NUMERICAL SOLUTION OF PRICING OF EUROPEAN PUT OPTION WITH STOCHASTIC VOLITILITY

*U.S. Rana*and A. Ahmad*

Department of Mathematics, D.A.V. (P.G.) College
Dehradun, Uttarakhnd, (248001), India
drusrana@yahoo.co.in - *asadahmad1004@yahoo.com*

*Corresponding Author

(Received: December 13, 2009 – Accepted in Revised Form: April 23, 2011)

Abstract In this paper, European option pricing with stochastic volatility forecasted by well known GARCH model is discussed in context of Indian financial market. The data of Reliance Ltd. stock price from 3/01/2000 to 30/03/2009 is used and resulting partial differential equation is solved by Crank-Nicolson finite difference method for various interest rates and maturity in time. The sensitivity measures "Greeks" are also determined to validate the model. It is observed that the value of European put option increases with maturity time and decreases with interest rate.

Keywords European Option, Finite Difference Method, Stochastic Volatility, GARCH (1, 1), Greeks

چکیده در این مقاله قیمت گذاری اروپایی با استفاده از تغییرات تصادفی توسط مدل معروف GARCH در بازار مالی هند پیش بینی شده است. در این مقاله از داده های قیمت سهام شرکت Reliance از تاریخ 2002/1/3 تا 2009/3/30 استفاده شده و معادله دیفرانسیل جزئی حاصل توسط روش اختلاف محدود Crank-Nicolson برای نرخ سودهای متغیر و زمان سررسید حل شده است. میزان حساسیت "Greeks" نیز به منظور تایید مدل مشخص شده اند. مشاهده شده است که ارزش سهام اروپایی با میزان سررسید افزایش و با نرخ سود کاهش یافته است.

1. INTRODUCTION

It is widely acknowledged by financial researchers Black, et al [1], Merton [2] that the valuation of options leads to mathematical model which has long been an intriguing problem in different ways. The most fundamental input into an option pricing model is volatility, a measure of how much the underlying asset price is likely to vary over time. In financial markets, volatility presents a strange paradox to the market participants, academicians and policy makers Nelson [3],even volatility estimation is by no means an exact science but a lot of efforts have been expended in improving volatility model since better forecasts transforms into better pricing of option Bollerslev, et al [4], and Loudon, et al [5].

Recently, Loudon, et al [5], Mc-Millan, et al [6], Yu [7], Klaassen [8], Vilasuso [9] and Balaban [10] investigated the forecasting models in various markets and found that ARCH class of models provide better forecast in terms of statistical error and evidence in favor of GARCH model over shorter intervals. In all these studies, various methods for the estimation of volatility and their performances were discussed in terms of statistical error but none of them used volatility forecasting in the valuation of option pricing governed by Black-Scholes partial differential equation.

Hull, et al [11] determined the numerical solution of Black-Scholes partial differential equation regarding constant volatility. Later on, an alternative approach has been proposed by Avellaneda, et al [12-13] in which it is assumed that volatility is uncertain but lies within a known range of values. Recently, Chawla, et al [14], Mayo [15], Tangman, et al [16] and Hu, et al [17] developed more efficient finite difference numerical methods for pricing of option giving better accuracy than normal difference method. But, the option prices obtained using Black-Scholes model with constant volatility is not consistent with observed option prices. One possible remedy for this is to make the volatility to be a function of time and strike price, which leads to a model in terms of parabolic partial differential equation in two variables i.e. volatility and underlying asset value.

In this paper an alternative approach is developed for the pricing of option by Black-Scholes partial differential equation regarding variable volatility which is forecasted by GARCH (1, 1) method and the resulting one dimensional parabolic partial differential equation is solved by implicit finite difference method [18].

2. FORMULATION AND SOLUTION OF MODEL

In the modeling of option pricing, the physical system is the financial market place and the particular object of observation is the price of the option's underlying asset. In order to develop a model tractable by mathematical and computational techniques, it is assumed that price of European option V is a function of current value of the underlying asset 'S' and the time t i.e. $V = V(S,t)$. The continuous-time, Black-Scholes model to European option is

$$\frac{\partial V}{\partial t} + rS\frac{\partial V}{\partial S} + \frac{1}{2}s^2S^2\frac{\partial^2 V}{\partial S^2} - rV = 0 \qquad t\in[0,T], S\in[0,\infty] \tag{1}$$

where, T is time of expiration, r is a risk-free interest rate and s is volatility of stock returns.

Suppose K and S_T be strike price and price of underlying asset on the date of expiration T, respectively. In case of $S_T < K$, it has a financial sense (in-the-money for holder of put option) and encourage to the holder of put option for exercise, because holder can sell the asset of worth S_T at the cost of K. Thus, the gain of holder from the call option is $(K - S_T)$. In case of $S_T \geq K$, the holder will forfeit the right to exercise the option because he can buy the asset at a cost, less than or equal to predetermined strike price K. Similarly, in-the-money position for put option it is $S_T < K$, under which, asset is sold at higher price of K instead of S_T. Thus, the terminal payoffs from the long position in a European call and put options are defined as:

$$V(S,T) = \max\{K - S, 0\} \tag{2}$$

In put options, the terminal payoffs are non-negative, which reflect the vary nature of the options. This condition is defined at a future point in time and we wish to determine values backwards to an earlier point in time.

The option pricing problem (1) is posed on the domain $[0,\infty]\times[0,T]$ with final condition (2). To complete the option pricing model, we prescribe two spatial boundary conditions at $S = 0$ and $S = \infty$. But, in case of numerical solution, infinite grids cannot be represented in the computer so we truncate the solution domain artificially at point $S = S_{\max}$ and replace the deleted portions with boundary conditions that minimize the deleterious effects of the truncation. The truncation point $S_{\max}$ has to be sufficiently far from the region of interest in order to avoid the excessive error due to truncation and even if the imposed boundary conditions are imperfect, it does not materially affect the solution. On the other hand, unnecessarily large value of $S_{\max}$ increases the computational cost. The choice of $S_{\max}$ is consider in [19]. Hence, the solution value for the option pricing model (1) is $[0, S_{\max}]\times[0,T]$.

Boundary conditions for put option are as:

- When $S = 0$ for some $(t < T)$, S will stay at zero at all subsequent times so that the option is sure to expire in-the-money. Hence

$$V(0,t) = Ke^{-r(t-T)} - S_{\max} \tag{3}$$

- When $S = S_{\max}$, it becomes almost certain that the put value will be in out-of-the–money. Hence

$$V(S_{\max}, t) = 0 \tag{4}$$

More and more researchers have found that the option pricing with stochastic volatility is more realistic than Black-Scholes model with constant volatility. Thus, volatility is an important issue to be addressed properly. From computational point of view, it is convenient to handle the pricing problem by forecasting the valuation by well known GARCH method than regarding volatility as function of strike price and time. Now we discuss the GARCH model.

2.1. GARCH (1, 1) Model In 1986, Bollerslev [4] proposed GARCH (1, 1) model as

$$y_t = e_t s_t \tag{5}$$

with conditional variance $s_t^2 = a_0 + a_i y_{t-1}^2 + b_i s_{t-1}^2$ with

$$a_0 > 0,\ and\ a_1,\ \ b_1 \geq 0, \tag{6}$$

Where s_t^2, s_{t-1}^2 are volatilities on the day t and previous day, y_{t-1} is return on the previous day and e_t denotes a real-valued discrete-time stochastic process as $e_t \approx N(0, s_t^2)$, y_t is the dependent variable of return x_t at a time t and a_0, a_1 *and* b_1 are weighted assigned to conditional and unconditional variances. The GARCH (1,1) regression model is obtained by assuming the e_ts be innovation in a linear regression

$$y_t = bx_t + e_t \tag{7}$$

The GARCH (1,1) process as defined in Equations 5 and 6 is stationary with $E(y_t) = 0$, $\mathrm{var}(y_t) = \dfrac{a_0}{1 - a_1 - b_1}$ and $\mathrm{cov}(y_t, y_s) = 0 \quad for\ t \neq s$ if and only if $a_1 + b_1 < 1$ or characteristic roots of GARCH (1,1) process are outside the unit circle. The likelihood function for estimation of parameters is defined as:

$$\mathbf{l}_q = -\frac{1}{2}\left(T \ln(2p) + \sum_{t=1}^{T} \left(\ln(s_t^2) + \frac{y_t^2}{s_t^2} \right) \right) \tag{8}$$

2.2. Discretization of Equation For numerical solution of partial differential Equation 1 the rectangular domain $[0, S_{\max}] \times [0, T]$ is divided into $(N+1) \times (M+1)$ uniform grid points. The step width ΔS and Δt are in general independent, where

$$S_{i+1} - S_i = \Delta S = \frac{S_{\max} - 0}{M}; \quad i = 0, 1, 2, ...M$$

$$t_{i+1} - t_i = \Delta t = \frac{T}{N}; \quad j = 0, 1, 2, ...N$$

and $V_{i,j}$ denotes the numerical approximation of $V(i\Delta S, j\Delta t)$ i.e. at value of V grid point (i, j).

