THE DETERMINISTIC GENERATION OF EXTREME SURFACE WATER WAVES BASED ON SOLITON ON FINITE BACKGROUND IN LABORATORY

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Abstract This paper aims to describe a deterministic generation of extreme waves in a typical towing tank. Such a generation involves an input signal to be provided at the wave maker in such a way that at a certain position in the wave tank, say at a position of a tested object, a large amplitude wave emerges. For the purpose, we consider a model called a spatial-NLS describing the spatial propagation of a slowly varying envelope of a signal. Such model has an exact solution known as (spatial) Soliton on Finite Background (SFB) that is a non-linear extension of Benjamin-Feir instability. This spatial-SFB is characterized by wave focusing leading to almost time periodic extremes that appear between phase singularities. Although phase singularities and wave focusing has been the subject of number of studies, this spatial-SFB written in the field variables has many interesting properties among which are the existence of many critical values related to the modulation length of the monochromatic signal in the far fields. These properties will be used to in choosing parameters for a deterministic generation of extreme waves. Some example of such a generation in realistic variables will be displayed.

Keywords NLS Equation, Spatial SFB, KdV Equation, Extreme Waves, MTA

چکیده هدف از این مقاله توصیف نسل معینی از موج‌های عظیم در یک مخزن یدک نمونه است. این نسل شامل یک سیگنال ورودی است که باید طوری در موج‌ساز وارد شود که در یک موقعیت خاص در مخزن موج، مثلا در موقعیت یک شی تحت تست، دامنه موج بزرگی ایجاد شود. برای این هدف ما مدلی به نام مدل فضایی NLS (یا spatial-NLS) را فرض می‌کنیم که انتشار فضایی یک بسته سیگنال در حال تغییرات کند را تعریف می‌کند. این مدل حل دقیقی به نام spatial-SFB ارائه می‌کند که بسط غیرخطی ناپایداری بنجامین-فیر است. این spatial-SFB با تمرکزموج مشخص می‌شود که سبب می‌شود نقاط نهایی تقریبا به صورت پریودیک زمانی بین تکینگی فازها ظاهر شوند. گرچه نقاط تکین فازها و تمرکز موج موضوع تحقیقات بسیاری بوده‌اند، spatial-SFB که برای متغیرهای میدانی نوشته شده، خواص جالب بسیاری دارد که ازجمله آنها می‌توان وجود مقادیر بحرانی متعدد وابسته به طول مدولاسیون سیگنال مونوکروماتیک در میدان‌های دور را نام برد. این خواص در انتخاب پارامترها برای انتخاب نسل معینی از موج‌های عظیم به‌کار برده خواهندشد. چند مثال از این نسل در متغیرهای واقعی نشان داده خواهد شد.

1. INTRODUCTION

The motivation of this activity stems from the need of hydrodynamic laboratories to generate "extreme waves", also often called rogue or freak waves that do not break while running downward in the wave tank. In a realistic situation involving spatial large spatial and temporal interval, such generation is not easy. This is due to the physical limitation of the wave makers as well as the non linear behaviour that dominate the deformations of propagating signals from the wave makers. The dominant non linear effects in large wave generations can be seen from the previous theoretical [1-3], numerical [5,6] as well as experimental investigation on bichromatic waves such as in [7]. Depending on the input amplitudes as well as frequencies, large deformations and amplitude increase can be found. The location of the maximum amplitude increase within the wave tank also depends on these parameters; see [2-4].

In a generation of extreme waves in hydrodynamic

laboratories, it is desirable that the position for which the extreme waves emerge be located at the position of tested object. A typical question in such a generation is that having the position within the wave tank for the extreme signal to appear and the amplitude of such a signal within some frequency region, a signal can be provided as an input to the wave makers (with their limitation) such that requested signal appears in the desired positions.

