N-POLICY FOR M/G/1 MACHINE REPAIR MODEL WITH MIXED STANDBY COMPONENTS, DEGRADED FAILURE AND BERNOULLI FEEDBACK

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Abstract In this paper, we study N-policy for a finite population Bernoulli feedback queuing model for machine repair problem with degraded failure. The running times of the machines between breakdowns have an exponential distribution. The repair times of the machines are independent and identically distributed random variables. If at any time a machine fails, it is sent to the repairman for repairing, the repairman restores the machine to the state as before failure. When the failed machine finds the repairman busy upon its failure, it has to wait until its turn as repairman stores only one machine at a time. When all the standby components are used, the failure of components occurs in a degraded fashion. To obtain the steady-state probabilities, the supplementary variable is introduced and a recursive method is employed. Some performance measures viz. expected number of down machines, expected number of machines waiting for repair in the queue, expected number of operating machines, expected number of spare machines, machine availability, etc. are established. Some special cases are deduced that match with the earlier existing results. To provide sensitivity analysis, numerical experiment is performed.

Key Words M/G/1 Queue, N-Policy, Machine Repair, Mixed Standby, Degraded Failure, Bernoulli Feedback

چکیده در این مقاله، مساله تعمیر ماشینی که خرابی آن در حال نقصان است توسط سیستم صف با جمعیت محدود دارای برگشت با تابع برنولی، مورد بررسی قرار می‌گیرد. زمان کارکرد ماشین در فاصله بین دو خرابی دارای توزیع نمایی است. زمان های تعمیر ماشین، متغیرهای تصادفی مستقل با توزیع یک شکل هستند. هر زمانی که ماشین خراب می‌شود، برای تعمیر فرستاده شده و تعمیر کار آن را تعمیر نموده و به صورت قبل از خرابی در می‌آورد. چون تعداد تعمیر کار یک نفر است، اگر هنگام فرستادن ماشین، تعمیر کار مشغول باشد، اجباراً باید صبر کرد تا نوبت تعمیر فرا رسد. برای به دست آوردن احتمالات در حالت پایدار، متغیر های کمکی تعریف می‌شوند و از روش برگشتی استفاده می‌شود. چند معیار عملکرد مثل تعداد ماشین های خراب شده در حال انتظار، میانگین تعداد ماشین هایی که برای تعمیر در صف انتظار هستند، میانگین تعداد ماشین‌های در حال کار، دسترس پذیری ماشین ها و ... محاسبه می شوند. چند حالت خاص که با محاسبات مَوجود و انجام شده در تطابق است، استخراج می‌شود. برای تحلیل حساسیت، محاسبات عددی نیز انجام شده است.

1. INTRODUCTION

In the production units, one of the commonly used objective functions is to minimize the expected total cost while maintaining the production up to a desired level. The other motto is to finish the repair of machines as early as possible so as to avoid the loss of production and increase the utilization of the resources. In this paper, we study M/G/1 machine repair model consisting of M operating machines along with S cold standby and Y warm standby components. Whenever any operating machine fails, it is replaced by cold standby if available. When all the cold standbys are exhausted, then warm standbys are used to replace failed machines. There is provision of single repairman who turns on; when N failed machines are accumulated, to renew these machines in order of their breakdowns.

The supplementary variable technique to analyze M/G/1 queue was applied by Keilson and Kooharian [1], Takacs [2], Hokstad [3]. The characterization and computation of optimal policies for operating an M/G/1 queuing system with removable server was also given by Bell [4]. Courtois and Georges [5] discussed a single-server finite queuing model with state-dependent arrival and service processes. Herzog et al. [6] gave the solution of M/G/1 queuing problem by a recursive technique. Gupta and Srinivasa Rao [7] discussed a recursive method for M/G/1/K model to compute the steady-state probabilities. Zhang and Love [8] suggested the threshold policy for the M/G/1 queue with an exceptional first vacation. For optimal control of a removable and non-reliable server, Wang et al. [9] developed $M/H_2/1$ queuing system. Lillo and Martin [10] developed optimal exhaustive policies for M/G/1 queue. Wang and Ke [11] suggested a recursive method for optimal control of an M/G/1 queuing system with finite capacity and infinite capacity.

The concept of N-Policy was introduced by Yadin and Naor [12]. According to this threshold policy, the repairman turns on when N failed machines are accumulated in the system and turns off if no failed machine is present there. $M/E_k/1$ queuing system, which is optimally controlled by removable service station, was suggested by Wang and Huang [13]. Recently, Jain [14] discussed N-policy for redundant repairable system with additional repairmen. Jain et al. [15] also studied degraded machining system with spares and server breakdowns by using N-policy.