Keeping view for stability of finite difference scheme in mind, we used Crank and Nicolson (1947) method, which incorporates both explicit and implicit features [20]. The Crank-Nicolson scheme for Equation 1 is

$$aV_j^{i-1} + bV_j^i + cV_j^{i+1} = xV_{j+1}^{i-1} + yV_{j+1}^i + zV_{j+1}^{i+1} \tag{9}$$

where

$$a = \frac{1}{4}\left(ri - s^2 i^2\right)\Delta t \quad b = \left(1 + \frac{s^2 i^2 \Delta t}{2} + \frac{r\Delta t}{2}\right) \quad c = -\frac{1}{4}\left(ri + s^2 i^2\right)\Delta t$$

$$x = \frac{1}{4}\left(s^2 i^2 - ri\right)\Delta t \quad y = \left(1 - \frac{s^2 i^2 \Delta t}{2} - \frac{r\Delta t}{2}\right) \quad z = \frac{1}{4}\left(ri + s^2 i^2\right)\Delta t$$

2.3. Discretization of Boundary and Initial Conditions We need the discretization of boundary and initial conditions for the European option. For a put option, boundary conditions are

$$V_j^0 = K\ and \quad V_j^M = 0 \qquad for\ \ j = 0, 1, 2, ...N$$

and initial condition is:

$$V_N^i = \max\left(K - i\Delta S, 0\right) \qquad for \quad i = 0, 1, 2, ...M$$

2.4. Stability of Scheme It is unconditionally stable and the amplification factor of Crank-Nicolson scheme for Equation 1 is:

$$G(b) = \frac{1 - \frac{1}{2}\frac{s^2 S^2}{\Delta S^2}\Delta t(1 - \cos b) - \frac{r}{2}\Delta t}{1 + \frac{1}{2}\frac{s^2 S^2}{\Delta S^2}\Delta t(1 - \cos b) + \frac{r}{2}\Delta t}$$

TABLE 1. 60 Days Forecasted Volatility of Reliance Ltd. using Parameters of GARCH (1,1) Model.

No. Days	Volatility s_i	No. Days	Volatility s_i	No. Days	Volatility s_i	No. Days	Volatility s_i
1	0.4247	16	0.4332	31	0.4339	46	0.4339
2	0.4262	17	0.4333	32	0.4339	47	0.4339
3	0.4277	18	0.4334	33	0.4339	48	0.4339
4	0.4284	19	0.4335	34	0.4339	49	0.4339
5	0.4293	20	0.4336	35	0.4339	50	0.4339
6	0.4300	21	0.4336	36	0.4339	51	0.4339
7	0.4306	22	0.4337	37	0.4339	52	0.4339
8	0.4311	23	0.4337	38	0.4339	53	0.4339
9	0.4315	24	0.4338	39	0.4339	54	0.4339
10	0.4319	25	0.4338	40	0.4339	55	0.4339
11	0.4322	26	0.4338	41	0.4339	56	0.4339
12	0.4325	27	0.4338	42	0.4339	57	0.4339
13	0.4327	28	0.4338	43	0.4339	58	0.4339
14	0.4319	29	0.4338	44	0.4339	59	0.4339
15	0.4330	30	0.4338	45	0.4339	60	0.4339

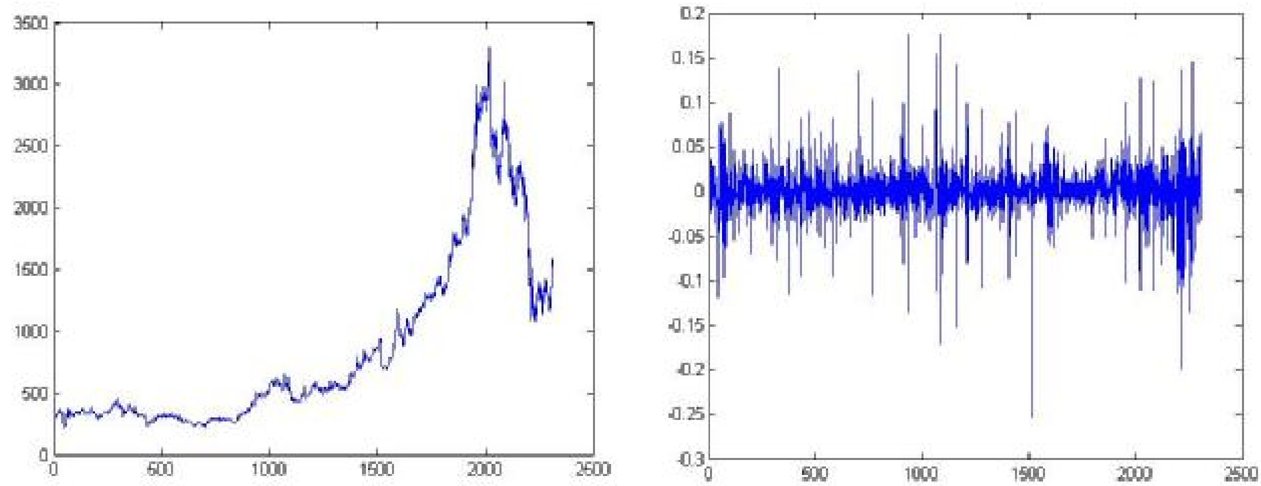

Figure 1. Stock price and return of Reliance Ltd. from 30/03/2009 to 01/01/2000.

2.5. Sensitivities by Finite Difference Scheme

Finite difference method provides the price of an

TABLE 2. Option Price for Different Stock Prices S, Strike Prices K = $30, $35, $40 and $45, Interest Rate r = 8 %, 9 %, 10 %, 11 % with One and Two Months Maturity Period.