Using an exact solution of a spatial NLSE called spatial Soliton on Finite Background (SFB), it is indeed possible to generate waves of reasonably large the wave tank of 200m. This SFB is a non linear extension of Benjamin-Feir instability [15] characterized by wave focusing that is almost time periodic. It is recently observed that an exact solution has numerous interesting properties when it is written in the field variable. Phase singularity; the phenomena merging and splitting of waves; for which the extreme elevations are sandwiched between them are among these properties. Similar studies on wave focusing and phase singularities have been carried out previously such as in [8-13]. We, however, are interested in its direct application on the extreme wave generation in a hydrodynamic laboratory.

The content of the paper is as follows. In section 2 we present a brief overview of the model used in this paper as well as the explicit formula for the spatial SFB. Some interesting features of spatial SFB will be presented in this section. In section 3, we will show a direct extreme wave generation based on the properties of the spatial SFB and transformation in to laboratory coordinates as well as its interpretation. Examples on realistic time and spatial laboratory scales of this wave generation will be presented there. In the last section, we present some concluding remarks are made.

2. THE MATHEMATICAL MODEL FOR SURFACE WAVES

Korteweg de Vries (KdV) equation is known as an asymptotic model for uni-directional surface gravity waves. In normalized variables with full dispersion [6] the equation has the form

$$\partial_t \eta + i\Omega(-i\partial_x)\eta + \frac{3}{4}\partial_x \eta^2 = 0\,, \qquad (1)$$

With η is the surface wave elevation. The symbol Ω is the operator that produces the dispersion relation between frequency ω and wave number k for small amplitude waves given by $\omega = \Omega(k) = \sqrt{k \tanh k}$. The laboratory variables for the wave elevation, horizontal space and time $\eta_{lab}, x_{lab}, t_{lab}$ are related to the normalized variables by $\eta_{lab} = h\eta, x_{lab} = hx$ and $t_{lab} = \sqrt{h/g}$, where h is the uniform water depth and g is the gravity acceleration.

In this paper we are looking for a solution of (1) in form of harmonic mode centered at a frequency ω_0 modulated by a slowly varying envelope $\psi(\xi,\tau)$. The complex amplitude $\psi(\xi,\tau)$ satisfies spatial Non-linear Schrodinger Equation (NLS) in the form

$$\partial_\xi \psi + i\tilde{\beta}\partial_{\tau\tau}\psi + i\tilde{\gamma}|\psi|^2\psi = 0\,. \qquad (2)$$

Here, the slow variable $\xi = x$ and the shifted time variable $\tau = t - x/\Omega'(k_0)$ are introduced while parameters $\tilde{\beta}$ and $\tilde{\gamma}$ depend on the wavenumber of the monochromatic and the central frequency ω_0 related to the wave number k_0 by the dispersion relation

$$\omega_0 = \Omega(k_0) = k_0\sqrt{\frac{\tanh k_0}{k_0}}\,, \qquad (3)$$

see [14].

Different from the temporal NLS that is openly used to describe evolution of ocean waves, spatial NLS is suitable to describe propagation of envelope in signaling problem such as in the problem of wave generation. In [15], it is shown that the temporal NLS has an exact solution called Soliton on Finite Background (SFB). Writing in the complex amplitude of such SFB in the form

$$\psi(\xi,\tau) = a(\xi,\tau)a_0\, e^{-i\tilde{\gamma}a_0^2\xi}\,, \qquad (4)$$

With

$$a(\xi,\tau) = \frac{(\hat{v}^2-1)\cosh\sigma\xi + \sqrt{1-\hat{v}^2/2}\cos v\tau - i\hat{v}\sqrt{2-\hat{v}^2}\sinh\sigma\xi}{\cosh\sigma\xi - \sqrt{1-\hat{v}^2/2}\cos v\tau}$$

and

$$\sigma = \tilde{\beta}\, v\sqrt{2v_*^2 - v^2}\ .$$

If

$$\hat{\upsilon} = \frac{\upsilon}{\upsilon^*}$$

With

$$\upsilon^* = a_0\sqrt{\frac{\tilde{\gamma}}{\tilde{\beta}}}\ ,$$

Then $\hat{\upsilon}$ lies in the region of Benjamin-Feir instability $0 < \hat{\upsilon} < \sqrt{2}$. This parameter υ lies in the region of Benjamin-Feir instability. The last parameter is the frequency of monochromatic ω_0 absorbed in the coefficients of NLS equation.

