(\frac{d_I}{c_I})^{2k}]-\exp[-(\frac{r_I}{c_I})^{2k}]}{1-\exp[-(\frac{r_I}{c_I})^{2k}]} & 0\le d_I\le r_I\\ 0 & d_I\ge r_I\end{cases} \tag{14}$$

where $d_I=|x-x_I|$ is the distance from node x_I to the point x; c_I is a constant controlling the sharp of the weight function w_I, and therefore the relative weights; and r_I is the size of the support for the weight function w_I and thus determines the support of node x_I. For simplicity, $k=1$ may be chosen.

3. USING THE MLS IN IMAGE MAGNIFICATION

The zooming process requires two steps: the creation of new pixel locations, and the assignment of gray levels to those new locations. Let us start with simple examples (Fig. 2). Fig. 2a is the portion of cameraman image, Fig. 2b is the portion of football image, and Fig. 2c, is the portion of Lena image.

Suppose that we have an image of size 50×50 pixels and want to enlarge it 3.94 times to 197×197 pixels. Conceptually, one of the easiest ways to visualize zooming is laying an imaginary 197×197 grid over the original image. Obviously, the spacing in the grid would be less than one pixel because we are fitting it over a smaller image. After interpolation, when we are done with all points in the overlay grid, simply expand it to the original specified size to obtain the zoomed image. In these examples, we use of both the quadratic spline and Gaussian-type weight functions.

(a) (b) (c)

Fig. 2. Original images

It is noticeable that the resultant after interpolation will be rounded (Fig. 3, Fig. 4 and Fig. 5). Fig. 3 shows the results for Fig. 2a such that Fig. 3b depicts zoom by MLS (exponential), Fig. 3c zoom by MLS (spline), Fig. 3d zoom by bilinear interpolation and Fig. 3e zoom by bicubic interpolation. Fig. 4 illustrates the results for Fig. 2m such that Fig. 4b shows the zoom by MLS (exponential), Fig. 4c zoom by MLS (spline), Fig. 4d zoom by bilinear interpolation and Fig. 4e zoom by bicubic interpolation. Fig. 5 depicts the results for Fig. 2n such that Fig. 5b is zoom by MLS (Gaussian-type), Fig. 5c zoom by MLS (spline), Fig. 5d zoom by bilinear interpolation and Fig. 5e zoom by bicubic interpolation.

This technique is used for a color image. In RGB images we act as 3 gray-scale images. In this model, each color is a combination of 3 colors (red, green and blue). For other models like HSI, the simple way is to convert HSI model to RGB model and follow the RGB procedure [1]. The MLS interpolation (quadratic basis with m=6) is used to determine the pixels unknown intensity.

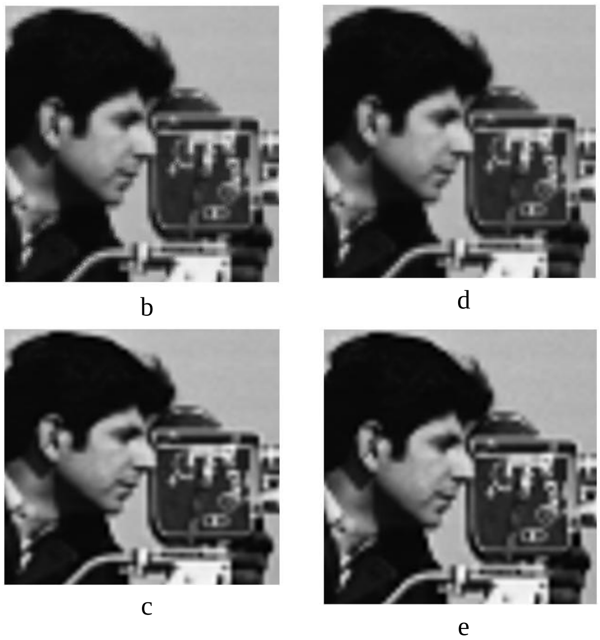

Fig. 3. Portion of cameraman image: (Fig.2a) original image, and, (b) zoom by MLS (exponential), (c) zoom by MLS (spline), (d) zoom by bilinear interpolation and (e) zoom by bicubic interpolation