(r In %)	(K) ($) / (S) ($)	6.00	12.00	18.00	24.00	30.00	36.00	42.00	48.00	54.00
	Maturity Time T = 30 Days									
r = 8 %	**30.00**	23.7927	17.7927	11.7928	5.8632	1.4259	0.1322	0.0060	0.0000	0.0000
	35.00	28.7595	22.7595	16.7595	10.7626	5.0242	1.2587	0.1551	0.0111	0.0006
	40.00	33.7262	27.7262	21.7262	15.7263	9.7515	4.3471	1.1472	0.1757	0.0173
	45.00	38.6930	32.6930	26.6930	20.6930	14.6947	8.7936	3.8167	1.0696	0.1909
r = 9 %	**30.00**	23.7669	17.7669	11.7670	5.8385	1.4129	0.1302	0.0058	0.0000	0.0000
	35.00	28.7295	22.7295	16.7295	10.7327	4.9977	1.2463	0.1527	0.0108	0.0005
	40.00	33.6922	27.6922	21.6922	15.6923	9.7179	4.3200	1.1351	0.1730	0.0170
	45.00	38.6548	32.6548	26.6548	20.6548	14.6565	8.7570	3.7897	1.0577	0.1879
r = 10 %	**30.00**	23.7411	17.7411	11.7412	5.8139	1.400	0.1281	0.0057	0.0002	0.0000
	35.00	28.6996	22.6996	16.6996	10.7028	4.9713	1.2339	0.1503	0.0106	0.0005
	40.00	33.6581	27.6581	21.6581	15.6582	9.6843	4.2930	1.1231	0.1703	0.0166
	45.00	38.6166	32.6166	26.6166	20.6166	14.6184	8.7205	3.7628	1.0460	0.1850
r = 11 %	**30.00**	23.7153	17.7153	11.7154	5.7893	1.3872	0.1261	0.0056	0.0002	0.0000
	35.00	28.6697	22.6697	16.6697	10.6730	4.9450	1.2216	0.1480	0.0104	0.0005
	40.00	33.6241	27.6241	21.6241	15.6242	9.6508	4.2661	1.1112	0.1676	0.0163
	45.00	38.5784	32.5784	26.5784	20.5784	14.5802	8.6839	3.7360	1.0344	0.1821
	Maturity Time T = 60 Days									
r = 8 %	**30.00**	23.5868	17.5868	11.5924	5.8965	1.9643	0.4259	0.0671	0.0086	0.0010
	35.00	28.5205	22.8505	16.5209	10.5695	5.2658	1.8848	0.4948	0.1024	0.0174
	40.00	33.4543	27.4543	21.4543	15.4608	9.6394	4.8000	1.8450	0.5602	0.1332
	45.00	38.3881	32.3881	26.3881	20.3889	14.4268	8.8368	4.4562	1.8209	0.5631
r= 9 %	**30.00**	23.5355	17.5355	11.5413	5.8510	1.9385	0.4176	0.0654	0.0083	0.0009
	35.00	28.4611	22.4611	16.4614	10.5113	5.2181	1.8585	0.4853	0.0998	0.0168
	40.00	33.3866	27.3866	21.3866	15.3933	9.5755	4.7512	1.8181	0.5494	0.1300
	45.00	38.3122	32.3122	26.3122	20.3130	14.3519	8.7692	4.4068	1.7934	0.5521
r = 10 %	**30.00**	23.4843	17.4843	11.4903	5.8056	1.9130	0.4095	0.0637	0.0081	0.0009
	35.00	28.4017	22.4017	16.4021	10.4531	5.1706	1.8325	0.4759	0.0974	0.0163
	40.00	33.3190	27.3190	21.3190	15.3259	9.5119	4.7027	1.7915	0.5388	0.1269
	45.00	38.2365	32.2364	26.2364	20.2372	14.2772	8.7018	4.3577	1.7661	0.5413
r = 11 %	**30.00**	23.4332	17.4332	11.4394	5.7604	1.8876	0.4015	0.0621	0.0078	0.0009
	35.00	28.3424	22.3424	16.3428	10.3951	5.1234	1.8068	0.4666	0.0950	0.0159
	40.00	33.2515	27.2515	21.2516	15.2586	9.4485	4.6544	1.7652	0.5283	0.1238
	45.00	38.1607	32.1607	26.1607	20.1616	14.2025	8.6346	4.3090	1.7392	0.5306

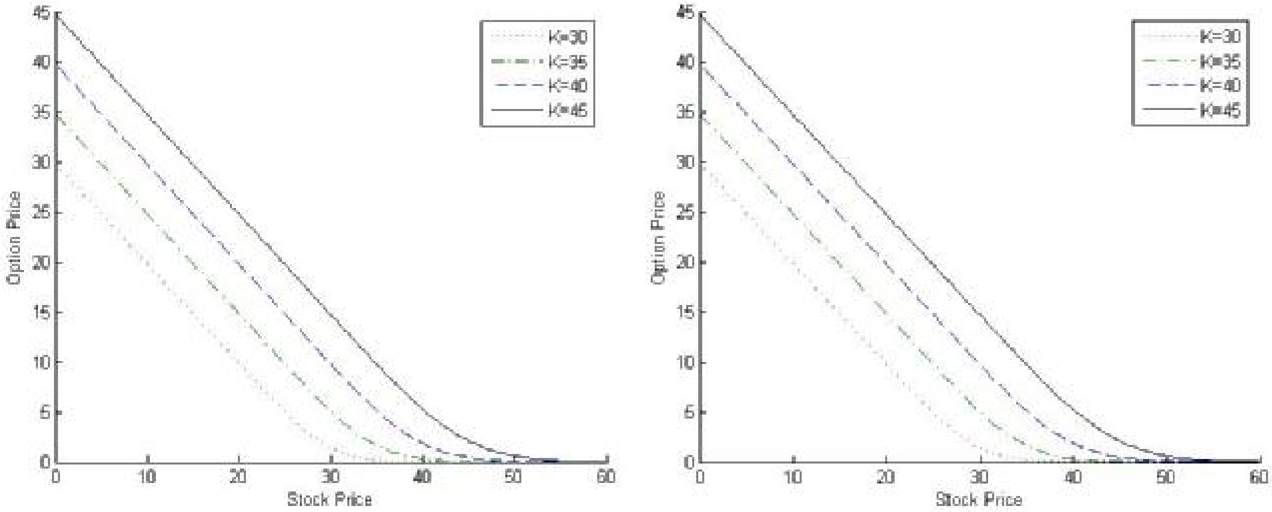

Figure 2. Option price for different stock prices S, strike prices K=$30, $35, $40, $45, interest rates r = 8 %, 9 % with one month maturity.

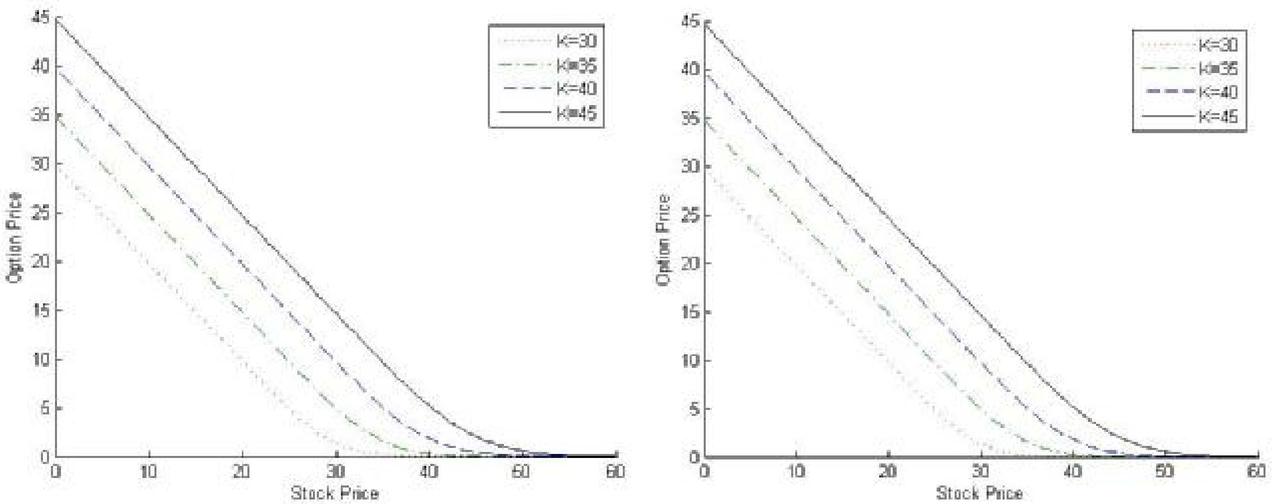

Figure 3. Option price for different stock prices S, strike prices K = $30, $35, $40, $45, interest rates r = 10 %, 11 % with one month maturity.

stock price (S) and present a concavity upward. The point S = K (i.e. at-the-money) the curve are more concave which referred that option lead to either in-the-money or out-of-the-money immediately. In case of in-the-money-(S<K) the value of option are changing in constant way with respect to the stock price (S) decrease, while the exponential change occurs when option become at-the-money as well as out-of-the money (Table 2, Figures 2-5).

