RESEARCH NOTE

SENSITIVITY ANALYSIS OF PARAMETER CHANGES IN NONLINEAR HYDRAULIC CONTROL SYSTEMS

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(Received: June 22, 2004)

Abstract In this research, the sensitivity analysis is applied to an electrohydraulic servovalve which is a nonlinear system. This system sensitivity study differs from previous studies by considering the dynamic behaviour and nonlinearity of the system performance. Different sensitivity analysis methods are compared to each other by studying the sensitivity of the actuator piston velocity of above servovalve with respect to eighteen parameters. By using the best method among the above mentioned methods, the sensitivity of the state variables of the sample system have been studied.

Keywords Sensitivity Analysis – Nonlinear Systems – Hydraulic Control Systems.

چکیده در این مقاله، آنالیز حساسیت بر روی یک سیستم شیر الکتروئیدرولیکی غیر خطی اعمال شده است. مطالعه حساسیت سیستم به خاطر در نظر گرفتن رفتار و عملکرد دینامیکی و غیر خطی سیستم با مطالعات قبلی متفاوت است. روش های مختلف آنالیز حساسیت با اعمال بر سیستم فوق و بررسی سرعت پیستون نسبت به هجده پارامتر با هم مقایسه شده اند. با استفاده از بهترین روش از میان روش های ذکر شده، حساسیت متغیرهای حالت سیستم فوقمورد بررسی قرار گرفته اند.

1. INTRODUCTION

Dynamic system can be characterized in several ways: in the time domain, in the frequency domain, or in terms of a performance index. There is evidently an adequate number of ways to define the sensitivity function of a dynamic system. The definition that is actually used depends on the form of the mathematical model as well as on the purpose of consideration. For example, if the system is represented by a transfer function, the sensitivity will be defined on the basis of the parameter-induced change of the transfer function; whereas in case of a space representation, the natural basis of the sensitivity definition will be the parameter-induced change of the trajectory.

Thus, the sensitivity function can be classified into the following three categories [1]:

- Sensitivity function in the time domain,
- Sensitivity function in the frequency or z-domain,
- Performance-index sensitivity.

Besides these sensitivity function, there are so-called sensitivity measures that are defined on the entirety of the sensitivity function, and, therefore, allow for a global characterization of the sensitivity by a single number. These Entirety measures are used as an index in this research.

The oldest definition of sensitivity function was given by Bode [2]. This definition is based on the transfer function and was restricted to infinitesimal parameter deviation. In the sequel, Horowitz [3] gave a different interpretation of Bode's sensitivity function and also used it with great success for the design of control systems in the frequency domain [4,5]. Perkins and Cruz [6] extended Bode's sensitivity function in different direction, also establishing its significance for time domain consideration.

In connection with simulation on network analyzer and analog computers, the output sensitivity functions were introduced in the fifties mainly by Bykhovsky [7] and Miller and Murray [8]. In the early sixties this definition was extended to the state space, resulting in the so-called trajectory sensitivity function [9,10,11]. The discussion of the merit of the time domain sensitivity function has not yet come to an end [4]. However, there is no doubt that they play an important role in the comparison of open- and closed-loop system as well as in the design of optimal controls. In 1963 Dorato [12] introduced the so-called performance-index sensitivity.

Beside the sensitivity function mentioned above there are various special sensitivity definitions, such as the sensitivity of the overshoot in the time or frequency domain, the eigenvalue (pole or zero) sensitivity, and so on. Definitions such as these may be very helpful in the characterization of the sensitivity of a system in a certain aspect such as its relative stability.

2. BASIC THEORY

Nonlinearities in the system models of electro-hydraulic control servos complicate the application of the sensitivity analysis.The basis for the first order sensitivity models that can be applied to electro-hydraulic position control servos can be introduced as follow [13]:

$$\dot{\bar{x}} = \bar{f}(\bar{x}, \bar{u}, \bar{\alpha}) \qquad (1)$$

where $\bar{x}$ is n-dimensional state vector

$\bar{u}$ is the r-dimensional input vector

$\bar{\alpha}$ is the p-dimensional parameter vector

It is assumed that unique solution of (1) exists for all initial conditions and for all values of $\bar{\alpha}$. Furthermore, it is assumed that $\bar{f}$ is continuously twice differentiable with respect to $\bar{x}$ and $\bar{\alpha}$.

Denote the nominal solution of equation (1):

$$x_n(t) = \bar{\varphi}(t, \bar{\alpha}_n) \qquad (2)$$

where $\bar{\alpha}_n$ is the nominal value (subscript n referring to nominal values) of $\bar{\alpha}$.

Denote the vector sensitivity function:

$$\bar{\lambda}^j = \left(\frac{\partial \bar{x}}{\partial \alpha_j}\right)_n \quad j = 1 \dots p \qquad (3)$$

Assuming that $\bar{u}$ is independent of $\bar{\alpha}$ and differentiating equation (1) partially with respect to $\bar{\alpha}$ we obtain the sensitivity equation in the form:

$$\dot{\bar{\lambda}}^j = \left(\frac{\partial \bar{f}}{\partial \bar{x}}\right)_n \bar{\lambda}^j + \left(\frac{\partial \bar{f}}{\partial \alpha_j}\right)_n \quad j = 1 \dots p \qquad (4)$$

where $(\partial f/\partial x)_n$ is the Jacobian matrix evaluated on the nominal solution.

The initial condition for (3) are :

$$\bar{\lambda}_0^j = \left(\frac{\partial \bar{x}_0}{\partial \alpha_j}\right)_n \quad j = 1 \dots p \qquad (5)$$

where $\bar{x}_0 = \bar{\varphi}(t_0, \bar{\alpha}_n)$ is the initial condition of (1).

