COMPUTATION OF EARTHQUAKE RESPONSE VIA FOURIER AMPLITUDE SPECTRA

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Abstract A theoretical relation is presented between the seismological Fourier amplitude spectrum and the mean squared value of the elastic response, which is defined by Gaussian distribution. By shifting a general process to its mean value, spectrum of the mean squared value of the displacement is computed from the Fourier amplitude spectrum and the real part of the relative displacement transfer function of the single-degree-of-freedom elastic oscillator. It is shown that the relation presented in this work opens the door for a better understanding of the relationship between time-invariant mean squared value of linear response of a single degree freedom system and seismological variables, such as magnitude, focal distance, and path and soil conditions. For illustrating the proposed theoretical relation, the mean squared values of a drift have been calculated for earthquake ground motions with different magnitude, focal distance and soil.

Keywords Response, Frequency Domain, Seismological Method, Soil Condition

چکیده یک رابطه ریاضی بین طیف دامنه فوریه حاصل از روش های تولید زمین لرزه و مقدار متوسط مربع پاسخ الاستیک ارائه شده است که در این رابطه فرض گردیده است مقادیر پاسخ سازه دارای توزیع گوسین است. با تغییر متغیر پاسخ های حاصل به مقدار متوسط آنها، واریانس پاسخ ها بر اساس طیف دامنه فوریه و بخش حقیقی تابع تبدیل جابجایی نسبی یک سیستم الاستیک یک درجه آزادی محاسبه می گردد. به کمک رابطه ارائه شده در این مطالعه، می توان درک بهتری از تغییرات طیف مقادیر متوسط مربع پاسخ الاستیک نسبت به پارامترهای نظیر بزرگای زلزله، فاصله کانونی و اثرات مسیر و سایت داشت. به ازای زمین لرزه های مختلف نظیر بزرگاها، فواصل کانونی و طبقه بندی های مختلف خاک بر اساس رابطه ارائه شده در این مطالعه، طیف مقادیر متوسط مربع پاسخ الاستیک محاسبه شده است.

1. INTRODUCTION

It is well recognized that earthquake ground motions involves various uncertain factors and does not appear easy to predict forthcoming events precisely at a specific site both in time and frequency [1,2]. Some of the uncertainties result from the lack of information due to the low occurrence rate of large earthquakes and this problem can not be resolved in a practical time span. It is, therefore, strongly desirable to develop a structural design method taking into account these uncertainties with limited information, enabling the design of a safer structure for a broader class of design earthquake.

The need for a stochastic dynamic of engineering systems, which stems from the fact that earthquake exhibits strong variability in both intensity and frequency content [3]. Earthquakes which result in loading processes are not only mathematically complex, but also exhibit a strong element of randomness.

The method of critical excitation was proposed by Drenick, et al [4] for linear elastic single-degree-of-freedom (SDOF) system in order to take into account inherent uncertainties in ground motions. This method is aimed at finding the excitation, producing the maximum response from a class of allowable inputs. Shinozuka, et al [5] discussed the same problem in the frequency domain and proved that, if an envelope function of Fourier amplitude spectra can be specified, a narrower upper bound of the maximum response can be derived.

In this study, a procedure for assessing the mean squared value of a drift at the specified region is presented. The procedure can be used at regions which lacks information about strong/moderate earthquake ground motions, and Fourier amplitude spectra of earthquake ground motions are calculated based on seismological method. This procedure is formulated based on the stochastic method of simulating ground motion and the random vibration theory for linear elastic systems subjected to stationary excitation. This expression reveals how the time-invariant mean squared value of linear system can be analytically related to the earthquake ground motion parameters.

2. MEAN SQUARED VALUE OF THE RESPONSE IN FREQUENCY DOMAIN

Spectral analysis deals with response under a continuous random fluctuating stochastic load process of linear systems. As long as the input is broad-banded (wide-banded) and the system is linear, the output (displacement, force, stress…) of many structural systems are narrow banded according to Gaussian processes [6].

