# HYDROMAGNETIC INSTABILITY OF STRATIFIED RIVLIN-ERICKSEN FLUID IN POROUS MEDIUM IN THE PRESENCE OF SUSPENDED PARTICLES

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**Abstract** The stability of stratified Rivlin-Ericksen fluid in porous medium in the presence of suspended particles and magnetic field has been investigated. Upon application of normal mode technique, the dispersion relation was obtained. The important results obtained in this paper include the instability of non-oscillatory modes and the stability of oscillatory modes. Also, it was found that the system is stable for  $\beta < 0$  and unstable for  $\beta > 0$  under certain conditions.

Keywords Hydromagnetic Instability, Stratified Fluid, Rivlin-Ericksen Fluid, Porous Medium

**چکیده** پایداری سیال ریولین\_ارکسون چینهبندی شدهٔ چگالی همراهبا ذرات معلق در محیط متخلخل با میدان مغناطیسی بررسی شده است. بااستفادهاز روش مد نرمال، رابطهٔ پراکندگی ذرات بهدست آمده است. از نتایج مهم این مقاله ناپایداری مدهای غیرنوسانی و پایداری مدهای نوسانی است. همچنین تحت شرایط خاص برای ο>β سامانهٔ پایدار و برای 0<β ناپایدار میشود.

## **1. INTRODUCTION**

The problem of dust and suspended particles in gas-particle medium has attracted wide attention in recent years due to its relevance in laboratory and astrophysical plasma as discussed by Alfven, et al [1]. Scanlon, et al [2] have discussed the problems of suspended particles on the onset of B'enard convection. Sharma, et al [3] have carried out the investigation on the effect of suspended particles on the onset of B'enard problem (statically unstable configuration) in hydromagnetics. They have found that the effect of suspended particles is to destabilize the layer and the magnetic field has a stabilizing effect. Sharma [4] has also investigated the effect of magnetic field on the gravitational instability of self gravitating homogeneous gasparticle medium with suspended particles.

Chhajlani, et al [5] have discussed suspended particles and the gravitational instability of rotating magnetized medium. Sharma, et al [6] have discussed the stability of stratified Rivlin-Ericksen fluid-particle mixture in hydromagnetics in porous medium. Sharma, et al [7] have discussed the effect of suspended particles on thermal instability in Rivlin-Ericksen elastico-visocus medium.

The stability derived from the equilibrium character of an incompressible heavy fluid of variable density under varying assumptions of hydrodynamics and hydromagnetics has been treated in detail by Chandrasekhar [8].

Kumar [9] has considered the Rayleigh-Taylor instability of Rivlin-Ericksen elastico-viscous fluids in the presence of suspended particles through porous medium. Bhatia [10] has considered the Rayleigh-Taylor instability of two viscous superposed conducting fluids in the presence of a uniform horizontal magnetic field. The stability of superposed fluids in the presence of a variable magnetic field has been studied by Sharma, et al [11]. Rayleigh [12] examined the equilibrium of an incompressible, non-viscous fluid of variable density stratified in the vertical direction.

The problem has been extensively investigated under various physical situations (such as for an

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electrically conducting fluid in the presence of a magnetic field, thermally conducting fluid with temperature variation and instability problems through porous medium, etc.). Sharma, et al [13] have discussed the thermosolutal instability of Rivlin-Ericksen rotating fluid in the presence of magnetic field and variable gravity field in porous medium and found that stable solute gradient has a stabilizing effect on the system while the magnetic field and rotation have stabilizing effect under certain conditions. The problem of stability of stratified visco-elastic dusty fluid (Walter's Model B') in porous medium has been studied by Pundir, et al [14]. They have found that the system is stable for  $\beta < 0$  and unstable for  $\beta > 0$  under certain conditions. Kumar, et al [15] have studied the effect of magnetic field on thermal instability of rotating Rivlin-Ericksen visco-elastic fluid. Sharma, et al [16] have discussed the stability of stratified elastico-viscous fluid (Walter's Model B') in the presence of horizontal magnetic field and rotation in porous medium. Gupta, et al [17] have studied Rivlin-Ericksen elastico-viscous fluid heated and soluted from below in the presence of compressibility rotation and Hall currents. Bhatia, et al [18] have discussed the problem of thermal instability of a visco-elastic fluid in hydromagnetics and found that the magnetic field has a stabilizing influence on a Maxwell fluid just as in the case of a Newtonian fluid. Sharma [19] has studied the instability of the plane interface between two Oldroydian visco-elastic superposed conducting fluids in the presence of an applied uniform magnetic field. Generally, the magnetic field has a stabilizing character; however, there are a few exceptions. For example, Kent [20], Gilman [21] and Jain, et al [22] have obtained unstable wave number ranges in the presence of a magnetic field which are known to be stable in its absence, showing thereby that a magnetic field acts as a catalyst for instability in certain situations. This dual character of a magnetic field has made the hydromagnetic stability of flows much more meaningful and interesting.

