# MODELING RISK OF LOSING A CUSTOMER IN A TWOECHELON SUPPLY CHAIN FACING AN INTEGRATED COMPETITOR: A GAME THEORY APPROACH 

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#### Abstract

In a competitive market, customer decision is made to maximize his utility. It can be assumed that risk of losing a supply chain's customer can be defined based on products utility from customer point of view. This paper takes account of product's price and service level as competition criteria. The proposed model is based on non-cooperative game theory, for one-manufacturer and one-retailer supply chain facing an outside integrated-competitor. The aim of the paper is to investigate the trade-offs of responsiveness and efficiency in a supply chain. Therefore, we consider product utility from the customer point of view as an objective, along with traditional profit objective function. Three scenarios are proposed in the paper: competition based on profit gained, competition based on responsiveness to customer needs, and finally, competition based on the profit gained and responsiveness to customer needs, concurrently. Numerical examples are presented including sensitivity analysis of key parameters. We illustrate that the relative importance that player considers for profit compared to the risk of losing customer plays a critical role in supply chain prosperity.


Keywords Risk of Losing Customer; Utility Function; Non-Cooperative Game; Supply Chain; Responsiveness.








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## 1. INTRODUCTION

In the competitive markets, firms present products with various characteristics; however, it is the customer that decides which products are appropriate to purchase. With the development of technology and the globalization of economy, the competition among firms is evolving into the competition among supply chains (SCs) [1]. Furthermore, with advancement of new ways for
selling products, such as multi channel distribution or internet selling, the SCs competition becomes more severe. This situation makes the SCs to provide more appealing product for each customer segment in order to assure the prosperity of products in the target markets.

Supply chain (SC) responsiveness implies its capabilities to provide product characteristics and service level according to customer requirements. The extra responsiveness of a SC increases the
probability that a customer choose chain's product among other substitutive products in the market. Responsiveness, however, comes at costs. For instance, for replying to short lead time requested by a customer, more investment on service level is needed, which raises costs. Therefore, the increases in cost bring about reduction in SC efficiency [2]. For every strategic choice to increase responsiveness, additional costs that lower efficiency should be carefully considered.

The customers take purchasing decisions based on utility maximization with respect to product "attractions" [3]. In other words, in the competitive market, a customer chooses a product with the highest utility. Therefore, SC competitive strategies are required to specify the product attractions relative to other substitutive products, which result in the foremost utility for the customers.

Pricing is a significant business behavior and competing firms often make a price war to attract customers. In addition to price, service is also an important criterion affecting the purchasing decisions of customers. For instance, in auto industries, financial services such as auto loan, insurance, and maintenance services play a significant role in selecting a brand for customers [1]. Therefore, price and service level are two critical criteria that indicate SC responsiveness to the customers.

In this paper, we propose three models based on the game theory approach for three different scenarios of SCs competition. A novel idea in the paper is taking account of the customer utility in selecting the product brand in the market and afterwards, modeling the competition based on this utility. In the models presented, a SC and a competitor firm simultaneously provide products for a customer. In scenario one, it is assumed that SC and its competitor, regulate product price and service level, with respect to profit objective, regardless of the utility of products for customer. We find that prosperity of the SC under this scenario conditions highly depends on its financial parameters compared to the competitor. The second scenario is related to modeling a severe competitive market in which SC and competitor only consider product utility from customer's point of view. By offering more favorable products, each of them attempts to prevent another one from attracting the customer. Nevertheless, this strategy
can cause immense decrease in profit gained and each party that has better financial capability is able to win the competition.

In scenario three, the two previous scenarios are combined. We study the condition that SC and competitor consider a relative importance for the profit objective with respect to product utility. The results show that this relative importance is a critical parameter in SC prosperity.

The paper is organized as follows: In Section 2, the related literature is reviewed. Section 3 includes a discussion of the problem and the related notation. The basic models are presented through the three separate scenarios within Section 4. Section 5 concerns some computational results of the numerical examples including the optimal price and service level for the players and their sensitivity analysis. Furthermore, the indifference utilities for a customer with respect to SC and competitor products are presented in this section. Finally, the paper concludes in Section 6 with some directions for future research in this context. Proofs are given in Appendix.

## 2. LITERATURE REVIEW

Supply chain consists of independent decision making agents. That is, the SC is typically decentralized which implies that participants are independent firms each with its own conflicting goals spanning production, service, purchasing, inventory, transportation, marketing and other functions. Collaboration and coordination of these agents will hopefully result in adding benefits [4]. In the literature, several researchers reviewed and studied the applying game theory in order to model these conflicting goals (see [4-7]). This paper is closely related to SCs competition based on price and service criteria, and attractive-based market share competition.

### 2.1. Supply Chains Competitions Beside

 competition and collaboration within a SC, in a specific product market, there are some kinds of competition among SCs to attract final customers. Some papers explicitly modeled these SCs interactions. Bernstein and Federguen [8] developed a stochastic inventory model for an oligopoly where demand was a function of all retailers' prices and service levels. Only threescenarios were studied in the paper: price competition, simultaneous price and service level competition, and two stage competition. They have shown that in each of these scenarios, a Nash equilibrium of infinite-horizon stationary strategies exists under which each retailer adopts a stationary price, fill rate, and base-stock policy.

Recently, a stream of SCs competition literature exists that deals with structuring and contracting of chains. Rezapour and Farahani [9] considered an equilibrium model for strategic design of a centralized SCN in markets with deterministic demand encounters a rival chain. The two chains present competitive and substitutable products to the markets, and demand of each market is a function of products' prices. Zhao and Shi [10] took two competitive supply SCs into account, each with multiple upstream suppliers presenting substitutable products to a single buyer. The uncertain demand of the buyer is sensitive to prices of the products. They analyzed the structure and contracting strategy of the rival chains. These two researchers did not consider service level of retailers, nor did they identify utility of products from customers' view point. Hafezalkotob et al. [11] studied a network design problem in a competition of two SCs where uncertain markets' demands depend on price, service level, and marketing expenditure of chains. They assumed that the risk of participants is derived from the uncertainty of market's demands. However, our decision structure in this paper is different, because we consider that prosperity and risk of SCs depend on utility of products from customer's point of view.

Wu et al. [12] considered two manufactures, each producing a substitutable product and selling it by either a decentralized retail store or an integrated one, which was modeled as a pricesetting newsvendor. In the paper, the effect of demand uncertainty and product substitutability on SC configuration, i.e. integrated or non-integrated structure, and also on equilibrium stocking factor was investigated. Mirzahosseinian et. al. [13] investigated a dual channel inventory model in a manufacturer-retailer SC consisting a traditional retailer-channel that competes with a direct channel. The proposed model was structured based on queuing theory where stocks are kept in both upper and lower echelons.

Bernstein and Fedegruen [14] assumed a
general model of two-echelon SCs with several competing retailers served by a common supplier. In their study, the demand of retailers is a stochastic function and depends on all of the firm's prices as well as a measure of their service levels, e.g., the steady-state availability of the products. By applying three different demand functions, the equilibrium Nash price and service level were computed. Hafezalkotob and Makui [15] studied the competition of two SCs which their participants have different risk attitudes. In the proposed model, both chains are internally involved in vertical pricing; however, they externally engage upon vertical pricing and service level competition. They investigated how investment on marketing efforts could reduce the risk of participants derived from demand uncertainty. They did not consider that customer's purchasing behavior coming from products' utilities, may result in risk of SCs.

Xiao and Yang [16] developed an information revelation mechanism model of two-echelon SC facing an outside competitor to investigate the effect of the risk sharing rule on revelation mechanism under demand uncertainty, where the risk sensitivity of the retailer is private information. Xiao and Yang [1] developed a price and service competition model of one manufacturer and one retailer SCs to study the optimal decisions of the players under demand uncertainty. They analyzed the effect of the retailer risk sensitivity on the player's optimal strategies, and the effect of investment efficiency of the retailer on the optimal price-service decisions of his rival, as well.

### 2.2. Price and Service Competition in Supply

 Chains Product price and service level offered by SCs are two significant factors affecting the purchasing decisions of the customers [1]. Several papers considered price and service competition [ $1,8,11,14,15,16,17,18]$. Among these researchers Bernstein and Federgruen [8,14] developed a price and service competition model based on market share computed by attractions and linear demand function, as well. On the other hand, Xiao and Yang [1,16], Allon and Federgruen [17], and Hafezalkotob et. al. [15] proposed models where retailer's demand is a linear combination of product price and service level offered by a SC and another retailer. Correspondingly, Hafezalkotoband Makui [11] investigated the competition of two SCs in prices, service levels, and advertising expenditures based on market's nonlinear demand. Tsay and Agrawal [18] investigated a onemanufacture and two-retailer SC which offered a common product to customers. The competition between retailers in service level and price was modeled based on customer's linear demand function.

### 2.3. Attraction-Based Market Share

 Competition Market share models utilize product attraction divided by total products attractions in the market [19]. Beckmann and Funk [3] presented a theory of household purchasing decisions based on utility maximization which was responsive to product attractions. They illustrated that market share which indirectly shows individual customers purchasing decisions can be considered as a function of these attractions. Cooper [20] reviewed the market share models and introduced several commonly used market share functions. He studied the specification of elasticity of these models and then analyzed the relationships between market shares and individual choice probabilities by considering the choice probabilities and purchase frequency of individual buyers. He also investigated the random base utility models that for extreme-value distribution led to a Multinomial Logit (MNL) market share. The MNL market share represents an aggregate concept that the consumer choice-based utility theory developed by McFadden [21] and others which may serve as a foundation for individual rational decision making in formulating market shares in the aggregate [22]. Gruca and Sudharshan [23] showed that in applying game theory approach for modeling market competition, the Multiplicative Competitive Interaction (MCI) and MNL market share models encounter some convergence problems in finding an equilibrium solution. Therefore, Mesak and Means [24] proposed simple transformations that prepare the MNL model for equilibrium analyses. Market share models are important aspects of competition in the markets and several papers have attempted to develop the competition based on maximization market share by applying the game theory [22,25,26]. These models take into account market share as an aggregate estimate of customers'behaviors.
Although it is the customer that designates conqueror of the market competition, to best of our knowledge through literature review, individual customer preferences are not directly modeled in the SCs competition. Nevertheless, in the market segmentation models, the customers' populations are divided into groups which have similar behaviors and utilities. Therefore, SCs competition can be effectively considered in each market segment. Our paper complements the literature by investigating the effect of customer choice based on utility. Similar to Beckmann and Funk [3], in the present paper, we consider the customer's utility of purchasing product that can be estimated from product characteristics. These characteristics include product price and service level offered by the SC and its competitor. This approach to competition may lead to additional cost for parties; however, it can decrease the risk of losing customer. As a result, by utilizing the model, SC would be able to analyze its product specifications to obtain the least risk of losing customer and the maximum feasible profit.

## 3. PROBLEM DESCRIPTION AND NOTATION

Consider a SC comprising of one manufacturer $\left(M_{I}\right)$ and one retailer ( $R$ ), facing an outside manufacturer $\left(M_{2}\right)$. Let the SC and outsidemanufacturer be indexed by player (1) and (2), respectively. Manufacturer 2 as an integrate-SC is able to sell products directly to customers; however, manufacturer 1 uses the retailer. The SC and outside-manufacturer compete with each other to attract customers in a market. The market includes independent customers each of which has different demand and product attractions. The competition mechanism can be considered for each customer separately. Consequently, the SC and outside-manufacturer competition for attracting a single customer is modeled in this paper. The developed models can be extended for all customers in the market.

Competitors, simultaneously declare retail price and service level to the customer. On the other hand, customer is capable of purchasing products from one of the SC or manufacturer 2, which has higher utility from his point of view. The SC and
manufacturer 2 want to maximize their profit objective and minimize the risk of losing costumer. The two terms manufacturer 2 and outsidemanufacturer are used interchangeably throughout the paper.

### 3.1. Assumptions

1. The parameters are deterministic and known in advance.
2. It is assumed that a competition between a decentralized SC and an integrated one is possible and both are in a same competition position upon the customer.
3. It is assumed that both SCs offer a unique price and service level to each customer.
4. Manufacturer 1 has an ample capacity and the planning period is longer than its lead time which implies that the manufacturer is able to deliver any quantity ordered by the retailer on time. Moreover, it is assumed that manufacturer 2 is an integrated SC, i.e. a manufacturer and a retailer are merged and organized as a single company which has an ample capacity.
5. The demand and the utility function of the customer depend upon product price and offered service level, according to customer's demand and utility function defined in Section 3.3. These functions are revealed for the retailer and manufacturers 1 and 2 in advance.
6. It is assumed that the buyer (customer) selects one of the brands in the market and orders with respect to product's price and offered service level. This assumption is rational in many real long-term contracts such as supplier selection problem.
7. Product types of SC and outsidemanufacturer are completely substitutable and differ only in price, and offered service level.
8. The sequence of the game is as follows:

Stage 1: Manufacturer 1 offers a wholesale price to the retailer over a given planning period of time.
Stage 2: The retailer and manufacturer 2 decide on retail price and service level. They simultaneously present their products to the
customer; one can interpret this situation as a non-cooperative game between the SC and manufacturer 2.
Stage 3: The customer's purchasing decision is based on utility maximization. Thus, at the purchasing time, customer selects the products of SC or outside manufacturer, whichever has high utility costs.
Stage 4: The customer orders products to the selected party based on announced demand function. The selected party delivers the amount ordered by customer.
Stages 1 to 2 indicate decision-making procedure of product providers; however, stage 3 refers to customer selecting procedure. In Stage 4, the real trade between customer and the selected product provider takes place.

