NUMERICAL SIMULATION OF FORCED CONVECTION OF NANOFLUIDS BY A TWO-COMPONENT NONHOMOGENIOUS MODEL

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Abstract Nanofluids, in which nano-sized particles (typically less than 100 nm) are suspended in liquids, have emerged as a possible effective way of improving the heat transfer performance of common fluids. In this paper a numerical study is performed to analyze the wall shear stress and heat transfer coefficient of γAl_2O_3-water nanofluids under laminar forced convection through a circular pipe. It is assumed that the distribution of nanoparticles in the flow field is nonhomogeneous. The results obtained show that addition of γAl_2O_3 nanoparticles to pure water effectively enhances the convective heat transfer. Moreover, the wall shear stresses are increased. The increasing rate of heat transfer depends on the volume concentration such that for the lowest and highest values of particle volume concentration 0.03 and 0.05, considered in this study, the heat transfer enhancement is approximately 23% and 40%, respectively. Also, compared with the available experimental data, the model used in this work is capable to predict the increasing rate of heat transfer of nanofluids properly.

Keywords Nanofluids, Heat Transfer Enhancement, Non-Uniform Particle Distribution, Two-Component Model, Constant Wall Temperature

چکیده نانوسیالات که از پراکندن ذرات در ابعاد نانو در مایعات حاصل می‌شوند، به‌عنوان راهی موثر در افزایش انتقال حرارت به شمار می‌روند. در این مقاله به بررسی عددی و مطالعه تنش برشی درسطح و ضریب انتقال حرارت نانوسیال آب–γAl_2O_3 در انتقال حرارت جابه‌جایی اجباری جریان آرام درون لوله دایروی پرداخته‌شده است. در حل فرض شده که توزیع ذرات در میدان جریان غیر یکنواخت باشد. نتایج بدست آمده حاکی از آن است که اضافه کردن نانوذرات γAl_2O_3 به آب سبب افزایش چشمگیر انتقال حرارت می‌گردد به‌طوری که این افزایش برای کمترین و بیشترین مقادیر کسر حجمی متوسط ۰/۰۳ و ۰/۰۵ ذرات، که در اینجا موردبررسی‌قرار گرفته‌است، به ترتیب ۲۳ و ٤۰ درصد است. همچنین، تنش برشی در سطح نیز افزایش می‌یابد. مقایسه نتایج بدست آمده با نتایج آزمایشگاهی موجود، بیان‌گر قابلیت خوب روش استفاده شده در این مقاله برای پیش‌بینی میزان افزایش انتقال حرارت در نانوسیال است.

1. INTRODUCTION

The inherently poor thermal conductivity of conventional fluids puts a fundamental limit on heat exchange processes. Therefore, for more than a century since Maxwell [1], scientists and engineers have made great efforts to break this fundamental limit by dispersing millimeter- or micrometer-sized particles in liquids. However, the major problems involved with the use of such large particles are the rapid settling of these particles in fluids as well as increasing the pumping power. Furthermore, these conventional suspensions do not work with the emerging miniaturized devices because they can clog the tiny channels of such devices.

Recent developments in nanotechnology and related manufacturing techniques have made the production of nanosized particles possible. Fluids with nanoparticles suspended in them are called nanofluids. Nanofluids (nanoparticle fluid suspensions) is the term coined by Choi [2] to describe this new class of nanotechnology-based heat transfer fluids that exhibit thermal properties superior to those of their base fluids or conventional particle fluid suspensions. Nanoparticles used in nanofluids have been made of various materials, such as oxide ceramics (Al_2O_3, CuO), nitride ceramics (AlN, SiN), carbide ceramics (SiC, TiC), metals (Cu, Ag, Au), semiconductors (TiO_2, SiC), and carbon nanotubes. Also, many types of liquids, such as water, ethylene glycol, and oil, have been used as base liquids in nanofluids. The volumetric fraction of the nanoparticles, φ, is usually below 5 %.

The presence of the nanoparticles in the fluids increases appreciably the effective thermal conductivity of the fluid and consequently enhances the heat transfer characteristics. Many researchers have reported experimental studies on the thermal conductivity of nanofluids [3-9]. Moreover, various mechanisms and models have been proposed for explaining the enhanced thermal conductivity of nanofluids using various assumptions. A few groups have proposed that the enhancement is due to the ordered layering of liquid molecules near the solid particles [10-12]. Koo, et al [13] considered the kinetic energy of the nanoparticles due to the Brownian movement and proposed a model for the effective thermal conductivity of nanofluids. Prasher, et al [14] introduced a Brownian-motion-based convective-conductive model which predicted the right trend with respect to different parameters such as nanoparticle volume fraction, nanoparticle diameter, and temperature. Jang, et al [15,16] proposed and modeled the Brownian-motion induced nanoconvection as a key nanoscale mechanism governing the thermal behavior of nanofluids. In their model effects of various parameters such as the ratio of the thermal conductivity of nanoparticles to that of a base fluid, volume fraction, nanoparticle size, and temperature on the effective thermal conductivity of nanofluids was included. Comparison of their model predictions with published experimental data showed good agreement for nanofluids containing oxide, metallic, and carbon nanotubes.