A single-server queue with feedback was initially suggested by Takacs [16]. Optimal operating policies for M/G/1 queuing system were considered by Heyman [17]. Disney et al. [18] studied M/G/1 queue with instantaneous Bernoulli feedback. The sojourn time in M/G/1 queue with Bernoulli feedback was provided by Disney [19]. Fontana and Berzosa [20] gave stationary queue-length distribution in an M/G/1 queue with two non-preemptive priorities and general feedback. Simon [21] studied priority queues with feedback. Takine et al. [22] considered sojourn times in vacation and polling systems with Bernoulli feedback. Gong et al. [23] suggested M/G/1 queue with queue-length dependent arrival rate. The response time in M/G/1 queue with service in random order and Bernoulli feedback was investigated by Takagi [24]. Boxma and Yechiali [25] considered an M/G/1 queue with multiple types of feedback and gated vacations. Medhi [26] discussed the response time for M/G/1 queuing system with Bernoulli feedback.

In previous studies, mixed standbys have been considered only for simple Markovian model (cf. Wang and Kuo [27]). In this paper, we introduce the concept of N-policy and Bernoulli feedback for degraded machine repair problem with the provision of mixed standbys. The concept of degraded components included in investigation makes our model closer to real time machining system in different frame-work of production, manufacturing, computer and communication system, etc. The rest of the paper is organized as follows: The model description along with underlying assumptions, notations and applications are described in section 2. In section 3, governing equations and their analysis by using recursive method is provided. Several performance measures are obtained in section 4. In section 5, we discuss some special cases. Numerical results to validate the analytical results are provided in section 6. In final section 7, the concluding remarks and future direction for research are given.

2. MODEL DESCRIPTION AND APPLICATION

The reliability and availability ratings of machining system with degraded components have necessitated the provision of mixed standbys due to cost and space constraints. In the operation of manufacturing, the failed components may return to get repair again because in some cases, the repair is not done successfully first time. Bernoulli feedback model based on queue theoretic approach may give proper insight to system manager to deal with such situation. In computer and communication system also, an interruption due to degradation in the components can be realized. The performance modeling of such systems based on optimal control policy pertaining to mixed standbys, threshold level and repair facility can play important role for the suitable choice of system parameter. The proper combination of warm and cold spares may be used to improve the productivity of the manufacturing industry.

We consider M/G/1 degraded machine repair model having M operating machines along with S cold standby and Y warm standby components. A supplementary variable technique is developed to analyze the system. For modeling purpose, following assumptions are made:

In case of failed operating machine, it is replaced by a cold standby component when there are less than S failed machines.

The life times of operating and standby components are exponentially distributed with rates λ and α respectively.

The failed machine is sent to the repairman for repair. If repairman is busy, it has to wait in the queue until the repair of previously failed machines is completed.

The repair times of the failed components are independent and identically distributed. For repair FIFO discipline is followed.

The switchover times from failure state to repair state, from repair state to standby state and from standby state to operating state are assumed to be negligible.

The repairman turns on when there are N failed components and turns off as soon as the system becomes empty.

When all standby (warm and cold) components are used, then due to stress, the components may fail in degraded manner with failure rate, λ_{n-S-Y}.

When the repair of the machine is unsatisfactory, it is again sent back to repair with probability (1-σ) so that Bernoulli feedback mechanism is considered.

After the satisfactory repair, the machine again becomes as good as new one and joins the set of operating (standby) machines if all standbys are exhausted (not exhausted).

The following notations are also used to develop the mathematical model:

$P_{0,0}(t)$	Probability that there is no failed machine in the system at time t.
$P_{0,n}(t)$	Probability that there are n failed machines in the system at time t when repairman is in turned off state.
$P_{1,n}(u,t)$	Probability of being n failed machines in the system at time t when repairman is turned on and working.
U(t)	Remaining service time for the machine under repair at time t.
H	Random variable denoting the service time for a super customer in Bernoulli feedback.
H(u)	Distribution function of H.
h(u)	Probability density function of H.
$H^*(\theta)$	Laplace Stieltjes transform of H.
$H^{*(1)}(\theta)$	1st order derivative of H with respect to θ.
B	Random variable denoting service time.
B(u)	Distribution function of B.
b(u)	Probability density function of B.
b_1	Mean repair time.
$B^*(\theta)$	Laplace Stieltjes transform of B.
$B^{*(1)}(\theta)$	1st order derivative of B^* with respect to θ.
$P^*_{1,n}(\theta)$	Laplace Stieltjes transform of $P_{1,n}(u)$.

In the next section, we formulate the mathematical model to obtain the steady-state probabilities that can be further employed to predict the various performance metrics. The modeling and analysis of the M/G/1 machining system under N-policy and Bernoulli feedback may be helpful to uncover the control issues so as to improve the grade of service (GoS) to desired extent subject to economic constraints.