### 3.2. Input Parameters

$\theta_{1}$ : the normalized attraction of price from the customer point of view, $\left(\theta_{1}>0\right)$;
$\theta_{2}$ : the normalized attraction of service level
$=\quad$ from the customer point of view, $\left(\theta_{2}>0\right)$;
$P$ : the maximum price that is acceptable for the customer, $P>0$;
$\alpha$ : the customer's base demand for the SCs, ( $\alpha>0$ );
$\beta$ : the demand sensitivity to service level offered to the customer, $(\beta>0)$;
$\gamma$ : the demand sensitivity to price offered to the customer, $(\gamma>0)$;
$c_{i}$ : the unit production purchasing price of $M_{1}$ and $M_{2},\left(c_{i}>0\right)$;
$\eta_{i}$ : the service investment efficiency coefficient of player $i, i=R, M_{2}$. The larger coefficient $\eta_{i}$, the lower the service investment efficiency $\left(1 / \eta_{i}\right)$ of player $i$ will be, $\left(\eta_{i}>0\right)$;
$\lambda_{i}$ : the relative importance coefficient of the profit objective with respect to the product utility objective from customer point of view for player $i, i=R, M_{1}, M_{2}$, ( $0 \leq \lambda_{i} \leq 1$ );
$F C_{i}$ : the fixed operation cost of player $i, i=R, M_{1}, M_{2},\left(F C_{i} \geq 0\right) ;$

### 3.3. Decision Variables

$w$ : the wholesale price of manufacture 1 offered to the retailer, $\left(w \geq c_{M_{1}}\right)$;
$m$ : the retailer's margin profit, $(m \geq 0)$;
$p_{2}$ : the retail price of manufacturer 2 product offered to customer, $\left(p_{2} \geq c_{M_{2}}\right)$;
$p_{1}$ : the retail price of SC product offered to the customer, $\left(p_{1}=m+w\right)$;
$s_{i}$ : the service level of player

$$
i, i=R, M_{2},\left(s_{i} \geq 0\right)
$$

$\mathrm{y}_{i}$ : binary variables which indicate customer purchasing decision, $i=1,2$. If the customer selects product of SC then $\mathrm{y}_{1}=1$, and if he select outside-manufacturer products then $y_{2}=1$.
Figure 1 illustrates the decision variables concerning the SC and outside-manufacturer competition.


Figure 1. Competition schema and related variables

Similar to Xiao and Yang [1,16], Allon and Federgruen [17], Tsay and Agrawal [18], we assume that the demand function of the SCs is affected mainly by the price $(p)$ and service level $(s)$ offered to customer. Let $D$ represents the customer's demand function which is a decreasing function of product price but an increasing function of service level offered to him. This demand function is considered as follows:
$D=\alpha-\gamma p+\beta s$
In this paper, it is assumed that the customer selects one of the SC or manufacturer 2 to purchase from; afterwards, the customer's demand is affected only by price and service level offered by the selected party, i.e. if the customer prefers SC product, then $D\left(p_{1}, s_{R}\right)=a-\gamma p_{1}+\beta s_{R}$ and otherwise, $D\left(p_{2}, s_{M_{2}}\right)=a-\gamma p_{2}+\beta s_{M_{2}}$.

Beckmann and Funke [3] indicated that attractions incorporate properties of a product into the utility function of a customer. This utility function was considered as follows:

$$
\begin{equation*}
U=u(\theta, x) \tag{2}
\end{equation*}
$$

where
$x$ : the vector of product quantities $\left(x_{1}, \ldots, x_{i}, \ldots, x_{n}\right)$,
$\theta$ : the vector of product attractions $\left(\theta_{1}, \ldots, \theta_{i}, \ldots, \theta_{n}\right)$.
Several utility functions can represent relationship between product quantities and product attraction. Since it is assumed that the price and service level are two main attractive factors affecting the customer utility, by applying the linear function form for the customer's utility introduced by Beckmann and Funke [3], we have:
$U=\theta_{1}(\overline{\bar{P}}-p)+\theta_{2} s$
where $\overline{\bar{P}}$, or veto price, is the maximum price which the customer is convinced to purchase a specific product, thus if the product price is higher than $\overline{\bar{P}}$, he would reject purchasing regardless of the service level offered by seller. Therefore, the customer's utility of products of SC and manufacturer 2 are $U_{1}=U\left(p_{1}, s_{R}\right)=\theta_{1}\left(\overline{\bar{P}}-p_{1}\right)+\theta_{2} s_{R}$ and $U_{2}=U\left(p_{2}, s_{R}\right)=\theta_{1}\left(\overline{\bar{P}}-p_{2}\right)+\theta_{2} s_{M_{2}}$, respectively. If $U_{1} \geq U_{2}$ the customer selects SC's product;otherwise, he prefers products of manufacturer 2.

## 4. THE BASIC MODELS

### 4.1. Scenario One: Profit Maximization in the

 Simultaneous Game In scenario one, it is considered that the players, i.e. retailer andmanufacturers 1 and 2 , only regard profit to maximize without any attention to how the product will be assessed by the customer.

### 4.1.1. The Supply Chain Problem Based On

 Scenario One Manufacturer 1 incurs margin unit production cost $c_{M_{1}}$ and offers products at wholesale price $w$, i.e., the manufacturer's margin is $w-c_{M_{1}}$. On the other hand, the retailer sets retail price $p_{1}=m+w$, where $m$ is the retailer's profit margin. The retailer sets service level $s_{R}$, to appeal customer to choose the SC's product and purchase more amounts.Similar to Xiao and Yang [1,16], Tsay and Agrawal [18], and Gilbert and Cvsa [27], we assume that when a player provides service level $s$, the service cost of the player is $\frac{1}{2} \eta s^{2}$, i.e., improving service level has a diminishing return on service expenditure. In this scenario, both SC partners, i.e. manufacturer 1 and the retailer want to maximize profit-margin expressed as $\pi_{M_{1}}(w)$ and $\pi_{R}\left(m, s_{R}\right)$, without concerning the product price and service level of manufacturer 2. The SC problem is formulated as follows:

$$
\begin{gathered}
\max _{w} \pi_{M_{1}}(w)=\left(w-c_{M_{1}}\right)\left(a-\gamma(m+w)+\beta s_{R}\right)-F C_{M_{1}} \\
\max _{m, R} \pi_{R}\left(m, s_{R}\right)== \\
m\left(a-\gamma(m+w)+\beta s_{R}\right) \\
-\frac{1}{2} \eta_{R} s_{R}{ }^{2}-F C_{R},
\end{gathered}
$$

s.t.
$w \geq c_{M_{1}}$
$m \geq 0$
$s_{R} \geq 0$
$\left(w-c_{M_{1}}\right)\left(a-\gamma(m+w)+\beta s_{R}\right)-F C_{M_{1}} \geq 0$
$m\left(a-\gamma(m+w)+\beta s_{R}\right)-\frac{1}{2} \eta_{R} s_{R}^{2}-F C_{R} \geq 0$.

Constraints (9) and (10) state that financial loss is not acceptable for manufacturer 1 and the retailer, respectively. Note that these constraints can be
extended in a situation that each player in the SC has a minimum acceptable profit.
4.1.2. Manufacturer 2 Problem Based On Scenario One Manufacturer 2, as an integrated SC , incurs margin unit production cost $c_{M_{2}}$ and offers products at price $p_{2}$ to the customer, i.e. the manufacturer margin is $p_{2}-c_{M_{2}}$. He sets service level $s_{M_{2}}$, to attract the customer to select its product and to encourage him to buy more amounts. It is assumed that he encounters fixed operation cost $F C_{M_{2}}$. Manufacturer 2 desires to maximize profit-margin times demand minus service level and fixed operation costs which is represented as $\pi_{M_{2}}\left(p_{2}, s_{M_{2}}\right)$. Manufacturer 2 problem can be modeled as follows:

$$
\begin{align*}
\max _{p_{2}, s_{M_{2}}} \pi_{M_{2}}\left(p_{2}, s_{M_{2}}\right)= & \left(p_{2}-c_{M_{2}}\right)\left(a-\gamma p_{2}+\beta s_{M_{2}}\right. \\
& -\frac{1}{2} \eta_{M_{2}} s_{M_{2}}{ }^{2}-F C_{M_{2}} \tag{11}
\end{align*}
$$

s.t.
$p_{2} \geq c_{M_{2}}$
$s_{M_{2}} \geq 0$
$\left(p_{2}-c_{M_{2}}\right)\left(a-\gamma p_{2}+\beta s_{M_{2}}\right)-\frac{1}{2} \eta_{M_{2}} s_{M_{2}}{ }^{2}$
$-F C_{M_{2}} \geq 0$
Constraint (14) ensures that the retailer and manufacturer 1 profit in trading.
Hessian matrices of $\pi_{R}$ and $\pi_{M_{2}}$ are:
$H_{R}=\left[\begin{array}{cc}-2 \gamma & \beta \\ \beta & -\eta_{R}\end{array}\right]$ and $H_{M_{2}}=\left[\begin{array}{cc}-2 \gamma & \beta \\ \beta & -\eta_{M_{2}}\end{array}\right]$.

Therefore, the retailer and manufacturer profit functions are concave on $\left(m, s_{M_{1}}\right)$ and $\left(p_{2}, s_{M_{2}}\right)$, if and only if their Hessian matrices are negatively definited [28]. Let us denote by $B_{i}=2 \gamma \eta_{i}-\beta^{2}, i=R, M_{2}$, the determinant of the Hessian matrices.

Proposition 1. If $B_{R}, B_{M_{2}}>0$, then the optimal decisions for price and service level of the players are:
$w^{n}=\frac{\eta_{R}\left(\alpha+2 \gamma c_{M_{1}}\right)-\beta^{2} c_{M_{1}}}{3 \eta_{R} \gamma-\beta^{2}}$,
$m^{n}=\frac{\eta_{R}\left(\alpha-\gamma c_{M_{1}}\right)}{3 \eta_{R} \gamma-\beta^{2}}$,
$s_{M_{1}}^{n}=\frac{\beta\left(\alpha-\gamma c_{M_{1}}\right)}{3 \eta_{R} \gamma-\beta^{2}}$,
$P_{2}^{n}=\frac{\eta_{M_{2}}\left(\alpha+\gamma c_{M_{2}}\right)-\beta^{2} c_{M_{2}}}{2 \eta_{M_{2}} \gamma-\beta^{2}}$,
$s_{M_{2}}^{n}=\frac{\beta\left(\alpha-\gamma c_{M_{2}}\right)}{2 \eta_{M_{2}} \gamma-\beta^{2}}$.
Note that the condition $B_{i}>0$ in Proposition 1 assures that the optimum solution satisfying the first order condition of $\pi_{R}$ and $\pi_{M_{2}}$ is optimal and each party obtains a positive profit. The parties excessively invest in service if $B_{i} \leq 0$, which incurs a negative profit to them. The condition $B_{i}>0$ means that the service investment should not be too inexpensive, which is consistent with those considered in [1,18, 27]. Therefore, in the present paper we assume $B_{R}, B_{M_{2}}>0$. Moreover, since the negative profit margin and service level are not feasible for the retailer, i.e. $m^{n}, s_{R}^{n}>0$, and the negative service level is not possible for manufacturer 2 as well, i.e. $s_{M_{2}}^{n}>0$, and with taking $B_{R}, B_{M_{2}}>0$ into account, we assume $\alpha>\max \left\{\gamma c_{M_{1}}, \gamma c_{M_{2}}\right\} \quad$ throughout this paper. Substituting Eqs. (15) into Eq. (3), gives
$U_{1}^{n}=\frac{\theta_{1}\left(\eta_{R} \gamma+B_{R}\right)\left(\overline{\bar{P}}-c_{M_{1}}\right)}{\eta_{R} \gamma+B_{R}}-\frac{\left(\alpha-\gamma c_{M_{1}}\right)\left(2 \theta_{1} \eta_{R}-\theta_{2} \beta\right)}{\eta_{R} \gamma+B_{R}}$,
$U_{2}^{n}=\frac{\theta_{1} B_{M_{2}}\left(\overline{\bar{P}}-c_{M_{2}}\right)}{B_{M_{2}}}-\frac{\left(\alpha-\gamma c_{M_{2}}\right)\left(\theta_{1} \eta_{M_{2}}-\theta_{2} \beta\right)}{B_{M_{2}}}$.
$U_{1}^{n}$ and $U_{2}^{n}$ are customer's utilities for SC and outside-manufacturer's products, respectively, corresponding to the Nash equilibrium solution (15).

### 4.1.3. The Customer Problem Based on

 Scenario One The customer makes his purchasing decision based on utility maximization, therefore when SC and manufacturer 2 declare their service level and product price, he selects product which has a higher utility. Hence, the customer purchasing decision model can be formulated as follows:$$
\begin{equation*}
\max _{y_{1}, y_{2}} U\left(y_{1}, y_{2}\right)=y_{1} U_{1}^{n}+y_{2} U_{2}^{n} \tag{17}
\end{equation*}
$$

s.t.
$\overline{\bar{P}}-w^{n}-m^{n}+M\left(1-y_{l}\right)>0$,
$\overline{\bar{P}}-p_{2}^{n}+M\left(1-y_{2}\right)>0$,
$y_{1}+y_{2}<1$,

$$
\begin{align*}
\pi_{R}^{\text {real }}= & m^{n} \cdot y_{1}\left(a-\gamma\left(m^{n}+w^{n}\right)+\beta s_{R}^{n}\right)  \tag{21}\\
& -\frac{1}{2} \eta_{R} s_{R}^{n 2}-F C_{R}, \\
\pi_{M_{1}}^{\text {real }}= & \left(w^{n}-c_{M_{1}}\right) y_{1}\left(a-\gamma\left(m^{n}+w^{n}\right)+\beta s_{R}\right)-F C_{M_{1}} \tag{22}
\end{align*}
$$

$$
\begin{align*}
\pi_{M_{2}}^{\text {real }}= & \left(p_{2}^{n}-c_{M_{2}}\right) y_{2}\left(a-\gamma p_{2}^{n}+\beta s_{M_{2}}^{n}\right)-1 / 2 \eta_{M_{2}} s_{M_{2}}^{n}{ }^{2}  \tag{23}\\
& -F C_{M_{2}}, \\
y_{1}, y_{2} & \in\{0,1\} \tag{24}
\end{align*}
$$