The downward concavity conformed that a decrease in the asset price (S) will increase probability of a positive terminal payoff, resulting in a higher value of option. By keeping r small, t constant and increase of strike price from K = $30 to K = $45, the increment in option price has same trends with respect to stock price (S) and option is become in-the-money. But, as interest rate increases from = 8 % to r = 9 %, 10 % and 11 % the interval of positive terminal payoff reduces and

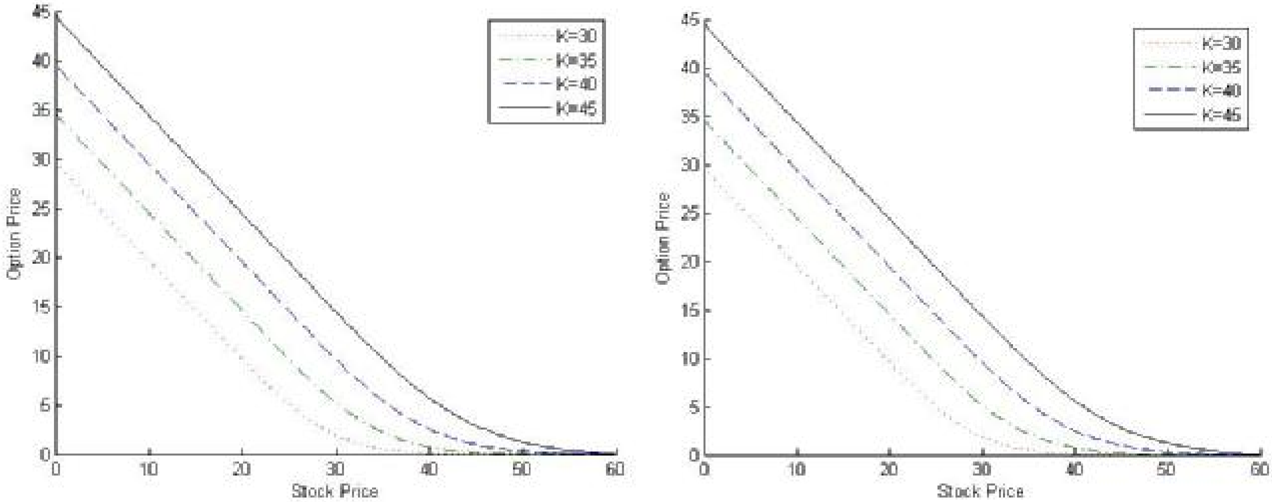

Figure 4. Option price for different stock prices S, strike prices K = \$30, \$35, \$40, \$45, interest rates r = 8 %, 9 % with two month maturity.

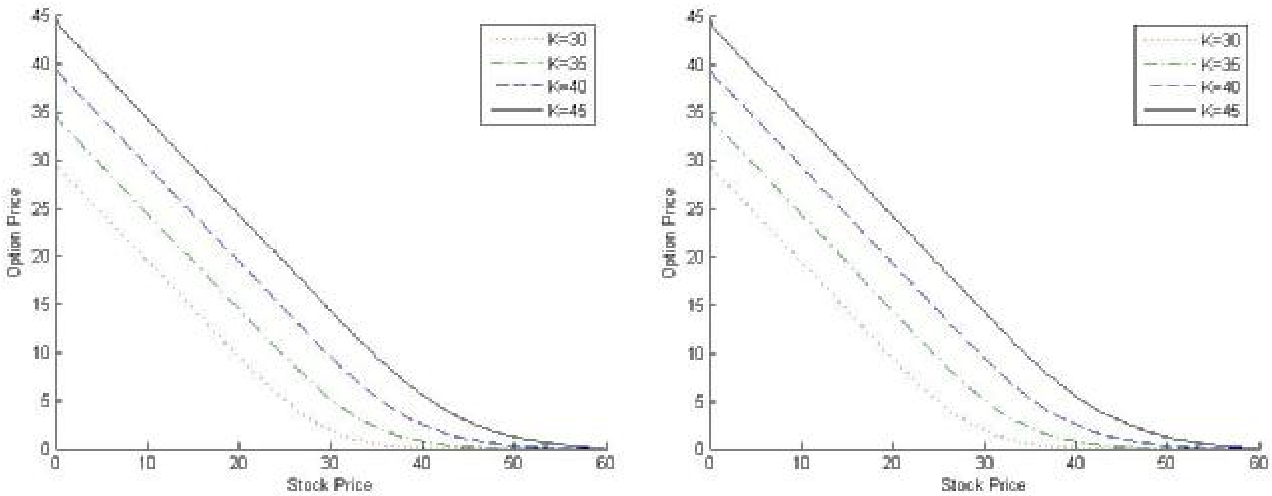

Figure 5. Option price for different stock prices S, strike prices K = \$30, \$35, \$40, \$45, interest rates r = 10 %, 11 % with two month.

the option prices also reduce that means the possibility of positive terminal payoff decreases and portfolio become more sensitive with respect to interest rate increment, resulting a less option price. Hence, as the interest rate (r) increases, the option price decreases and the positive terminal payoff increase at only deep in-the-money. But, comparison of maturity life i.e. a short lived option with a long lived option depicts very strange results (Table 2). If option become deep in-the-money, the option price decreases when maturity time increases, while in case of at-the-money and out-of the money option price increases with maturity time increase. Intuitively, in case of put option, probability of positive payoff increases as maturity time and interest rate increases.

The delta (Δ_p) of put option shows the negative values and lies (-1, 0). The negativity of delta (Δ_p) for put option function confirmed the decreasing nature of the function with stock price (S) i.e. the

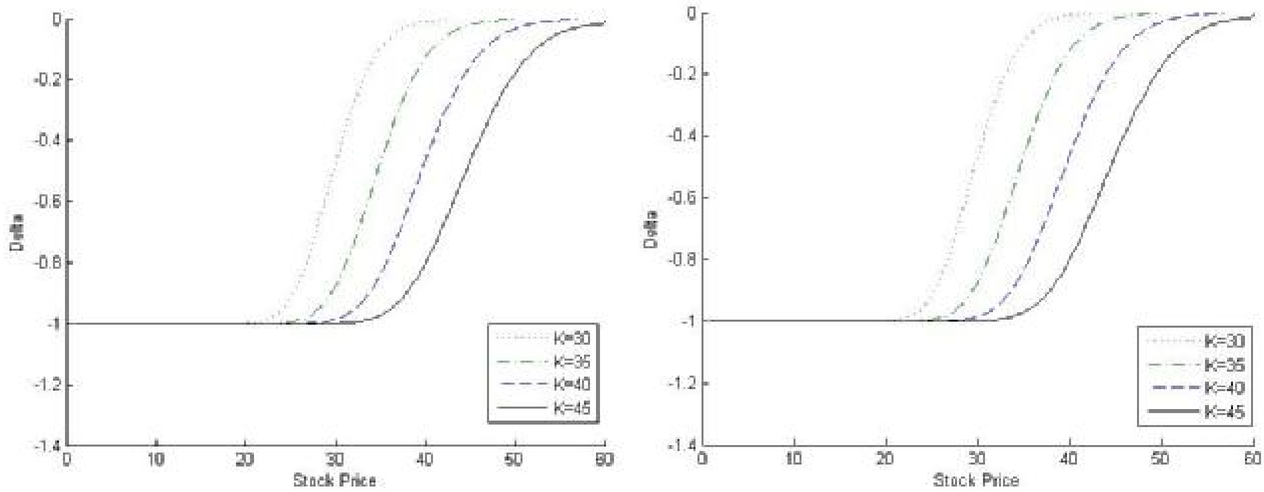

Figure 6. Delta (Δ_p) of the put option for different stock prices S, strike prices K = \$30, \$35, \$40, \$45, interest rates r = 8 %, 9 % with one month maturity.