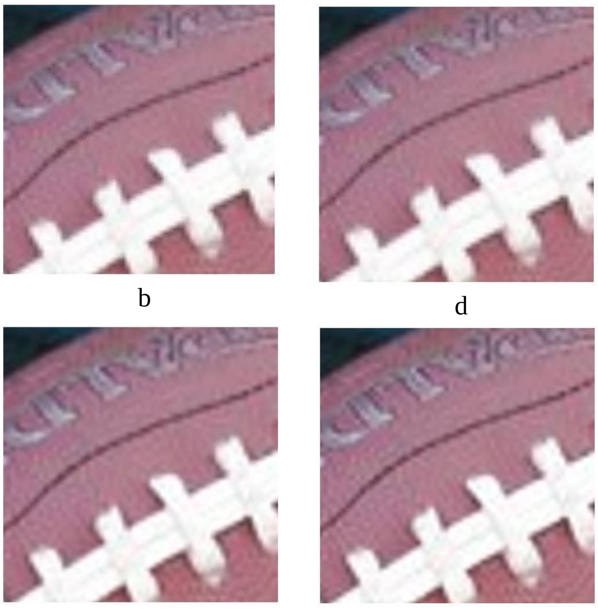

Fig. 4. Portion of football image: (Fig.2m) original image and (b) zoom by MLS (exponential), (c) zoom by MLS (spline), (d) zoom by bilinear interpolation and (e) zoom by bicubic interpolation

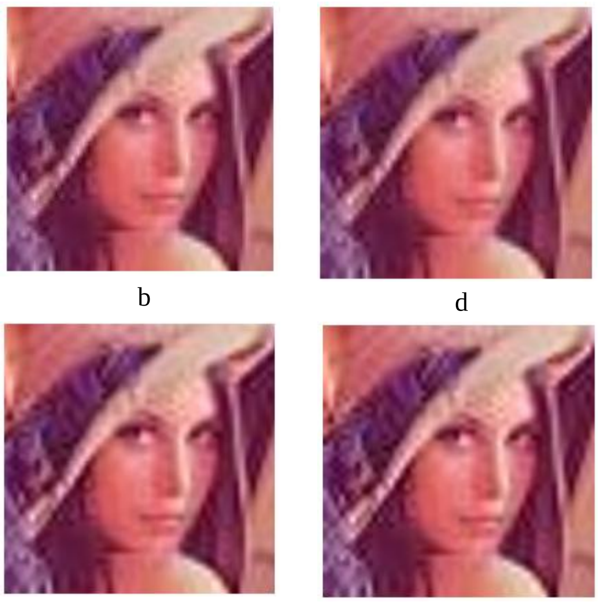

Fig. 5. Portion of Lena image: (Fig.2n) original image, and (b) zoom by MLS (Gaussian-type), (c) zoom by MLS (spline), (d) zoom by bilinear interpolation and (e) zoom by bicubic interpolation

4. CONCLUSION

In this paper, we proposed an image magnification method based on moving least squares for gray-scale and color image. In this method; when weight function is quadratic spline, the results are superior bilinear interpolation in quality, and comparable to bicubic interpolation. In this case, blocking effects are less than bilinear and bicubic interpolations. When weight function is exponential, the results are comparable with bilinear interpolation.

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چکیده

در این مقاله، یک روش جدید بر مبنای داده های محلی، جهت زوم کردن بر روی عکس های سیاه– سفید و رنگی پیشنهاد شده است. در روش پیشنهادی، مقادیر نامعین مربوط به پیکسل های جدید با استفاده از تقریب حداقل مربعات بازگشتی و بر پایه وزن های منحنی و گوسی محاسبه می شوند. نتایج عددی حاکی از برتری کیفی این روش بر روش دو خطی (برای زمانی که از تابع وزن منحنی استفاده شود)، و قابل مقایسه با روش دو مکعبی (برای زمانی که از تابع وزن گوسی استفاده شود) می باشند.

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