Eqs. (15) and (16).
Here, $M$ represents a very large positive number, thus constraints (18) and (19) state that if the price offered was higher than the veto price then the customer refuses to purchase. Constraint (20) expresses that customer will choose one of the products offered by SC and manufacturer 2 . When customer selects SC product, i.e. $y_{1}=1$, there exists no demand for manufacturer 2 , and vice versa. Therefore, according to constraints (21)-(23) actual profits of the players depend on customer selections. In the competitive market, if $U_{1}^{n}<U_{2}^{n}$ and $p_{2}^{n}<\overline{\bar{P}}$ then manufacturer 2 will attract the customer i.e. $y_{2}=1$. Substituting Eqs. (15) into Eq. (23) yields:

$$
\begin{equation*}
\pi_{M_{2}}^{\text {real }}=\frac{\eta_{M_{2}}\left(\alpha-\gamma c_{M_{2}}\right)^{2}-2 F C_{M_{2}} B_{M_{2}}}{2 B_{M_{2}}} \tag{25}
\end{equation*}
$$

Conversely, if $U_{1}^{n}>U_{2}^{n}$ and $w^{n}+m^{n}<\overline{=}$ then $y_{1}=1$. Replacing Eqs. (15) in Eqs. (21) and (22) gives:

$$
\begin{align*}
& \pi_{R}^{\text {real }}=\frac{\eta_{R} B_{R}\left(\alpha-\gamma c_{M_{1}}\right)^{2}}{\left(\eta_{R} \gamma+B_{R}\right)^{2}}-\frac{2 F C_{R}\left(\eta_{R} \gamma+B_{R}\right)^{2}}{\left(\eta_{R} \gamma+B_{R}\right)^{2}},  \tag{26}\\
& \pi_{M_{1}}^{\text {real }}=\frac{\eta_{R}^{2} \gamma\left(\alpha-\gamma c_{M_{1}}\right)^{2}}{\left(\eta_{R} \gamma+B_{R}\right)^{2}}-\frac{F C_{M_{1}}\left(\eta_{R} \gamma+B_{R}\right)^{2}}{\left(\eta_{R} \gamma+B_{R}\right)^{2}} . \tag{27}
\end{align*}
$$

Since the players only maximize their own profit, the values obtained for the profits are higher than the profits values in the twosubsequent scenarios. Thus, the expected profits corresponding the optimal solution, i.e. $\pi_{M_{2}}^{\text {real }}, \pi_{M_{1}}^{\text {real }}$, and $\pi_{R}^{\text {real }}$ are considered as upper bounds for the profit in the two succeeding scenarios, and they are represented as $\overline{\pi_{M_{2}}}, \overline{\pi_{M_{1}}}$, and $\overline{\pi_{R}}$, respectively. According to Eqs. (9), (10), and (14), these ideal profits need to be positive values, otherwise players refuse to trade with the customer. This condition means that the fixed operation costs should not be too high. More specifically, it is required to have $F C_{M_{2}}<\eta_{M_{2}}\left(\alpha-\gamma c_{M_{2}}\right)^{2} / 2 B_{M_{2}}$,
$F C_{R}<\eta_{R} B_{R}\left(\alpha-\gamma c_{M_{1}}\right)^{2} / 2\left(\eta_{R} \gamma+B_{R}\right)^{2}, \quad$ and $F C_{M_{1}}<\eta_{R}^{2} \gamma\left(\alpha-\gamma c_{M_{1}}\right)^{2} /\left(\eta_{R} \gamma+B_{R}\right)^{2}$.

Proposition 2. If the SC and manufacturer 2 are assumed identical, i.e. all parameters including $F C_{i}, c_{i}$, and $\eta_{i}$ are considered the same (represented by $F C, c$, and $\eta$, respectively), we have:
i. The SC's service level is constantly less than outside-manufacturer's service level.
ii. The SC's product price is higher than the outside-manufacturer's product price if $\beta^{2}<\eta \gamma$ and vice versa if $\eta \gamma<\beta^{2}<2 \eta \gamma$.
iii. The SC will win the competition if $\frac{-\left(\eta \gamma-\beta^{2}\right)}{\beta \gamma}>\frac{\theta_{2}}{\theta_{1}}$ and vice versa, if
$\frac{-\left(\eta \gamma-\beta^{2}\right)}{\beta \gamma}<\frac{\theta_{2}}{\theta_{1}}$

Note when the SC's product price is higher than the outside-manufacturer's one, i.e. if $\quad \beta^{2}<\eta \gamma$ then we have $-\left(\eta \gamma-\beta^{2}\right) / \beta \gamma<\theta_{2} / \theta_{1}$; thus the SC will certainly lose in the competition. Otherwise, i.e. $\eta \gamma<\beta^{2}<2 \eta \gamma$, according to the conditions $-\left(\eta \gamma-\beta^{2}\right) / \beta \gamma>\theta_{2} / \theta_{1} \quad$ or $\quad-\left(\eta \gamma-\beta^{2}\right) / \beta \gamma<\theta_{2} / \theta_{1}$, either the SC or manufacturer 2 will attract the customer, respectively.

### 4.2. Scenario Two: Risk Evasion Strategies in the Simultaneous Game In the highly

 competitive market, the profit margin decreases with respect to severity of the competition among the parties. In this scenario, it is considered that the SC partners and the outside-manufacturer endeavor to present more favorable product to the customer, without attention to the profit of trading. Thus, they are able to decrease the risk of losing the customer. According to Eq. (3), offering more favorable product involves lower product price accompanied by higher service level. Since the retailer and outside-manufacturer simultaneously offer products to the customer, they are not aware of each other's price and service level; hence, they attempt to offer more suitable products with lower price supporting by more services, regardless of the decision taken by the other party. In this scenario, the models of SC partners and outsidemanufacturer are developed, afterward; the customer decision effects on players prosperity are investigated.
### 4.2.1. The Supply Chain Problem in Scenario

 Two In the SC, both players, i.e. the manufacturer and retailer, desire to offer more favorable products to the customer. Thus, taking account of customer's utility function (3), the manufacturer problem can be expressed mathematically as:$$
\begin{equation*}
\max _{w} U\left(m+w, s_{R}\right)=\theta_{1}(\overline{\bar{P}}-m-w)+\theta_{2} s_{R}, \tag{28}
\end{equation*}
$$

s.t.

Constraints (6), (7), (8) and (9).
Moreover, the retailer problem can be formulated as follows:

$$
\begin{equation*}
\max _{m, s_{R}} U\left(m+w, s_{R}\right)=\theta_{1}(\overline{\bar{P}}-m-w)+\theta_{2} s_{R} \tag{29}
\end{equation*}
$$

s.t.

Constraints (6), (7), (8), and (10).
Proposition 3. When manufacturer 1 and the retailer are profitable, and the retailer offers a service level, i.e. $s_{R}>0$, then the minimum risk strategy, i.e. $\underline{m}, \underline{w}$, and $\underline{s_{R}}$, for the SC can be computed by solving the following nonlinear equations:

$$
\begin{align*}
& \left(\underline{w},-c_{M_{1}}\right)\left(a-\gamma(\underline{m}+\underline{w})+\beta \underline{s_{R}}\right)-F C_{M_{1}}=0,  \tag{30}\\
& \left(\beta \theta_{1}-2 \gamma \theta_{2}\right) \underline{m}+\left(\beta \theta_{2}-\eta_{R} \theta_{1}\right) \underline{s_{R}}-\gamma \theta_{2} \underline{w}+\alpha \theta_{2}=0,  \tag{31}\\
& \underline{m}\left(a-\gamma(\underline{m}+\underline{w})+\beta \underline{s_{R}}\right)-1 / 2 \eta_{R} \underline{s}_{R}^{2}-F C_{R}=0 . \tag{32}
\end{align*}
$$

Proposition 3 states that the SC partners are ready to spend all profit margins to increase their product utility from the customer point of view. Thus, the customer utility with respect to this strategy, i.e. ( $\underline{m}, \underline{w}, s_{R}$ ), reaches a peak, and the corresponding utility can be interpreted as an upper bound or ideal value for the customer utility from the SC's product. Let us denote by $\overline{U_{1}}$, the upper bound for SC's product utility.

### 4.2.2. Manufacturer 2 Problem in Scenario Two

 Similar to the SC, under the conditions of scenario 2 , manufacturer 2 is willing to make price and service level decisions in order to reduce the risk of losing customer. The retailer and manufacturer 2 simultaneously offer products to the customer. Therefore, the manufacturer needs to increase customer utility, regardless of what price and services offered by SC. Hence, the utility maximization problem faced by manufacturer 2 is given by:$\max _{p_{2}, s_{M_{2}}} U\left(p_{2}, s_{M_{2}}\right)=\theta_{1}\left(\overline{\bar{P}}-p_{2}\right)+\theta_{2} s_{M_{2}}$,
s.t.

Constraints (12), (13), and (14).
Proposition 4. When the outside-manufacturer is
profitable and has a service level, i.e. $s_{M_{2}}>0$, then the minimum risk strategy, i.e. $\underline{p_{2}}$ and $s_{M_{2}}$, can be computed by solving the following nonlinear equations:

$$
\begin{align*}
& \left(\beta \theta_{1}-2 \gamma \theta_{2}\right) \underline{p_{2}}+\left(\beta \theta_{2}-\eta_{M_{2}} \theta_{1}\right) \underline{s_{M_{2}}}-  \tag{34}\\
& c_{M_{2}} \theta_{1} \beta+\theta_{2}\left(\alpha-\gamma c_{M_{2}}\right)=0, \\
& \left(\underline{p_{2}}-c_{M_{2}}\right)\left(a-\gamma \underline{p_{2}}+\beta \underline{s_{M_{2}}}\right)-\frac{1}{2} \eta_{M_{2}} \underline{s_{M_{2}}}{ }^{2}  \tag{35}\\
& -F C_{M_{2}}=0 .
\end{align*}
$$

Proposition 4 expresses that the competitor utilizes all financial resources to increase product utility from the customer viewpoint. Therefore, the customer utility with respect to this strategy, i.e. $\left(\underline{p_{2}}, \underline{s_{M_{2}}}\right)$, can be an upper bound for $U_{2}$ which is denoted by $\overline{U_{2}}$.

### 4.2.3. The Customer Problem in Scenario Two

The customer selects the product with higher attraction, thus his problem can be expressed mathematically as:

$$
\begin{equation*}
\max _{y_{1}, y_{2}} U\left(y_{1}, y_{2}\right)=y_{1} \overline{U_{1}}+y_{2} \overline{U_{2}}, \tag{36}
\end{equation*}
$$

s.t.

$$
\begin{equation*}
\overline{\bar{P}}-\underline{w}-\underline{m}+M\left(1-y_{1}\right)>0, \tag{37}
\end{equation*}
$$

$$
\begin{equation*}
\overline{\bar{P}}-\underline{p_{2}}+M\left(1-y_{2}\right)>0, \tag{38}
\end{equation*}
$$

$\pi_{R}^{\text {real }}=\underline{m} \cdot y_{1}\left(a-\gamma(\underline{m}+\underline{w})+\beta \underline{s_{R}}\right)-\frac{1}{2} \eta_{R} \underline{s_{R}}{ }^{2}-F C_{R}$,

$$
\begin{equation*}
\pi_{M_{1}}^{\text {real }}=\left(\underline{m}-c_{M_{1}}\right) y_{1}\left(a-\gamma(\underline{m}+\underline{w})+\beta \underline{s_{R}}\right)-F C_{M_{1}}, \tag{40}
\end{equation*}
$$

$$
\begin{align*}
\pi_{M}^{\text {real }}= & \left(\underline{p_{2}}-c_{M}\right) y_{2}\left(a-\gamma \underline{p_{2}}+\beta \underline{s_{M}}\right) \\
& -\frac{1}{2} \eta_{M_{2}} s_{M_{2}}{ }^{2}-F C_{M_{2}} \tag{41}
\end{align*}
$$

Constraints (20) and (24).
Objective function (36) states that the customer prefers more favorable products. Constraints (37) and (38) guarantee that the customer does not select product with the price higher than veto price. Constraints (39)-(41) represent the actual profit of the retailer, manufacturer 1 and 2, respectively, which depend on the customer purchasing decision. The model expresses that each party that has better capability to present more attractive product to the customer, will succeed in the competition. Note, in the extreme risk evasion scenario, none of the players profit in the market.
For instance, if $\overline{U_{1}}>\overline{U_{2}}$ and $\overline{\bar{P}}>\underline{w}+\underline{m}$, then SC will sell products to the customer, i.e. $y_{1}=1$ and $y_{2}=0$. According to Proposition 3, the price $\underline{w}+\underline{m}$ and service level $s_{R}$ have zero profits for the retailer and manufacturer 1 . On the other hand, since $y_{2}=0$, it is obvious from Eq. (41) that manufacturer 2 loses $0.5 \eta_{M_{2}} \underline{s_{M_{2}}}{ }^{2}+F C_{M_{2}}$.

### 4.3. Scenario Three: Risk Evasion and Profit Seeking Strategies in the Simultaneous Game

Optimizing profit, regardless of utility of product from customer point of view can diminish product competitive advantage in comparison with substitute products. Therefore, parties in the market can reduce risk of losing customers by offering more satisfying products. On the other hand, as showed in scenario two, increasing product utility without attention to the profit objective, makes the profit margin go to a deep. However, in the real market competition, these two extreme scenarios are rare conditions. In scenario three, by combining the two previous scenarios, we investigate risk evasion and profit seeking strategies. Hence, for the retailer and manufacturer 1 and 2, a relative importance coefficient, i.e. $\lambda_{i}, 0 \leq \lambda_{i} \leq 1$, is considered which represents the relative importance of profit gained with respect to the importance of product utility. $\quad \lambda_{i}=1$ means that player $i$ only takes profit objective into account, and the model situation will be identical to scenario one. In the same way, $\lambda_{i}=0$ will yield the model of scenario two. In the highly competitive markets, it is expected that parties
choose $\lambda_{i}$ near to zero, means that they emphasize increasing product utility and decreasing risk of losing customer.

This relative importance plays a significant role in the marketing strategies and states how competitors take their strategic decisions to lower their risks and obtain profit in the market. In other words, by regulating $\lambda_{i}$, firms specify their effectiveness with respect to responsiveness to the customer needs.