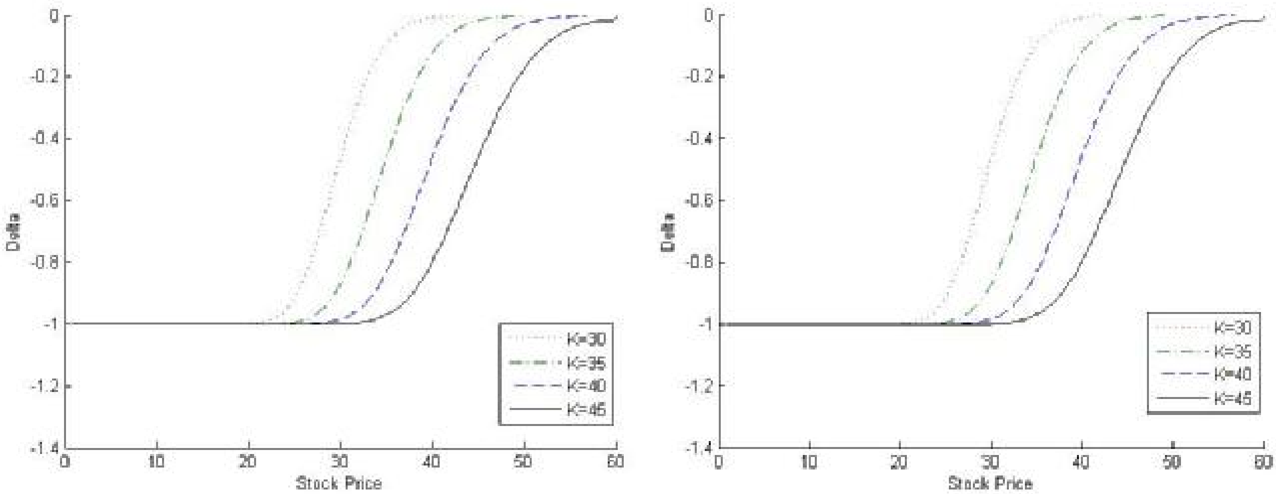

Figure 7. Delta (Δ_p) of the put option for different stock prices S, strike prices K = \$30, \$35, \$40, \$45, interest rates r = 10 %, 11 % with one month maturity.

increment in the asset price causes a decrease in the value of put option prices and probability of a positive terminal payoff and then a long position for put option will be hedged by a continuously varying long position in the underlying asset.

The effect of strike price is also significant to hedge to the portfolio. Figures 6-9 show the curves of delta (Δ_p) against stock price (S), change in concavity appears where option is at-the-money $(S = K)$ so that the curve concave upward for in-the-money $(0 < S < K)$ and downward for out-of-the-money $(K < S < \infty)$.When option values become out-of-the-money ,the concavity increases as strike price (K) increases, which means the option price for a higher strike price (K) has a small changes in corresponding delta (Δ_p) i.e. the delta (Δ_p) hedge dynamic for a higher strike price (K) is less than as compared with low strike price

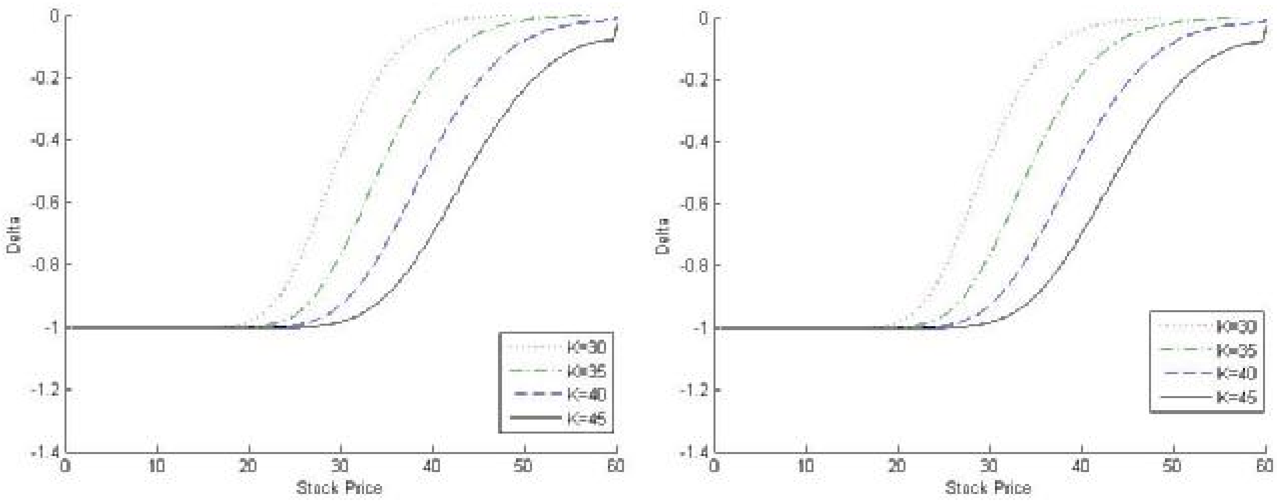

Figure 8. Delta (Δ_p) of the put option for different stock prices S, strike prices K=$30, $35, $40, $45, interest rates r = 8 %, 9 % with two month maturity.

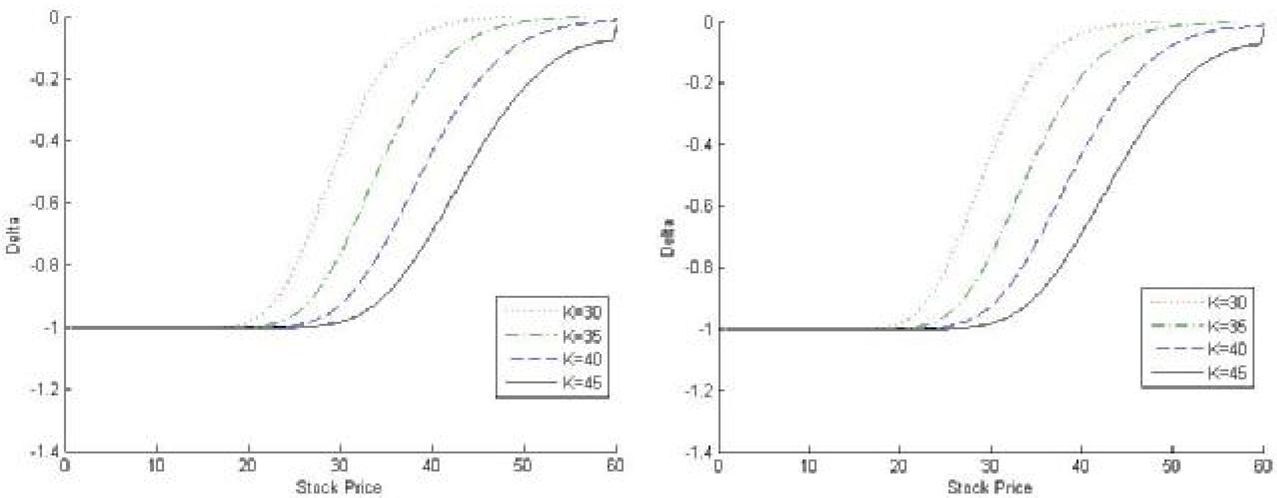

Figure 9. Delta (Δ_p) of the put option for different stock prices S, strike prices K = $30, $35, $40, $45, interest rates r = 10 %, 11 % with two month maturity.

(K).The effect of interest rate on delta (Δ_p) also represents an interesting result. It is observed that when strike price is sufficiently small (K=$30) the interest rate (r) has very small changes as r = 8 % to 9 %, 10 % and 11 % then the delta (Δ_p) increases only when option become at-the-money, but it remains same for option being deep in-the-money and out-of-the-money. If we peer in the data of delta for K=$35, $40 and $45 for different interest rate, it has enough change with respect to interest rate.

The impact of maturity life of option is countable in this problem as maturity life increases then the absolute values of delta decreases for each case of stock price, strike price and interest rate as result option is less sensitive i.e. increments in strike price, interest rate and maturity time increase the probability of positive terminal payoff increase. Further, the delta can be shown easily that

$\lim_{t \to \infty} \frac{\partial P}{\partial S} = 0$ for all values of stock price (S).