The number of studies available in convective heat transfer area is limited compared to that in the area of thermal conductivity of nanofluids. Wen, et al [17] reported heat transfer enhancement due to the addition of γ-Al_2O_3 nanoparticles to deionized water flowing through a copper tube in the laminar flow regime. Their results indicated that the Nusselt number increased up to 47 % when 1.6 % volume fraction of γ-Al_2O_3 nanoparticles was added to water. Zeinali Heris, et al [18] conducted an experimental investigation to study a laminar flow forced convection heat transfer of Al_2O_3-water nanofluid inside a circular tube with constant wall temperature. Their data showed that adding nanoparticles with 2.5 % volume concentration could enhance averaged wall heat transfer coefficient up to 40 %. Yang, et al [19] reported experimental results which illustrated the convective heat transfer coefficient of graphite nanoparticles dispersed in liquid for laminar flow in a horizontal tube heat exchanger. The experimental results illustrated that the heat transfer coefficient increased with the Reynolds number and the particle volume fraction. For example, at 2.5 wt % the heat transfer coefficient of the nanofluids was 22 % higher than the base fluid at 50°C fluid temperature and 15 % higher at 70°C.

Also, there are few studies that assumed certain modeling concepts to be valid and performed a numerical analysis to present the details of the convective heat transfer phenomenon in nanofluids. Maiga, et al [20] presented a mathematical formulation and numerical method to determine the forced convective heat transfer and wall shear stress for the laminar and turbulent regions of water–γAl_2O_3 and ethylene glycol–γAl_2O_3 flowing inside a uniformly heated tube. For the turbulent flow region, the Reynolds-averaged Navier-Stokes equation and $k-\varepsilon$ turbulent model were used to describe the stresses and heat flux of the nanofluids.

Palm, et al [21] extended the radial channel flow problem with consideration of temperature dependent properties. They evaluated temperature-dependent properties by fitting curves to the experimental data of Putra, et al [22]. They observed that use of a variable property model predicted higher thermal and hydraulic performance. Raisee, et al [23] performed a numerical study to investigate the hydrodynamic and thermal characteristics of water–γAl_2O_3 mixture flows through circular pipes. Their observations showed that temperature dependency (Brownian motion) had a significant effect on the heat transfer characteristics of nanofluids.

To investigate the heat transfer enhancement by very fine particles suspended in a fluid, two main approaches have been adopted in the literature. The first one is the two-phase model that takes into account the fluid and solid phase role in the heat transfer process. The second one is the single-phase model where both the fluid phase and the solid particles are in thermal equilibrium state and flow with the same local velocity. This approach is simpler and requires less computational time. Also, if the main interest is focused on the heat transfer process, the modified single-phase, is more convenient than the two-phase model. Recently, Buongiorno [24] presented seven slip mechanisms and concluded that Brownian diffusion (movement of nanoparticles from the high concentration site to the low concentration site) and thermophoresis (movement of nanoparticles from the high temperature site to the low temperature site) are important slip mechanism in nanofluids with order of magnitude analysis. Based on this finding, he developed a two-component four-equation nonhomogeneous equilibrium model for mass, momentum, and heat transport in nanofluids.

In this paper, the model proposed by Buongiorno [24] is used for the first time to investigate the heat transfer of water-γAl_2O_3 mixture flows through circular pipe. The flow is laminar and the wall temperature is assumed to be constant.

2. THEERMOPHYSICAL PROPERTIES OF NANOFLUIDS

Thermophysical properties of nanofluids are calculated by using the following formulas.

The density of nanofluids is predicted by mixing theory as [24],

$$\rho = \varphi\rho_p + (1-\varphi)\rho_{bf} \tag{1}$$

It should be noted that for calculating the specific heat of nanofluid, some of prior researchers have used the following correlation [25,26],

$$C_p = \varphi C_{p,p} + (1-\varphi)C_{p,bf} \tag{2}$$

The modified form of the above equation as suggested by Buongiorno [24], expected to be more accurate, is used in the present work, that is,

$$C_p = \frac{\varphi\rho_p C_{p,p} + (1-\varphi)\rho_{bf} C_{p,bf}}{\rho} \tag{3}$$

Among the various formulas of the thermal conductivity for nanofluid presented in the literature, in this paper, Jang, et al model [15,16] which can predict the enhancement of the effective thermal conductivity for nanofluids in terms of nanoparticles concentration, size and temperature-dependency is used for evaluating the effective thermal conductivity of nanofluids. This model can predict well the effective thermal conductivity of water-γAl_2O_3 nanofluids over a very wide concentration range from 0.01 to 5 Vol. % [27]. The effective thermal conductivity of nanofluids presented by Jang, et al [15,16] is given as

$$k = k_{bf}(1-\varphi) + \beta k_p \varphi + C_1 \frac{d_{bf}}{d_p} k_{bf} \, Re_{d_p}^2 \, Pr \, \varphi \tag{4}$$

Where C_1, β, and φ are proportional constants, related to Kapitza resistance and volume fraction of nanoparticle, respectively. The details of the above model can be seen in [16].