The sensitivity equations of (4) are linear differential equation with time-varying coefficient. There will be n(p+1) equation (n state variable equations and $n \times p$ sensitivity equations) to be solved to produce the system states and the sensitivity function. These equations can be solved using a computer simulation.

In the system models of electro-hydraulic control servos the function $\overline{f}$ is continuous everywhere. On the other hand, in the corner of some nonlinearity its first derivative is discontinuous. Between these discontinuity points, $\overline{f}$ is continuously twice differentiable with respect to $\overline{x}$ and. $\overline{\alpha}$ So in the intermediate areas the sensitivity equation can be defined in the form of (4). In solving the vector sensitivity functions one has to change the form of the

state function and sensitivity equations as one moves from one area to another. On the other hand, in the new area the initial condition of the altered equations are replaced by the final condition of the previous area.In the computer simulation this is done simply by the control logic, which recognizes the area in which we operate during the solution, and in moving from one area to another the structure of the state function and sensitivity equations is changed automatically to represent the conditions in the new area. So the initial conditions of the equations in the new area automatically given the values of the final conditions in the previous area. In the sensitivity model of this case, parameter influence on the discontinuity of the first derivate is not taken into consideration.

In addition to the sensitivity function, the complete differential variation of the nominal solution (2), which is:

$$\delta\overline{x}_t = \overline{\varphi}(t,\overline{\alpha}_n) - \overline{\varphi}(t,\overline{\alpha}) \qquad (6)$$

has to be known, and is due to the parameter variation:

$$\delta\overline{\alpha} = \overline{\alpha} - \overline{\alpha}_n \qquad (7)$$

using Taylor's theorem, equation (6) may be written:

$$\delta\overline{x} = \left(\frac{\partial\overline{x}}{\partial\overline{\alpha}}\right)_n \delta\overline{\alpha} + \text{higher order terms} \qquad (8)$$

where $(\partial\overline{x}/\partial\overline{\alpha})_n$ is the $n \times p$ matrix of the sensitivity functions. The vector sensitivity functions are the columns of the sensitivity matrix. Once the vector sensitivity function have been known, according to equation (8), the first order approximation of the variation δx can be calculated.

3. VILENIUS METHOD

This method was introduced by Professor M.J. Vilenius [13] and has been applied to an electro-hydraulic position control servo. The main idea in this method is that, once one knows the size of the parameter variation $\delta\alpha$, one is able to calculate the size of the variation of the nominal step response of x_i by only taking into account the first order terms in equation (8) as follows:

$$\delta x_i = \sum_{j=1}^{p} \lambda_i^j \, \delta\alpha_j \qquad i=1 \ldots n \qquad (9)$$

With equation (9) we are able to calculate the size of any influence of parameter variation on the step response of x_i at every time instant. To simplify the comparison between the different parameters, equation (9) can be scaled with the steady state value and consider only the maximum values of $\delta x_i / x_{is}$ and also change one parameter at a time. Thus the equation for comparison will be as follows:

$$\frac{\delta x_i}{x_{is}}\Big|_{\max j} = \frac{\lambda_i^j \big|_{\max} \delta\alpha_j}{x_{is}} \qquad (10)$$

By means of simulation studies it has been found [13] that the first order sensitivity model is still very accurate when the variations in the parameter vector α are 10 percent. By comparison, Daniels, Lee and pal [14] noticed that first order sensitivity function give satisfactory results up to 20 percent parameter variations, so if one is looking at the influences of 1 percent parameter changes, he can be sure that the first order sensitivity model gives results accurate enough for comparisons. Giving 1 percent change for the parameter ($\delta\alpha_j = 0.01\alpha_{jn}$)

and taking the maximum values $\lambda_i^j \big|_{max}$ according to simulation programs, the maximum variations $\delta x_i / \delta x_{is} \big|_{max\, j}$ can be computed by equation (10).

3. REVISED VILENIUS METHODS

This method is the same as the first method except that instead of computing $\delta x_i / \delta x_{is}$ where x_{is} is the steady state step size,one should calculate $\delta x_i(t)/x_i(t)$ instantaneously, and then capture the maximum value. This gives a better index comparison. Note that:

$$\delta x_i(t) = \sum_{j=1}^{p} \lambda_i^j \delta\alpha_j$$

then,

$$\frac{\delta x_i(t)}{x_j(t)} = \sum_{j=1}^{p} \frac{\lambda_i^j(t)}{x_i(t)} \delta\alpha_j \quad (11)$$

Consequently, it is sufficient to compute

$$\frac{\lambda_1^j(t)}{x_1(t)} \qquad j = 1 \text{ to } p$$

$$\frac{\lambda_2^j(t)}{x_2(t)} \qquad j = 1 \text{ to } p$$

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and then capture the maximum in each case. The only problem which remain yet, is the calculation of $\delta x_i(t)/x_i(t)$ when $x_i(t) \approx 0.0$.
To overcome this problem we can consider only the case where $x_i > \xi x_{is}$ where ξ can be $0 < \xi < 1.0$.

4. INDIVIDUAL CHARACTERISTICS METHOD

As it was mentioned in the first two methods, they choose only λ_i^j at one instant of the time, which is the maximum in one or the other way. This λ_i^j not only does not have information about other instants of the time but also its information at that special point of time is combination of different characteristics of the system performance changes (e.g. amplitude, frequency, …changes).
To make these problems more clear, the system performances before and after change of parameter α_j are presented in Figure.1. Amplitude differences between the two curves at different points of times show the $\lambda_i^j \delta\alpha_j$, values. As it can be seen at some points this value ($\lambda_i^j \delta\alpha_j$) is equal to zero. But it dose not mean that the parameter change has no effect on the system performance rather, it means different characteristics changes of the system performance neutralize effects of each other at that point of time. Hence it will be better to study individual characteristics of the system performance separately.