Recent spacecraft observations have confirmed that dust particles play an important role in the dynamics of atmosphere as well as in the diurnal and surface variations in the temperature of the Martian weather. It is therefore, of interest to study the presence of suspended particles in astrophysical situations. Further motivation for this study is the fact that knowledge concerning fluid-particle mixtures is not commensurate with their industrial and scientific importance.

The flow through porous medium is of considerable interest for petroleum engineers and for geophysical fluid dynamicists. When the fluid slowly percolates through the pores of macroscopically homogeneous and isotropic porous medium, the gross effect is represented by Darcy's law according to which the usual viscous term in the equation of fluid motion is replaced by

the resistance term  $[(-\frac{\mu}{k_1})]q$ , where  $\mu$  is the

viscosity of the fluid,  $k_1$  is permeability of the medium and q is the Darcian (Filter) velocity of the fluid.

The instability in a porous medium of a plane interface between viscous (Newtonian) and viscoelastic (Walter's B') fluid containing suspended particles may be of interest in geophysics, chemical technology and biomechanics.

In view of the fact that the study of visco-elastic fluids in a porous medium may find applications in geophysics and chemical technology, a number of researchers have contributed in this direction. The effect of magnetic field on the stability of stratified Rivlin-Ericksen fluid in porous medium in the presence of suspended particles may find application in many modern technologies. However, the stability of stratified Rivlin-Ericksen fluid in porous medium in the presence of suspended particles seems to the best of our knowledge uninvestigated thus far.

In this paper, therefore, we have made an attempt to critically examine the stability of stratified Rivlin-Ericksen fluid in porous medium in the presence of suspended particles and magnetic field. It can be looked upon as an extension of work on the stability of stratified visco-elastic dusty fluid (Walter's Model B') in porous medium by Pundir, et al [14].

# 2. CONSTITUTIVE EQUATION AND THE EQUATIONS OF MOTION

Consider a static state in which an incompressible Rivlin-Ericksen visco-elastic fluid is arranged in an isotropic and homogeneous porous medium confined between two infinite horizontal planes situated at axis z = 0 and having density  $\rho$  as a function of the vertical co-ordinates z only. The fluid is under the action of gravity g(0,0,-g) and magnetic field H(0,0,H). This fluid layer is assumed to be flowing through an isotropic and homogeneous porous medium of porosity  $\varepsilon$  and medium permeability  $k_1$ .

Let  $\mu$ ,  $\mu'$  and q(u,v,w) denote respectively viscosity, visco-elasticity and the velocity of the hydromagnetic fluid in the x, y, z direction respectively. V(x,t) and N(x,t) denote the velocity and the number density of particles respectively, where x is the position vector at time t.  $k = 6\pi\mu\eta$ , where  $\eta$  is the particle radius, is a constant at a point X(x,y,z).

Hence, the governing equation is

$$\frac{\rho}{\varepsilon} \left[ \frac{\partial q}{\partial t} + \frac{1}{\varepsilon} (q.\nabla) q \right] = -\nabla p + g\rho\lambda - \frac{1}{k_1} \left( \mu + \mu' \frac{\partial}{\partial t} \right) q +$$

$$\frac{kN_0}{\varepsilon 0} (\nabla - q) + \frac{\mu_e}{4\pi} \left[ (\nabla \times H) \times H \right],$$
(1)
$$\nabla \cdot q = 0 , \qquad (2)$$

$$\varepsilon \frac{\partial r}{\partial t} + (q.V) \rho = 0, \qquad (3)$$

$$\varepsilon \frac{\partial \mathbf{H}}{\partial t} = \nabla \times (\mathbf{q} \times \mathbf{H}) \tag{4}$$

and

$$\nabla . \mathbf{H} = 0 \,. \tag{5}$$

Where the magnetic permeability  $\mu_e$  is assumed to be constant.

The presence of dust particles adds an extra force term proportional to the velocity difference between the particles and the fluid. Since there is a force exerted by the particles on the fluid, there must exist an extra force term equal in magnitude, but opposite in sign, in the equation of motion of the particles. Also, the distance between the particles is quite large as compared to their diameter.