By using (4) and after simplification, solution of (1) can be written as

$$\eta(x,t) = \psi(\xi,\tau)\, e^{i(k_0 x - \omega_0 t)} + C.C = \frac{2a_0\sqrt{u^2+\lambda^2}}{\cosh\sigma\xi - \sqrt{1-\hat{v}^2/2}\cos v\tau}\cos\phi(x,t), \qquad (5)$$

With c.c denotes the complex conjugate of the previous term. In this solution

$$u = (\hat{v}^2 - 1)\cosh\sigma\xi + \sqrt{1-\hat{v}^2/2}\cos v\tau,$$
$$\lambda = \hat{v}\sqrt{2-\hat{v}^2}\ \sinh\sigma\xi\ ,$$

and $\phi(x,t) = k_0 x - \omega_0 t - \tilde{\gamma}a_0^2\xi + \tan^{-1}(\lambda/u)$, with ω_0 and k_0 are the frequency and wave number of the monochromatic, respectively. We can see that the solution has three parameters (a_0,v,ω_0); the real amplitude in the far field $2a_0$, the modulated frequency v describing the perturbation on the monochromatic by a long time signal and the monochromatic frequency ω_0.

The largest elevation of the SFB (4) is at $(\xi,\tau) = (0,0)$ and at far distance from the position $\xi = 0$, $a(\xi,\tau) \approx a_0$ where the surface wave is monochromatic in time. Some properties of SFB is that compared to the amplitude of monochromatic in the far field $1 < a(\xi,\tau) < 3$ and in limiting case $\lim_{\upsilon\to 0}\frac{a(\xi,\tau)}{a_0} = 3$ giving the largest possible amplification factor is 3 from this monochromatic background.

If M denotes the extreme (maximum) value of the η, then

$$M = \eta(0,0) = \frac{2a_0\sqrt{u^2(0,0)+\lambda^2(0,0)}}{1-\sqrt{1-\hat{v}/2}}\cos(0,0)$$

It is clear that $\upsilon^2(0,0)$ and cos (0,0)=1, then

$$M = 2a_0\frac{\hat{v}^2 - 1 + \sqrt{1-\frac{\hat{v}^2}{2}}}{1-\sqrt{1-\frac{\hat{v}^2}{2}}} = 2a_0\left[1 + 2\sqrt{1-\frac{\hat{v}^2}{2}}\right] \qquad (6)$$

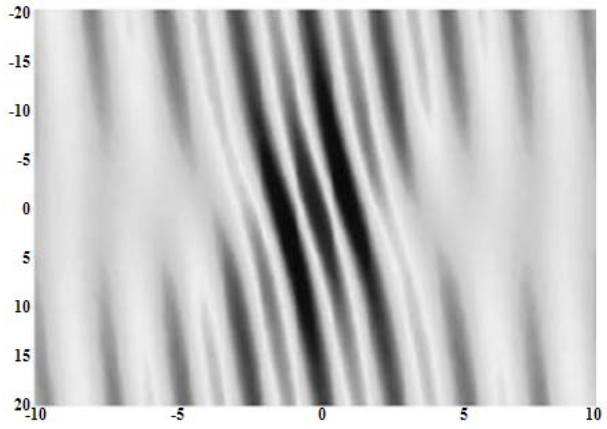
From this expression, we can see that the extreme elevation M depends on α_0 v and ω_0.

It has been known that these extreme elevations appear between two phase singularities; a phenomenon when two waves are merging into one wave or one wave is splitting into two. The phenomenon can be seen in a close up contour plot of the elevation in Figure 1a. The envelope corresponding to this spatial SFB is shown in Figure 1b, where the (time) locations (or instants) of the phase singularities clearly visible; they are the instants when the envelope touches the x-t plane.