Within the context of structural dynamics, the external excitation vector is balanced through the combined action of the inertial, damping and restoring forces. Consider the equation of motion of the single-degree-of-freedom elastic oscillator subjected to ground acceleration $a(t) = \ddot{u}_g(t)$:

$$\ddot{y} + 2\xi\omega_n \dot{y} + \omega_n{}^2 y = -a(t) \tag{1}$$

Where y(t) is the relative displacement (output) under an excitation (input) a(t), ζ is the damping ratio and ω_n is the natural frequency of the oscillator. Since the input a(t) is a random process, the output y(t) will also be a random process. A direct solution of Equation 1 to output y(t) in time domain is not feasible in this study and, therefore, it is more appreciated to concentrate on the alternative method of representing the relationship between a(t) and y(t). The frequency response method is applied for this purpose, in which both input and output process a(t) and y(t) are represented by harmonic functions. In the frequency domain, the displacement can be expressed as:

$$Y(\omega) = H_D(\omega;\omega_n,\xi).A(\omega) \tag{2}$$

Where $Y(\omega)$ is the Fourier transform of y(t), $A(\omega)$ is Fourier transform of ground acceleration and $H_D(\omega;\omega_n,\xi)$ is the transfer function of base acceleration to relative displacement and it is called the complex frequency response, given by:

$$H_D(\omega;\omega_n,\xi) = -\frac{1}{\omega_n{}^2 - \omega^2 + 2\xi_n.i.\omega_n.\omega} \tag{3}$$

Implementation of the above deterministic technique is feasible in a routine fashion, provided that both structural properties and excitation vectors can be precisely described. There are certain cases, however, for which the excitation process and/or certain structural characteristics are either not known accurately or are random in nature [7]. In fact, the theory behind stochastic dynamics essentially integrates conventional deterministic dynamic analyses within the theoretical framework of stochastic processes [8,9].

The power spectral of the output process, $S_{YY}(\omega)$, can readily be obtained in terms of the power spectral of input process, $S_{AA}(\omega)$. The scalar statement of the spectral result can be written as [6]:

$$\begin{aligned} S_{YY}(\omega) &= H_D{}^*(\omega;\omega_n,\xi)\,H_D(\omega;\omega_n,\xi)\,S_{AA}(\omega) \\ &= \left|H_D(\omega;\omega_n,\xi)\right|^2 S_{AA}(\omega) \end{aligned} \tag{4}$$

In which $H_D{}^*(\omega;\omega_n,\xi)$ is complex frequency response evaluated at frequency-ω and $H_D(\omega;\omega_n,\xi)$ is a complex but even function. Schematic diagram of the critical excitation for finding the power spectral density (PSD) function has been shown in Figure 1.

In engineering application, It is assumed that response Y(t) has a normal (or Gaussian) distribution [10,11]. For computation of this distribution, two quantities of response process are interesting in the structural analysis, e.g. mean values and variances of the response. By shifting a general process to its mean value a zero mean value process can be obtained, in which the mean

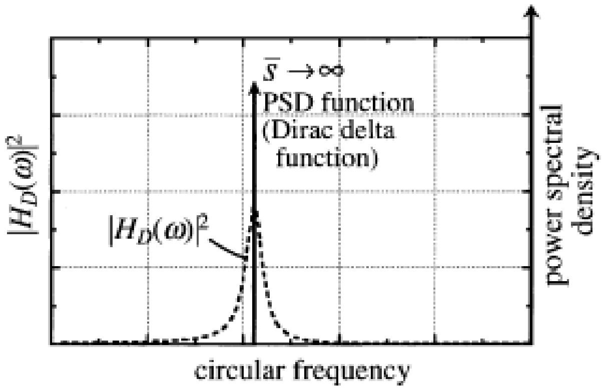

Figure 1. Schematic diagram of critical excitation for finding the power spectral density (PSD) function.

