A NEW METHOD FOR CALCULATING PROPAGATION MODES OF A ONE-DIMENSIONAL PHOTONIC CRYSTAL

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(Received: May 9, 2002 – Accepted: November 5, 2003)

Abstract Photonic band-gap (PBG) crystals offer new dimensions of freedom in controlling propagation of electromagnetic waves. The existence of stop-bands in the transmission characteristic of these crystals makes them a suitable element for the realization of many useful microwave and optical subsystems. In this paper, we calculate the propagation constant of a one-dimensional (1-D) photonic crystal by using an equivalent transmission-line network. The method is expanded for structures with arbitrary variation in relative permittivity. The effect of filling factor and relative permittivity on the width of the band-gap and its frequency is also investigated.

Key Words One-Dimensional Photonic Crystals, Wave Propagation in Photonic Crystals, Transmission Line Equation

چکیده در طبیعت ساختارهای فراوانی وجود دارد که در برابر امواج الکترومغناطیسی خاصیتهایی چون باند توقف و باند عبور فرکانسی از خود نشان می‌دهند. به این ساختارها، ساختارهای با باند شکاف الکترومغناطیسی و یا کریستالهای فتونی گویند. در این مقاله ثابت انتشار کریستال فتونی یک بعدی به دو روش محاسبه می‌شود. روش اول که به نام مدل خط انتقال نامگذاری شده، فقط به ساختارهای متناوب یک بعدی که در آنها تغییرات ناگهانی ε_r وجود دارد، قابل اعمال است. در این روش لایه های ساختمان متناوب تا زمانی که امواج صفحه ای فرض شوند، قابل مدل شدن به صورت خط انتقال می باشد. روش دوم که تعمیم یافته روش اول است، قابل اعمال به هر نوع تغییرات دلخواه یک بعدی ε_r است. در اینجا علاوه بر محاسبه ثابت انتشار ساختار متناوب یک بعدی، تاثیر تغییرات طول و جنس عایق ساختار متناوب بر اولین باند شکاف فتونی نیز بررسی می‌شود.

1. INTRODUCTION

Electromagnetic Band-Gap (EBG) structures are 3-D periodic objects that prevent the propagation of electromagnetic waves in a specific band of frequency for all angles and all polarization states. However, in practice, it is very hard to obtain such complete band-gaps. So partial bang-gaps are used instead. Photonic Band-Gap crystals are one of EBG structures which typically cover in-plane angles of arrival and also sensitive to polarization states [1].

The concept of photonic band structure is introduced relatively recently. A number of technical papers have been published with emphasis on the occurrence of the PBG [2-5]. In the present work, a transmission line equivalent network is proposed for calculating the propagation constant of a 1-D PBG structure. This method is applicable to structures with abruptly varying dielectric constants, as shown in Figure 1. In section 3, the introduced method is generalized for structures with arbitrary variation in relative permittivity.

2. A TRANSMISSION LINE EQUIVALENT NETWORK

Assume a one-dimensional periodic structure of period ‘p’ as shown in Figure 1. The width of the dielectric layer is ‘l’. The structure is infinite and homogeneous in the *xz* plane, so there is no

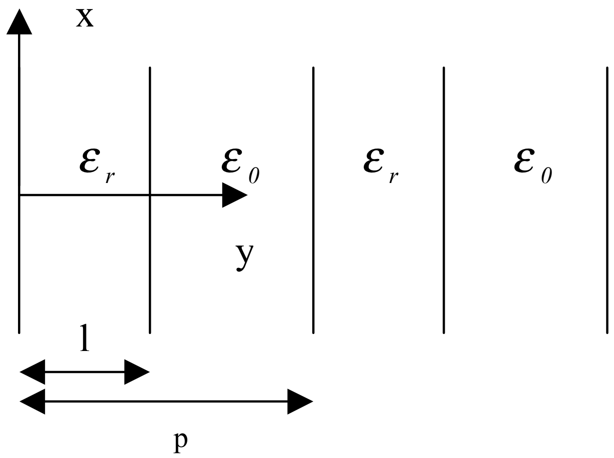

Figure 1. One-dimensional periodic structure.