5. ENTIRETY-INDEX METHOD

As it noticed, the different methods for the sensitivity analysis (methods 2-4) have various advantages and disadvantages. Individual characteristics method gives most of the system performance properties but it makes it difficult to use directly these result for some special purpose such as system optimization, and insensitivity. Then a simple method which has all or most of the system performance characteristics sensitivity has to be introduced.
For this purpose instead of capturing maximum value of sensitivity, we integrate $\left|\lambda_i^j \delta\alpha_j\right|$, over some period of time ($0:T_1$), figure 1. The integral of $\left|\lambda_i^j \delta\alpha_j\right|$ simply represent the area between two state variable of the system.

6. SAMPLE SYSTEM

To apply the sensitivity analysis a novel electro-hydraulic servovalve has been chosen [15]. Considering a conventional hydraulic servovalve circuit instead of connecting backpressure of actuator to drain through servo-valve, it will be connected to drain through a metering valve and a

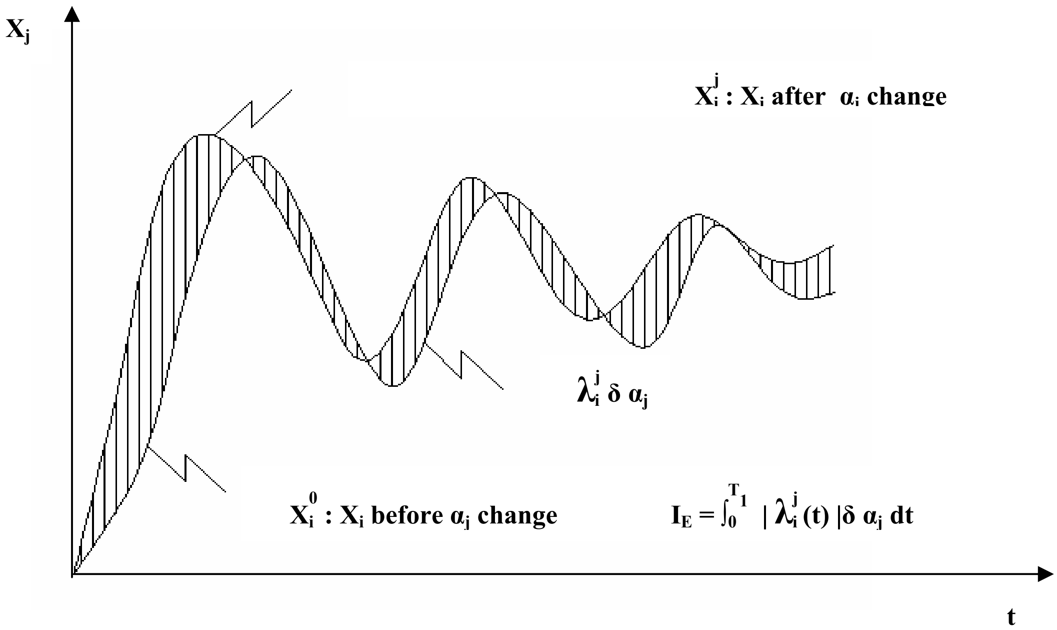


Figure 1. Entirty Index of state variable X_i

relief valve (Figure. 2). In this way the backpressure and drain orifice area are controllable.

The system is defined by state model as below:

$\dot{\bar{x}} = \bar{f}(\bar{x},\bar{u})$ where $\bar{x} = \{a_v, p_1, p_2, u_p\}$

Using the informal equation of the various components of the system, the function $\bar{f}$ can be derived as bellow:

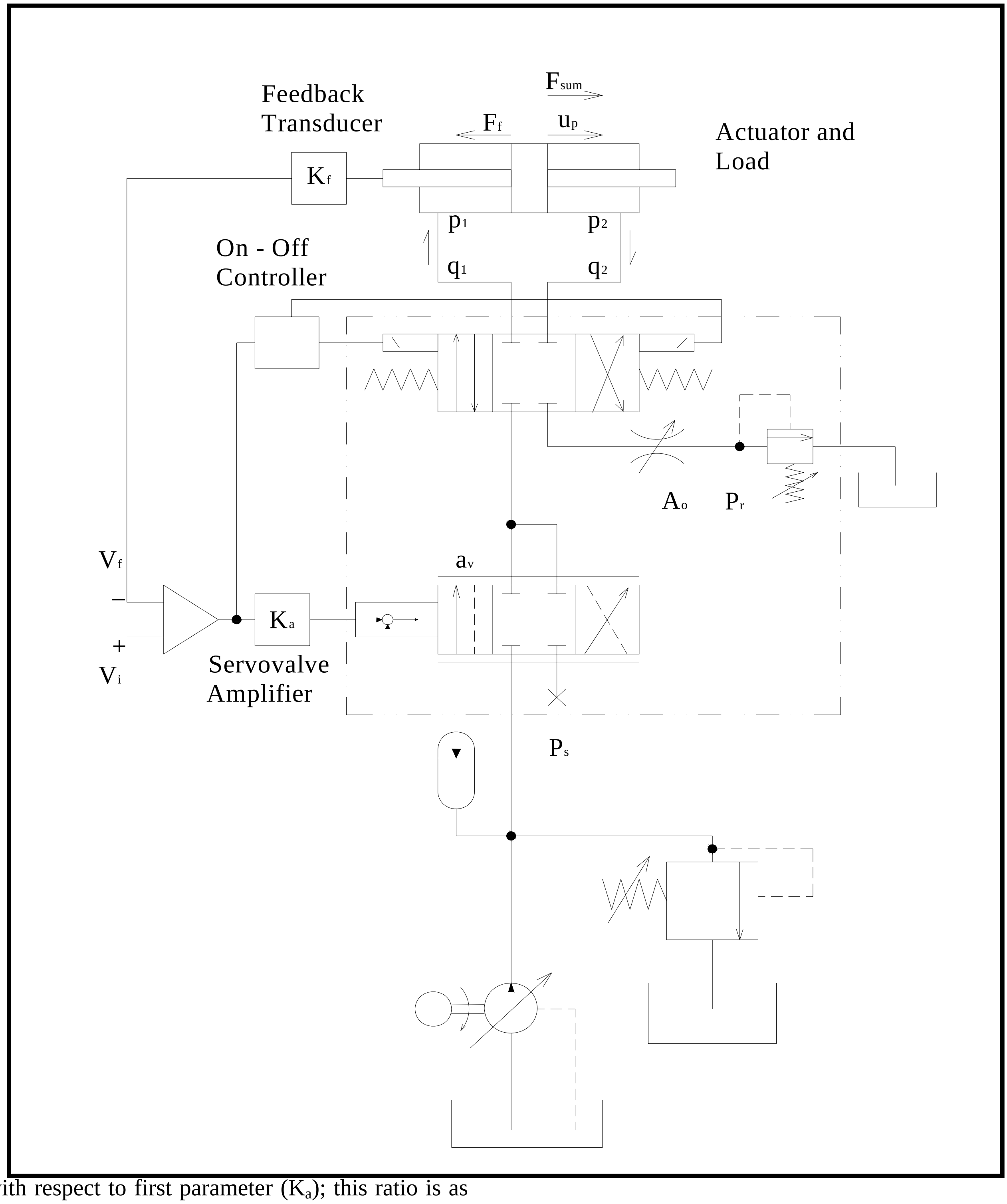
$$\bar{f}(\bar{x},\bar{u}) = \begin{cases} \dot{x}_1 = \dfrac{da_v}{dt} = \dfrac{-1}{\tau_v}x_1 - \dfrac{1}{\tau_v}K_xK_vK_aK_f x_4 + \dfrac{1}{\tau_v}K_xK_vK_aV_{in} \\ \dot{x}_2 = \dfrac{dp_1}{dt} = \dfrac{\beta}{v_s}\left(q_1 - A_px_4 - c_1(x_2 - x_3)\right) \\ \dot{x}_3 = \dfrac{dp_2}{dt} = \dfrac{\beta}{v_r}\left(-q_2 + A_px_4 + c_1(x_2 - x_3)\right) \\ \dot{x}_4 = \dfrac{du_p}{dt} = \left((x_2 - x_3)A_p - F_f\right)/M \end{cases}$$

Where q_1,q_2 (oil flows) and v_s, v_r (supply, return oil volume) depend on position of the direction control valve which is controlled by input error voltage $(V_i - V_f)$ and F_f (friction force) depends on x_4 (actuator piston velocity). Other parameter defination and their values are given at the end.

7. APPLICATION OF THE METHODS

Four different methods are applied to the chosen hydraulic system and the best index selected. To compare different methods, only sensitivity of the state variable u_p which is most important output of the system is considerd. But to study the sensitivity of other state variables the last method (entirety-index) is used to derive their sensitivity histograms with respect to all parameters change.

In the first method the ratio of the maximum change of amplitude of state variable $(\lambda_i^j\delta\alpha_j)$ to steady state value of amplitude of state variable is studied, For example, for the last state variable (u_p)



with respect to first parameter (K_a); this ratio is as

Figure 2. Schematic of servomechanism with new servovalve configuration.

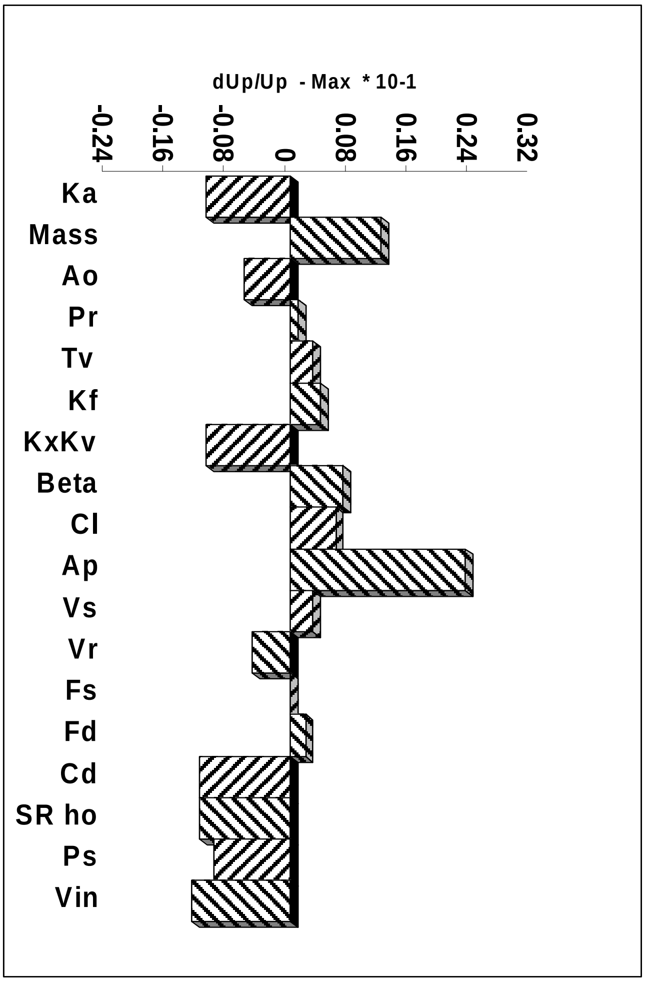


Figure 3. Vilenius Method Histograms

below:

$$\frac{\lambda_4^1(t)\delta\alpha_1}{x_{4s}}\Big|_{max} = \frac{\left(\frac{\partial u_p(t)}{\partial K_a}\right)\delta K_a}{U_{ps}}\Big|_{max}$$

The value of $\delta\alpha_j$ is $0.01\,\alpha_j$ for all the histograms.