As a general and accurate model for prediction of the viscosity of a nanofluid, μ, is not available at this moment, the effective dynamic viscosity of nanofluids is approximated by means reported in the literature. On one hand, it can be calculated using several existing formulas that have been derived for two-phase mixtures like one proposed by Brinkman [28]. Other relationships found in available literature could also be used taking into account their limitations and applications [29]. In this paper the effective dynamic viscosity of water-γAl_2O_3 nanofluid is calculated by using two models. First, a correlation proposed by Maiga, et al [20] based on fitting curves through regression analysis of experimental data, is used:

$$\mu = \mu_{bf}(123\varphi^2 + 7.3\varphi + 1) \tag{5}$$

Second, based on the experimental data published by Putra, et al [22], a correlation was developed that relates nanofluid viscosity with particle volume concentration and the temperature of the nanofluid.

$$\mu = (-0.206667\,\varphi + 0.159867)e^{(0.033333\,\varphi - 0.0173333)T} \tag{6}$$

In the above formula T is the temperature in Kelvin and φ is the volumetric concentration of nanoparticles. The function was assumed to vary continuously with the particle volume fraction because data for two volume concentration cases were available.

3. MATHEMATICAL FORMULATION

The problem under consideration is a two-dimensional (axisymmetric) steady, forced laminar convection flow of nanofluid flowing inside a straight circular tube. The fluid enters the circular tube with a developed velocity profile and a uniform temperature. The flow and thermal fields are assumed to be axisymetric with respect to the horizontal plane parallel to x-axis as shown in Figure 1.

The nanofluid is treated as a two-component mixture (base fluid+nanoparticles) with the following assumptions [24]:

- incompressible flow,
- no chemical reactions,
- negligible external forces,
- dilute mixture (φ<<1),
- negligible viscous dissipation,
- negligible radiative heat transfer,
- nanoparticles and the base fluid are in thermal equilibrium locally.

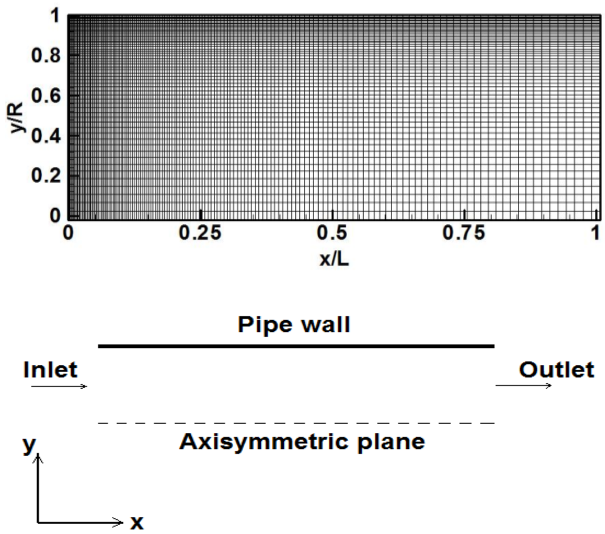

Figure 1. Grid layout and geometry of problem.

Therefore, under the above assumptions, the continuity equation for the nanofluid, nanoparticle continuity equation, nanofluid momentum equation, and nanofluid energy equation, can be expressed, respectively, as [24];

$$\nabla \cdot V = 0 \tag{7}$$

$$V \cdot \nabla\varphi = \nabla \cdot \left[D_B \nabla\varphi + D_T \frac{\nabla \cdot T}{T} \right] \tag{8}$$

$$\rho(V \cdot \nabla V) = -\nabla P + \nabla \cdot \mu \nabla V \tag{9}$$

$$(\rho C_p) V \cdot \nabla T = \nabla \cdot k \nabla T \tag{10}$$

Equation 8 states that nanoparticles not only can move homogeneously with the fluid (left-hand side), but also possess a slip velocity relative to the fluid (right-hand side), which is due to Brownian diffusion and thermophoresis. Brownian motion is proportional to the volumetric fraction of nanoparticles, in the direction from high to low concentration, whereas the thermophoresis is proportional to the temperature gradient, from hot to cold. In Equation 8, D_B represents the Brownian diffusion coefficient, given by the Einstein-Stokes's equation, and D_T represents the thermophoretic diffusion coefficient of the nanoparticles:

$$D_B = \frac{k_B T}{3\pi\mu d_p} \tag{11}$$

$$D_T = \left(\frac{0.26k}{2k + k_p} \right) \left(\frac{\mu}{\rho} \right) \varphi \tag{12}$$

Note that the expression for D_T (Equation 12) was established for particles greater than 1 μm in diameter [30]. In the absence of thermophoretic data, however, it has also been extended to nanoparticles.

The following boundary conditions were used for solving the coupled non-linear partial differential equations given above. At the tube inlet section, a parabolic velocity profile, uniform temperature, T_{in}, and uniform particle distribution, φ_m, are specified. The length of the computational domain is chosen large enough so that the flow exits the pipe with fully developed velocity and temperature profiles. Therefore, at the outlet section, the zero normal gradient condition is applied.

On the upper wall of the tube, the no-slip boundary condition and constant wall temperature were imposed. Due to Impermeability, the total nanoparticle mass flux at the wall is zero, i.e.,

$$\left[D_B \nabla\varphi + D_T \frac{\nabla \cdot T}{T}\right]_{wall} = 0 \qquad (13)$$

On the lower wall of the domain, the symmetry boundary condition, the zero normal gradient for all variables, was applied.