While $\lim_{t \to 0} \frac{\partial P}{\partial S} = \begin{cases} 0 & if \quad S > K \\ -\frac{1}{2} & if \quad S = K \\ -1 & if \quad S < K \end{cases}$

Figures 10-13 represent the values of theta (Δ_p) for a put option against stock prices (S) are negative and positive. The curve of theta (Δ_p), tends asymptotically as option become deep in-the-money. This means the results discussed above through Table 2 (i.e. the value of option decreases when option becomes out-of-the-money (S>K) and increases if option is in-the-money (S<K).While option becomesdeep in-the-money then the change in option price appears in a constant way) are validated from these figures. The negative sign of

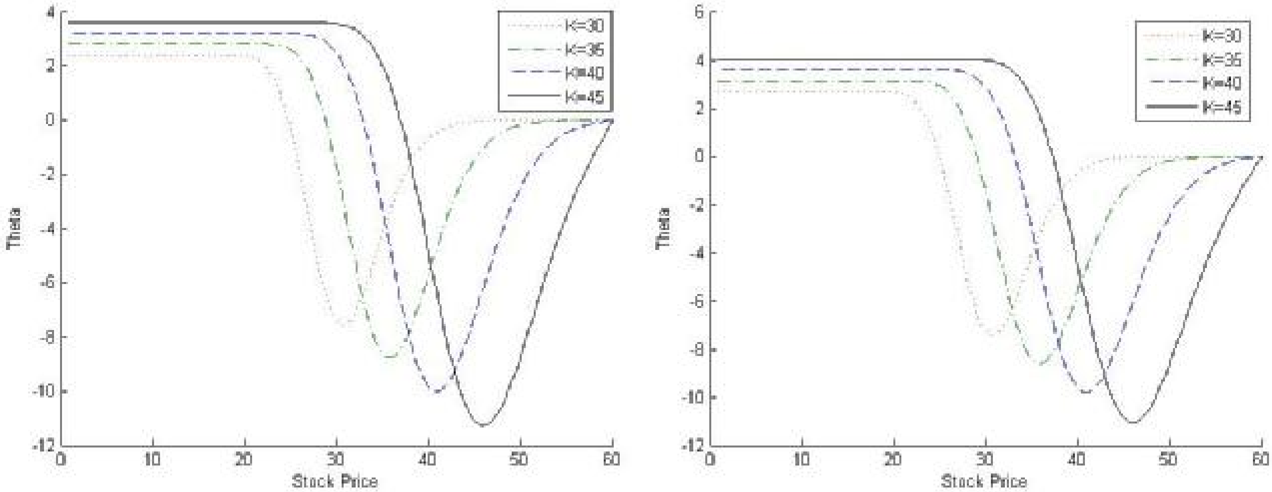

Figure 10. Theta (Δ_p) of the put option for different stock prices S, strike prices K = \$30, \$35, \$40, \$45, interest rates r = 8 %, 9 % with one month maturity.

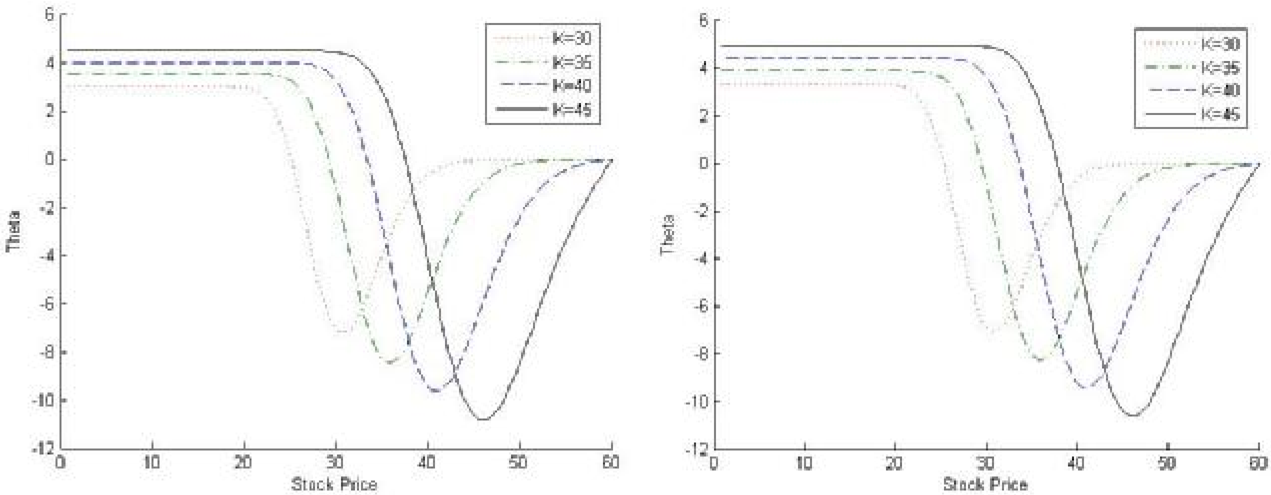

Figure 11. Theta (Δ_p) of the put option for different stock prices S, strike prices K = \$30, \$35, \$40, \$45, interest rates r = 10 %, 11 % with one month maturity.

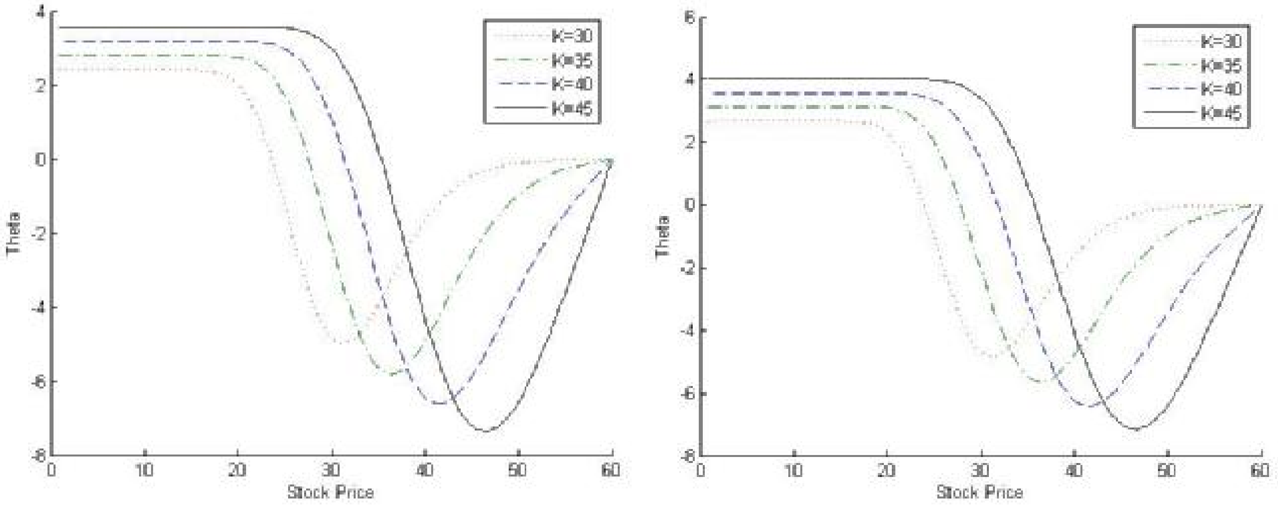

Figure 12. Theta (Δ_p) of the put option for different stock prices S, strike prices K = \$30, \$35, \$40, \$45, interest rates r = 8 %, 9 % with two month maturity.

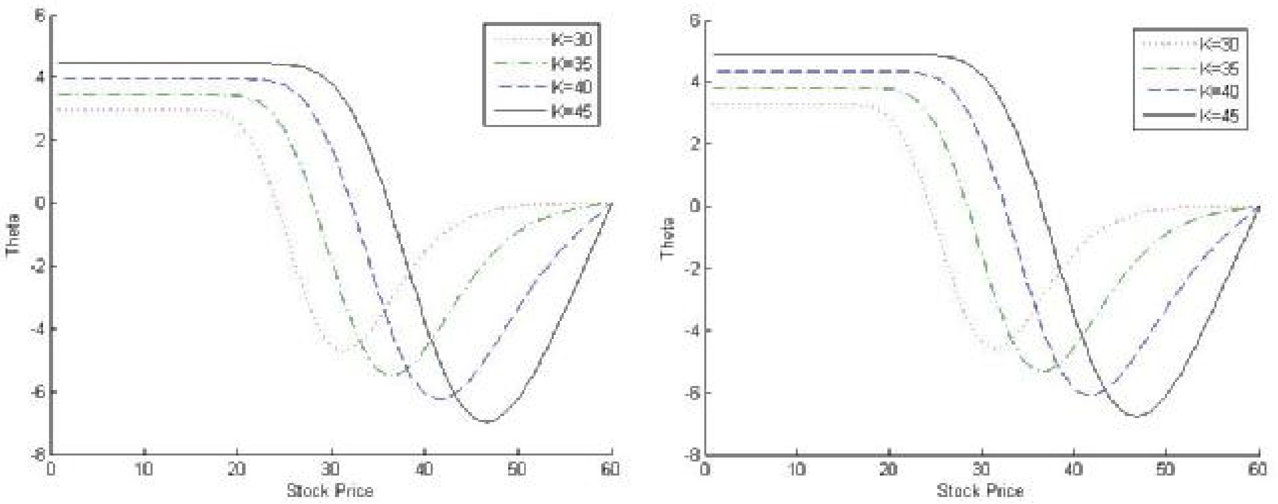

Figure 13. Theta (Δ_p) of the put option for different stock prices S, strike prices K = \$30, \$35, \$40, \$45, interest rates r = 10 %, 11 % with two month maturity.