### 4.3.1. The Supply Chain Problem in Scenario

 Three In the SC, manufacturer 1 and the retailer optimize the combination of profit and product utility from customer viewpoint. Hence, manufacturer 1 problem is as follows:$$
\begin{align*}
& \min _{w} \operatorname{Payoff}_{M_{1}}(w)= \\
& \lambda_{M_{1}} \frac{\overline{\pi_{M_{1}}}-\left[\left(w-c_{M_{1}}\right)\left(a-\gamma(m+w)+\beta s_{R}\right)-F C_{M_{1}}\right]}{\overline{\pi_{M_{1}}}} \\
& +\left(1-\lambda_{M_{1}}\right) \frac{\overline{U_{1}}-\left[\theta_{1}(\overline{\bar{P}}-m-w)+\theta_{2} s_{R}\right]}{\overline{U_{1}}} \tag{42}
\end{align*}
$$

s.t.

$$
\begin{equation*}
0 \leq \lambda_{M_{1}} \leq 1, \tag{43}
\end{equation*}
$$

Constraints (6), (7) and (8).

The objective function includes two parts: profit and utility objectives. The first part refers to the difference between ideal profit and manufacturer's profit. The second one indicates the difference between ideal utility and utility of SC product. Optimizing the manufacturer's payoff leads to non-dominated solutions corresponding to profit and utility objectives. Note that for investigating the effect of changing $\lambda_{i}$ on the profit objectives, the constraints which assure players to be profitable, i.e. constraints (9), (10) and (14) are relaxed in this scenario.

In the SC , losing the customer makes the same risk for the manufacturer and retailer. Thus, the retailer like the manufacturer desires to provide more satisfactory products for the customer. The retailer problem is formulated as follows:

$$
\begin{align*}
& \min _{m, s_{R}} \operatorname{Payoff}_{R}\left(m, s_{R}\right)= \\
& \quad \overline{\pi_{R}}-\left[m\left(a-\gamma(m+w)+\beta s_{R}\right)-\frac{1}{2} \eta_{R} s_{R}^{2}-F C_{R}\right] \\
& \lambda_{R} \frac{\overline{\pi_{R}}}{\overline{U_{1}}-\left[\theta_{1}(\overline{\bar{P}}-m-w)+\theta_{2} s_{R}\right]} \\
& +\left(1-\lambda_{R}\right) \frac{\overline{U_{1}}}{} \\
& \text { s.t. }  \tag{45}\\
& 0 \leq \lambda_{R} \leq 1,
\end{align*}
$$

Constraints (6), (7) and (8).
The utility function part of the retailer payoff is identical with the manufacturer's one. Therefore, the retailer like the manufacturer can increase the competitive advantage of product by selecting small values for $\lambda_{R}$.

### 4.3.2. Manufacturer 2 Problem in Scenario

Three Manufacturer 2 problem can be expressed as:

$$
\begin{align*}
& \min _{p_{2}, s_{2}} \operatorname{Payoff}_{M_{2}}\left(p_{2}, s_{2}\right)= \\
& \lambda_{M_{2}} \frac{\overline{\pi_{M_{2}}}-\left[\left(p_{2}-c_{M_{2}}\right)\left(a-\gamma p_{2}+\beta s_{M_{2}}\right)\right]}{\overline{\pi_{M_{2}}}} \\
& +\lambda_{M_{2}} \frac{\frac{1}{2} \eta_{M_{2}} s_{M_{2}}{ }^{2}-F C_{M_{2}}}{\overline{\pi_{M_{2}}}}  \tag{46}\\
& +\left(1-\lambda_{M_{2}}\right) \frac{\overline{U_{2}}-\left[\theta_{1}\left(\overline{\bar{P}}-p_{2}\right)+\theta_{2} s_{M_{2}}\right]}{\overline{U_{2}}}
\end{align*}
$$

s.t.
$0 \leq \lambda_{M_{2}} \leq 1$,
Constraints (12) and (13).
$\overline{\pi_{M_{1}}}$ and $\overline{U_{M_{2}}}$ are ideal values for the profit and product utility, respectively. Therefore, the model optimizes the difference of profit and utility objectives from their ideal values. The higher $\lambda_{M_{2}}$, the higher the importance of the profit objective and the lower the importance of the risk objective will be.

Proposition 5. If $B_{R}, B_{M_{2}}>0$, then the optimal retail price and the optimal service level of the retailer and manufacturers 1 and 2 in the risk
evasion and profit seeking scenario are:

$$
\begin{align*}
& w^{n}= \frac{\eta_{R}\left(\alpha+2 \gamma c_{M_{1}}\right)-\beta^{2} c_{M_{1}}}{3 \eta_{R} \gamma-\beta^{2}} \\
&+\frac{\overline{\pi_{R}} \lambda_{M_{1}}\left(\lambda_{R}-1\right) \eta_{R} \theta_{1} \gamma}{\overline{U_{1}} \lambda_{M_{1}} \lambda_{R} \gamma\left(B_{R}+\eta_{R} \gamma\right)} \\
&+\frac{\overline{\pi_{M_{1}}} \lambda_{R}\left(1-\lambda_{M_{1}}\right) \theta_{1} B_{R}}{\overline{U_{1}} \lambda_{M_{1}} \lambda_{R} \gamma\left(B_{R}+\eta_{R} \gamma\right)} \\
&+\frac{\overline{\pi_{R}} \lambda_{M_{1}}\left(1-\lambda_{R}\right) \beta\left(\beta \theta_{1}-\gamma \theta_{2}\right)}{\overline{U_{1}} \lambda_{M_{1}} \lambda_{R} \gamma\left(B_{R}+\eta_{R} \gamma\right)}, \\
& m^{n}= \frac{\eta_{R}\left(\alpha-\gamma c_{M_{1}}\right)}{\frac{3 \eta_{R} \gamma-\beta^{2}}{\pi_{R}} \lambda_{M_{1}}\left(1-\lambda_{R}\right)\left(2 \eta_{R} \theta_{1}-\beta \theta_{2}\right)} \\
&+\frac{\overline{U_{1}} \lambda_{M_{1}} \lambda_{R} \gamma\left(B_{R}+\eta_{R} \gamma\right)}{\pi_{M_{1}} \lambda_{R}\left(1-\lambda_{M_{1}}\right) \eta_{R} \theta_{1}}, \\
&-\frac{\overline{U_{1}} \lambda_{M_{1}} \lambda_{R} \gamma\left(B_{R}+\eta_{R} \gamma\right)}{s_{r}^{n}=}  \tag{48}\\
& \beta \frac{\beta\left(\alpha-\gamma c_{M_{1}}\right)}{3 \eta_{R} \gamma-\beta^{2}} \\
&+ \frac{\frac{\pi_{R}}{\lambda_{M_{1}}}\left(1-\lambda_{R}\right)\left(2 \theta_{1} \beta-3 \gamma \theta_{2}\right)}{\overline{U_{1}} \lambda_{M_{1}} \lambda_{R}\left(B_{R}+\eta_{R} \gamma\right)} \\
& \frac{\frac{\pi_{M_{1}}}{\lambda_{R}}\left(1-\lambda_{M_{1}}\right) \theta_{1} \beta}{U_{1} \lambda_{M_{1}} \lambda_{R}\left(B_{R}+\eta_{R} \gamma\right)},
\end{align*}
$$

$$
\begin{aligned}
P_{2}^{n}= & \frac{\eta_{M_{2}}\left(\alpha+\gamma c_{M_{1}}\right)-\beta^{2} c_{M_{1}}}{2 \eta_{M_{2}} \gamma-\beta^{2}} \\
& +\frac{\overline{\pi_{M_{2}}}\left(1-\lambda_{M_{2}}\right)\left(\theta_{1} \eta_{M_{2}}-\theta_{2} \beta\right)}{\overline{U_{2}} \lambda_{M_{2}} B_{M_{2}}} \\
s_{2}^{n}= & \frac{\beta\left(a-\gamma c_{M_{2}}\right)}{2 \eta_{M_{2}} \gamma-\beta^{2}} \\
& +\frac{\frac{\overline{\pi_{M_{2}}}\left(1-\lambda_{M_{2}}\right)\left(\theta_{1} \beta-2 \theta_{2} \gamma\right)}{\overline{U_{2}} \lambda_{M_{2}} B_{M_{2}}}}{}
\end{aligned}
$$

The optimal values consist of two parts. The first parts are independent of $\lambda_{i}$ and they are similar to optimal values computed in scenario one. Nevertheless, the second parts depend on $\lambda_{i}$ and ideal profit and utility values. For $\lambda_{R}=\lambda_{M_{1}}=\lambda_{M_{2}}=1$, the players only consider the profit objective and the optimum values are identical to what are obtained in scenario one. Substituting Eqs. (48) into Eq. (3) yields

$$
\begin{align*}
& U_{1}^{n}=\frac{\theta_{1}\left(\eta_{R} \gamma+B_{R}\right)\left(\overline{\bar{P}}-c_{M_{1}}\right)}{\eta_{R} \gamma+B_{R}} \\
&- \frac{\left(\alpha-\gamma c_{M_{1}}\right)\left(2 \theta_{1} \eta_{R}-\theta_{2} \beta\right)}{\eta_{R} \gamma+B_{R}} \\
&--\overline{\pi_{R}} \lambda_{M_{1}} \eta_{R} \theta_{1}^{2} \gamma\left(\lambda_{R}-1\right) \\
&-\overline{U_{1}} \lambda_{M_{1}} \lambda_{R} \gamma\left(\eta_{R} \gamma+B_{R}\right)  \tag{49}\\
&-\frac{\pi_{M_{1}} \lambda_{R}\left(1-\lambda_{M_{1}}\right) \theta_{1}\left(2 \eta_{R} \gamma \theta_{1}\right)}{\overline{U_{1}} \lambda_{M_{1}} \lambda_{R} \gamma\left(\eta_{R} \gamma+B_{R}\right)} \\
&-\frac{\overline{\pi_{M_{1}}} \lambda_{R}\left(1-\lambda_{M_{1}}\right) \theta_{1}\left(-\beta^{2} \theta_{1}-\eta_{R} \theta_{1}+\beta \theta_{2}\right)}{\overline{U_{1}} \lambda_{M_{1}} \lambda_{R} \gamma\left(\eta_{R} \gamma+B_{R}\right)} \\
&-\frac{\overline{\pi_{R}} \lambda_{M_{1}}\left(1-\lambda_{R}\right)\left(\beta^{2} \theta_{1}^{2}-\beta \gamma \theta_{1} \theta_{2}\right)}{\overline{U_{1}} \lambda_{M_{1}} \lambda_{R} \gamma\left(\eta_{R} \gamma+B_{R}\right)} \\
&-\frac{\overline{\pi_{R}} \lambda_{M_{1}}\left(1-\lambda_{R}\right)\left(2 \eta_{R} \theta_{1}^{2}-3 \beta \theta_{1} \theta_{2}+3 \gamma \theta_{2}^{2}\right)}{\overline{U_{1}} \lambda_{M_{1}} \lambda_{R} \gamma\left(\eta_{R} \gamma+B_{R}\right)} .
\end{align*}
$$

By differentiating $U_{1}^{n}$ and $U_{2}^{n}$ with respect to the relative importance coefficients, we have

$$
\begin{align*}
& \frac{\partial U_{1}^{n}}{\partial \lambda_{M_{1}}}=\frac{\overline{\pi_{M_{1}}} \theta_{1}\left(\theta_{1} \beta^{2}-\eta_{R} \gamma \theta_{1}-\beta \gamma \theta_{2}\right)}{\overline{U_{1}} \lambda_{M_{1}}^{2} \gamma\left(3 \eta_{R} \gamma-\beta^{2}\right)}, \\
& \frac{\partial U_{1}^{n}}{\partial \lambda_{R}}=\frac{\overline{\pi_{R}}\left(-\theta_{1}^{2} \beta^{2}-3 \theta_{2}^{2} \gamma^{2}-\eta_{R} \gamma \theta_{1}^{2}+4 \theta_{1} \theta_{2} \beta \gamma\right)}{\overline{U_{1}} \lambda_{R}^{2} \gamma\left(3 \eta_{R} \gamma-\beta^{2}\right)},  \tag{50}\\
& \frac{\partial U_{2}^{n}}{\partial \lambda_{M_{2}}}=\frac{\overline{\pi_{M_{2}}}\left(-\theta_{1}^{2} \eta_{M_{2}}+2 \theta_{1} \theta_{2} \beta-2 \theta_{2}^{2} \gamma\right)}{\overline{U_{2}} \lambda_{M_{2}}^{2}\left(2 \eta_{M_{2}} \gamma-\beta^{2}\right)} .
\end{align*}
$$

Proposition 6 summarizes the behavior of the
product utility of the SC and outside manufacturer with respect to the variations of the relative importance coefficients.

Proposition 6. Taking $B_{R}, B_{M_{2}}>0$ into account, we have
i. If $\eta_{R} \theta_{1} \leq \beta \theta_{2}$ then $\partial U_{1} / \partial \lambda_{M_{1}} \leq 0$,
ii. If $\eta_{R} \theta_{1} \geq(4-2 \sqrt{3}) \theta_{2} \beta$ then $\partial U_{1} / \partial \lambda_{R} \leq 0$,
iii. $\partial U_{2} / \partial \lambda_{M_{2}} \leq 0$.

Proposition 6 and Eqs. (50) give the following insights:

- When player $i$ concentrates on the competition and improvement of the product utility, $\lambda_{i}$ decreases. It is expected that this condition leads to more attractive and appealing products from the customer point of view, which can mathematically be stated as $\partial U / \partial \lambda_{i} \leq 0$. According to Part (iii) of Proposition 6, this condition is true for an integrated SC such as manufacturer 2. In the case of non-integrated SC i.e. SC 1, according to Part (i) and (ii), at least one of $\partial U_{1} / \partial \lambda_{M_{1}} \leq 0$ and $\partial U_{1} / \partial \lambda_{R}$ are negative and in the special case $(4-2 \sqrt{3}) \beta \theta_{2} \leq \theta_{1} \eta \leq \beta \theta_{2}$ both of them are negative as well.
- With respect to Eqs. (50), both of $\lambda_{M_{1}}$ and $\lambda_{R}$ affect the SC's product utility, however because of $\partial^{2} U_{1}^{n} / \partial \lambda_{M_{1}} \partial \lambda_{R}=0$, there is no interactive effect between these two coefficients and the product utility. In other words, by regulating the importance coefficients, the manufacturer and retailer in the SC can independently cause the product utility to increase or decrease.