4. NUMERICAL METHOD AND VALIDATION

The system of governing Equations 7-10 constitute a complete set of equations from which V (velocity vector field), P (pressure field), φ (particle volume fraction distribution), and T (temperature field) can be calculated, once the boundary conditions are known, and the nanofluid transport coefficients $(\rho, C, \mu, k, D_B, D_T)$ are specified as functions of φ and temperature.

Note that the conservation equations are strongly coupled. That is, V depends on φ via viscosity; φ depends on T mostly because of thermophoresis; T depends on φ via thermal conductivity and also via the Brownian and thermophoretic terms in the energy equation; φ and T obviously depend on **V** because of the convection terms in the nanoparticle continuity and energy equations, respectively.

In the present study, the system of governing Equations 7-10 were solved using the finite volume method. In this technique, integrating the governing equations over finite control-volumes, results in a set of algebraic equations that can be solved numerically. Staggered grids have been used where the velocity components are calculated at the centre of the volume interfaces while the pressure as well as other scalar quantities such as temperature are computed at the centre of the control-volume. The algebraic discretization equations have been solved sequentially as well as iteratively throughout the physical domain by combining the line-by line procedure and the well-known TDMA technique. Pressure and velocity were coupled using Semi Implicit Method for Pressure Linked Equations [SIMPLE] [31]. Convergence of the iterative solution was ensured when the residual of all variables was less than 10^{-4}.

As shown in Figure 1, the grid used for computations consists of 150 × 60 grid nodes in the streamwise (x) and radial (y) directions, respectively. Finer grid did not improve the accuracy appreciably. As can be seen, the grid points are highly packed near the pipe entrance, as well as in the vicinity of the wall in order to resolve steep gradients of variables in these regions.

The computer program was successfully validated by comparing the calculated results with the available data in the literature for the classical problem of the laminar flow of pure water in tubes. To do this, first developing laminar flow and heat transfer of a pure water through a pipe at Re = 1000 is computed. It is assumed that flow enters the pipe with uniform velocity and temperature. A constant temperature is applied as the wall thermal boundary condition.

Figure 2 shows the velocity profile obtained in the fully developed region of the pipe. As can be seen, this profile is in complete agreement with the well-known quadratic profile, i.e., $u/U_{in} = 2[1-(y/R)^2]$, obtained from the analytical solution of the Navier–Stokes equations. Moreover, the local Nusselt number obtained from our computer program is compared with the following Shah equation [32,33] of the local Nusselt number in a circular tube for a constant wall temperature as shown in Figure 3,

$$Nu_x = \begin{cases} 1.077 x_*^{-1/3} - 0.70 & x_* \le 0.01 \\ 3.657 + 6.874(10^3 x_*)^{-0.488} e^{-57.2 x_*} & x_* > 0.01 \end{cases} \qquad (14)$$

Where

$Nu_x = h(x)d/k_{water}$, $x_* = [(x/d)/(Re_d\, Pr)]$.

Again, a good agreement is apparently displayed between the results obtained by our computer code and that of Shah equation [32,33]. Thus, these results confirm the validity of the computational scheme used in the present investigation.

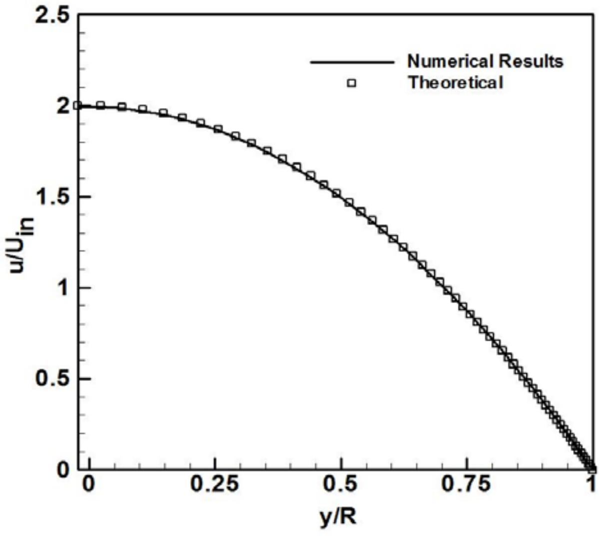

Figure 2. Recent seismicity map of iran (Tavakoli, et al [1]).

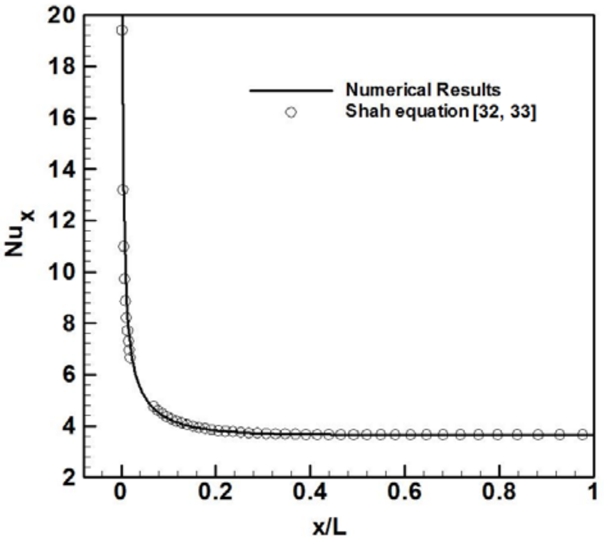

Figure 3. Recent seismicity map of iran (Tavakoli, et al [1]).