$\frac{\partial P}{\partial t}$ confirms the long lived counterparts (Table 2).

The positive sign of $\frac{\partial P}{\partial t}$ depicts the put values should be below their intrinsic values (K-S) and option should be grown to (K-S) at expiry. The tendency of asymptotic of curve also validatesthe change in option price, when option become deep in-the-money, constant way. It is also clear that the theta (q_p) has its greatest absolute value when the option is at-the-money (S<K) since the option may become in-the-money (S<K) or out-of-the-money (S>K) at an instant later. Also, the theta (q_p) has a minimum absolute value when the option is sufficiently out-of-the-money (S>K). Hence, the absolute value of theta (q_p) increases as strike price increases but decreases as interest rate (r) and maturity time increase (Figures 10-13).

The gamma (g_p) for put option are positive, this explain why the curves of the option price function with respect to stock price (S) are concave upward. From the figures, it is clear that, generally the absolute values of gamma are maximum as option become at-the-money (S=K) for each strike price (K), interest rate (r) and maturity time (T). This implies that the concavity of the curve for the option prices should be maximum as option become at-the-money (S=K) (Figures 2-6). The absolute values of the gamma (g_p) decreases as strike price increases i.e. the change of delta (Δ_p) is small when strike price increases, this means a portfolio could be hedged by taking long position in put option. From Figures 14-17, it is clear that the gamma (g_p) shows a bell shape curve which have a left long tail and right long tail when option becomes in-the-money (S<K) and out-of-the-money (S>K),

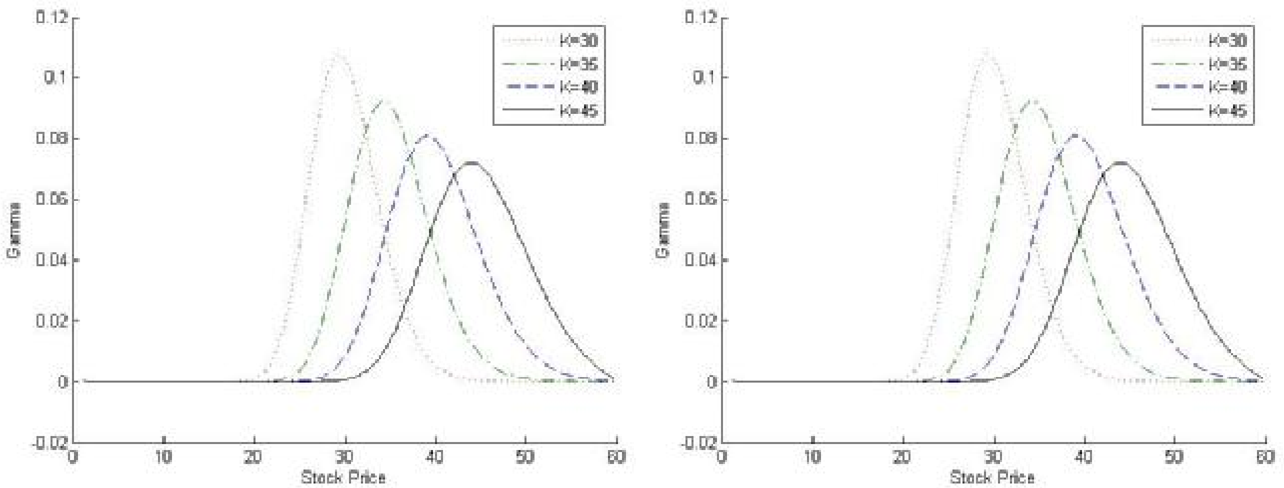

Figure 14. Gama (γ_p) of the put option for different stock prices S, strike prices K = \$30, \$35, \$40, \$45, interest rates r = 8 % , 9 % with one month maturity.

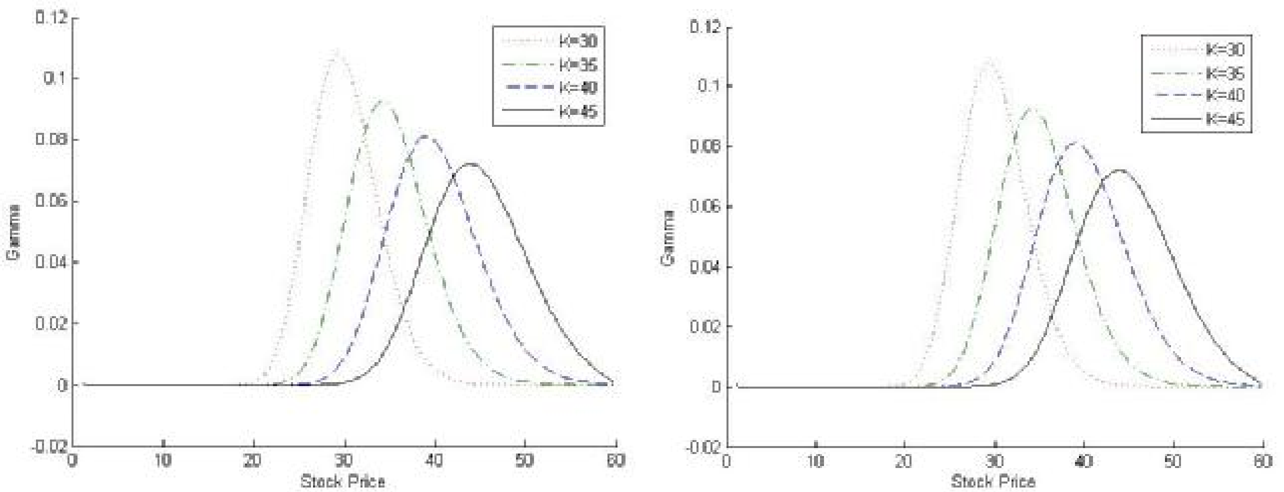

Figure 15. Gama (γ_p) of the put option for different stock prices S, strike prices K = \$30, \$35, \$40, \$45, interest rates r = 10 %, 11 % with one month maturity.

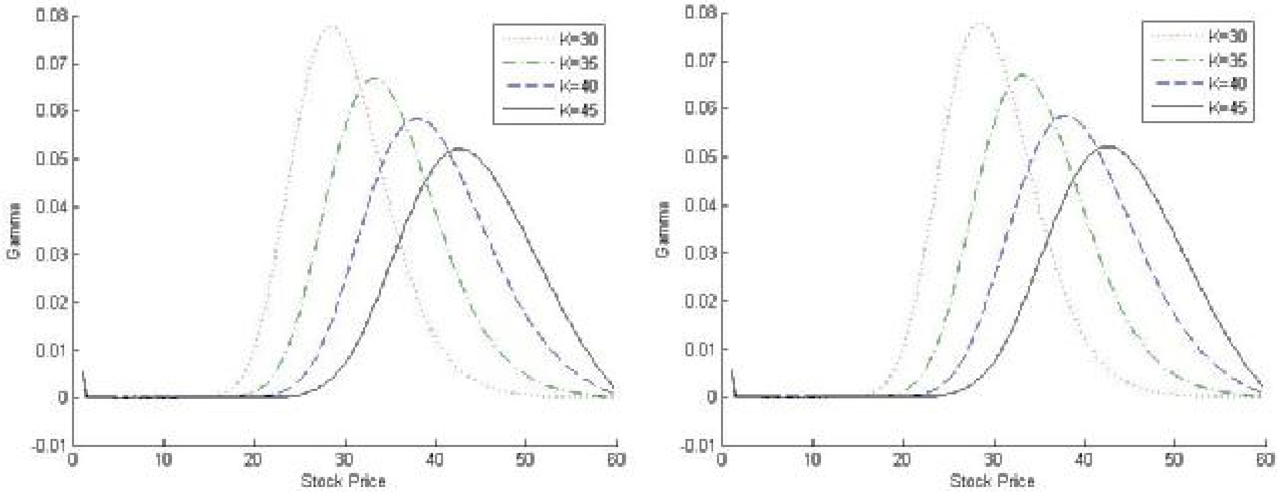

Figure 16. Gama (γ_p) of the put option for different stock prices S, strike prices K = \$30, \$35, \$40, \$45, interest rates r = 8 %, 9 % with two month maturity.