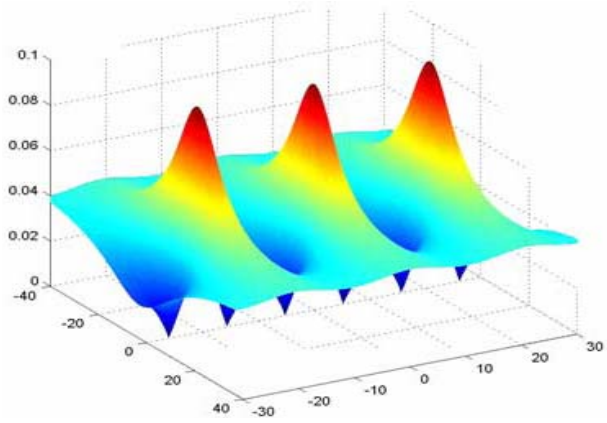
In Figure 2, we display a time signal with an SFB envelope at $\xi = 0$ for three parameter values $\omega_0 = 2.5$, $a_0 = 0.0207$, $\upsilon = 0.3014$ and so $T = 2\pi/v = 28.8458$. We observe that the extreme elevation is almost repeated in time with periodicity T.

To give an easier interpretation related to the problem of wave generation later, in what follows, we require that at $(\xi,\tau) = (0,0)$, $\psi(0,0) = m_0$. Suppose that the frequency of the monochromatic ω_0 is given, which is the center frequency of the wave group being generated. If the amplification factor α is defined as a quotient between the elevation at the extreme position and in the far field, that is $m_0 = aa_0$ then the parameters of SFB (a_0,v,ω_0) can be recovered from $a_0 = m_0/a$ and

$$v = v(\omega_0,\alpha) = a_0\sqrt{\frac{2-(\alpha-1)^2}{2}}\ \sqrt{\frac{\tilde{\gamma}(\omega_0)}{\tilde{\beta}(\omega_0)}}\ . \qquad (7)$$



(a)



(b)

Figure 1. Contour plot of the elevation where the extreme wave appears between the phase singularities signified by merging of two waves into one and (a) splitting one wave into two and (b) the corresponding envelope.

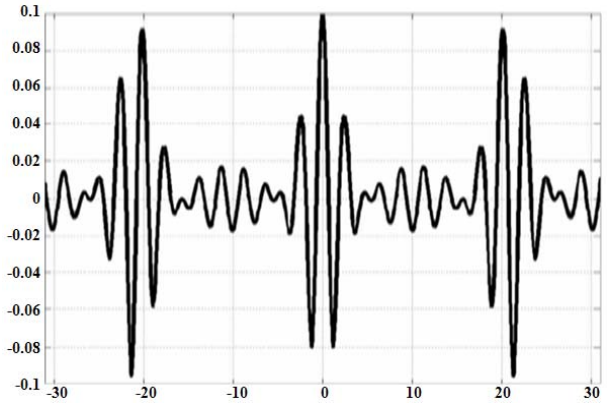


Figure 2. Signal at extreme position $\xi = 0$ for $\omega_0 = 2.5$, $a_0 = 0.0207$, $\upsilon = 0.3014$.

3. RESULTS AND DISCUSSIONS

In what follows, we use the laboratory coordinate for the case of a wave tank with depth H of 5 m. The wave tank at MARIN (Maritime Research Institute Netherlands, Waginingen) is taken as reference tank; see [6]. The length of that tank is 200 m, and those extreme waves and the amplitude amplification should appear within a distance of 200 m from the wave maker. It is our attention to compare the results here with numerical computation obtained from a numerical wave tank as well as measurement in the near future. Although, here we do not describe precisely on the technical restriction of wave maker, our aims is that the required signal can be fed easily to the wave makers.