of the second moment equals to the variance of the process. In stationary process, the second moment is equal to the value of the correlation function at τ = 0, e.g. the variance of the process Y(t) with zero-mean is stated as:

$$\sigma_Y{}^2 = R_{YY}(0) = E[Y^2] \tag{5}$$

The correlation $R_{YY}(0)$ can be calculated from the frequency integration of the corresponding power spectrum as:

$$\sigma_Y{}^2 = \int_{-\infty}^{\infty} S_{YY}(\omega)d\omega \tag{6}$$

Variance $\sigma_Y{}^2$, which equals to the time-invariant mean squared value of the response of the SDOF system (relative displacement), based on Equation 4 can be described by:

$$\sigma_Y{}^2 = \int_{-\infty}^{\infty} \left|H_D(\omega;\omega_n,\xi)\right|^2 S_{AA}(\omega)d\omega \tag{7}$$

Where $S_{AA}(\omega)$, power spectral density (PSD) function of input ground motion, is defined as:

$$S_{AA}(\omega) = |F(\omega)|^2 / T \tag{8}$$

Where F(ω) is Fourier spectrum of a ground motion acceleration and T is earthquake ground motion duration.

3. FOURIER AMPLITUDE SPECTRA IN VIEW OF SEISMOLOGY

There is a vast amount of research aimed to predict amplitude Fourier spectra, coming especially from the engineering seismology field. In fact, the amplitude Fourier spectrum has been, so far, the most widely used form of specifying ground-motion characteristics in engineering seismology. Take, for instance, the ground-motion descriptions, always given in terms of Fourier amplitude spectra, which comes from the use of theoretical models of the radiated spectrum plus attenuation, diminution and amplification functions. This approach has been used in the past to predict peak motion values and response spectra [12]. One of the essential characteristics of this method is that, it distills what is known about the various factors affecting ground motions (source, path, and site) into simple functional forms.

Brune, et al [13] assumes that the far-field accelerations on an elastic half space, are band-limited, finite-duration, white Gaussian noise, and that the source spectra are described by single corner-frequency model whose corner frequency depend on earthquake size. The Fourier amplitude spectrum, F(ω), used in a seismological model [12,13] can be broken into contributions from earthquake Brune’s source model, typical geometric, anelastic whole path and upper crust attenuation, and site functions, so that:

$$\left|F(\omega)\right| = \frac{R_P F_S P}{4\pi\rho\beta^3}\frac{1}{R}E(\omega)An(\omega)P(\omega)A(\omega) \tag{9}$$

Where R is the focal distance, R_P is the wave radiation factor (taken here as 0.55), F_S is the free surface amplification factor (taken equal to 2), P is the factor partitioning energy into the orthogonal directions (taken equal to $\sqrt{2}/2$). ρ is the density of rock within the top 10 km of the earth crust, is typically 2.8 ton/m^3 and β is the shear-wave velocity in the vicinity of the source. E(ω) is Brune’s source spectrum, given by:

$$E(\omega) = \frac{M_0\,\omega^2}{1+(\omega/\omega_c)^2} \tag{10}$$

Where M_0 is the seismic moment and ω_c is the corner frequency, taken as:

$$\omega_c = (2\pi) \times 4.9 \times 10^6 \beta_s (\Delta\sigma / M_0)^{1/3} \tag{11}$$

Where $\Delta\sigma$, in bar, is the stress drop and in this equation ω_c is in Hz., β_s in km/s, and M_0 in dyne-cm. The seismic moment, M_0 is often expressed in terms of the moment magnitude (M_w) which is defined as follows [14]:

$$M_w = \frac{2}{3} \log M_0 - 10.7 \tag{12}$$

The loss of energy along the wave travel path is very complex. The An(ω) factor, by definition, includes all the losses which have not been accounted for by the geometrical attenuation factor, and is defined by the exponent expression, given by [12]:

$$An(\omega) = \exp(-0.5\omega R \Big/ \beta . Q_0 (\frac{1}{2\pi}\omega)^n) \tag{13}$$

Q_0 and n are the regional dependent factors of the wave transmission quality factor, Q, which is defined by the exponent expression.