3. EQUATIONS AND ANALYSIS

Constructing governing equations as follow does the system modeling:

$$\frac{\partial}{\partial t}P_{0,0}(t) = -M\lambda P_{0,0}(t) + P_{1,1}(0,t) \tag{1}$$

$$\frac{\partial}{\partial t}P_{0,n}(t) = -M\lambda P_{0,n}(t) + M\lambda P_{0,n-1}(t), \quad 1 \le n \le N\text{-}1 \tag{2}$$

$$\left(\frac{\partial}{\partial t} - \frac{\partial}{\partial u}\right)P_{1,1}(u,t) = -M\lambda P_{1,1}(u,t) + P_{1,2}(0,t)h(u) \tag{3}$$

$$\left(\frac{\partial}{\partial t}-\frac{\partial}{\partial u}\right)P_{1,n}(u,t)=-M\lambda P_{1,n}(u,t)+M\lambda P_{1,n-1}(u,t)$$
$$+P_{1,n+1}(0,t)h(u)$$

$1\leq n\leq N\text{-}1$ (4)

$$\left(\frac{\partial}{\partial t}-\frac{\partial}{\partial u}\right)P_{1,N}(u,t)=-M\lambda P_{1,N}(u,t)+M\lambda P_{1,N-1}(u,t)$$
$$+M\lambda P_{0,N-1}(u,t)+P_{1,N+1}(0,t)h(u) \quad (5)$$

$$\left(\frac{\partial}{\partial t}-\frac{\partial}{\partial u}\right)P_{1,n}(u,t)=-M\lambda P_{1,n}(u,t)+M\lambda P_{1,n-1}(u,t)$$
$$+P_{1,n+1}(0,t)h(u)$$

$N<n<S$ (6)

$$\left(\frac{\partial}{\partial t}-\frac{\partial}{\partial u}\right)P_{1,S}(u,t)=-(M\lambda+Y\alpha)P_{1,S}(u,t)+ \quad (7)$$
$$M\lambda P_{1,S-1}(u,t)+P_{1,S+1}(0,t)h(u)$$

$$\left(\frac{\partial}{\partial t}-\frac{\partial}{\partial u}\right)P_{1,n}(u,t)=-[M\lambda+(S+Y-n)\alpha]P_{1,n}(u,t)$$
$$+[M\lambda+(S+Y-n+1)\alpha]P_{1,n-1}(u,t)+P_{1,n+1}(0,t)h(u)$$

$S<n<S+Y$ (8)

$$\left(\frac{\partial}{\partial t}-\frac{\partial}{\partial u}\right)P_{1,n}(u,t)=-(K-n)\lambda_{n-S-Y}P_{1,n}(u,t)$$
$$+(K-n+1)\lambda_{n-1-S-Y}P_{1,n-1}(u,t)+P_{1,n+1}(0,t)h(u)$$

$S+Y\leq n<K\ (=M+S+Y)$ (9)

$$\left(\frac{\partial}{\partial t}-\frac{\partial}{\partial u}\right)P_{1,K}(u,t)=\lambda_{M-1}P_{1,K-1}(u,t) \quad (10)$$

For steady-state we define

$P_{0,n}=\lim_{t\to\infty}P_{0,n}(t)$, $n = 0, 1, 2, \ldots, N\text{-}1$

$P_{1,n}=\lim_{t\to\infty}P_{1,n}(t)$, $n = 1, 2, \ldots, K$

$P_{1,n}(u)=\lim_{t\to\infty}P_{1,n}(u,t)$, $n = 1, 2, \ldots, K$

Further define

$$P_{0,N-1}(u)=\lim_{t\to\infty}P_{0,N-1}h(u) \quad , \quad (11)$$

The steady-state equations are given as

$$0=-M\lambda P_{0,0}+P_{1,1}(0) \quad (12)$$

$$0=-M\lambda P_{0,0}+P_{1,1}(0) \ , \quad 1\leq n\leq N\text{-}1 \quad (13)$$

$$-\frac{d}{du}P_{1.1}(u)=-M\lambda P_{1,1}(u)+P_{1,2}(0)h(u) \quad (14)$$

$$-\frac{d}{du}P_{1.n}(u)=-M\lambda P_{1,n}(u)+M\lambda P_{1,n-1}(u)+P_{1,n+1}(0)h(u)$$

$2\leq n\leq N\text{-}1$ (15)

$$-\frac{d}{du}P_{1.N}(u)=-M\lambda P_{1,N}(u)+M\lambda P_{1,N-1}(u)+M\lambda P_{0,N-1}(u)$$
$$+P_{1,N+1}(0)h(u) \quad (16)$$

$$-\frac{d}{du}P_{1.n}(u)=-M\lambda P_{1,n}(u)+M\lambda P_{1,n-1}(u)+P_{1,n+1}(0)h(u)$$

$N<n<S$ (17)

$$-\frac{d}{du}P_{1.S}(u)=-(M\lambda+Y\alpha)P_{1,n}(u)+M\lambda P_{1,S-1}(u)+$$
$$P_{1,S+1}(0)h(u) \quad (18)$$

$$-\frac{d}{du}P_{1.n}(u)=-[M\lambda+(S+Y-n)\alpha]P_{1,n}(u)+$$
$$[M\lambda+(S+Y-n+1)\alpha]P_{1,n-1}(u)+P_{1,n+1}(0)h(u)$$