Let $x_{wm} = 0$ be the location of the wave maker and x_{sh} be the location in the tank such that the signal generated at the wave maker achieves its extreme. If $\eta_{lab}(x_{lab},t_{lab})$ is the elevation of a signal with SFB envelope in the laboratory variables at x_{lab}, let us define at a given position, the maximum temporal amplitude (MTA) in the form

$$M(x_{lab}) = \max_{t_{lab}} \eta_{lab}(x_{lab}, t_{lab}). \qquad (8)$$

Here, x_{sh} corresponding to $\xi = 0$ in NLS spatial variable and so the translation from laboratory to the NLS variables are as follows.

$$\xi = x - x_{sh} \text{ and } \tau = t - \frac{x - x_{sh}}{\Omega'(k_0)},$$

With $\omega_0 = \sqrt{k_0 \tanh k_0}$ and $\omega_0 = \omega_{lab}\sqrt{H/g}$, for the laboratory frequency ω_{lab} and gravitational acceleration g. The MTA measures at each location in the wave tank the maximum over time of the surface wave elevation. As reported in [5], the location where the MTA curve achieves its maximum is the extreme position: there the largest waves will be found.

The amplification factor

$$\alpha = \frac{M(x_{sh})}{2a_0^{lab}}$$

With $2a_0^{lab}$ is the amplitude in the laboratory variable of the monochromatic in the far field. The

actual amplification factor is

$$\alpha_{actual} = \frac{M(x_{sh})}{M(0)}$$

In what follow we list a number of cases of extreme waves generated based on SFB. The position of ship is taken to be 125 m from the wave maker while the maximum temporal amplitude of the signal at the ship position is $M(x_{sh}) = 0.5$ m and $\omega_{lab} = 3.5$ Hz.

Further, we plot the signal at the wave maker and at the ship position 125 m away from the wave maker for two cases from Table 1, namely for $\alpha = 2.4142$ and $\alpha = 2.7321$. The maximum temporal amplitude $M(x_{lab})$ as well as the corresponding the maximum temporal steepness for both cases is provided in the last figure.

4. CONCLUDING REMARKS

We have considered a model called spatial Non-linear Schrodinger (NLS) equation that is suitable to describe the propagation of slowly varying envelopes in signaling problems such as in the wave generation often performed in hydrodynamic laboratory. This equation has an exact solution called spatial Soliton on Finite Background (SFB) with interesting properties related to wave focusing and phase singularity. Written in the field variables, it is interesting to see that the extreme waves appear between the phase singularities signified by merging of two waves in to one or one wave splitting into two. We have used properties of SFB in the generation of extreme waves in realistic laboratory coordinates. Cases on realistic time and spatial laboratory scales of this wave generation

TABLE 1. Samples of Extreme Wave Generation Based on SFB. The Position of Ship is Taken to be 125 m from the Wave Maker and the Maximum Temporal Amplitude at the Ship Position is $M(x_{sh}) = 0.5$ m.

$x_{sh} = 125$ m	α	$\hat{v}$	$2a_0^{lab}$	V_{lab}	T_{lab}	$M(0)$	α_{actual}
Case 1	2.0000	$\sqrt{3/2}$	0.2500	0.6238	10.0723	0.2707	1.8468
Case 2	2.2642	1.0959	0.2208	0.4930	12.7439	0.2481	2.0151
Case 3	2.4142	1	0.2071	0.4220	14.8906	0.2391	2.0916
Case 4	2.7321	$\sqrt{1/2}$	0.1830	0.2636	23.8337	0.2276	2.1967
Case 5	2.9412	0.3405	0.1700	0.1179	53.2941	0.2247	2.2253

have been presented. The results showed that it was possible to generate such extreme waves in hydrodynamic laboratories. Future research will focus on both numerical and experimental evidences to validate the theoretical prediction (Figures 3-5).