Inter-particle reactions are also not considered as we assume that the distances between the particles are large as compared to their diameter.

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The equations of motion and continuity, under the above assumptions are

$$mN\left[\frac{\partial V}{\partial t} + \frac{1}{\varepsilon}(V.\nabla)V\right] = kN(q-V)$$
(6)

and

$$\varepsilon \frac{\partial N}{\partial t} + \nabla .(NV) = 0.$$
<sup>(7)</sup>

Here, m is the mass of the dust particles.

# 3. BASIC STATE AND PERTURBATION EQUATIONS

The initial stationary state whose stability we wish to examine is that of an incompressible, Rivlin-Ericksen, visco-elastic fluid of varying density and variable viscosity arranged in horizontal strata in a homogeneous and isotropic porous medium. The system is acted upon by a variable horizontal magnetic field H(0,0,H) and the gravity field g(0,0,-g). The character of equilibrium is examined by supposing that the system in slightly disturbed and then by following its further evolution.

Let  $\delta\rho$ ,  $\delta p$ , q(u,v,w), V(l,r,s) and  $H(h_x,h_y,H+h_z)$ denote respectively the perturbation in density  $\rho$ , in pressure p, fluid velocity (0,0,0), filter velocity (0,0,0) and the magnetic field (0,0,H). Linearizing the equation in perturbations and breaking down the perturbations into normal modes, we seek the solution whose dependence on x, y and t is given by

$$\exp\left[i\left(k_{x}x+k_{y}y\right)+nt\right]$$
(8)

Where  $k_x$  and  $k_y$  are the wave numbers in the x and y direction respectively,  $k = \sqrt{k_x^2 + k_y^2}$  is the resultant wave number and n is the frequency which is complex, in general.

On expanding Equations 1 to 7 along x, y and z coordinates and analyzing the perturbation into normal modes (i.e., using (8)), we get:

$$\frac{\rho}{\epsilon}n'u = -ik_x\delta p - \frac{1}{k_1}(\mu + \mu'n) u + \frac{\mu_e H}{4\pi}(Dh_x - ik_x h_z),$$
(9)

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$$\frac{\rho}{\epsilon}n'\upsilon = -ik_y\delta p - \frac{1}{k_1}(\mu + \mu'n)\upsilon + \frac{\mu_e H}{4\pi}(Dh_y - ik_y h_z),$$
(10)

$$\frac{\rho}{\epsilon}n'w = -D\delta p - g\delta \rho - \frac{1}{k_1}(\mu + \mu' n)w, \qquad (11)$$

$$ik_{x}u - ik_{y}v + Dw = 0, \qquad (12)$$

 $\varepsilon n \delta p = -w D \rho, \tag{13}$ 

 $\operatorname{enh}_{\mathbf{X}} = \mathrm{HDu},$  (14)

 $\operatorname{enh}_{y} = HDv,$  (15)

$$\operatorname{enh}_{z} = \operatorname{HDw}$$
 (16)

and

$$ik_{x}h_{x} + ik_{y}h_{y} + Dh_{z} = 0.$$
(17)

# Here, D is the derivative w.r.t. z. From Equation 6

$$\rho nq = kN (V - q) = kN\left(\frac{kq}{mn + k} - q\right) = N\left(-\frac{mn}{1 + \frac{mn}{k}}\right)q \Rightarrow \rho nq + N\left(\frac{mn}{1 + \frac{mn}{k}}\right)q = 0$$

or

$$\left[\rho n + \frac{mnN}{1 + \frac{mn}{k}}\right] q = 0 \Rightarrow$$

$$\rho n \left[1 + \frac{mN}{\rho \left(1 + \frac{mn}{k}\right)}\right] q = 0 \Rightarrow \rho n' q = 0$$

Where,

$$n' = n \left[ 1 + \frac{mN}{\rho \left( 1 + \frac{mn}{k} \right)} \right]$$
(18)

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Multiplying Equation 9 by  $ik_x$  and (10) by  $ik_y$  and adding, we get:

$$\frac{\rho_{\varepsilon}n'Dw = -k^2\delta p - \frac{1}{k_1}(\mu + \mu'n)Dw +$$

$$\frac{\mu_e H}{4\pi}(D^2 - k^2)h_z$$
(19)