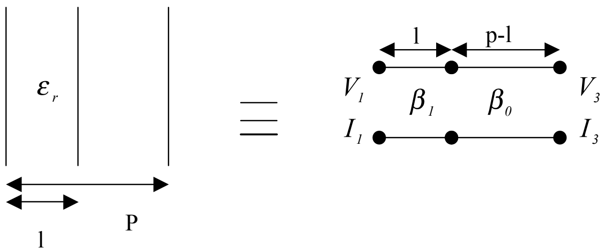

Figure 2. Transmission line model for one period.

variation in the x and z directions. Therefore each unit cell of this periodic structure can be modeled as two cascaded transmission lines. The propagation constants of these lines are $\beta_1 = \sqrt{\varepsilon_r}k_0$ and $\beta_0 = k_0$ as shown in Figure 2 where $k_0 = \omega_0\sqrt{\mu_0\varepsilon_0}$. The voltage V_3 and current I_3 can be related to V_1 and I_1 as follows:

$$\begin{bmatrix} V_3 \\ I_3 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_1 \\ I_1 \end{bmatrix} \tag{1}$$

In which A, B, C and D are:

$$A = \cos\beta_0(p-l)\cos\beta_1 l - \frac{Z_0}{Z_1}\sin\beta_0(p-l)\sin\beta_1 l \tag{2-a}$$

$$B = -jZ_1\cos\beta_0(p-l)\sin\beta_1 l - Z_0\sin\beta_0(p-l)\cos\beta_1 l \tag{2-b}$$

$$C = -j\frac{1}{Z_1}\cos\beta_0(p-l)\sin\beta_1 l - \frac{1}{Z_0}\sin\beta_0(p-l)\cos\beta_1 l \tag{2-c}$$

$$D = \cos\beta_0(p-l)\cos\beta_1 l - \frac{Z_1}{Z_0}\sin\beta_0(p-l)\sin\beta_1 l \tag{2-d}$$

According to the Floquet theorem [6], we can represent V_3 and I_3 in terms of V_1 and I_1 as:

$$\begin{bmatrix} V_3 \\ I_3 \end{bmatrix} = \begin{bmatrix} e^{-\gamma p} & 0 \\ 0 & e^{-\gamma p} \end{bmatrix} \begin{bmatrix} V_1 \\ I_1 \end{bmatrix} \tag{3}$$

Considering Equations 1 and 3, it could be concluded:

$$\begin{bmatrix} A - e^{-\gamma p} & B \\ C & D - e^{-\gamma p} \end{bmatrix} \begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \underline{0} \tag{4}$$

To have a none-zero solution, the determinant of the coefficient matrix must be equal to zero.

3. PBG CALCULATION FOR ARBITRARY VARIATION OF ε_R

The method of Section 2 is applicable to abruptly changing dielectric constants, that is a region of ε_1 and a region of ε_2. In this section, a new method is introduced. For this method any arbitrary one-dimensional variation of ε_r is allowed.

It is straightforward to prove that the electric and magnetic fields can be expanded to the following pseudo-Fourier series:

$$E(y) = \lim_{M\to\infty}\sum_{m=-M}^{M} E_m e^{-j(\frac{2\pi m}{p}+\beta_y)y} \tag{5-a}$$

$$H(y) = \lim_{M\to\infty}\sum_{m=-M}^{M} H_m e^{-j(\frac{2\pi m}{p}+\beta_y)y} \tag{5-b}$$

in which β_y is the propagation constant of the structure in the y direction. It is proven that the equivalent matrix expression of the constitutive relation, $D(y) = \varepsilon_0\varepsilon_r E(y)$, is as follows [7]:

$$[D] = \varepsilon_0 N^2 [E] \tag{6}$$

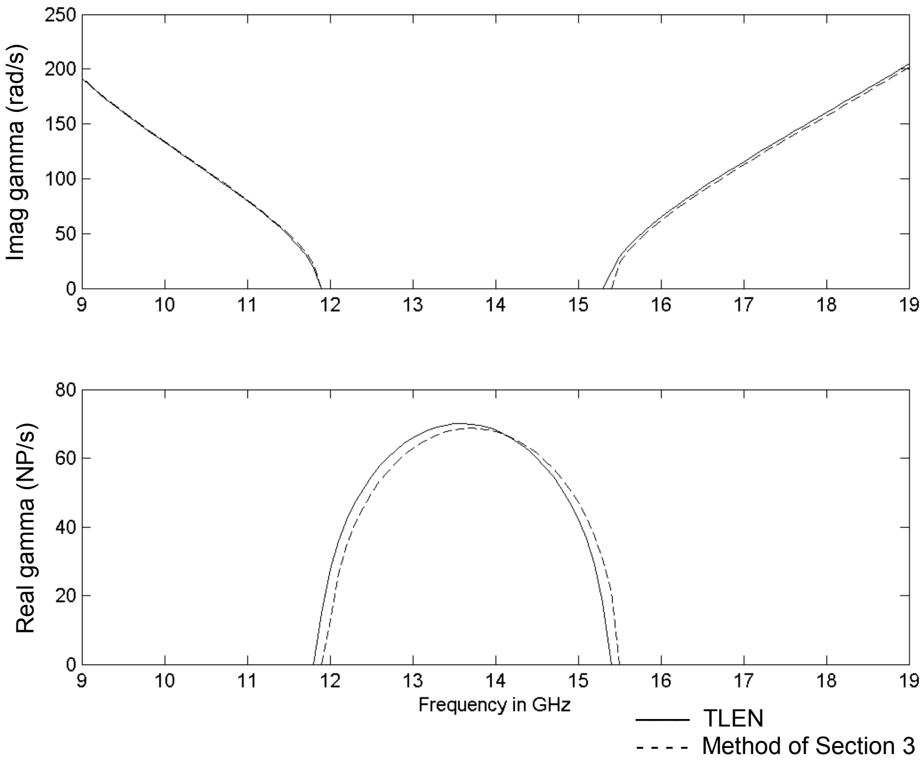

Figure 3. Propagation and attenuation constant.