The time when these maximum changes of amplitudes occur, are approximately equal to the peak-time (t_p = 0.024 s; where $u_p = U_{pmax}$). The difference between these times and peak time depends on the individual characteristics changes of state variable, especially rise time (T_r) and frequency (F_r) changes. The value of these maxima depends mostly on the steady state (X_{ss}) and the overshoot (P_o) changes.

As the histogram of figure.1 indicate, the most sensitive parameter is A_p and the insensitive parameter is F_s some other relativity sensitive parameters are V_{in}, C_d, $\sqrt{1/\rho}$, M, K_a, K_xK_v and P_s respectively. Two special parameters A_o and P_r which this sample system has been designed according to them, are not as sensitive as other parameters.

In the second method the maximum amplitude changes are considered with respect to the performance value, u_p, at the same instant of time instead of steady state value. For example, for the first parameter (K_a) this ratio is as bellow:

$$\frac{\lambda_4^1(t)\delta\alpha_1}{x_{4s}(t)}\Big|_{\max} = \frac{\left(\frac{\partial u_p(t)}{\partial K_a}\right)\delta K_a}{U_{ps}}\Big|_{\max}$$

But unlike the previous method which does not care for the value of the state variable in the time when maximum of $\lambda^j_i\delta\alpha_i$ occurs, this methods has a tendency to choose the maximum of $\delta\lambda^j_i\alpha_i^j$ where the state variable value is less than its steady state value. Hence the sign of the most sensitivity indices with respect to each parameter change are different in two histograms (figure.4). The same as the pervious method, A_p is the most sensitive parameter and F_s is the insensitive parameter. The difference between the two methods is that the absolute values of all sensitivity indices change a few percentages, except the absolute value of $\partial u_p/\partial M$ index which increases about 33%. The difference between this parameter (mass) and others is that according to the next method (individual characteristics) mass is the only parameter which is sensitive according to all individual characteristics and absolutely insensitive according to the steady state value.

The third method is the most detailed method considers different characteristics individually. Using the analytical method, the sensitivity of the five different characteristics (x_{ss}, T_r, P_o, F_r, D_c) of performance, u_p are evaluated. As it can be see from the histogram (Figure. 5), each parameter has different effect on various characteristics. It is difficult to recognize this difference by using other methods. General survey of all the five histograms show that, as usual, F_s is insensitive V_{in} and A_p is one of the most sensitive parameter, according to all performance characteristics, although in the two pervious methods V_{in} was not as sensitive as A_p. Sensitivity of the parameter in some performance characteristics such as the frequency surpasses the sensitivity of other parameters even parameter A_p. This means that the sensitivity of the parameter V_{in} according to different performance characteristics somewhat nullify each other. Another interesting point is that in different histogram the order of the sensitivity of the parameter are different. For example in the first histogram (X_{ss}), V_{in} C_d and $\sqrt{1/\rho}$ are the most sensitive parameters and F_s, V_r, V_s, β, τ_v, and mass are absolutely insensitive parameters, although in second histogram (T_r), mass is one of most sensitive parameters (third one) and C_d and $\sqrt{1/\rho}$ almost insensitive parameters. Entirety-Index Method gives an index ($I=\int_0^T \left|\lambda_4^{j(t)}\right|\delta\alpha_j dt$) which is good combination of all the performance characteristics sensitivity with respect to each parameter. So it makes very easy to locate to an insensitive system. Unlike other methods, in this method we have only the absolute value of the sensitivity and it does not show that this sensitivity whether improves or deteriorates the performance.

As it can be seen in histogram (Figure. 6) parameters V_{in}, C_d, $\sqrt{1/\rho}$, A_p, K_a, and K_xK_v which cause the most sensitivity on all the performance characteristics also are most sensitive parameters in this method and F_s which was insensitive parameter in all previous methods according to all performance characteristics, also is insensitive in this method. Other parameters have also a sensitivity which is combination of all different performance characteristics sensitivity with respect to that parameter.

8. CONCLUSION

A survey of all method shows that sensitivity parameters K_a and K_xK_v on the one hand and also parameters C_d and $\sqrt{1/\rho}$ on the other are the same according to all the methods and performance characteristics. This is because they appear in all the expression of simulation in the same position. Therefore it is possible to consider $K_a\times K_xK_v$ and $C_d\times\sqrt{1/\rho}$ as combination parameters.