Now, multiplying (11) by  $k^2$ , we get:

$$\frac{\rho}{\epsilon}n'k^{2}w = -D(k^{2}\delta p) + \frac{gk^{2}w}{\epsilon n}(D\rho) - \frac{1}{k}(\mu + \mu'n)k^{2}w$$
(20)

Subtracting Equation 20 from (19), we get:

$$\begin{split} &\frac{n'}{\epsilon} [D(\rho Dw) - \rho k^2 w] + \frac{gk^2 (D\rho)w}{\epsilon n} - \\ &\frac{1}{k_1} (\mu + \mu' n) (D^2 - k^2) w - \frac{\mu_e H}{4\pi} (D^2 - k^2) h_z = 0 \end{split}$$

Using (18) and (16), we get:

$$\frac{n}{\varepsilon} \left[ 1 + \frac{mN}{\rho \left( 1 + \frac{mn}{k} \right)} \right] [D(\rho Dw) - \rho k^2 w] + \frac{gk^2 (D\rho)w}{\varepsilon n} - \frac{1}{k_1} (\mu + \mu'n) (D^2 - k^2) w - \frac{\mu_e H^2}{4\pi} (D^2 - k^2) D^2 w = 0$$
(21)

or

$$\frac{n}{\varepsilon} [D(\rho Dw) - \rho k^2 w] + \frac{n}{\varepsilon(1+\tau n)} [D(mNDw) - mNk^2 w] + \frac{gk^2(D\rho)w}{\varepsilon n} + \frac{1}{k_1} (\mu + \mu'n) (D^2 - k^2) w - \frac{\mu_e H^2}{4\pi\varepsilon n} (D^2 - k^2) D^2 w = 0.$$
(22)

Let us assume:

$$\rho = \rho_0 e^{\beta z} , \mu = \mu_0 e^{\beta z} , \mu' = \mu'_0 e^{\beta z} ,$$

$$N = N_0 e^{\beta z} \text{ and } H^2 = H_0^2 e^{\beta z}$$
(23)

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Where,  $\rho_0$ ,  $\mu_0$ ,  $\mu'_0$ ,  $N_0$ ,  $H_0$  and  $\beta$  are constants. Equation 23 shows that the coefficient of kinematic viscosity  $\upsilon$  and coefficient of kinematic visco-elasticity  $\upsilon'$  are constant everywhere. Substituting the value of  $\rho$ ,  $\mu$ ,  $\mu'$ , N and H in Equation 22 and neglecting the effect of heterogenicity on inertia, we obtain

$$\left[\frac{n}{\epsilon} + \frac{nmN}{\rho_0 \epsilon (1+\tau n)} + \frac{\upsilon_0 + \upsilon'_0 n}{k_1}\right] (D^2 - k^2)w +$$

$$\frac{g\beta k^2 w}{\epsilon n} - \frac{V_A^2}{\epsilon n} (D^2 - k^2) D^2 w = 0$$
(24)

Where,

$$\tau = \frac{m}{k}, \ \upsilon_0 = \frac{\mu_0}{\rho_0}, \ \ \upsilon_0' = \frac{\mu_0'}{\rho_0} \text{ and } V_A^2 = \frac{\mu_e H_0^2}{4\pi}.$$

Assuming that the system is confined between two planes z = 0 and z = d and that both the boundaries are free, the boundary conditions for the case of two free surfaces are given by

$$w = D^2 w = 0$$
 at  $z = 0$  and  $z = d$ . (25)

The proper solution of Equation 24 satisfying (25) is given by

$$w = A \sin \frac{m\pi z}{d}$$
(26)

Where, A is a constant and m is any integer.

Substituting (26) in (25) and after simplification, we get

$$\begin{bmatrix} 1 + \frac{\varepsilon \upsilon_{0}'}{k_{1}} \end{bmatrix} \tau n^{3} + \begin{bmatrix} 1 + \frac{mN_{0}}{\rho_{0}} + \frac{\varepsilon}{k_{1}} (\tau \upsilon_{0} + \upsilon_{0}') \end{bmatrix} n^{2} + \\ \begin{bmatrix} \frac{\varepsilon \upsilon_{0}}{k_{1}} + V_{A}^{2} \tau (L - k^{2}) - \frac{\tau g \beta k^{2}}{L} \end{bmatrix} n +$$
(27)
$$\begin{bmatrix} V_{A}^{2} (L - k^{2}) - \frac{g \beta k^{2}}{L} \end{bmatrix} = 0$$

Where,

 $L = \left[ \left( \frac{m\pi}{d} \right)^2 + k^2 \right].$ 

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#### **4. RESULTS AND DISCUSSION**

Depending upon various physical parameters, we obtain a number of results stating clearly the role of these parameters.