$S<n<S+Y$ (19)

$$-\frac{d}{du}P_{1.n}(u)=-(K-n)\lambda_{n-S-Y}P_{1,n}(u)+$$
$$(K-n+1)\lambda_{n-1-S-Y}P_{1,n-1}(u)+P_{1,n+1}(0)h(u)$$

$S+Y\leq n<K$ (20)

$$-\frac{d}{du}P_{1,K}(u)=\lambda_{M-1}P_{1,K-1}(u) \quad (21)$$

Using Equations 12 and 13, we have

$$P_{1,1}(0)=M\lambda P_{0,n} \qquad 0\le n\le N\text{-}1 \quad (22)$$

Further define

$$H^*(\theta)=\int_0^\infty e^{-\theta u}dH(u)=\int_0^\infty e^{-\theta u}h(u)du$$

and $P_{1,n}^*(\theta)=\int_0^\infty e^{-\theta u}P_{1,n}(u)$

and $\int_0^\infty e^{-\theta u}\frac{\partial}{\partial u}P_{1,n}(u)du=\theta P_{1,n}^*(\theta)-P_{1,n}(0)$

Taking Laplace transform of Equations 14-21 and applying above definitions, it is found that

$$(M\lambda-\theta)P_{1,1}^*(\theta)=P_{1,2}(0)H^*(\theta)-P_{1,1}(0) \quad (23)$$

$$(M\lambda-\theta)P_{1,n}^*(\theta)=M\lambda P_{1,n-1}^*(\theta)+P_{1,n+1}(0)H^*(\theta)-P_{1,n}(0)$$

$$2\le n\le N\text{-}1 \quad (24)$$

$$(M\lambda-\theta)P_{1,N}^*(\theta)=M\lambda P_{1,N-1}^*(\theta)+P_{1,N+1}(0)H^*(\theta)+ M\lambda P_{0,N-1}(0)H^*(\theta)-P_{1,N}(0) \quad (25)$$

$$(M\lambda-\theta)P_{1,n}^*(\theta)=M\lambda P_{1,n-1}^*(\theta)+P_{1,n+1}(0)H^*(\theta)-P_{1,n}(0)$$

$$N<n<S \quad (26)$$

$$[(M\lambda+Y\alpha)-\theta]P_{1,S}^*(\theta)=M\lambda P_{1,S-1}^*(\theta)+P_{1,S+1}(0)H^*(\theta)-P_{1,S}(0) \quad (27)$$

$$[M\lambda+(S+Y-n)\alpha-\theta]P_{1,n}^*(\theta)=[M\lambda+(S+Y-n+1)\alpha]P_{1,n-1}^*(\theta)+P_{1,n+1}(0)H^*(\theta)-P_{1,n}(0)$$

$$S<n<S+Y \quad (28)$$

$$[(K-n)\lambda_{n-S-Y}-\theta]P_{1,n}^*(\theta)=(K-n+1)\lambda_{n-S-Y}P_{1,n-1}^*(\theta)+P_{1,n+1}(0)H^*(\theta)-P_{1,n}(0)$$

$$S+Y\le n<K \quad (29)$$

$$-\theta P_{1,K}^*(\theta)=\lambda_{M-1}P_{1,K-1}^*(\theta)-P_{1,K}(0) \quad (30)$$

Now Equation 25 becomes

$$(M\lambda-\theta)P_{1,N}^*=M\lambda P_{1,N-1}^*(\theta)+P_{1,N+1}(0)H^*(\theta)+P_{1,1}(0)H^*(\theta)-P_{1,N}(0) \quad (31)$$

Adding Equations 23-26, we have

$$\sum_{n=1}^{S-1}P_{1,n}^*(\theta)=\left\{\frac{1-H^*(\theta)}{\theta}\right\}\sum_{n=1}^{S-1}P_{1,n}(0)+\frac{H^*(\theta)}{\theta}P_{1,S}(0) \quad (32)$$

Taking limit $\theta\to 0$, Equation 32 yields

$$\mathrm{Lim}_{\theta\to 0}\sum_{n=1}^{S-1}P_{1,n}^*(\theta)=\mathrm{Lim}_{\theta\to 0}\left[\left\{\frac{1-H^*(\theta)}{\theta}\right\}\sum_{n=1}^{S-1}P_{1,n}(0)+\frac{H^*(\theta)}{\theta}P_{1,S}(0)\right]$$

or $$\sum_{n=1}^{S-1}P_{1,n}^*(0)=\mathrm{Lim}_{\theta\to 0}-H^{*(1)}(\theta)\sum_{n=1}^{S-1}P_{1,n}(0) \quad (33)$$