5. RESULTS AND DISCUSSION

The working nanofluid used in this simulation is γAl_2O_3-water mixture. The thermophysical properties of nanoparticles adopted in this study are constant as tabulated in Table 1, while the thermophysical properties of water as the base fluid are temperature-dependant and computed by the correlations proposed in [34].

Numerical simulation has been carried out for the following values and ranges: Re = 600-2000, φ_m = 0.03, 0.04, 0.05, T_w = 373K, and T_{in} = 293K. Before presentation of the results, first examine the validity of the models used in this work. To do this, water–γAl_2O_3 nanofluid flow and heat transfer through the pipe under a constant wall temperature has been numerically investigated and numerical results are compared with the experimental data of Zeinali Heris, et al [18]. The difference between two models of the present work is that in one the correlation proposed by Maiga, et al [20] is used for calculating the effective dynamic viscosity of nanofluid (model I), whereas in another the correlation (6) which relates nanofluid viscosity with particle volume concentration and the temperature of the nanofluid is used (model II). As is seen in Figure 4, prediction by the two models is

TABLE 1. Thermophysical Properties of Nanoparticles.

Property	Aluminum Oxide
C_p (J/kg K)	880
ρ (kg/m^3)	3920
k (W/m K)	42.34

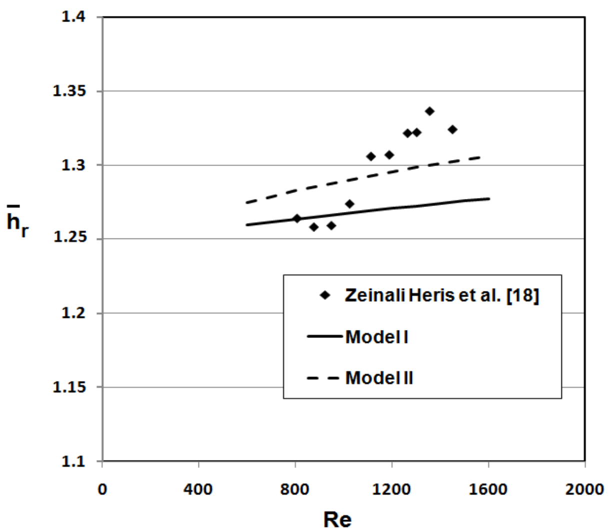

Figure 4. Comparison between the two models and experimental data in prediction of the averaged wall heat-transfer coefficient ratio at φ_m = 0.025.

in good agreement with experimental results. The average wall heat transfer coefficient ratio is defined as $\overline{h}_r = \overline{h}/\overline{h}_{bf}$ (Ratio of the averaged wall heat transfer coefficient for nanofluid to the averaged wall heat transfer coefficient for pure water).

As is seen in Figure 5, addition of γAl_2O_3 nanoparticles to pure water effectively enhances convective heat transfer in laminar regime. The level of heat transfer increase depends on the volume concentration. For the lowest value of $\varphi_m = 0.03$ the heat transfer increase is approximately 23 % while at the upper limit of φ_m (φ_m=0.05) the level of heat transfer augmentation reaches to about 40 %. Also, computations with both models show that the sensitivity of the heat transfer enhancement to Reynolds number for a fixed value of volume concentration is negligible.

In Figures 6 and 7 the effects of particle volume concentration on the axial wall heat-transfer coefficient ratio distribution of water–γAl_2O_3 nanofluid at Re = 600 are presented. The influence of the addition of nanoparticles to the base fluid on heat transfer enhancement is also visible in this figure.

In Figures 8 and 9 the heat transfer coefficient ratio along the axial direction at Re = 1500 is illustrated. It is seen that in the two models, first h_r decreases and then due to temperature dependent properties, it increases slightly along the fully developed region. Comparison of the results obtained for Re = 600 in Figures 6 and 7, with those obtained for Re = 1500 in Figures 8 and 9, shows that h_r curves have nearly the same behavior. This is especially true in the fully developed region where they reach the same values which indicate the independency on Reynolds number in a specified particle volume concentration. Moreover, these variations of local heat transfer coefficient are consistent with the averaged ones in Figure 5.

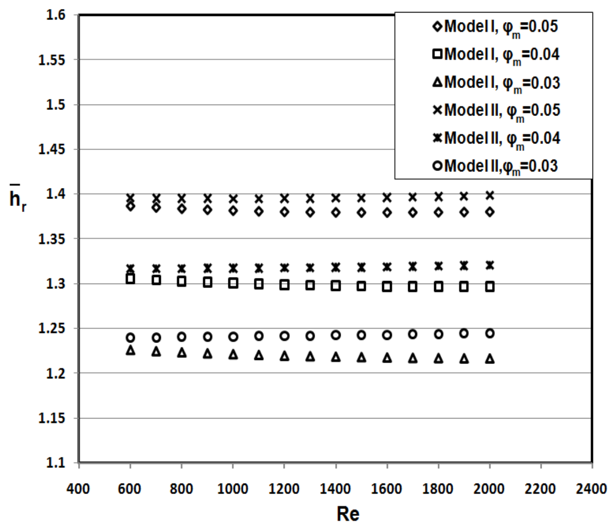

Figure 5. Effects of particle volume concentration on the averaged wall heat transfer coefficient ratio.