### 4.3.3. The Customer Problem in Scenario Three

Afterward the SC and manufacturer 2 declare their service level and product price, customer selects product based on his utility function. The customer problem is

$$
\begin{align*}
& \max _{y_{1}, y_{2}} U\left(y_{1}, y_{2}\right)=y_{1} U_{1}^{n}+y_{2} U_{2}^{n},  \tag{51}\\
& \text { s.t. } \\
& \pi_{R}^{\text {real }}= m^{n} y_{1}\left(a-\gamma\left(m^{n}+w^{n}\right)+s_{R}^{n}\right) \\
&-\frac{1}{2} \eta_{R} s_{R}^{n 2}-F C_{R},  \tag{52}\\
& \pi_{M_{1}}^{\text {real }}=\left(w^{n}-c_{M_{1}}\right) y_{1}\left(a-\gamma\left(m^{n}+w^{n}\right)+\beta s_{R}^{n}\right) \\
&-F C_{M_{1}},  \tag{53}\\
& \pi_{M_{2}}^{\text {real }}=\left(p_{2}^{n}-c_{M_{2}}\right) y_{2}\left(a-\gamma p_{2}^{n}+\beta s_{M_{2}}^{n}\right) \\
&-\frac{1}{2} \eta_{M_{2}} s_{M_{2}}^{n}{ }^{2}-F C_{M_{2}}, \tag{54}
\end{align*}
$$

Constraints (18), (19), (20), (24), (48) and (49).
Constraints (52)-(54) express the actual profit of the players which depends on customer purchasing decisions.

## 5. EXTENSION OF THE MODEL IN THE REAL MULTI-BUYERS ENVIRONMENT

In the real competition in the markets, SCs often compete for several buyers. In this circumstance, the utility and demand functions should be individually estimated for each buyer. For instance, in market segmentation, the heterogeneous market is divided into groups of individual markets made up of people or organizations with one or more specific characteristics. The market may be segmented according to lifestyle, psychographic behavior, gender, religion, income, or even geographical locations [29]. The participants of the segment are relatively homogeneous, that is they have analogous needs, wants, and psychographic behaviors regarding special product type. The presented model for the SCs competition can effectively be developed for these distinctive market segments. To this end, let $N$ represent the set of market segments $\{1,2, . ., n, . .|N|\}$, and assume $a_{n}, \beta_{n}, \gamma_{n}, \theta_{n 1}$, and $\theta_{n 2}$ are corresponding coefficients for market segment $n$. Now, we are able to develop the payoffs of the manufacturer, retailer, and rival manufacturer as follows:

$$
\begin{aligned}
& \min _{w} \operatorname{Payoff}_{M_{1}}(w)=\lambda_{M_{1}} \frac{\left[\left(w-c_{M_{1}}\right) \sum_{n \in N}\left(a_{n}-\gamma_{n}(m+w)+\beta_{n} s_{R}\right)-F C_{M_{1}}\right]}{\overline{\pi_{M_{1}}}} \\
& +\left(1-\lambda_{M_{1}}\right) \frac{\overline{U_{1}}-\sum_{n \in N}\left[\theta_{n 1}(\overline{\bar{P}}-m-w)+\theta_{n 2} s_{R}\right]}{\overline{U_{1}}},
\end{aligned}
$$

$$
\min _{m, s_{R}} \operatorname{Payoff}_{R}\left(m, s_{R}\right)=\lambda_{R} \frac{\left[m \sum_{n \in N}\left(a_{n}-\gamma_{n}(m+w)+\beta_{n} s_{R}\right)\right]}{\overline{\pi_{R}}}
$$

$$
+\lambda_{R} \frac{\frac{1}{2} \eta_{R} s_{R}^{2}+F C_{R}}{\overline{\pi_{R}}}
$$

$$
+\left(1-\lambda_{R}\right) \frac{\overline{U_{1}}-\sum_{n \in N}\left[\theta_{n 1}(\overline{\bar{P}}-m-w)+\theta_{n 2} s_{R}\right]}{\overline{U_{1}}}
$$

$$
\min _{p_{2}, s_{2}} \operatorname{Payoff}_{M_{2}}\left(p_{2}, s_{2}\right)=\frac{\left[\left(p_{2}-c_{M_{2}}\right) \sum_{n \in N}\left(a_{n}-\gamma_{n} p_{2}+\beta_{n} s_{M_{2}}\right)\right]}{\overline{\pi_{M_{2}}}}
$$

$$
+\lambda_{M_{2}} \frac{\frac{1}{2} \eta_{M_{2}} s_{M_{2}}{ }^{2}+F C_{M_{2}}}{\overline{\pi_{M_{2}}}}
$$

$$
+\left(1-\lambda_{M_{2}}\right) \frac{\overline{U_{2}}-\sum_{n \in N}\left[\theta_{n 1}\left(\overline{\bar{P}}-p_{2}\right)+\theta_{n 2} s_{M_{2}}\right]}{\overline{U_{2}}} .
$$

Optimal decisions of the SC and rival manufacturer $w^{n}, m^{n}, s_{r}^{n}, P_{2}^{n}$ and $s_{2}^{n}$, are computed similar to Preposition 5. Since the real profits of competitors depend on decisions of each market segment, the real profits are as follows:

$$
\begin{aligned}
\pi_{R}^{\text {real }}= & m^{n} \sum_{n \in N} y_{n 1}\left(a_{n}-\gamma_{n}(m+w)+\beta_{n} s_{R}\right) \\
& -\frac{1}{2} \eta_{R} s_{R}^{n 2}-F C_{R}, \\
\pi_{M_{1}}^{\text {real }}= & \left(w^{n}-c_{M_{1}}\right) \sum_{n \in N} y_{n 1}\left(a_{n}-\gamma_{n}(m+w)+\beta_{n} s_{R}\right) \\
& -F C_{M_{1}}, \\
\pi_{M_{2}}^{\text {real }}= & \left(p_{2}^{n}-c_{M_{2}}\right) \sum_{n \in N} y_{n 2}\left(a_{n}-\gamma_{n} p_{2}^{n}+\beta_{n} s_{M_{2}}^{n}\right) \\
& -\frac{1}{2} \eta_{M_{2}} s_{M_{2}}^{n}{ }^{2}-F C_{M_{2}},
\end{aligned}
$$

where $y_{n 1}$ and $y_{n 2}$ are binary variables that indicate decisions of each market segment (buyer) with respect to products' utility.

## 6. COMPUTATIONAL RESULTS

Now, we illustrate the effect of the risk evasion strategies on the optimal profit and utility of the products offered by the SC and outsidemanufacturer. In this numerical example, we consider the buyer attaches higher importance to the service level as compared to product price, that is $\theta_{1}=0.25$ and $\theta_{2}=0.75$. For simplicity and following Xiao and Yang [1] and Hafezalkotob et al. [11], we assume the production price and service investment efficiency coefficients are identical in both chains. The default values of the parameters are as follows:
$\theta_{1}=0.25, \theta_{2}=0.75, \alpha=5000, \gamma=10, \beta=0.5$,
$\eta_{R}=\eta_{M_{2}}=0.1, c_{M_{1}}=c_{M_{2}}=100, \overline{\bar{P}}=500$,
$F C_{R}=F C_{M_{1}}=100000$, and $F C_{M_{2}}=200000$.
These default values fulfill the conditions $B_{R}, B_{M_{2}}>0$. In Scenario one that the SC's partners only consider profit objective according to Proposition 1, the optimal decisions are computed as below:
$w^{n}=245.4545, m^{n}=145.4545, s_{R}^{n}=727.2727$,
$P_{2}^{n}=328.5714$, and $s_{2}^{n}=1142.8571$
Both the SC's product prices, i.e. $w^{n}+m^{n}$ and $P_{2}^{n}$, are feasible because they are less than the veto price. According to Proposition 2 Part (iii), we have $\quad-\left(\eta_{i} \gamma-\beta^{2}\right) / \beta \gamma=-0.15<\theta_{2} / \theta_{1}=3$, therefore, the SC will lose in the competition and the customer chooses the competitor's product to purchase, i.e. $y_{2}=1$. Solving the customer problem in Scenario 1 results in:
$y_{1}=0, y_{2}=1, U_{1}^{n}=572.7273, U_{2}^{n}=900$,
$\pi_{R}^{\text {real }}=-126446.281, \pi_{M_{1}}^{\text {real }}=-100000$, and
$\pi_{R}^{\text {real }}=257142.8572$.

The potential upper bound values for the profit gained by the retailer, manufacturers 1 and 2 are derived from Constraints (26), (27), and (25), which equal to $85123.9669,111570.2478$, and 257142.8572, respectively.

Under the conditions of scenario two, the players attempt to promote their product utility regardless of the profit gained in trade. According to Proposition 3, the SC strategies are found by solving Eqs. (30)-(32) simultaneously, which leads to $\quad \underline{m}=267.0074, \quad \underline{w}=136.4412, \quad$ and $\underline{s_{R}}=3557.2685$. Similarly, according to Proposition 4, the outside-manufacturer's price and service level are found by solving Eqs. (34) and (35) simultaneously, which results in $\underline{p_{2}}=385.6012$ and $\underline{s_{2}}=3566.6243$.

Obviously, products prices of the SC and outside-manufacturer in the risk evasion strategy are lesser than product prices in the profit seeking strategy (in scenario one). Conversely, service levels offered by players are increased dramatically through the Scenario 2. The maximum products utilities of the SC and outsidemanufacturer are $\overline{U_{1}}=2492.0892$ and $\overline{U_{2}}=2792.0892$, respectively. Note when players choose the risk evasion strategy, the utility of product from the customer viewpoint rises considerably. This consequence is consistent with many industrial situations which SCs endeavor to improve products competitive advantage by offering more satisfying products and services to customers. Since $\overline{U_{2}}>\overline{U_{1}}$, from customer problem (36)-(41), it follows that $y_{1}=0$ and $y_{2}=1$.

In scenario three, each partner regulates the relative importance coefficients to obtain the profit and diminish the risk in the competition. These coefficients are critical controlling parameters that can distinguish the prosperity of a SC against other competitors. Thus, we intend to investigate the effect of changing these coefficients on the profit gained and the product utility for each player. Tables 1-4 present the optimum price and service level decisions of the SC and its competitor and also the customer's utility value corresponding to these decisions. As mentioned earlier, the customer selects the SC product with higher utility.

TABLE 1. Sensitivity analysis of the risk evasion and profit seeking model with respect to $\lambda_{M_{1}}, \lambda_{R}$, and $\lambda_{M_{2}}=0.25$

| $N o$. | $\lambda_{M_{1}}$ | $\lambda_{R}$ | $w^{n}$ | $m^{n}$ | $S_{R}^{n}$ | $P_{2}^{n}$ | $s_{2}^{n}$ | $U_{1}$ | $U_{2}$ | $y_{1}$ | $y_{2}$ | $\pi_{M_{1}}$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 0.25 | 0.25 | 256.54 | 157.25 | 1466.12 | 274.03 | 3461.04 | 1246.14 | 2777.27 | 0 | 1 | -100000.00 | -207475.72 |
| 2 | 0.25 | 0.5 | 247.89 | 150.11 | 977.16 | 274.03 | 3461.04 | 883.37 | 2777.27 | 0 | 1 | -100000.00 | -147741.72 |
| 3 | 0.25 | 0.75 | 245.01 | 147.73 | 814.17 | 274.03 | 3461.04 | 762.44 | 2777.27 | 0 | 1 | -100000.00 | -133143.45 |
| -104531.60 |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 4 | 0.25 | 1 | 243.56 | 146.53 | 732.67 | 274.03 | 3461.04 | 701.98 | 2777.27 | 0 | 1 | -100000.00 | -126840.51 |
| 5 | 0.5 | 0.25 | 257.80 | 156.53 | 1462.52 | 274.03 | 3461.04 | 1243.31 | 2777.27 | 0 | 1 | -100000.00 | -206948.50 |
| -104531.60 |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 6 | 0.5 | 0.5 | 249.15 | 149.39 | 973.56 | 274.03 | 3461.04 | 880.53 | 2777.27 | 0 | 1 | -100000.00 | -147390.55 |
| 7 | 0.5 | 0.75 | 246.27 | 147.00 | 810.57 | 274.03 | 3461.04 | 759.61 | 2777.27 | 0 | 1 | -104531.60 |  |
| 8 | 0.5 | 1 | 244.82 | 145.81 | 729.07 | 274.03 | 3461.04 | 699.14 | 2777.27 | 0 | 1 | -10000.00 | -132850.96 |
| 9 | 0.75 | 0.25 | 258.22 | 156.29 | 1461.32 | 274.03 | 3461.04 | 1242.36 | 2777.27 | 0 | 1 | -100000.00 | -126577.37 |
| 10 | 0.75 | 0.5 | 249.57 | 149.15 | 972.36 | 274.03 | 3461.04 | 879.59 | 2777.27 | 0 | 1 | -100000.00 | -206773.05 |
| 11 | -104531.60 |  |  |  |  |  |  |  |  |  |  |  |  |
| 12 | 0.75 | 0.75 | 246.69 | 146.76 | 809.37 | 274.03 | 3461.04 | 758.66 | 2777.27 | 0 | 1 | -100000.00 | -132753.76 |
| 12 | -104531.60 |  |  |  |  |  |  |  |  |  |  |  |  |
| 13 | 1 | 0.25 | 258.43 | 145.57 | 727.87 | 274.03 | 3461.04 | 698.20 | 2777.27 | 0 | 1 | -100000.00 | -126489.94 |
| 14 | 1 | 0.5 | 249.78 | 156.17 | 1460.72 | 274.03 | 3461.04 | 1241.89 | 2777.27 | 0 | 1 | -100000.00 | -206685.38 |
| 15 | 1 | 0.75 | 246.90 | 146.64 | 971.76 | 274.03 | 3461.04 | 879.12 | 2777.27 | 0 | 1 | -100000.00 | -147215.45 |
| 16 | 1 | 1 | 245.45 | 145.45 | 708.77 | 274.03 | 3461.04 | 758.19 | 2777.27 | 0 | 1 | -100000.00 | -132705.21 |
| -104531.60 |  |  |  |  |  |  |  |  |  |  |  |  |  |