We know that

$$H^*(\theta)=\frac{\sigma B^*(\theta)}{1-(1-\sigma)B^*(\theta)}$$

so that $$H^{*(1)}(\theta)=\frac{\sigma B^{*(1)}(\theta)}{\{1-(1-\sigma)B^*(\theta)\}^2}$$

and $$H^{*(1)}(0)=\frac{\sigma B^{*(1)}(0)}{\{1-(1-\sigma)\}^2}=\frac{B^{*(1)}(0)}{\sigma}$$

Now from Equation 33, we get

$$\sum_{n=1}^{S-1}P_{1,n}^*(0)=-\frac{B^{*(1)}(0)}{\sigma}\sum_{n=1}^{S-1}P_{1,n}(0)=\frac{b_1}{\sigma}\sum_{n=1}^{S-1}P_{1,n}(0) \quad (34)$$

where $b_1 = -B^{*(1)}(0)$.

Equations 27 and 28 give:

$$P^*_{1,n}(\theta)=\frac{\left[M\lambda+(S+Y-n+1)\alpha\right]P^*_{1,n-1}(\theta)+P_{1,n+1}(0)H^*(\theta)-P_{1,n}(0)}{\left[M\lambda+(S+Y-n)\alpha\right]}$$

$S\le n<S+Y$ (35)

Also from Equations 29 and 30, we obtain

$$P^*_{1,n}(\theta)=\frac{(K-n+1)\lambda_{n-1-S-Y}P^*_{1,n-1}(\theta)+P_{1,n+1}(0)H^*(\theta)-P_{1,n}(0)}{(K-n)\lambda_{n-S-Y}}$$

$S+Y\le n\le K$ (36)

Now using $\theta=M\lambda$ in Equation 23, we have

$$P_{1,2}(0)=\frac{P_{1,1}(0)}{H^*(M\lambda)}$$

Setting $\theta=M\lambda$ in Equations 24, 25 and 26, we find

$$P_{1,n+1}(0)=\frac{P_{1,n}(0)-M\lambda P^*_{1,n-1}(M\lambda)}{H^*(M\lambda)}$$

$1\le n<N$ (37)

$$P_{1,N+1}(0)=\frac{P_{1,N}(0)-M\lambda P^*_{1,N-1}H^*(M\lambda)-P_{1,1}(0)H^*(M\lambda)}{H^*(M\lambda)}$$

and $$P_{1,n+1}(0)=\frac{P_{1,n}(0)-M\lambda P^*_{1,n-1}(M\lambda)}{H^*(M\lambda)}$$

$N<n<S$ (38)

Putting $\theta=M\lambda+Y\alpha$ in (27), we have

$$P_{1,S+1}(0)=\frac{P_{1,S}(0)-[M\lambda+(Y-1)\alpha]P^*_{1,S-1}(M\lambda)}{H^*(M\lambda+Y\alpha)}$$

On substituting $\theta=M\lambda+(S+Y-n)\alpha$ in Equation 28, we get

$$P_{1,n+1}(0)=\frac{P_{1,n}(0)-[M\lambda+(S+Y-n+1)\alpha]P^*_{1,n-1}[M\lambda+(S+Y-n)\alpha]}{H^*[M\lambda+(S+Y-n)\alpha]}$$

$S<n<S+Y$ (39)

Now putting $\theta=(K-n)\lambda_{n-S-Y}$ in Equation 29, we obtain

$$P_{1,n+1}(0)=\frac{P_{1,n}(0)-(K-n+1)\lambda_{n-1-S-Y}P^*_{1,K-1}[(K-n)\lambda_{n-S-Y}]}{H^*[(K-n)\lambda_{n-S-Y}]}$$

$S+Y<n<K$ (40)

On setting $\theta=0$ in (23), we have

$$P^*_{1,1}(0)=\frac{P_{1,2}(0)H^*(0)-P_{1,1}(0)}{M\lambda}=\frac{P_{1,2}(0)-P_{1,1}(0)}{M\lambda}\quad(41)$$

Similarly we obtain

$$P^*_{1,2}(0)=\frac{P_{1,3}(0)H^*(0)-P_{1,1}(0)}{M\lambda}=\frac{P_{1,3}(0)-P_{1,1}(0)}{M\lambda}$$

Now substituting $\theta=0$ in (24)-(30), we find

$$P^*_{1,n}(0)=\frac{M\lambda P^*_{1,n-1}(0)+P_{1,n+1}(0)-P_{1,n}(0)}{M\lambda}$$

$1\le n<N$ (42)

$$P^*_{1,N}(0)=\frac{M\lambda P^*_{1,N-1}(0)+P_{1,N+1}(0)H^*(0)-P_{1,N}(0)}{M\lambda}\quad(43)$$

$$P^*_{1,n}(0)=\frac{M\lambda P^*_{1,n-1}(0)+P_{1,n+1}(0)-P_{1,n}(0)}{M\lambda}$$