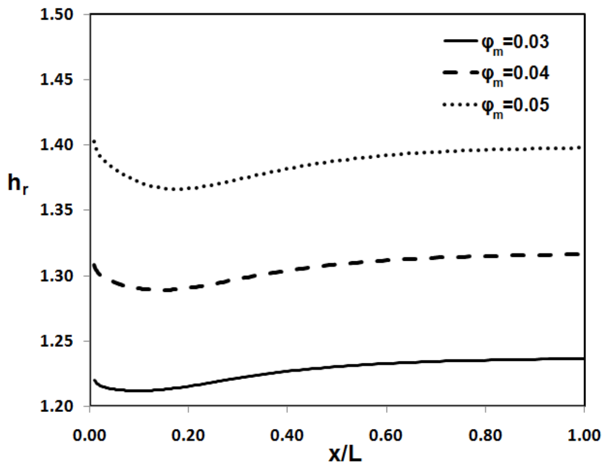

Figure 6. Wall heat-transfer coefficient ratio distribution at Re=600, Model I.

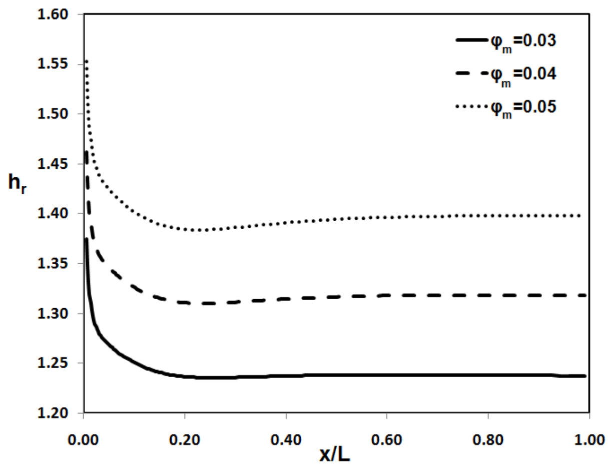

Figure 7. Wall heat-transfer coefficient ratio distribution at Re=600, Model II.

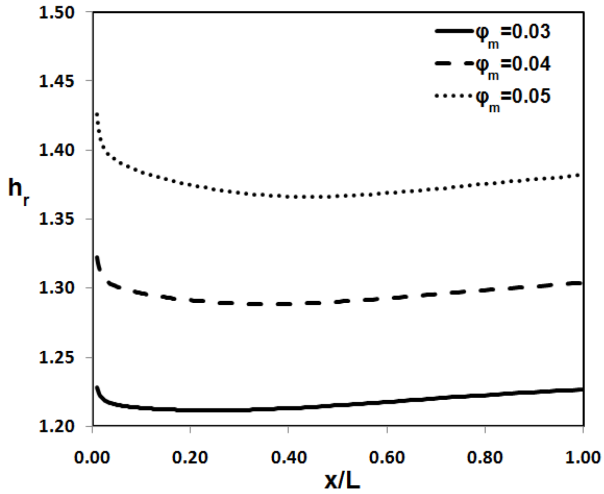

Figure 8. Wall heat-transfer coefficient ratio distribution at Re=1500, Model I.

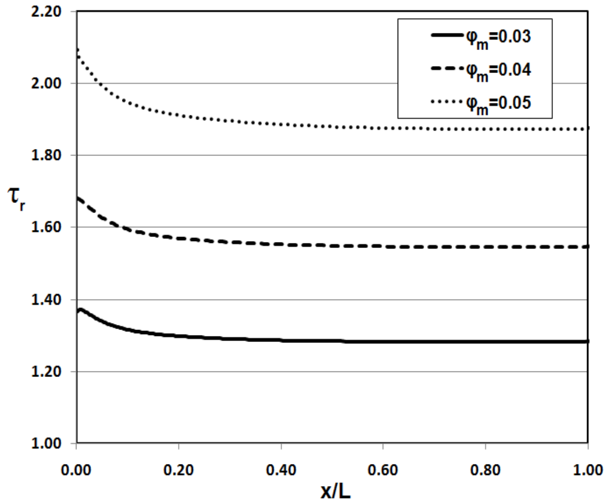

Figure 10. Wall shear stress ratio at Re = 600, Model I.