TABLE 2. Sensitivity analysis of the risk evasion and profit seeking model with respect to $\lambda_{M_{1}}, \lambda_{R}$, and $\lambda_{M_{2}}=0.5$

| No. | $\lambda_{M_{1}}$ | $\lambda_{R}$ | $w^{n}$ | $m^{n}$ | $s_{R}^{n}$ | $P_{2}^{n}$ | $s_{2}^{n}$ | $U_{1}$ | $U_{2}$ | $y_{1}$ | $y_{2}$ | $\pi_{M_{1}}$ | $\pi_{R}$ | $\pi_{M_{1}}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 0.25 | 0.25 | 256.54 | 157.25 | 1466.12 | 310.39 | 1915.59 | 1246.14 | 1609.09 | 0 | 1 | -100000.00 | -207475.72 | 216956.81 |
| 2 | 0.25 | 0.5 | 247.89 | 150.11 | 977.16 | 310.39 | 1915.59 | 883.37 | 1609.09 | 0 | 1 | -100000.00 | -147741.72 | 216956.81 |
| 3 | 0.25 | 0.75 | 245.01 | 147.73 | 814.17 | 310.39 | 1915.59 | 762.44 | 1609.09 | 0 | 1 | -100000.00 | -133143.45 | 216956.81 |
| 4 | 0.25 | 1 | 243.56 | 146.53 | 732.67 | 310.39 | 1915.59 | 701.98 | 1609.09 | 0 | 1 | -100000.00 | -126840.51 | 216956.81 |
| 5 | 0.5 | 0.25 | 257.80 | 156.53 | 1462.52 | 310.39 | 1915.59 | 1243.31 | 1609.09 | 0 | 1 | -100000.00 | -206948.50 | 216956.81 |
| 6 | 0.5 | 0.5 | 249.15 | 149.39 | 973.56 | 310.39 | 1915.59 | 880.53 | 1609.09 | 0 | 1 | -100000.00 | -147390.55 | 216956.81 |
| 7 | 0.5 | 0.75 | 246.27 | 147.00 | 810.57 | 310.39 | 1915.59 | 759.61 | 1609.09 | 0 | 1 | -100000.00 | -132850.96 | 216956.81 |
| 8 | 0.5 | 1 | 244.82 | 145.81 | 729.07 | 310.39 | 1915.59 | 699.14 | 1609.09 | 0 | 1 | -100000.00 | -126577.37 | 216956.81 |
| 9 | 0.75 | 0.25 | 258.22 | 156.29 | 1461.32 | 310.39 | 1915.59 | 1242.36 | 1609.09 | 0 | 1 | -100000.00 | -206773.05 | 216956.81 |
| 10 | 0.75 | 0.5 | 249.57 | 149.15 | 972.36 | 310.39 | 1915.59 | 879.59 | 1609.09 | 0 | 1 | -100000.00 | -147273.79 | 216956.81 |
| 11 | 0.75 | 0.75 | 246.69 | 146.76 | 809.37 | 310.39 | 1915.59 | 758.66 | 1609.09 | 0 | 1 | -100000.00 | -132753.76 | 216956.81 |
| 12 | 0.75 | 1 | 245.24 | 145.57 | 727.87 | 310.39 | 1915.59 | 698.20 | 1609.09 | 0 | 1 | -100000.00 | -126489.94 | 216956.81 |
| 13 | 1 | 0.25 | 258.43 | 156.17 | 1460.72 | 310.39 | 1915.59 | 1241.89 | 1609.09 | 0 | 1 | -100000.00 | -206685.38 | 216956.81 |
| 14 | 1 | 0.5 | 249.78 | 149.03 | 971.76 | 310.39 | 1915.59 | 879.12 | 1609.09 | 0 | 1 | -100000.00 | -147215.45 | 216956.81 |
| 15 | 1 | 0.75 | 246.90 | 146.64 | 808.77 | 310.39 | 1915.59 | 758.19 | 1609.09 | 0 | 1 | -100000.00 | -132705.21 | 216956.81 |
| 16 | 1 | 1 | 245.45 | 145.45 | 727.27 | 310.39 | 1915.59 | 697.73 | 1609.09 | 0 | 1 | -100000.00 | -126446.28 | 216956.81 |

TABLE 3. Sensitivity analysis of the risk evasion and profit seeking model with respect to $\lambda_{M_{1}}, \lambda_{R}$, and $\lambda_{M_{2}}=0.75$

| No. | $\lambda_{M_{1}}$ | $\lambda_{R}$ | $w^{n}$ | $m^{n}$ | $s_{R}^{n}$ | $P_{2}^{n}$ | $s_{2}^{n}$ | $U_{1}$ | $U_{2}$ | $y_{1}$ | $y_{2}$ | $\pi_{M_{1}}$ | $\pi_{R}$ | $\pi_{M_{1}}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 0.25 | 0.25 | 256.54 | 157.25 | 1466.12 | 322.51 | 1400.43 | 1246.14 | 1219.70 | 1 | 0 | 149709.81 | 43357.20 | -298060.66 |
| 2 | 0.25 | 0.5 | 247.89 | 150.11 | 977.16 | 322.51 | 1400.43 | 883.37 | 1219.70 | 0 | 1 | -100000.00 | -147741.72 | 252677.74 |
| 3 | 0.25 | 0.75 | 245.01 | 147.73 | 814.17 | 322.51 | 1400.43 | 762.44 | 1219.70 | 0 | 1 | -100000.00 | -133143.45 | 252677.74 |
| 4 | 0.25 | 1 | 243.56 | 146.53 | 732.67 | 322.51 | 1400.43 | 701.98 | 1219.70 | 0 | 1 | -100000.00 | -126840.51 | 252677.74 |
| 5 | 0.5 | 0.25 | 257.80 | 156.53 | 1462.52 | 322.51 | 1400.43 | 1243.31 | 1219.70 | 1 | 0 | 150583.60 | 41608.65 | -298060.66 |
| 6 | 0.5 | 0.5 | 249.15 | 149.39 | 973.56 | 322.51 | 1400.43 | 880.53 | 1219.70 | 0 | 1 | -100000.00 | -147390.55 | 252677.74 |
| 7 | 0.5 | 0.75 | 246.27 | 147.00 | 810.57 | 322.51 | 1400.43 | 759.61 | 1219.70 | 0 | 1 | -100000.00 | -132850.96 | 252677.74 |
| 8 | 0.5 | 1 | 244.82 | 145.81 | 729.07 | 322.51 | 1400.43 | 699.14 | 1219.70 | 0 | 1 | -100000.00 | -126577.37 | 252677.74 |
| 9 | 0.75 | 0.25 | 258.22 | 156.29 | 1461.32 | 322.51 | 1400.43 | 1242.36 | 1219.70 | 1 | 0 | 150870.84 | 41027.81 | -298060.66 |
| 10 | 0.75 | 0.5 | 249.57 | 149.15 | 972.36 | 322.51 | 1400.43 | 879.59 | 1219.70 | 0 | 1 | -100000.00 | -147273.79 | 252677.74 |
| 11 | 0.75 | 0.75 | 246.69 | 146.76 | 809.37 | 322.51 | 1400.43 | 758.66 | 1219.70 | 0 | 1 | -100000.00 | -132753.76 | 252677.74 |
| 12 | 0.75 | 1 | 245.24 | 145.57 | 727.87 | 322.51 | 1400.43 | 698.20 | 1219.70 | 0 | 1 | -100000.00 | -126489.94 | 252677.74 |
| 13 | 1 | 0.25 | 258.43 | 156.17 | 1460.72 | 322.51 | 1400.43 | 1241.89 | 1219.70 | , | 0 | 151013.70 | 40737.77 | -298060.66 |
| 14 | 1 | 0.5 | 249.78 | 149.03 | 971.76 | 322.51 | 1400.43 | 879.12 | 1219.70 | 0 | 1 | -100000.00 | -147215.45 | 252677.74 |
| 15 | 1 | 0.75 | 246.90 | 146.64 | 808.77 | 322.51 | 1400.43 | 758.19 | 1219.70 | 0 | 1 | -100000.00 | -132705.21 | 252677.74 |
| 16 | 1 | 1 | 245.45 | 145.45 | 727.27 | 322.51 | 1400.43 | 697.73 | 1219.70 | 0 | 1 | -100000.00 | -126446.28 | 252677.74 |

TABLE 4. Sensitivity analysis of the risk evasion and profit seeking model with respect to $\lambda_{M_{1}}, \lambda_{R}$ and $\lambda_{M_{2}}=1$

| $N o$. | $\lambda_{M_{1}}$ | $\lambda_{R}$ | $w^{n}$ | $m^{n}$ | $S_{R}^{n}$ | $P_{2}^{n}$ | $s_{2}^{n}$ | $U_{1}$ | $U_{2}$ | $y_{1}$ | $y_{2}$ | $\pi_{M_{1}}$ | $\pi_{R}$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 0.25 | 0.25 | 256.54 | 157.25 | 1466.12 | 328.57 | 1142.86 | 1246.14 | 1025.00 | 1 | 0 | 149709.81 | 43357.20 | -265306.12 |
| 2 | 0.25 | 0.5 | 247.89 | 150.11 | 977.16 | 328.57 | 1142.86 | 883.37 | 1025.00 | 0 | 1 | -100000.00 | -147741.72 | 257142.86 |
| 3 | 0.25 | 0.75 | 245.01 | 147.73 | 814.17 | 328.57 | 1142.86 | 762.44 | 1025.00 | 0 | 1 | -100000.00 | -133143.45 | 257142.86 |
| 4 | 0.25 | 1 | 243.56 | 146.53 | 732.67 | 328.57 | 1142.86 | 701.98 | 1025.00 | 0 | 1 | -100000.00 | -126840.51 | 257142.86 |
| 5 | 0.5 | 0.25 | 257.80 | 156.53 | 1462.52 | 328.57 | 1142.86 | 1243.31 | 1025.00 | 1 | 0 | 150583.60 | 41608.65 | -265306.12 |
| 6 | 0.5 | 0.5 | 249.15 | 149.39 | 973.56 | 328.57 | 1142.86 | 880.53 | 1025.00 | 0 | 1 | -100000.00 | -147390.55 | 257142.86 |
| 7 | 0.5 | 0.75 | 246.27 | 147.00 | 810.57 | 328.57 | 1142.86 | 759.61 | 1025.00 | 0 | 1 | -100000.00 | -132850.96 | 257142.86 |
| 8 | 0.5 | 1 | 244.82 | 145.81 | 729.07 | 328.57 | 1142.86 | 699.14 | 1025.00 | 0 | 1 | -100000.00 | -126577.37 | 257142.86 |
| 9 | 0.75 | 0.25 | 258.22 | 156.29 | 1461.32 | 328.57 | 1142.86 | 1242.36 | 1025.00 | 1 | 0 | 150870.84 | 41027.81 | -265306.12 |
| 10 | 0.75 | 0.5 | 249.57 | 149.15 | 972.36 | 328.57 | 1142.86 | 879.59 | 1025.00 | 0 | 1 | -100000.00 | -147273.79 | 257142.86 |
| 11 | 0.75 | 0.75 | 246.69 | 146.76 | 809.37 | 328.57 | 1142.86 | 758.66 | 1025.00 | 0 | 1 | -100000.00 | -132753.76 | 257142.86 |
| 12 | 0.75 | 1 | 245.24 | 145.57 | 727.87 | 328.57 | 1142.86 | 698.20 | 1025.00 | 0 | 1 | -100000.00 | -126489.94 | 257142.86 |
| 13 | 1 | 0.25 | 258.43 | 156.17 | 1460.72 | 328.57 | 1142.86 | 1241.89 | 1025.00 | 1 | 0 | 151013.70 | 40737.77 | -265306.12 |
| 14 | 1 | 0.5 | 249.78 | 149.03 | 971.76 | 328.57 | 1142.86 | 879.12 | 1025.00 | 0 | 1 | -100000.00 | -147215.45 | 257142.86 |
| 15 | 1 | 0.75 | 246.90 | 146.64 | 808.77 | 328.57 | 1142.86 | 758.19 | 1025.00 | 0 | 1 | -100000.00 | -132705.21 | 257142.86 |
| 16 | 1 | 1 | 245.45 | 145.45 | 727.27 | 328.57 | 1142.86 | 697.73 | 1025.00 | 0 | 1 | -100000.00 | -126446.28 | 257142.86 |

From Tables 2 and 3, we find that although the parameters for the SC and the outside manufacturer are assumed similar, the outsidemanufacturer gets more competitive advantage than the SC, due to the fact that when the outsidemanufacturer chooses high or even medium importance for product utility, i.e. $\lambda_{M_{2}}=0.25$ or 0.5 , there is no chance for the SC to retain the customer. On the other hand, it is obvious from Tables 4 and 5 that the retailer has a key role in the SC prosperity, because when he elects a high utility importance coefficent to attract customer, i.e. $\lambda_{R}=0.25$, there will be a chance for the SC to beat the competitor. Thus, the manufacturer without cooprating with the retailer and even in the case of choosing a high importance for attracting cutomer, i.e. 0.25 , has no opportunity to attract the customer. One can conclude that the manufacturer needs to negotiate with the retailer and motivate him by offering a discount or persuasive payement to choose a higher utility importance coefficent.