$N\le n<S$…(44)

$$P^*_{1,n}(0)=\frac{\left[M\lambda+(S+Y-n+1)\alpha\right]P^*_{1,n-1}(0)+P_{1,n+1}(0)H^*(0)-P_{1,n}(0)}{\left[M\lambda+(S+Y-n)\alpha\right]}$$

$S\le n<S+Y$ (45)

$$P_{1,n}^*(0) = \frac{(K-n+1)\lambda_{n-1-S-Y}\, P_{1,n-1}^*(0) + P_{1,n+1}(0)H^*(0) - P_{1,n}(0)}{(K-n)\lambda_{n-S-Y}}$$

$S+Y \le n \le K$ (46)

The normalizing condition is used to determine the steady-state probabilities

$$\sum_{n=0}^{N-1} P_{0,n} + \sum_{n=1}^{K} P_{1,n}^*(0) = 1 \tag{47}$$

4. SOME PERFORMANCE MEASURES

The steady state probability $P_n^*(0)$, $0 \le n \le K\text{-}1$ can be obtained from $P_{1,n}(0)$, $1 \le n \le K$ and given by

$$P_n^*(0) = \frac{P_{n+1}(0)}{\sum_{n=1}^{K} P_{1,n}(0)} \qquad n = 0, 1, 2, \ldots, K\text{-}1 \tag{48}$$

Various measures characterizing the system performance can also be obtained by using queue size distribution which is given as below:

Probability of repairman being in idle state is given by

$$P(I) = \sum_{n=0}^{N-1} P_{0,n} = NP_{0.0} \tag{49}$$

The expected number of failed components in the system is obtained as

$$E(N) = \sum_{n=0}^{N-1} n\, P_{0,n}(0) + \sum_{n=1}^{K} n\, P_{1,n}^*(0) \tag{50}$$

The expected number of failed machines waiting for repair in the queue

$$E(N_q) = \sum_{n=0}^{N-1} (n-1)P_{0,n} + \sum_{n=1}^{K} (n-1)P_{1,n}^*(0) \tag{51}$$

The expected number of operating machines in the system is

$$E(O) = M - \sum_{n=S+Y}^{K} (n-S-Y)\, P_{1,n}^*(0) \tag{52}$$

The expected number of standby machines in the system

$$E(S) = \sum_{n=0}^{N-1} (S+Y-n)\, P_{0,n} + \sum_{n=N}^{S+Y} (S+Y-n)\, P_{1,n}^*(0) \tag{53}$$

5. SPECIAL CASES

Case I: M/G/1 model; When $N \to 1$, S = 0, Y = 0, $H^*(\theta) \cong B^*(\theta)$, our model reduces to M/G/1 model with state-dependent arrival rates investigated by Gupta and Srinivasan Rao [28].

Case II: The M/G/1 model with cold standbys; If Y=0, $N \to 1$ and $H^*(\theta) \cong B^*(\theta)$ we get the results for M/G/1 machine interference model having only cold standby spares.

Case III: The M/G/1 model with N-Policy; Setting S = 0, Y = 0 and $H^*(\theta) \cong B^*(\theta)$, our results match with N-Policy M/G/1 model with finite capacity developed by Wang and Ke [11].

6. NUMERICAL ILLUSTRATION

In this section, numerical experiment is performed to validate the analytical result by using Matlab software. For illustration purpose, we consider the exponential service time distribution so that $H^*(\theta) = \sigma\mu / \theta + \sigma\mu$. Also we fix $\sigma = .5$. Table 1 displays the effect of number of operating machines (M), warm standby components (Y) and cold standby components (S) on the expected number of failed components E(N), expected number of failed machines in the queue $E(N_q)$ and probability of repairman being in idle state P(I), simultaneously. We note that if the number of operating machines (M) increases while Y and S are constant, the value of E(N) and $E(N_q)$ increase while P(I) decreases. In the other situation wherein Y varies while M and S are constant, we observe

TABLE 1. Performance Indices by Varying (M,Y,S) ($N = 3$, $\lambda = 0.06$, $\alpha = 0.04$, $\mu = 2$).

(M,Y,S)	E (N)	E (N_q)	P (I)
(10,5,3)	1.57	1.20	0.630449
(12,5,3)	1.82	1.39	0.569960
(14,5,3)	10.63	9.90	0.271430
(16,5,3)	22.45	21.45	0.002391
(18,5,3)	24.75	23.75	0.000006
(20,5,3)	26.90	25.90	0.000000
(15,5,5)	13.43	12.66	0.228898
(15,6,5)	10.44	9.73	0.293817
(15,7,5)	7.85	7.19	0.339551
(15,8,5)	5.96	5.33	0.364287
(15,9,5)	4.72	4.09	0.372640
(15,10,5)	3.96	3.33	0.370250
(15,9,5)	4.72	4.09	0.372640
(15,9,6)	3.11	2.51	0.399579
(15,9,7)	2.55	1.96	0.408548
(15,9,8)	2.37	1.78	0.411352
(15,9,9)	2.32	1.73	0.412212
(15,9,10)	2.30	1.71	0.412473

TABLE 2: Performance Indices by Varying (K) and (λ) ($M = 7$, $Y = 5$, $S = 2$, $N = 3$, $\alpha = 0.04$, $\mu = 2$).