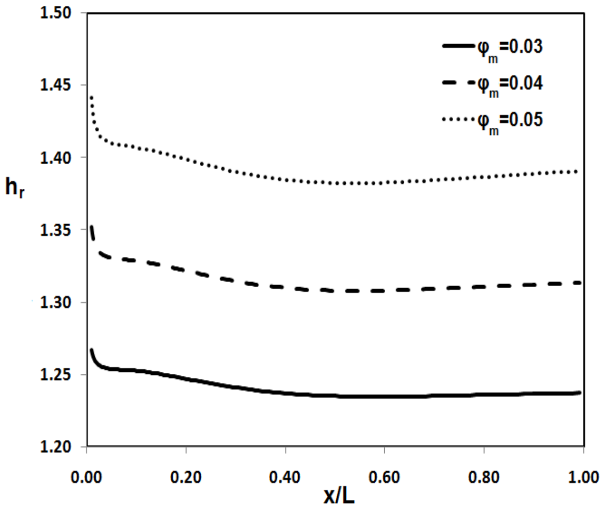

Figure 9. Wall heat-transfer coefficient ratio distribution at Re=1500, Model II.

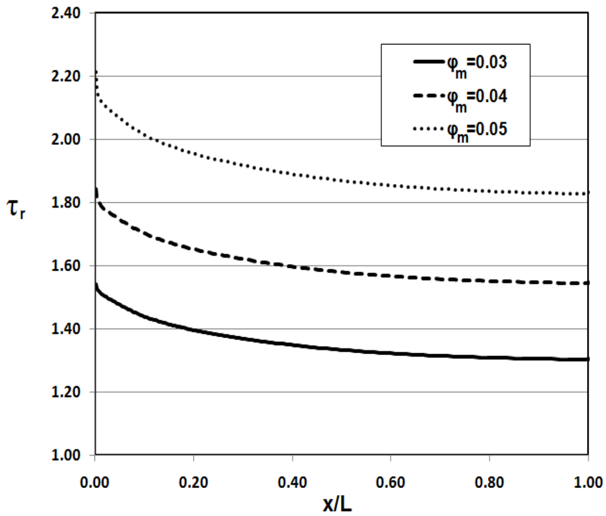

Figure 11. Wall shear stress ratio at Re = 600, Model II.

Figures 10 and 11, show the ratio of local wall shear stress of nanofluid to base fluid at a typical value of Re = 600 for two models. It is clear that addition of nanoparticles to the base fluid results in increasing the wall shear stresses. As can be seen from these figures, in the first model the wall shear stresses in the inlet of pipe is smaller than one obtained by the second model. Also, two models approach approximately to the same asymptotic values. But the slope of variation in the second model is higher. Moreover, the ratio of local wall shear stresses of nanofluid to base fluid at a higher typical value of Re = 1500 are illustrated in Figures 12 and 13 for two models. Compared with the Figures 10 and 11, increasing the Reynolds number causes the ratio of wall shear stresses become greater. It should be noted that in the work of Raisee, et al [23], the viscosity of nanofluid is increased by increasing the temperature in their second model (see Equations 6 and 12 in Raisee, et al [23]).

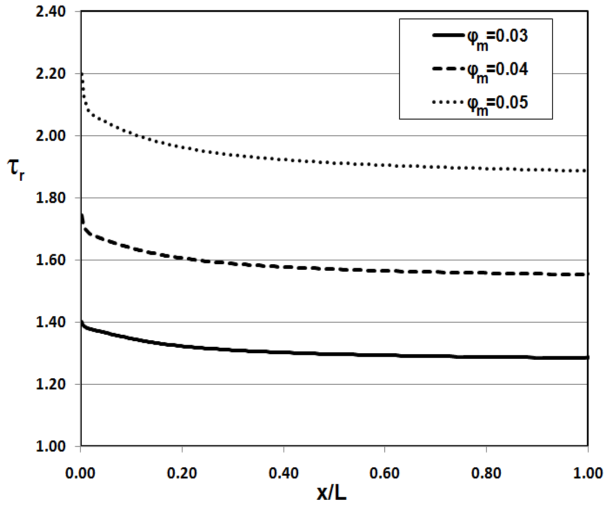

Figure 12. Wall shear stress ratio at Re = 1500, Model I.

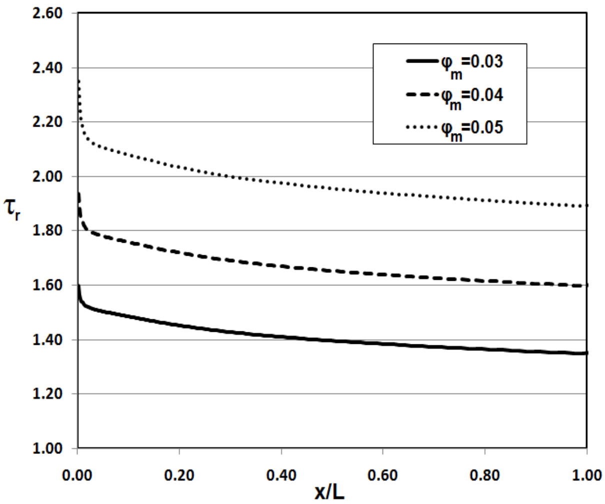

Figure 13. Wall shear stress ratio at Re = 1500, Model II.

But this behavior is in contrast with the experimental observations [22,35] which show that the viscosity decreases with increasing the temperature. Hence, the presented wall shear stresses in the present work are more reasonable than ones in [23], since a correlation based on the experimental data [22] has been used for the calculation of the viscosity of nanofluid.