The customer is indifferent to purchase from the SC or outside manufacturer, if the utility of both offered products be the same for him. Hence, the combination $\left(\lambda_{M_{1}}, \lambda_{R}, \lambda_{M_{2}}\right)$ makes an indifference point for the customer, if and only if $U_{1}\left(\lambda_{M_{1}}, \lambda_{R}\right)=U_{2}\left(\lambda_{M_{2}}\right)$. By considering $\lambda_{M_{1}}, \lambda_{R}$, and $\lambda_{M_{2}}$ as variables and from Eqs. (49), we have
$U_{1}\left(\lambda_{M_{1}}, \lambda_{R}\right)=\frac{514.9218 \lambda_{M_{1}} \lambda_{R}+181.387 \lambda_{R}+1.4176 \lambda_{R}}{\lambda_{M_{1}} \lambda_{R}}$,

$$
U_{2}\left(\lambda_{M_{1}}\right)=\frac{440.9084 \lambda_{M_{1}}+584.0916}{\lambda_{M_{1}}}
$$

Solving $U_{1}\left(\lambda_{M_{1}}, \lambda_{R}\right)=U_{2}\left(\lambda_{M_{2}}\right)$ leads to the indifference surface in Fig. 2. Space inside the surface demonstrates the combination of coefficients that the customer prefers the outsidemanufacturer's product rather than the SC's one, i.e. $\quad U_{1}\left(\lambda_{M_{1}}, \lambda_{R}\right)<U_{2}\left(\lambda_{M_{2}}\right)$, therefore the competitor will defeat the SC in this region.


Figure 2. Indifference surface for customer which none of the products has no preference to another

To analyze the effect of the relative importance coefficient on the profit gained by each player, we consider the case which the SC players choose the same value for this coefficient, i.e. $\lambda_{M_{1}}=\lambda_{R}=\lambda$. Figure 3 demonstrates the combinations of $\lambda$ and $\lambda_{M_{2}}$ that the SC overcomes the competitor. When $\lambda$ is very low (i.e. SC extremely emphasizes the utility of product) and on the other hand, the competitor selects medium or high $\lambda_{M_{2}}$, the SC will attract the customer.


Figure 3. The SC success in attracting the customer with respect to the profit-risk importance coefficients

Figs. 4 and 5 illustrate the manufacturer 1 and retailer profits with respect to the relative importance coefficient, respectively. Obviously, only in the region which the SC defeats the outside-manufacturer which is specified in Fig. 3 with value 1 for $v_{1}$, these profits may be positive. Figure 6 elucidates the outside-manufacturer profit that may be positive only in the region which he defeats the SC. This region is stated in Fig. 3 with value 0 for $y_{i}$. Since when the retailer and outsidemanufacturer consider very low values for $\lambda_{R}$ and $\lambda_{M_{2}}$; they need to reduce the product price and increase the service level investment. According to Figs. 4 and 5, this condition leads to the sharply fall of their profits.


Figure 4. The supplier profit with respect to the profitutility importance coefficients


Figure 5. The retailer profit with respect to the profitutility importance coefficients


Figure 6. The outside-manufacturer profit with respect to the profit-utility importance coefficients

## 7. EXTENSIONS AND MANAGERIAL INSIGHTS IN THE REAL COMPETITIVE MARKETS

- The presented model was restricted to two criteria affecting customers buying decisions. However, one can develop this model for other aspects of product such as advertising and products' quality. At first, as shown in Fig. 7 both utility and demand functions should be estimated for obtaining ideal profit and utility. Afterwards, the ideal values are employed in the model of scenario 3 to compute optimal decisions.


Figure 7. The model-building schema for the copetition of suppliers

- When an integrated-SC and a nonintegrated-SC have similar cost structures and offer similar products to a buyer, the product utility of the nonintegrated one is often lower than the integrated one. That is, the competition through a SC often deteriorates responsiveness of the chain to the buyer's needs.
- Unlike and integrated-SC, in a nonintegrated-SC, all behaviors of all participants affect the utility of product from buyer's viewpoint. Consequently, the coordination among SC's participants
would boost the chance of product prosperity in the market.
- In the practical applications of the model, paired comparison matrix according to AHP technique can be utilized for estimating utility function of the market segments. Moreover, demand function of the buyers may be estimated by multivariable regression on data obtained from buyer behavior analysis.
- The model may be employed in the supplier selection problem from the buyer's point of view. That is, when the buyer is a large manufacturer, the competition between the SC and his rival manufacturer can be considered as the competition between the suppliers and the buyer's problem will be supplier selection problem based on the maximum offered products' utilities.
- This paper considers each SC having no information regarding the rival chain. However, in the real competition, each SC may have an estimation of rivals' strategies. Therefore, symmetric or asymmetric information of rivals about each other would be a very interesting development of the paper, which leads to games under symmetric or asymmetric information.
- The paper assumes SCs pursue uniform pricing and service level strategies for different buyers. It can be extended to the case where SCs take the combination of uniform and buyer-specific decisions regarding some trade legislations. For instance, SCs may set uniform retail-price for all buyers considering anti-trust law; however, the offered service level may set differently regarding the majority, importance, or reputation of buyers.


## 8. CONCLUSION

Product utility from customer point of view is a critical parameter that specifies the purchasing decision in every competitive market. We define the risk of losing customer based on these product utilities. Taking this definition into account,
providing appealing and attractive products that have the most utility for costumer results in declining SCs risk in the markets. However, providing such products may be far from economical decisions of SCs. Therefore, SCs in every market environment need to determine their product characteristics, such as product price and service level, that simultaneously guarantee customer satisfaction and profit gained. In this research, we investigate two scenarios that SCs only consider profit or risk of losing costumer in competitive environment. Afterwards, by combining these two scenarios, the third one that SCs concurrently consider profit and risk of losing customer is generated. Our results imply that the importance which each player considers for profit with respect to risk is an importance coeffication that specifies the winner in the competition. In the case of nonintegrated SC, prosperity depends on this parameter for each partner; however the effect of the parameter for different partners of SC may vary based on problem situation.

## 9. APPENDIX

Proof of Proposition 1. Due to $\partial^{2} \pi_{M_{1}}(w) / \partial w^{2}=-2 \gamma \leq 0$, profit function of manufacturer 1 is concave on $w$. On the other hand, Hessian matrices of $\pi_{R}$ and $\pi_{M_{2}}$ are

$$
\mathbf{H}_{R}=\left[\begin{array}{cc}
-2 \gamma & \beta \\
\beta & -\eta_{R}
\end{array}\right] \text { and } \mathbf{H}_{M_{2}}=\left[\begin{array}{cc}
-2 \gamma & \beta \\
\beta & -\eta_{M_{2}}
\end{array}\right] .
$$

Therefore, the retailer and outside-manufacturer profits are concave functions on $\left(m, s_{R}\right)$ and $\left(p_{2}, s_{2}\right)$ respectively, if and only if the Hessian matrix is negative definite [28]. Due to $\gamma, \eta_{i} \geq 0$ and $B_{R}, B_{M_{2}}>0$, the retailer and outside-manufacturer profit objective functions are concave.

To determine the Nash pricing and service level equilibrium, which correspond to simultaneous moves of the retailer and manufacturers 1 and 2, we need to consider the optimality condition for their objective functions.

The Nash equilibrium, ( $w^{n}, m^{n}, s_{R}^{n}, p_{2}^{n}, s_{2}^{n}$ ), is found by solving the system of equations simultaneously (A.1)-(A.5) that results in Eqs. (15). Thus, Proposition 1 follows.

Proof of Proposition 2. Part (i): In the symmetric case where $c_{M_{1}}=c_{M_{2}}=c_{M}$ and $\eta_{R}=\eta_{M_{2}}=\eta$, since $\eta \gamma>0$, it follows $s_{2}^{n}>s_{R}^{n}$. Part (ii): In the symmetric case, we know that the competitor product price is

$$
P_{2}^{n}=\left[\eta\left(\alpha+\gamma c_{M}\right)-\beta^{2} c_{M}\right] /\left(2 \eta \gamma-\beta^{2}\right),
$$

furthermore the SC product price, i.e. $w^{n}+m^{n}$, is $\left[\eta\left(\alpha+\gamma c_{M}\right)-\beta^{2} c_{M}+\alpha \eta\right] /\left(3 \eta \gamma-\beta^{2}\right)$. Thus, we have

$$
w^{n}+m^{n}-P_{2}^{n}=\eta\left(\alpha-\gamma c_{M}\right)\left(\eta \gamma-\beta^{2}\right)
$$

$$
\begin{equation*}
/\left(2 \eta \gamma-\beta^{2}\right)\left(3 \eta \gamma-\beta^{2}\right) \tag{A.6}
\end{equation*}
$$

From $B=2 \eta \gamma-\beta^{2}>0$ and $\alpha>\gamma c_{M}$, it follows if $\eta \gamma>\beta^{2}$ then $w^{n}+m^{n}>P_{2}^{n}$.
Part (iii): From Eq. (16), we know in the symmetric case

$$
\begin{aligned}
& U_{1}^{n}=\frac{\theta_{1}(\eta \gamma+B)\left(\overline{\bar{P}}-c_{M}\right)-\left(\alpha-\gamma c_{M}\right)\left(2 \theta_{1} \eta-\theta_{2} \beta\right)}{\eta \gamma+B}, \\
& U_{2}^{n}=\frac{\theta_{1} B\left(\overline{\bar{P}}-c_{M}\right)-\left(\alpha-\gamma c_{M}\right)\left(\theta_{1} \eta-\theta_{2} \beta\right)}{B}
\end{aligned}
$$

$$
\begin{align*}
& \partial \pi_{M_{1}}(w) / \partial w=\alpha-\gamma(m+m)+\beta s_{R} \\
& -\gamma\left(w-c_{M_{1}}\right)=0,  \tag{A.1}\\
& \partial \pi_{R}\left(m, s_{R}\right) / \partial m=\alpha-\gamma(m+m)+\beta s_{R}  \tag{A.2}\\
& -\gamma m=0, \\
& \partial \pi_{R}\left(m, s_{R}\right) / \partial s_{R}=\beta m-\eta_{R} s_{R}=0,  \tag{A.3}\\
& \partial \pi_{M_{2}}\left(p_{2}, s_{2}\right) / \partial p_{2}=\alpha-\gamma p_{2}+\beta s_{2} \\
& -\gamma\left(p_{2}-c_{M_{2}}\right)=0 \text {, }  \tag{A.4}\\
& \partial \pi_{M_{2}}\left(p_{2}, s_{2}\right) / \partial s_{2}=\beta\left(p_{2}-c_{M_{2}}\right)-\eta_{2} s_{2}=0 . \tag{A.5}
\end{align*}
$$

Thus, we have

$$
\begin{equation*}
U_{1}^{n}-U_{2}^{n}=\frac{\eta\left(a-\gamma m c_{M}\right)\left[-\theta_{1}\left(\eta \gamma-\beta^{2}\right)-\theta_{2} \beta \gamma\right]}{\left(2 \eta \gamma-\beta^{2}\right)\left(3 \eta \gamma-\beta^{2}\right)} . \tag{A.7}
\end{equation*}
$$

From $\alpha>\gamma m c_{M}$ and $2 \eta \gamma-\beta^{2}>0$, it follows that $U_{1}^{n}-U_{2}^{n}$ has the same sign as $\left[-\theta_{1}\left(\eta \gamma-\beta^{2}\right)-\theta_{2} \beta \gamma\right]$ Thus, if $-\left(\eta \gamma-\beta^{2}\right) / \beta \gamma>\theta_{2} / \theta_{1}$ then $U_{1}^{n}>U_{2}^{n}$ and the customer will choose the SC product to purchase and Proposition 2 follows.

Proof of Proposition 3. Let $\left(w, m, s_{R}\right) \neq(0,0,0)$ be the optimal solution for the manufacturer problem, thus the Karush-Kuhn-Tucker, KKT, necessary optimality condition for the problem can be written in the following form:

$$
\begin{align*}
& -\theta_{1}-v_{1}\left[-2 \gamma \underline{w}+\alpha-\gamma \underline{m}+\beta \underline{s_{R}}+c_{M_{1}} \gamma\right] \leq 0  \tag{A.8}\\
& \underline{w}\left[-\theta_{1}-v_{1}\left(-2 \gamma \underline{w}+\alpha-\gamma \underline{m}+\beta \underline{s_{r}}+c_{M_{1}} \gamma\right)\right]=0  \tag{A.9}\\
& \left(\underline{w}-c_{M_{1}}\right)\left[\alpha-\gamma(\underline{m}+\underline{w})+\beta \underline{s_{R}}\right]-F C_{M_{1}} \geq 0  \tag{A.10}\\
& v_{1}\left[\left(\underline{w}-c_{1}\right)\left[\alpha-\gamma(\underline{m}+\underline{w})+\beta \underline{s_{R}}\right]-F C_{M_{1}}\right]=0 \tag{A.11}
\end{align*}
$$

Due to $\underline{w} \geq c_{M_{1}}>0$ and from Eq. (A.9), we have $\theta_{1}+v_{1}\left(-2 \gamma \underline{w}+\alpha-\gamma \underline{m}+\beta \underline{s_{r}}+c_{M_{1}} \gamma\right)=0$; hence, we obtain $v_{1}=-\theta_{1} /\left(-2 \gamma \underline{w}+a-\gamma \underline{m}+\beta \underline{s_{R}}+c_{M_{1}} \gamma\right)$.

Because of $\theta_{1}>0$, we derive that $v_{1} \neq 0$. From $v_{1} \neq 0$ and (A.11), it follows that
$\left(\underline{w}-c_{1}\right)\left[\alpha-\gamma(\underline{m}+\underline{w})+\beta \underline{s_{R}}\right]-F C_{M_{1}}=0$.