K	λ	E(N_q)	P(I)
12	0.10	1.64	0.3939
	0.20	3.68	0.1040
	0.30	6.18	0.0166
	0.40	7.68	0.0025
	0.50	8.46	0.0004
14	0.10	2.87	0.2334
	0.20	9.16	0.0100
	0.30	10.82	0.0004
	0.40	11.28	0.0000
	0.50	11.51	0.0000

TABLE 3: Performance Indices by Varying (K) and (α) ($M = 7$, $Y = 5$, $S = 2$, $N = 3$, $\lambda = 0.1$, $\mu = 1$).

K	α	E (N_q)	P (I)
12	0.00	1.53	0.4738
	0.02	1.65	0.4156
	0.04	1.76	0.3643
	0.06	1.87	0.3191
	0.08	1.99	0.2794
14	0.00	3.12	0.2681
	0.02	3.34	0.2288
	0.04	3.55	0.1957
	0.06	3.74	0.1677
	0.08	3.92	0.1440

that as Y increases, the values of E(N) and E(N_q) decrease while P(I) increases. In another case in which S varies but M and S remain constant, we also see that E(N) and E(N_q) decrease while P(I) increases. Thus the provision of cold/warm standbys reduces the queue length as we expect.

The results for E(N_q) and P(I) are summarized in tables 2-5 with the variation in λ ,α, μ and K. We see that as K increases, E(N_q) increases while P(I) decreases, on the contrary, as μ increases, E(N_q) decreases while P(I) increases. Table 5 displays the effect of K and N on E(N_q) and P(I). We have noted that as K and N increase, the value of E(N_q) increases while P(I) decreases.

From sensitivity analysis performed, overall we conclude that

- As M increases, E(N) and E(N_q) increase while P(I) decreases. However when Y and S increase, E(N) and E(N_q) decrease while P(I) increases S.
- As K, λ and α increase, the value of E(N_q) increases while P(I) decreases.
- As μ increases, the value of E(N_q) decreases while P(I) increases.
- As N increases, the value of both E(N_q) and P(I) increase.

7. CONCLUSION

In this paper, we have derived steady-state results for a finite M/G/1 model operating under N-policy. For computing the system state probabilities, a recursive method is employed which can be easily implementable. The degraded failure machining system not only affects the production while in operation but also increases the production cost. The quantitative assessment suggested for such systems may provide an insight in saving the huge cost due to blocking, delay and down time. The provision of mixed standby components along with Bernoulli feedback and degraded failure considered is helpful in achieving the desired output at minimum cost expenditure. Performance measures viz. expected number of down /operating machines, expected number of spare components, machine availability etc. may play crucial role for determining the management strategies in many manufacturing environments encountered in computer, communication and production systems.

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TABLE 4. Performance Indices by Varying (K) and (μ) (M = 7, Y = 5, S = 2, N = 3, λ = 0.06, α = 0.04).

K	μ	$E(N_q)$	P(I)
22	1.0	20.28	0.0000
	2.0	19.42	0.0005
	3.0	7.44	0.3286
	4.0	1.31	0.6218
	5.0	1.14	0.6982
24	1.0	22.37	0.0000
	2.0	21.69	0.0000
	3.0	20.53	0.0088
	4.0	5.13	0.4624
	5.0	1.29	0.6585

TABLE 5. Performance Indices by Varying (K) and (N) (M = 15, Y = 15, S = 5, λ = 0.06, α = 0.04, N = 2).

K	N	$E(N_q)$	P(I)
30	4	2.08	0.4489
	6	3.02	0.4676
	8	3.98	0.4865
	10	4.96	0.5057
	12	5.96	0.5256
34	4	2.55	0.3395
	6	3.44	0.3568
	8	4.35	0.3740
	10	5.30	0.3910
	12	6.37	0.4066