The attenuation, or diminution, operator P(ω) in Equation 9 accounts for the path independent loss of high-frequency in the ground motions.

$$P(\omega) = \exp(-\frac{\omega\kappa}{2}) \tag{14}$$

This loss may be due to a source effect or a site effect or by a combination of these effects and κ is the attenuation parameter to account for high-frequency cutoff [15].

In Equation 9, A(ω) is the upper crust amplification factor and it is a function of shear-wave velocity vs. depth. The corresponding frequency dependent upper crust amplification factor, A(ω), has also been estimated by "quarter wavelength approximation" method. The geometrical attenuation factor which represents geometrical damping is given by piecewise continuous series of straight lines [12]. For the sake of simplicity, in this study, R^{-1} has been accepted.

4. DESCRIPTION OF MEAN SQUARED VALUE OF THE RESPONSE IN A NEW MEASURE

In order to show a proposed measure for describing the time-invariant mean squared value of the relative displacement (drift) of the SDOF system, the following set of parameters have been used; $\Delta\sigma$ = 100 bar, ρ = 2.8 gr/cm^3, and β = 3.5 km/s. The kappa operator (κ) that is a function of distance below the site and the site condition of the station, is assumed to be 0.05 and site amplification factor is chosen according to Boore and Joyner results for two groups of rock and very hard rock sites [16]. We also have a set focal distances equal to 20, 40 and 80 km, and moment magnitude (M_w) equal to 6 and 7 Richter. The regional dependent factors of quality factor are chosen based on Atkinson and Silva study [17]. In this study, it is accepted that the duration of generated records, T, is equal 20 s.

Based on these parameters and Equations 8 and 9, the PSD functions $S_{xx}(\omega)$ can be calculated. These functions have been substituted into Equation 7 to evaluate the time-averaged standard deviation σ_y of the relative displacement of the SDOF model. These values for the damping ratio ζ = 0.05 are plotted in Figure 2a-d for different classes of ground motions with respect to the model natural period $T_0 = 2\pi/\omega_n$. It may be possible to evaluate the power of the ground motions by comparing the results in different focal distance (bold solid, solid and dotted lines). It can be observed that, for both M_w7.0 and M_w6.0, the response in rock site ($\overline{V}_{30} = 620$m/s.) is twice the response in hard rock site ($\overline{V}_{30} = 2900$m/s.).

The response spectral value is expressed as multiple of time-invariant mean squared value of the relative displacement of the SDOF system. Mean squared value of drift is multiplied by a coefficient which depends on an exceeding probability and earthquake ground motion duration [18,19]. It is worth mentioning that in seismological simulation techniques, the ground motion duration is the summation of source rupture duration which is proportional to the inverse corner frequency, and the propagating time of the radiated waves from source to the station [20].