Similarly, let $\left(w, m, s_{R}\right) \neq(0,0,0)$ be the optimal solution for the retailer's problem, therefore the KKT necessary condition for the
problem can be inscribed in the following form:

$$
\begin{align*}
& -\theta_{1}-v_{1}\left[-2 \gamma \underline{w}+\alpha-\gamma \underline{m}+\beta \underline{s_{R}}+c_{M_{1}} \gamma\right] \leq 0,  \tag{A.12}\\
& \theta_{2}-v_{2}\left(\beta \underline{m}-\underline{s_{R}} \eta_{R}\right) \leq 0,  \tag{A.13}\\
& \underline{m}\left[-\theta_{1}-v_{2}\left(-2 \gamma \underline{m}+\alpha-\gamma \underline{w}+\beta \underline{s_{R}}\right)\right]=0,  \tag{A.14}\\
& \underline{s_{R}}\left[\theta_{2}-v_{2}\left(\beta \underline{m}-\underline{s_{R}} \eta_{R}\right)\right]=0,  \tag{A.15}\\
& \underline{m}\left(\alpha-\gamma(\underline{m}+\underline{w})+\beta \underline{s_{R}}\right)-\frac{1}{2} \eta_{R} \underline{s_{R}}  \tag{A.16}\\
& \quad-F C_{R} \geq 0, \\
& v_{2}\left[\underline{m}\left(\alpha-\gamma(\underline{m}+\underline{w})+\beta \underline{s_{R}}\right)-\frac{1}{2} \eta_{R} \underline{s_{R}}{ }^{2}\right.  \tag{A.17}\\
& \left.-F C_{R}\right]=0, \\
& \underline{m}, \underline{s_{R}}>0 \text { and } v_{2} \geq 0 .
\end{align*}
$$

From $\underline{m}, \underline{s_{R}}>0$, Eqs. (A.14) and (A.15), it follows $\theta_{1}+v_{2}\left(-2 \gamma \underline{m}+\alpha-\gamma \underline{w}+\beta \underline{s_{R}}\right)=0 \quad$ and $\theta_{2}-v_{2}\left(\beta \underline{m}-\underline{s_{R}} \eta_{R}\right)=0$. Thus, we have

$$
\begin{align*}
v_{2} & =-\theta_{1} /\left(-2 \gamma \underline{m}+\alpha-\gamma \underline{w}+\beta \underline{s_{R}}\right) \\
& =\theta_{2} /\left(\beta \underline{m}-\underline{s_{R}} \eta_{R}\right) \tag{A.18}
\end{align*}
$$

From (A.18), Eq. (30) follows. Furthermore, due to $\theta_{1}, \theta_{2}>0$ and Eq. (A.18), we know that $v_{2} \neq 0$. From Eq. (A.17) and $v_{2} \neq 0$ we have
$\underline{m}\left(\alpha-\gamma(\underline{m}+\underline{w})+\beta \underline{s_{R}}\right)-\frac{1}{2} \eta_{R} \underline{s_{R}}{ }^{2}-F C_{R}$.
Thus, Proposition 3 follows.
Proof of Proposition 4. Let $\left(\underline{p_{2}}, \underline{s_{2}}\right) \neq(0,0)$ be the optimal solution for the outside manufacturer problem, thus the KKT necessary optimality condition for the problem can be written in the following form:

$$
\begin{equation*}
-\theta_{1}-v_{3}\left(-2 \gamma \underline{p_{2}}+\alpha-\gamma c_{M_{2}}+\beta \underline{s_{M_{2}}}\right) \leq 0 \tag{A.19}
\end{equation*}
$$

$$
\begin{align*}
& \theta_{2}-v_{3}\left(\beta \underline{p_{2}}-\beta c_{M_{2}}-\eta_{M_{2}} \underline{s_{2}}\right) \leq 0  \tag{A.20}\\
& \underline{p_{2}}\left[-\theta_{1}-v_{3}\left(-2 \gamma \underline{p_{2}}+\alpha-\gamma c_{M_{2}}+\beta \underline{s_{M_{2}}}\right)\right] \\
& \quad=0 \tag{A.21}
\end{align*}
$$

$$
\begin{equation*}
\underline{s_{2}}\left[\theta_{2}-v_{3}\left(\beta \underline{p_{2}}-\beta c_{M_{2}}-\eta_{M_{2}} \underline{s_{2}}\right)\right]=0 \tag{A.22}
\end{equation*}
$$

$$
\begin{equation*}
\left(\underline{p_{2}}-c_{M_{2}}\right)\left(\alpha-\gamma \underline{p_{2}}+\beta \underline{s_{2}}\right)-\frac{1}{2} \eta_{M_{2}}{\underline{s_{2}}}^{2} \tag{A.23}
\end{equation*}
$$

$$
-F C_{M_{2}} \geq 0
$$

$v_{3}\left[\left(p_{2}-c_{M_{2}}\right)\left(\alpha-\gamma \underline{p_{2}}+\beta \underline{s_{2}}\right)-\frac{1}{2} \eta_{M_{2}} \underline{s}_{2}{ }^{2}\right.$
$\left.-F C_{M_{2}}\right]=0$,
$\underline{p_{2}} \geq c_{M_{2}}, \underline{s_{2}}>0$ and $v_{3} \geq 0$.

Due to $\underline{p_{2}}, \underline{s_{2}}>0$, Eqs. (A.21) and (A.22), we get

$$
\begin{align*}
v_{3} & =\frac{-\theta_{1}}{-2 \gamma \underline{p_{2}}+a-\gamma c_{M_{2}}+\beta \underline{s_{2}}} \\
& =\frac{\theta_{2}}{\beta \underline{p_{2}}-\beta c_{M_{2}}-\eta_{2} \underline{s_{2}}} . \tag{A.25}
\end{align*}
$$

From Eq. (A.25), Eq. (35) follows. Furthermore, taking into account $\theta_{1}, \theta_{2}>0$ and Eq. (A.25), it derives that $v_{3} \neq 0$. From Eq. (A.24) and $v_{3} \neq 0$, we have

$$
\left(\underline{p_{2}}-c_{M_{2}}\right)\left(\alpha-\gamma \underline{p_{2}}+\beta \underline{s_{2}}\right)-\frac{1}{2} \eta_{M_{2}}{\underline{s_{2}}}^{2}-F C_{M_{2}}=0
$$

Thus, Proposition 4 follows.
$\begin{array}{llr}\begin{array}{l}\text { Proof of } \quad \text { Proposition }\end{array} & \text { 5. } & \text { Due to } \\ \frac{\partial^{2} \text { Payoff }_{M_{1}}(w)}{\partial w^{2}}=-\frac{2 \lambda_{M_{1}} \gamma}{\pi_{M_{1}}} \leq 0, & \text { the }\end{array}$
manufacturer payoff function is a concave function on $w$. On the other hand, Hessian matrices of Payoff $_{R}$ and Payoff $M_{M_{2}}$ are

$$
\mathbf{H}_{R}=\left[\begin{array}{cc}
-\frac{2 \lambda_{R} \gamma}{\pi_{R}} & \frac{\lambda_{R} \beta}{\overline{\pi_{R}}} \\
\frac{\lambda_{R} \beta}{\overline{\pi_{R}}} & -\frac{\lambda_{R} \eta_{R}}{\overline{\pi_{R}}}
\end{array}\right]
$$

$$
\mathbf{H}_{M_{1}}=\left[\begin{array}{cc}
-\frac{2 \lambda_{M_{1}} \gamma}{\overline{\pi_{M_{1}}}} & \frac{\lambda_{M_{1}} \beta}{\overline{\pi_{M_{1}}}} \\
\frac{\lambda_{M_{1}} \beta}{\overline{\pi_{M_{1}}}} & -\frac{\lambda_{M_{1} \eta_{M_{1}}}^{\overline{\pi_{M_{1}}}}}{}
\end{array}\right]
$$

The retailer and outside-manufacturer payoffs are concave functions on $\left(m, s_{i}\right)$ and $\left(p_{2}, s_{2}\right)$, respectively, if and only if the Hessian matrix is negatively definite. Due to $\gamma, \eta_{i} \geq 0$ and $\left(\lambda_{i} / \overline{\pi_{i}}\right)^{2} B_{i}>0, i=R, M_{2}$, the retailer and outsidemanufacturer payoffs functions are concave.

To determine the Nash pricing and service level equilibrium, which correspond to the simultaneous moves of the retailer and manufacturers 1 and 2, we need to consider the optimality condition for their objective functions.

$$
\begin{align*}
& \frac{\partial \text { Payoff }_{M_{1}}(w)}{\partial w}= \\
& \quad-\lambda_{M_{1}} \frac{\alpha-\gamma(m+w)+\beta s_{R}-\gamma\left(w-c_{M_{1}}\right)}{\pi_{M_{1}}} \\
& \quad+\left(1-\lambda_{M_{1}}\right) \frac{\theta_{1}}{\overline{U_{1}}}=0, \\
& \frac{\partial \text { Payoff }_{R}\left(m, s_{R}\right)}{\partial m}= \\
& \quad-\lambda_{r} \frac{\alpha-\gamma(m+w)+\beta s_{R}-\gamma m}{\overline{\pi_{R}}}  \tag{A.27}\\
& \quad+\left(1-\lambda_{R}\right) \frac{\theta_{1}}{\overline{U_{1}}}=0, \\
& \frac{\partial P a y o f f_{R}\left(m, s_{R}\right)}{\partial s_{R}}=  \tag{A.28}\\
& \quad-\lambda_{R} \frac{\beta m-\eta_{R} s_{R}}{\pi_{R}}-\left(1-\lambda_{R}\right) \frac{\theta_{2}}{\overline{U_{1}}}=0
\end{align*}
$$

$$
\begin{align*}
& \frac{\partial \text { Payoff }_{M_{2}}\left(p_{2}, s_{2}\right)}{\partial p_{2}}= \\
& \quad-\lambda_{M_{2}} \frac{\alpha-\gamma p_{2}+\beta s_{2}-\gamma\left(p_{2}-c_{M_{2}}\right)}{\overline{\pi_{M_{2}}}}  \tag{A.29}\\
& \quad+\left(1-\lambda_{M_{2}}\right) \frac{\theta_{1}}{\overline{U_{2}}}=0
\end{align*}
$$

$$
\begin{align*}
& \frac{\partial \text { Payoff }_{M_{2}}\left(p_{2}, s_{2}\right)}{\partial s_{2}}= \\
& -\lambda_{M_{2}} \frac{\beta\left(p_{2}-c_{M_{2}}\right)-\eta_{M_{2}} s_{2}}{\overline{\pi_{M_{2}}}}  \tag{A.30}\\
& -\left(1-\lambda_{M_{2}}\right) \frac{\theta_{2}}{\overline{U_{2}}}=0 .
\end{align*}
$$

Solving the linear system of equations (A.26)(A.30) leads to Eqs. (48), and Proposition 5 follows.

Proof of Proposition 6. Part (i): By considering $B_{R} \geq 0$, it follows that $\partial U_{1}^{n} / \partial \lambda_{M_{1}}$ has the same sign as $\left(\theta_{1} \beta^{2}-\eta_{R} \gamma \theta_{1}-\beta \gamma \theta_{2}\right)$. From $B_{R} \geq 0$, we know $2 \eta_{R} \gamma-\beta^{2} \geq 0$ and therefore we obtain
$\theta_{1} \beta^{2}-\theta_{1} \eta_{R} \gamma \leq \theta_{1} \eta_{R} \gamma$.

Subtracting $\beta \gamma \theta_{2}$ from Eq. (A.31) leads to
$\theta_{1} \beta^{2}-\theta_{1} \eta_{R} \gamma-\beta \gamma \theta_{2} \leq \theta_{1} \eta_{R} \gamma-\beta \gamma \theta_{2}$.

Thus, it follows that if $\theta_{1} \eta_{R} \leq \beta \theta_{2}$, then $\partial U_{1}^{n} / \partial \lambda_{M_{1}} \leq 0$.
Part (ii): Due to $B_{R} \geq 0$, it follows that the sign of $\partial U_{1}^{n} / \partial \lambda_{R}$ is equivalent to the sign of $-\theta_{1}^{2} \beta^{2}-3 \theta_{2}^{2} \gamma^{2}-\eta_{R} \gamma \theta_{1}^{2}+4 \theta_{1} \theta_{2} \beta \gamma$. As we know, $-\left(\theta_{1} \beta-\sqrt{3} \gamma \theta_{2}\right)^{2} \leq 0$ and thus we obtain

$$
\begin{equation*}
-\theta_{1}^{2} \beta^{2}-3 \theta_{2}^{2} \gamma^{2}+2 \sqrt{3} \theta_{1} \theta_{2} \beta \gamma \leq 0 \tag{A.33}
\end{equation*}
$$

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Adding $\quad\left(-\eta_{R} \gamma \theta_{1}^{2}+(4-2 \sqrt{3}) \theta_{1} \theta_{2} \beta \gamma\right)$ to Eq. (A.33), results in

$$
\begin{aligned}
-\theta_{1}^{2} \beta^{2}- & 3 \theta_{2}^{2} \gamma^{2}-\eta_{R} \gamma \theta_{1}^{2}+4 \theta_{1} \theta_{2} \beta \gamma \leq-\eta_{R} \gamma \theta_{1}^{2} \\
+ & (4-2 \sqrt{3}) \theta_{1} \theta_{2} \beta \gamma .
\end{aligned}
$$

Therefore, it follows that if
$-\eta_{R} \gamma \theta_{1}^{2}+(4-2 \sqrt{3}) \theta_{1} \theta_{2} \beta \gamma \leq 0$,
i.e.
$\eta_{R} \theta_{1} \geq(4-2 \sqrt{3}) \theta_{2} \beta$, then $\partial U_{1}^{n} / \partial \lambda_{R} \leq 0$.
Part (iii): Due to $B_{M_{2}} \geq 0$, it follows that the sign of $\partial U_{1}^{n} / \partial \lambda_{M_{2}}$ has the same sign as $\left(-\theta_{1}^{2} \eta_{M_{1}}+2 \theta_{1} \theta_{2} \beta-2 \theta_{2}^{2} \gamma\right)$. As we know, $-\left(\theta_{1} \sqrt{\eta_{M_{2}}}-\theta_{2} \sqrt{2 \gamma}\right)^{2} \leq 0$ and thus we get
$-\theta_{1}^{2} \eta_{M_{2}}+2 \theta_{1} \theta_{2} \sqrt{2 \gamma \eta_{M_{2}}}-2 \theta_{2}^{2} \gamma \leq 0$.

On the other hand, from $B_{M_{2}} \geq 0$, we have $\sqrt{2 \eta_{M_{2}} \gamma} \geq \beta$. The term $\sqrt{2 \eta_{M_{2}} \gamma}$ in Eq. (A.34) can be substituted for a smaller one, i.e. $\beta$, that leads to
$-\theta_{1}^{2} \eta_{M_{2}}+2 \theta_{1} \theta_{2} \beta-2 \theta_{2}^{2} \gamma \leq 0$.
Therefore, Part (iii) of Proposition 6 follows.

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