As it was mentioned the Entirety-Index is the best method, among four mentioned methods, for general optimization methods which consider all the performance characteristics together. Hence for this method all the four state variable are considered (Figure. 7). The effect of the parameters on sensitivity of the state variables a_v and u_p are the same except that A_p and C_d (also $\sqrt{1/\rho}$) are among the most sensitive

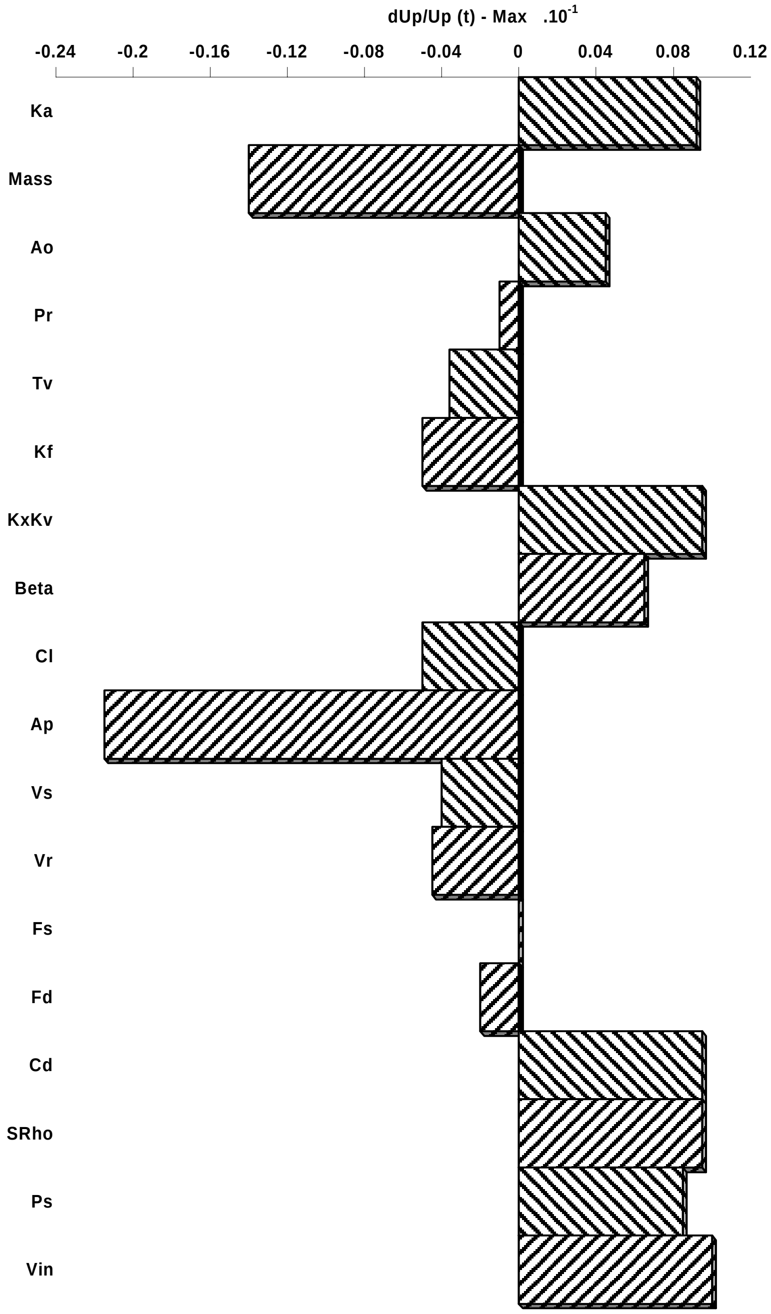


Figure 4. Revised Vilenius Method Histograms

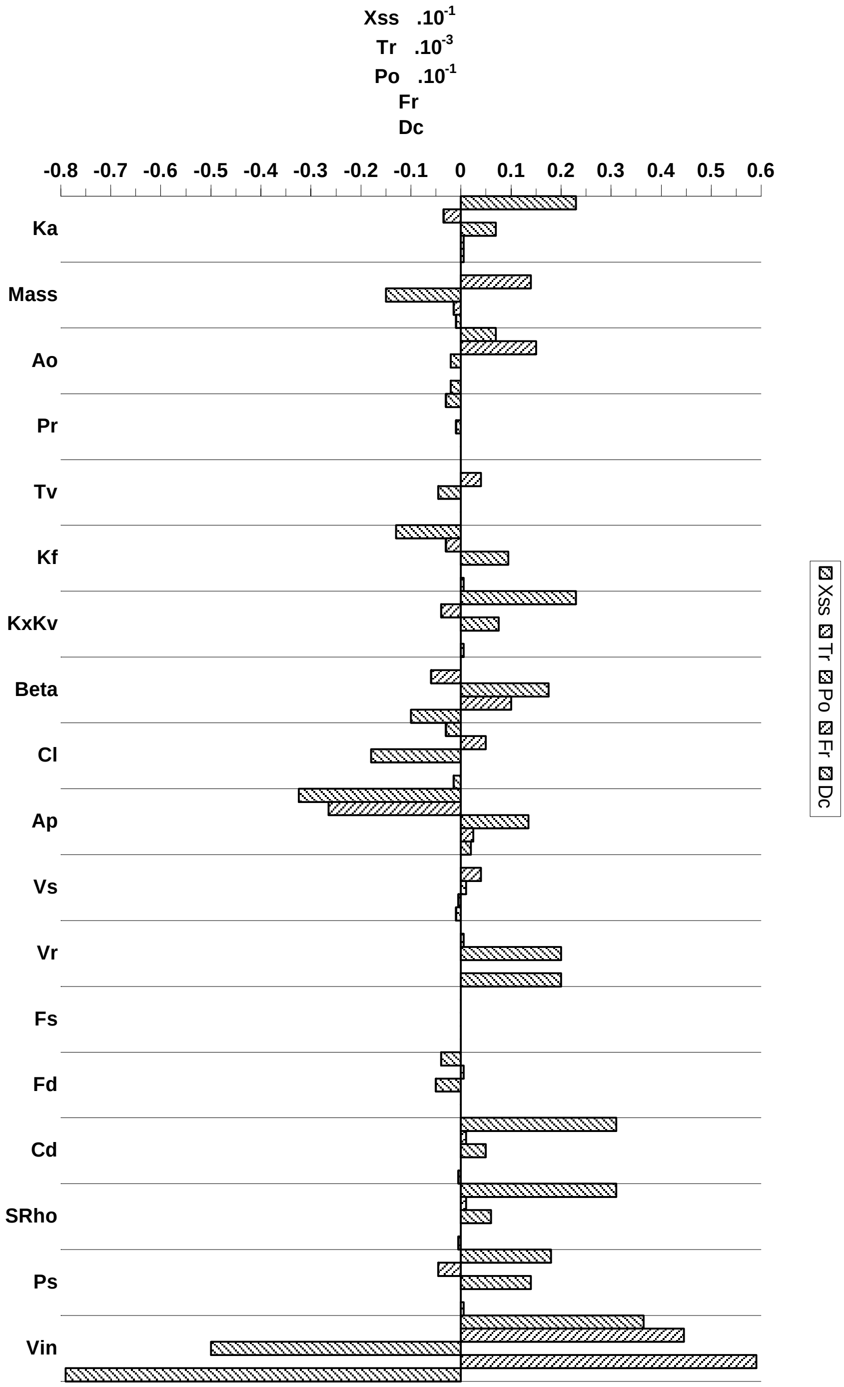


Figure 5. Individual Characteristic Method Histograms.