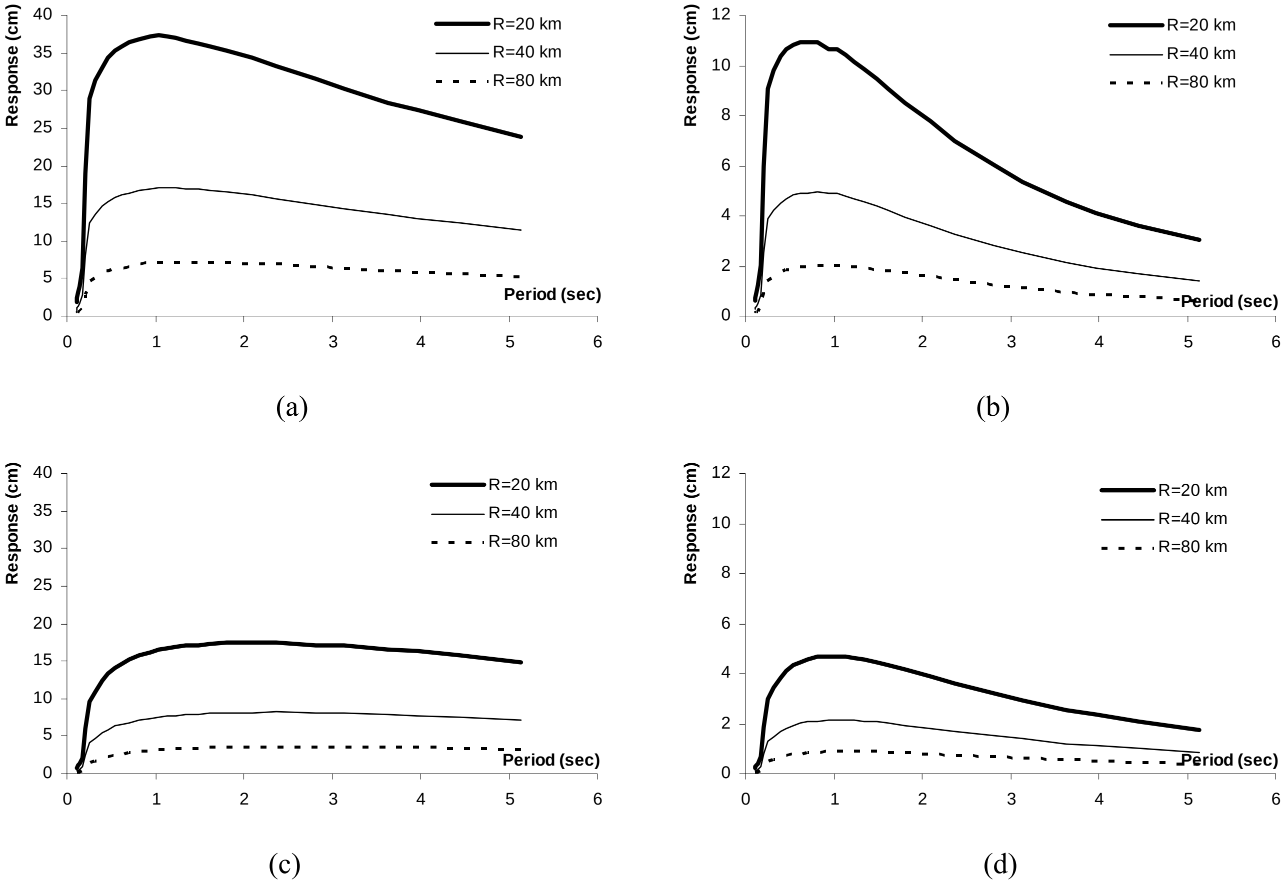

Figure 2. Time-averaged standard deviation of drift in a SDOF model subjected to different ground motions in 20, 40 and 80 km, (a) Rock site and $M_w = 7.0$, (b) Rock site and $M_w = 6.0$, (c) Hard rock site and $M_w = 7.0$ and (d) Hard rock site and $M_w = 6.0$.

5. CONCLUSIONS

A theoretical relation has been presented between the seismological Fourier amplitude spectrum and the mean squared value of the elastic response. As shown, its assumed response has a Gaussian distribution and by shifting a general process to its mean value, spectrum of the mean squared value of the displacement is computed from the Fourier amplitude spectrum, and the real part of the relative displacement transfer function of the single-degree-of-freedom elastic oscillator. The presented relation shows understanding of the relationship between mean squared value of linear response and seismological variables, such as magnitude, focal distance, path and site effects are easier. The response spectra, which can be calculated based on spectrum of the mean squared value of the displacement and the input energy, is an indication of the potential structural damage. The presented procedure in this study can be used at regions with lack of information about strong/moderate earthquake ground motion, which Fourier amplitude spectra of earthquake ground motions are calculated based on seismological method. It is possible to evaluate the power of the ground motions by comparing the results in different focal distance, magnitude, and different soil category.

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