5. ACKNOWLEDGMENT

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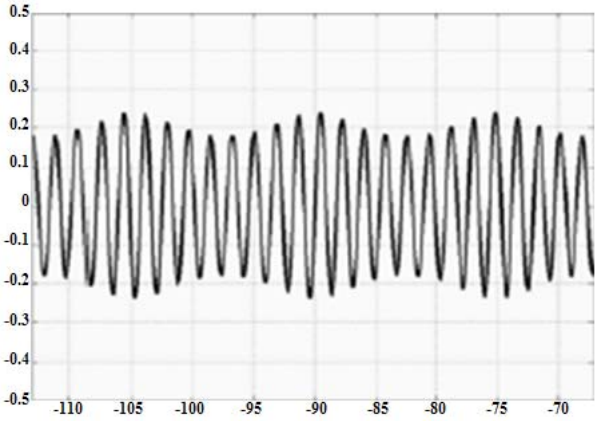
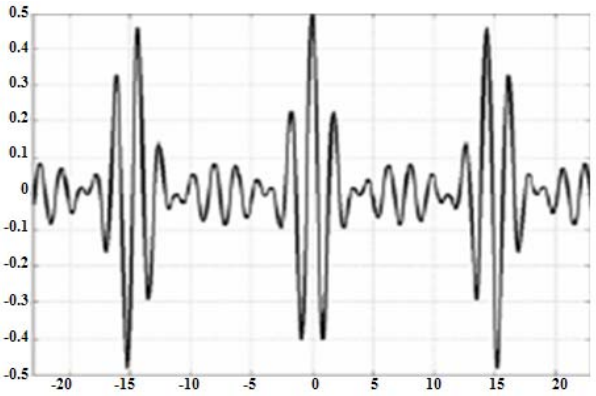
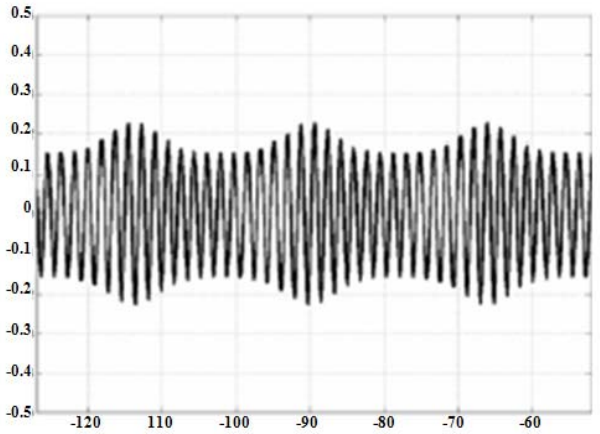


(a)

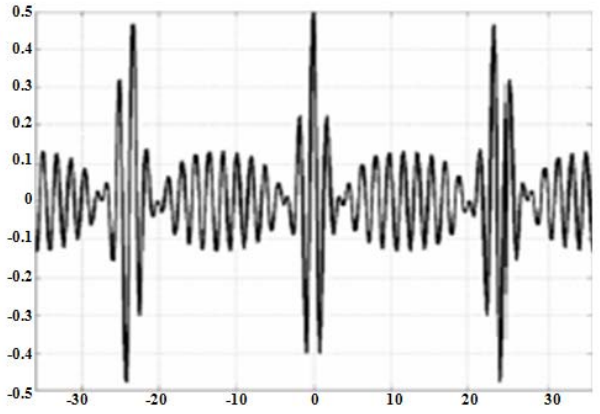


(b)

Figure 3. (a) Signal for α = 2.4142 at the wave maker and (b) at 125 m from the wave maker.



(a)



(b)

Figure 4. (a) Signal for α = 2.7321 at the wave maker and (b) at 125 m from the wave maker.

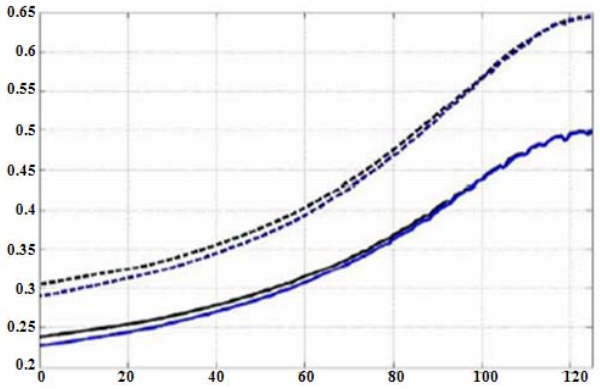


Figure 5. Maximum temporal amplitude (full line) and absolute steepness (dash line) for α = 2.4142 (black) and α = 2.7321 (blue).

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