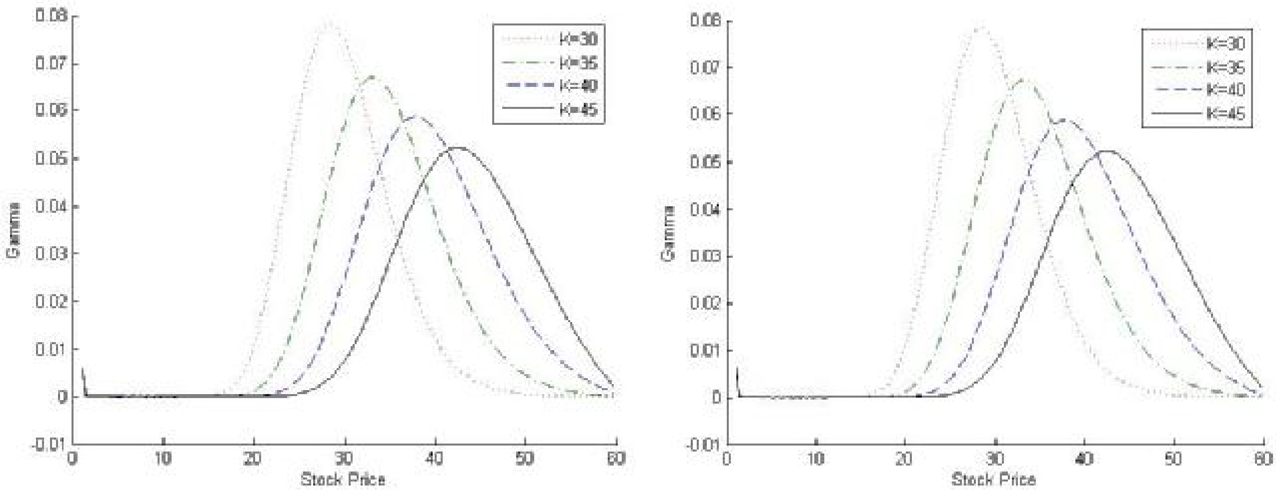

Figure 17. Gama (γ_p) of the put option for different stock prices S, strike prices K = \$30, \$35, \$40, \$45, interest rates r = 10 %, 11 % with two month maturity.

respectively. From the figures of gammas (g_p) it is quite clear that the area of left long tail is less than the area of right long tail. This means the changes of deltas (Δ_p) with respect to stock price (S) should be more dynamic when option becomes in-the-money whenever, option becomes out-of-the-money. This also shows that option has a positive terminal payoff in the case of in-the-money.

The results depict that the change of interest rate is insignificant on gamma values (Figures 14 and 15). But increase in the maturity time i.e. long lived option results in less values of gamma as compared to short lived option (Figures 14 and 16). So, the curvature of long lived option is less than short lived option, while the option prices for an option become at-the-money and out-of-the-money, long lived option are greater than short lived option which has a financial sense that hedging is less dynamic for long lived put option with short lived option.

4. CONCLUSION

Due to stochastic nature of financial market, the volatility is a crucial variable in option pricing and hedging strategies. The alternative approach for the stochastic volatility used in this paper form a one dimensional partial differential equation, where volatility is regarded as a function of stock price (S) and time (t), leads to a partial differential equation in two variables. From the computational point of view, the present model is more economic as compared to model with $\sigma(S, t)$ The results may be useful for the financial engineers in order to understand the effect of stochastic volatility on the hedging movement.

5. REFERENCES

1. Black, F. and Scholes, M., "The Pricing of Options and Corporate Liabilities", ***Journal of Political Economy***, Vol .27, (1973), 637-659.
2. Merton, R.C., "Theory of Rational Option Pricing", ***Journal of Economics and Management Science***, Vol. 4, (1973), 141-183.
3. Nelson D.B., "Conditional Heteroskedasticity in Asset Returns: A New Approach", ***Econometrica***, Vol. 59, No. 2, (1991), 347-370.
4. Bollerslev, T., "Generalized Autoregressive Conditional Heteroskedasticity", ***Journal of Economerics***, Vol. 31, (1986), 307-327.
5. Loudon, W, Watt, G. and Yadav, P., "An Empirical Analysis of Alternative Parametric ARCH Models", ***Journal of Applied Econometrics***, Vol. 15, (2000), 117-136.
6. Mc-Millan, D., Speight, A. and Gwilym, O., "Forecasting U.K. Stock Market Volatility", ***Applied Financial Economics***, Vol. 10, (2000), 435-448.
7. Yu, J., "Forecasting Volatility in the New Zealand Stock Market", ***Appl. Fin. Economics***, Vol. 12, (2002), 193-202.
8. Klassen, F., "Improving GARCH Volatility Forecasts", ***Empirical Economics***, Vol. 27, (2002), 363-394.
9. Vilasuso, J., "Forecasting Exchange Rate Volatility", ***Economic Letters***, Vol. 76, (2002), 59-64.
10. Balban, E., "Comparative Forecasting Performance of Symmetric and Asymmetric Conditional Volatility Models of an Exchange Rate", ***Economic Letters***, Vol. 83, (2004), 99-105.
11. Hull, J. and White, A., "Valuing Derivative Securities using the Finite Difference Method", ***J. Fin. and Quant. Analysis***, Vol. 25, (1990), 88-99.
12. Avellaneda, M., Levy, A. and Paras, A., "Pricing and Hedging Derivative Securities in Markets with Uncertain Volatilities", ***Appl. Math. Finance***, Vol. 2, (1995), 73-88.
13. Avellaneda, M., Levy, A. and Paras, A., "Managing Security Risk of Portfolios of Derivatives: The Lagrangian Uncertain Volatility Model", ***Appl. Math. Finance***, Vol. 3, (1996), 21-52.
14. Chawla, M.M. and Evans, D.J., "High Accuracy Finite Difference Methods for the Valuation of Options", ***Int. J. of Computer Mathematics***, Vol. 82, (2005), 1157-1165.
15. Mayo, A., "High Order Accurate Implicit Finite Difference Method for Evaluating American Options", ***The European Journal of Finance***, Vol. 10, No. 3, (2004), 212-237.
16. Tangman, D.Y., Gopaul, A. and Bhurut, M., "*Numerical Pricing of Options using High-Order Compact Finite Difference Schemes*", ***J. Comp. and Appl. Math***., Vol. 218, No. 2, (2008), 270-280.
17. Hu, B.L.J. and Jiang, L., "Optimal Convergence Rate of the Explicit Finite Difference Scheme for American Option Valuation", ***J. Comp. and Appl. Math***., Vol. 230, No. 2, (2009), 583-599.
18. Rana, U.S., Kumar, N. and Baloni, J., "Mathematical Modeling for Homogeneous Tumor with Delay in Time", ***International Journal of Engineering***, Vol. 22, (2009), 49-56.
19. Ikonen, S. and Toivanen, J., "Pricing American Option using LU Decomposition", ***Applied Mathematical Sciences***, Vol. 1 (2007), 2529-2551.
20. Tavella, D. and Randall, C., "Pricing Financial Instrument: The Finite Difference Method", John Wiley and Sons, Inc., New York, U.S.A., (2000).
21. Kwok, Y.K., "Mathematical Model of Financial Derivative", Springer, New York, U.S.A., (2001).