9. REFERENCES

1. Keilson, J. and Kooharian, A., "On Time Dependent Queuing processes", ***Annals. of Mathematical Statistics***, Vol. 31, (1960), 104-112.
2. Takacs, L., "Delay Distributions for one Line with Poisson Input, General Holding Times and Various Orders of Service", ***Bell System Technical Journal***, Vol. 42, (1963a), 487-504.
3. Hokstad, P., "A Supplementary Variable Technique Applied to the M/G/1 Queue", ***Scandinavian Journal of Statistics***, Vol. 2, (1975), 95-98.
4. Bell, C. E., "Characterization and Computation of Optimal Policies for Operating an M/G/1 Queuing System with Removable Server", ***Oper. Res.***, Vol. 19, (1971), 208-218.
5. Courtois, P. J. and Georges, J., "On a single-server finite queuing model with state-dependent arrival and service processes" ***Oper. Res.***, Vol. 19, (1971), 424-435.
6. Herzog, U., Woo, L. and Chandy, K. M., "Solution of Queuing Problems by a Recursive Technique". ***IBM J. Res. Develop***, Vol. 19, (1975), 295-300.
7. Gupta, U. C. and Srinivasa Rao, T. S. S., "A recursive Method to Compute the Steady-State Probabilities of the Machine Interference Model" ***(M/G/1)/K. Computers and Operations Research***, Vol. 21, (1994), 597-605.
8. Zhang, Z. G. and Love, C. E., "The Threshold Policy in the M/G/1 Queue with an Exceptional First Vacation", ***INFOR.***, Vol. 36, No. 4, (1998), 193-204.
9. Wang, K. H., Chang, K. W. and Sivazlian, B. D., "Optimal Control of a Removable and Non-Reliable Server in an Infinite and a Finite $M/H_2/1$ Queuing System", ***Appl. Math. Model***, Vol. 23, (1999), 651-666.
10. Lillo, R. E. and Martin, M., "On Optimal Exhaustive Policies for the M/G/1 Queue". ***Oper. Res. Lett***., Vol. 27, (2000), 39-46.
11. Wang, K. H. and Ke, J. C., "A Recursive Method to the Optimal Control of an M/G/1 Queuing System with finite Capacity and Infinite Capacity", ***Appl. Math. Model***, Vol. 24, (2000), 899-914.
12. Yadin, M. and Naor, P., "Queuing Systems with Removable Service Stations", ***Oper. Res. Quart.***, Vol. 14, (1963), 393-405.
13. Wang, K. H. and Huang, H. M., "Optimal Control of an $M/E_k/1$ Queuing System with a Removable Service Station", ***OPSEARCH***, Vol. 46, (1995), 1014-1022.
14. Jain, M., "N-Policy for Redundant Repairable System with Additional Repairmen", ***OPSEARCH***, Vol. 40, No. 2, (2003), 97-114.
15. Jain, M., Sharma, G. C. and Singh, M., "N-Policy for Degraded Machining System with Spares and Server Breakdowns", ***IJOMAS***., Vol. 18/19 (In press), (2003).
16. Takacs, L., "A Single-Server Queue with Feedback", ***The Bell System Technical Journal***, Vol. 42, No. 2, (1963b), 505-519.
17. Heyman, D. P, "Optimal Operating Policies for M/G/1 Queuing system". ***Oper. Res***., Vol. 16, (1968), 362-382.
18. Disney, R. L., McNickle, D. C. and Simon, B., "The M/G/1 Queue with Instantaneous Bernoulli Feedback" ***Naval Res. Logist. Quart.***, Vol. 27, (1980), 635-644.
19. Disney, R. L., "A Note on Sojourn Times in M/G/1

Queues with Instantaneous, Bernoulli Feedback" ***Nav. Res. Logist. Quart.***, Vol. 27, (1981), 679-684.

20. Fontana, B. and Berzosa, C. D., "Stationary Queue-Length Distribution in an M/G/1 Queue with Two Non-Preemptive Priorities and General Feedback. Performance of Computer-Communication Systems", Ed. W. Bux and H. Rudin, Elsevier, North-Holland, Amsterdam, (1984), 333-347.
21. Simon, B., "Priority Queues with Feedback", ***J. Assoc. Comput. Mach.,*** Vol. 31, (1984), 134-149.
22. Takine, T. A., Takagi, H. and Hasegawa, T., "Sojourn Times in Vacation and Polling Systems with Bernoulli Feedback", ***J. Appl. Prob***, Vol. 28, (1991), 422-432.
23. Gong, W. B., Yan, A. and Gassandras, C. G., "The M/G/1 Queue with Queue-Length Dependent Arrival Rate", ***Comm. Statist. Stochastic Models***, Vol. 8, No. 4, (1992), 733-741.
24. Takagi, H., "A Note on the Response Time in M/G/1 Queue with Service in Random Order and Bernoulli Feedback", ***J. Oper. Res. Soc. of Japan***, Vol. 39, No. 4, (1996), 486-500.
25. Boxma, O. J. and Yechiali, U., "An M/G/1 Queue with Multiple Types of Feedback and Gated Vacations", ***J. Appl. Prob.***, Vol. 34, (1997), 773-784.
26. Medhi, J., "Response Time in an M/G/1 Queuing System with Bernoulli Feedback", Recent Developments in Operational Research, Ed. M.L. Agarwal and K. Sen, Narosa Publishing House, New Delhi, India, (2001), 249-259.
27. Wang, K.H. and Kuo, C.C., "Cost and Probabilistic Analysis of Series Systems with Mixed Standby Components", ***Appl. Math. Model***, Vol. 24, (2000), 957-967.
28. Gupta, U. C. and Srinivasa Rao, T. S. S., "On the Machine Interference Model with Spares, Euro". ***J. Oper. Res***, Vol. 89, (1996), 164-171.