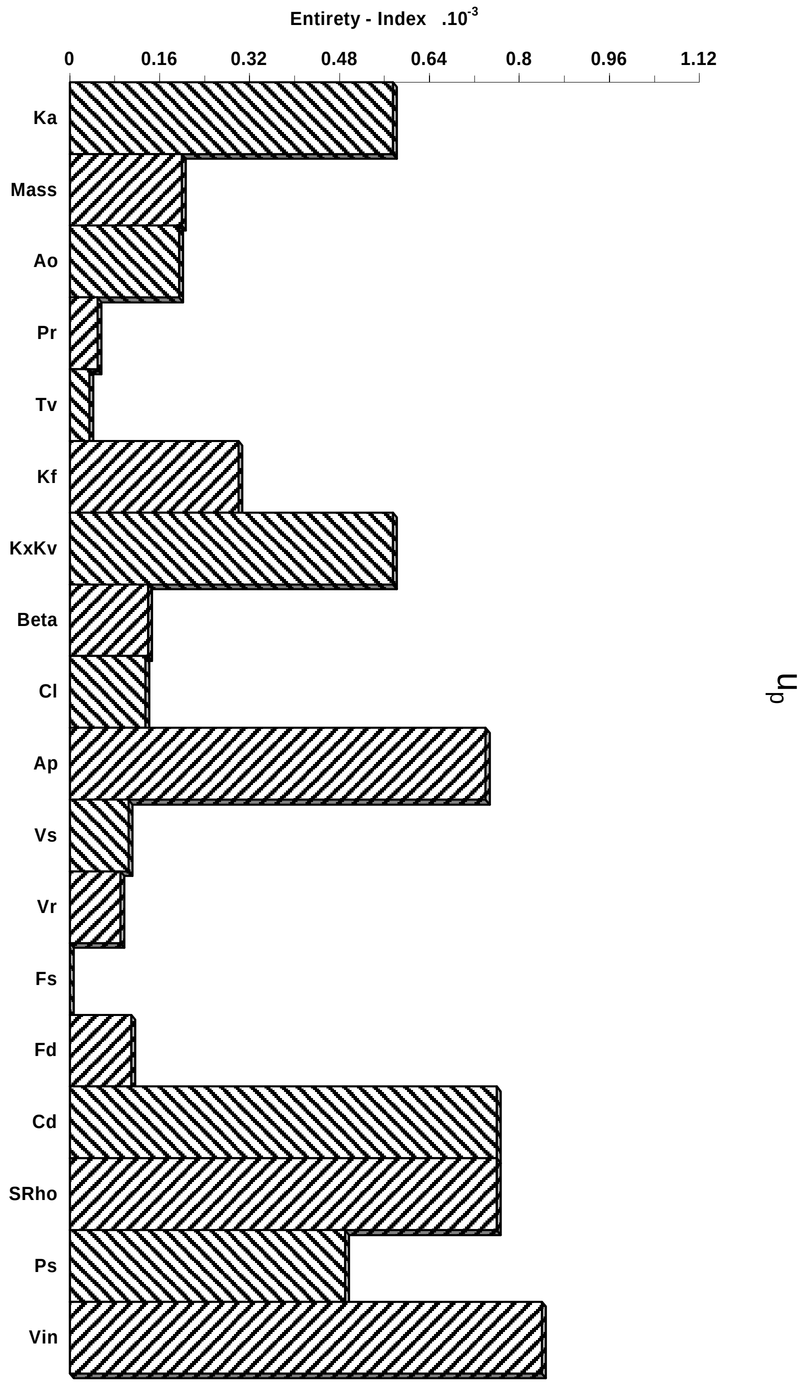


Figure 6. Entierty-Index Method Histogram u_p

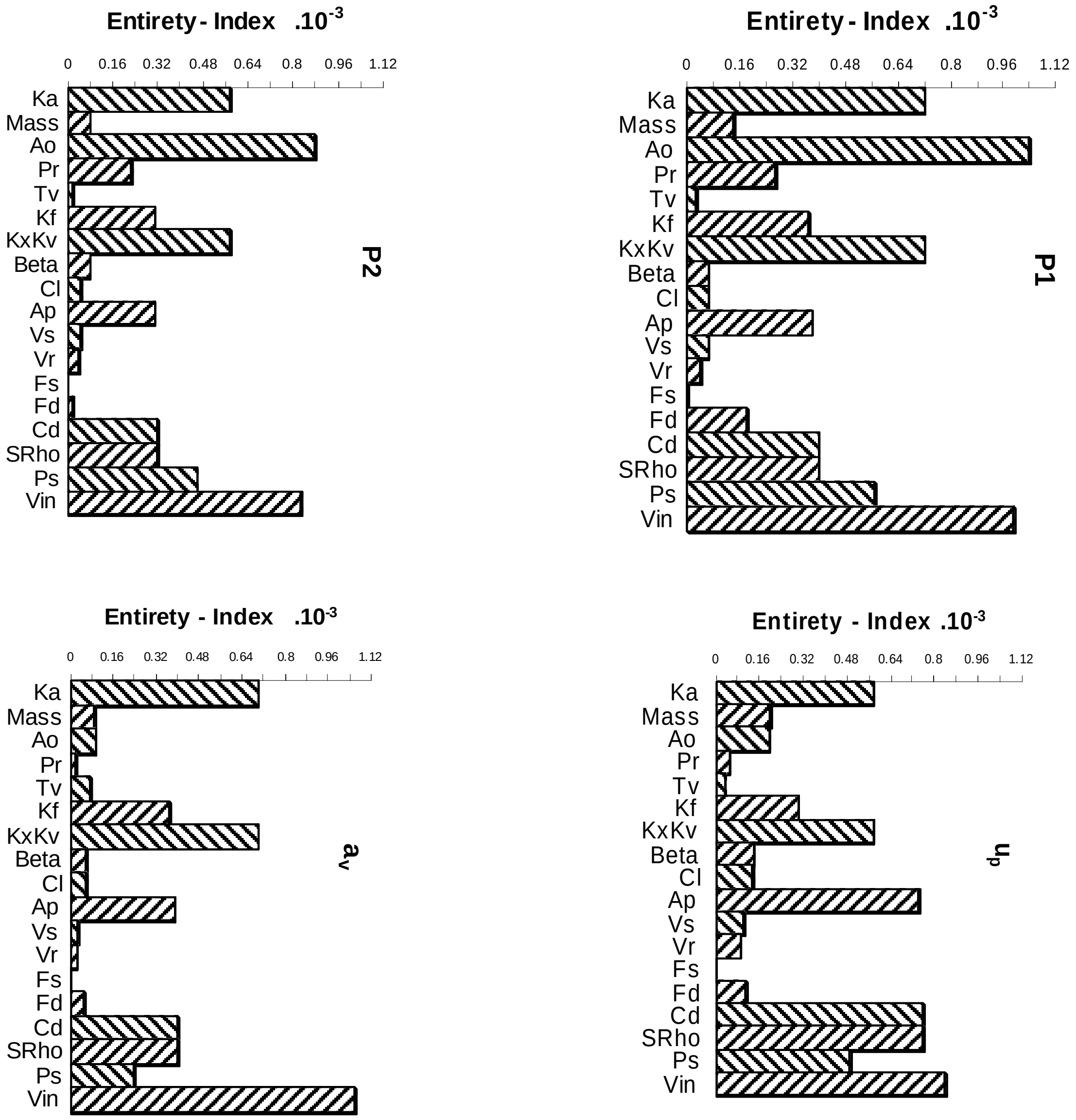


Figure 7. Entierty-Index Method Histogram p_1, p_2, a_v, u_p

parameters of state variable u_p, but they are among the least sensitive parameters of state variable a_v.

Although in actuator pressures ($p_1$and p_2), the sensitivity of parameters A_p and C_d are low too, parameter A_o which was one of the least sensitive parameter (w.r.t u_p and a_v) is the most sensitive parameter (w.r.t p_1 and p_2) even more sensitive than parameter V_{in}. Also p_1 about 20% more sensitive than p_2 to all the parameters except for two. The first one is the parameter β (bulk modulus of the oil) to which both pressure have the same sensitivity, and the second one is the parameter F_d (dynamic friction) to which pressure p_1 is ten times more sensitive than pressure p_2.

9. SYSTEM PARAMETERS AND THEIR VALUES

A_o (orifice opernning of metering valve) =2.066E-06(m^2).

A_p (actutor piston area) =6.340E- 04(m^2).

a_v (servovalve opening)

C_d (flow discharge coefficient) =6.000E-01

F_d (dynamic coulomb friction) =1.740E+02(N).

F_s (static coulomb friction) =4.500E+02(N).

K_a (servovalve amplifier gain) =1.000E+01(mA/V).

K_f (velocity feedback gain) =9.000E+00(V/m/s).

K_x (servovalve torque motor constant (m/mA)).

K_v (servovalve area constant (m^2/m)).

K_xK_v =8.325E-08(m^2/mA).

M (total mass in motion) =8.400E+01(Kg).

P_s (supply pressure) =4.826E+06(N/m^2).

P_r (relief valve setting pressure) =4.826E+05(N/m^2).

t_d (delay of direction control valve) =8.000E-03(s).

T (period of square wave input signal) =4.000E-01(s).

v_s (supply oil volume)

v_r (return oil volume)

V_0 (piston referance velocity) =1.000E-04(m/s).

V_{in} (input signal voltag) =2.150E+00(Volt).

β (bulk moodulus of fluid) =7.995E+09(N/m^2).

ρ (density of the oil) =8.580E+02(Kg/m^3).

τ_v (servovalve time constant) =4.000E-03(s).

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