The main difference between the model used in this study with traditional single-phase ones is existence of the nanoparticle continuity equation, i.e., Equation 8. This equation states that the nanoparticles not only can move homogeneously with the fluid, but also they possess a slip velocity relatively to the fluid, which is due to Brownian diffusion and thermophoresis [24]. Consequently, there is a non-uniform particle volume concentration in the flow field. The radial distribution of particle concentration in tree cross sections of pipe are shown in Figures 14 and 15. According to these

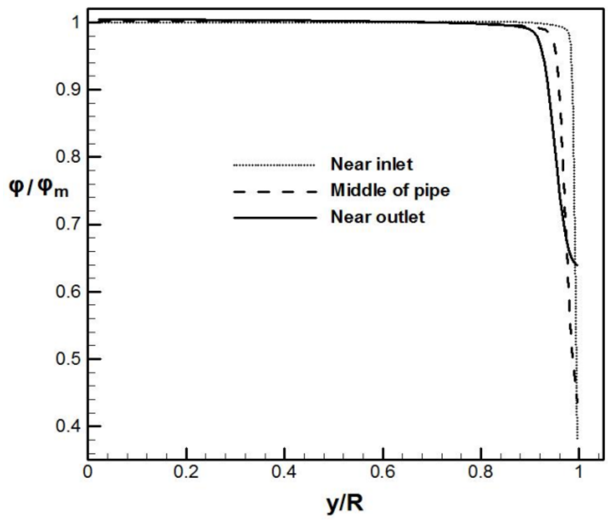

Figure 14. Radial particle concentration distribution at Re = 600 and φm = 0.05.

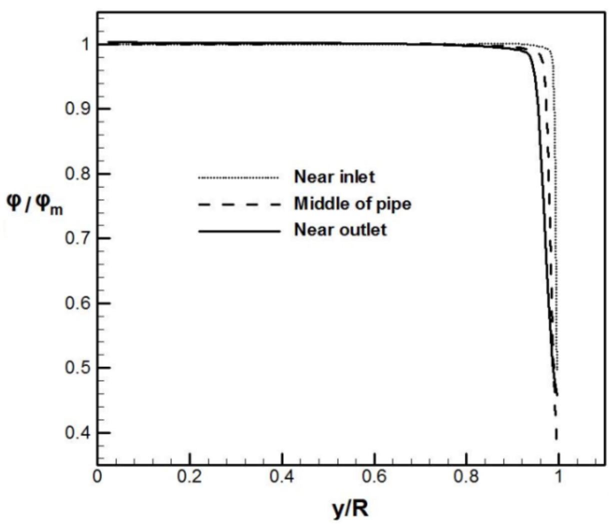

Figure 15. Radial particle concentration distribution at Re = 2000 and φm = 0.05.

figures, near the wall where the velocity is zero or close to it, convection terms on the left hand side of Equation 8 become negligible and the diffusion terms on the right hand side of this Equation become dominant. This causes particle migration towards the pipe centerline and the reduction of particle volume fraction near the wall.

6. CONCLUSIONS

In this paper a numerical study has been performed to investigate the effects of adding γAl_2O_3 nanoparticles to the pure water as a base fluid, on heat transfer and wall shear stresses of flow through a circular pipe. Moreover, the non-uniform distribution of nanoparticles due to thermophoresis and Brownian motion is taken into account. The obtained results show that the addition of nanoparticles increases both the wall heat transfer coefficient and wall shear stress and inclusion of nanoparticles has a more pronounced effect on the wall shear stress than on the heat transfer. For the lowest value of $\varphi_m = 0.03$ the heat transfer increase is approximately 23 % while at the upper limit of φ_m (φ_m=0.05) the level of heat transfer augmentation reaches to about 40%. The wall shear stress increases for the lower and upper limits of volume concentration are about 30% and 100%, respectively. The results gained indicate that the two models adopted in the present work can predict the flow and heat transfer behaviors effectively.

7. NOMENCLATURE

C	Nanofluid specific heat (J/kg K)
C_{bf}	Base fluid specific heat (J/kg K)
C_p	Nanoparticle specific heat (J/kg K)
d	Pipe diameter (m)
d_p	Nanoparticle diameter
D_B	Brownian diffusion coefficient (m^2/s)
D_T	Thermal diffusion coefficient (m^2/s)
h	Heat transfer coefficient (W/m^2 K)
k	Nanofluid thermal conductivity (W/m K)
k_B	Boltzmann constant (J/K)
k_p	Nanoparticle thermal conductivity (W/m K)
k_{water}	Thermal conductivity of water
L	Total length of pipe (m)
P	Pressure (Pa)
R	Radius of pipe (m)
Re	Reynolds number
T	Nanofluid temperature (K)
u	Nanofluid axial velocity (m/s)
V	Nanofluid velocity (m/s)
x	Axial coordinate (m)
y	Radial coordinate (m)

Greek Symbols

φ	Nanoparticle volumetric fraction
μ	Viscosity (Pa s)
ρ	Nanofluid density (kg/m^3)
ρ_{bf}	Base fluid density (kg/m^3)
ρ_p	Nanoparticle density (kg/m^3)
τ_w	Shear stress at the wall (Pa)

Subscripts

b	Bulk
bf	Base fluid
in	Inlet
m	Mean
p	Nanoparticle
r	Relative
w	Wall

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