**4.1. Theorem** If  $\beta < 0$  everywhere in the flow domain, then the system is stable.

**Proof** We observe that if  $\beta < 0$ , everywhere in the flow domain, then Equation 27 does not allow any positive value of n. Neither does it allow n to be zero so that n can take only negative values, implying thereby that the system is stable.

**4.2. Theorem** If  $\beta > 0$ , everywhere in the flow domain, then the system is stable provided

$$V_A^2(L-k^2) > \frac{g\beta k^2}{L}.$$

**Proof** If 
$$\beta > 0$$
 and  $V_A^2(L-k^2) > \frac{g\beta k^2}{L}$ 

everywhere in the flow domain, then Equation 14 does not allow any positive value of n. Neither does it allow n to be zero, so that n can take only negative values, implying thereby that the system is stable.

**4.3. Theorem** If  $\beta > 0$ , everywhere in the flow domain, then the system is unstable provided

$$V_A^2(L-k^2) < \frac{g\beta k^2}{L}$$

 $\label{eq:proof} {\mbox{ If } \beta > 0 \mbox{ and } V_A^2 \, (L-k^2) < \frac{g\beta k^2}{L} \, ,$ 

everywhere in the flow domain: Since the constant term of Equation 14 is negative, it therefore allows one change of sign and has one positive root. The occurrence of a positive root implies that the system is unstable.

In the absence of a magnetic field, the system is clearly unstable for  $\beta > 0$ . However, the system can be completely stabilized by a magnetic field if  $V_A^2 > \frac{g\beta k^2}{L}$ .

**4.4. Theorem** If  $\beta > 0$  everywhere in the flow domain, then the non-oscillatory modes are unstable when  $V_A^2(L-k^2) < \frac{g\beta k^2}{L}$ .

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**Proof** Let the modes be non-oscillatory so that  $n_i = 0$ , then Equation 27 is reduced to

$$An_{r}^{3} + Bn_{r}^{2} + Cn_{r} - D_{1} = 0$$
(28)

Where

$$A = \tau \left( 1 + \varepsilon \frac{\upsilon'_0}{k_1} \right),$$

$$B = \left[ 1 + \frac{mN_0}{\rho_0} + \frac{\varepsilon}{k_1} (\tau \upsilon_0 + \upsilon'_0) \right]$$

$$C = \left[ \frac{\varepsilon \upsilon_0}{k_1} + V_A^2 \tau (L - k^2) - \frac{\tau g \beta k^2}{L} \right]$$
and

 $D_1 = \left[\frac{g\beta k^2}{L} - V_A^2 (L - k^2)\right]$ 

Equation 28 is a cubic equation in  $n_r$  and if  $n_{r_1}, n_{r_2}$  and  $n_{r_3}$  are the roots of this equation, then

since 
$$\beta > 0$$
 and  $V_A^2(L-k^2) > \frac{g\beta k^2}{L}$ , we have

$$n_{r_1} \cdot n_{r_2} \cdot n_{r_3} = -\frac{D_1}{A} > 0$$

So that either all of the three roots are positive or one root is positive and two roots are negative. In both cases the system becomes unstable. It follows that the one non-oscillatory mode, if it exists, is unstable when the density increase in vertically upward direction.

**4.5. Theorem** If  $\beta > 0$  and the condition  $V_A^2(L-k^2) < \frac{g\beta k^2}{L}$  holds everywhere in the flow

domain, then exactly one non-oscillatory mode is unstable and the other two are stable.

**Proof** If  $n_{r_1}, n_{r_2}$  and  $n_{r_3}$  are the roots of Equation 28, then

 $n_{r_1} \cdot n_{r_2} \cdot n_{r_3} = -\frac{D_1}{A} > 0$ 

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and

$$n_{r_1} + n_{r_2} + n_{r_3} = -\frac{B}{A}$$

Clearly, when  $V_A^2 (L-k^2) < \frac{g\beta k^2}{L}$  and both A and

B are already positive so that the product of roots is positive. Also if  $\beta > 0$ , then the sum of the roots is negative implying one root is positive and the other roots are negative. Therefore, the possibility that all three non-oscillatory modes are unstable is ruled out. It follows that one non-oscillatory mode is unstable and the other two are stable.

**4.6. Theorem** The estimate of n, for the growth rate of oscillatory modes under the condition  $\beta < 0$  is given by  $|n|^2 > \frac{D_1}{B}$ .

**Proof** Equation 27 can be written as

$$An^{3} + Bn^{2} + Cn + D_{1} = 0$$
 (29)

Where,

$$\begin{split} \mathbf{A} &= \left(1 + \varepsilon \frac{\upsilon_0'}{k_1}\right) \tau, \\ \mathbf{B} &= \left[1 + \frac{mN_0}{\rho_0} + \frac{\varepsilon}{k_1} \left(\tau \upsilon_0 + \upsilon_0'\right)\right] \\ \mathbf{C} &= \left[\frac{\varepsilon \upsilon_0}{k_1} + \mathbf{V}_A^2 \tau (\mathbf{L} - \mathbf{k}^2) - \frac{g\beta \mathbf{k}^2 \tau}{\mathbf{L}}\right], \\ \text{and} \end{split}$$

$$D_{1} = \left[ V_{A}^{2} \left( L - k^{2} \right) - \frac{g\beta k^{2}}{L} \right]$$

Dividing Equation 29 by n and separating real and imaginary parts, we get:

$$A(n_{r}^{2} - n_{i}^{2}) + Bn_{r} + C + \frac{D_{1}n_{r}}{|n|^{2}} = 0$$
(30)

and

$$n_{i}\left[2An_{r} + B - \frac{D_{1}}{|n|^{2}}\right] = 0$$
 (31)

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Let the modes be oscillatory, i.e.,  $n_i \neq 0$ , then from Equation 31, we have:

$$2\mathrm{An}_{\mathrm{r}} = \frac{\mathrm{D}_{\mathrm{1}}}{\left| \mathbf{n} \right|^2} - \mathrm{B} \,.$$

Since A and B are definitely positive and  $\beta < 0$ , they ensure that  $D_1$  is positive. Therefore  $|n|^2 > \frac{D_1}{B}$ implying that  $n_r < 0$ , which yields the stability of the system. Hence, the estimate of n for the growth

rate of oscillatory modes under the condition  $\beta < 0$ are given by  $|n|^2 > \frac{D_1}{B}$ .

**4.7. Theorem** The estimate of n, for the growth rate of oscillatory unstable modes under the condition  $\beta < 0$  are given by  $|n|^2 > \frac{D_1}{B}$ .

**Proof** Consider Equation 31

$$n_{i}\left[2An_{r}+B-\frac{D_{1}}{\left|n\right|^{2}}\right]=0$$

Modes are oscillatory, i.e.,  $n_i \neq 0$ . Hence for unstable modes, i.e.,  $n_r > 0$  it implies  $|n|^2 < \frac{D_1}{B}$ .

Thus the estimate of n for the growth rate of oscillatory unstable mode under the condition  $\beta < 0$  are given by  $|n|^2 < \frac{D_1}{B}$ .

**4.8. Remarks** The above two theorems ensure the stability or instability of oscillatory modes under the given conditions of:

• 
$$|n|^2 > \frac{D_1}{B}$$
, i.e.,  $n_r^2 + n_i^2 > \frac{D_1}{B} \Rightarrow Oscillatory$ 

modes are stable.

• 
$$|n|^2 < \frac{D_1}{B}$$
, i.e.,  $n_r^2 + n_i^2 < \frac{D_1}{B} \Rightarrow Oscillatory$ 

# modes are unstable.

Hence, we can find a circle where the oscillatory modes are unstable inside the circle and stable outside the circle.

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## 6. NOMENCLATURE

d	Depth of layer
g(0,0,-g)	Gravity field
H(0,0,H)	Magnetic field
$h(h_x,h_y,h_z)$	Perturbation in magnetic field
K	Stoke's drag coefficient
k	Wave number
k <sub>x</sub> , k <sub>y</sub>	Wave number in x and y
-	directions
m	Mass of suspended particles
N	Number density of suspended
	particles
n	Stability parameter
р	Pressure
t	Time coordinate
q(u,v,w)	Velocity of fluid
V(l,r,s)	Velocity of suspended particles
X(x,y,z)	Space coordinates
бр	Perturbation in pressure

# **Greek Letters**

μ	Fluid viscosity
μ'	Fluid visco-elasticity
3	Medium porosity
ρ	Density
β	Uniform temperature gradient
v	Kinematic viscosity
ν'	Kinematic viscoelasticity
μ <sub>e</sub>	Magnetic permeability
δp	Perturbation in density

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