in which N^2 is:

$$N^2 = \begin{bmatrix} & & \vdots & & \\ & \varepsilon_0 & \varepsilon_{-1} & \varepsilon_{-2} & \\ \cdots & \varepsilon_1 & \varepsilon_0 & \varepsilon_{-1} & \cdots \\ & \varepsilon_2 & \varepsilon_1 & \varepsilon_0 & \\ & & \vdots & & \end{bmatrix} \tag{7}$$

and ε_m is the coefficient of the Fourier series expansion of ε_r calculated by the following equation:

$$\varepsilon_m = \frac{1}{p}\int_0^p \varepsilon_r(y)\, e^{j\frac{2\pi m}{p}y}\, dy \tag{8}$$

By substituting Equations 5 and 6 in Maxwell's equations and considering the fact that the structure is 1-D, Maxwell's equations will be summarized as follows:

$$-j[\alpha][H_z] = j\omega\varepsilon_0 N^2 [E_x] \tag{8-a}$$

$$-j[\alpha][E_x] = j\omega\mu\,[H_z] \tag{8-b}$$

where $[\alpha]$ is a diagonal matrix, equivalent to partial derivatives in the y direction, with elements $\alpha_{ii} = (\frac{2\pi i}{p} + \beta_y)$. By eliminating $[H_z]$ between Equations 8-a and 8-b it is concluded:

$$\left([\alpha]^2 - \omega^2\mu_0\varepsilon_0 N^2\right)[E_x] = \underline{0} \tag{9}$$

To have a non-zero solution, the determinant of coefficients matrix must be zero, from which propagation constants are calculated.

4. RESULTS

In this part both methods are applied to a simple

1-D periodic structure shown in Figure 1. The structure is assumed to have a 12.7 mm period, 4.8 mm dielectric width and $\varepsilon_r = 8.9$. Figure 3 shows the propagating modes of the 1-D photonic crystal. Excellent agreement is found between two methods.

The frequencies and bandwidth of PBG in PCs are determined by the crystal geometry, lattice constant, filling factor and the dielectric constant. For a given implant shape, the filling factor and the dielectric constant ratio determine the photonic band characteristics. PBG maps (band-gap zones versus filling factor and relative permittivity) are shown in Figures 4 and 5.

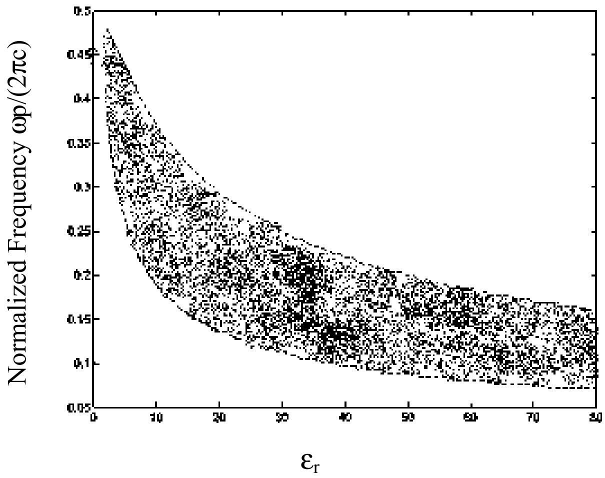

Figure 4. Band-gap zone versus ε_r.

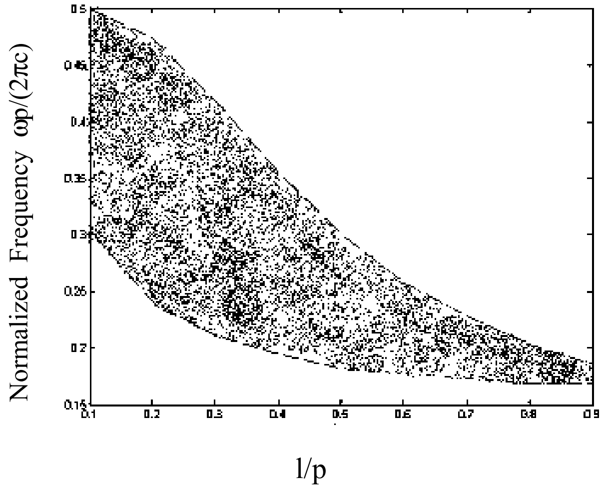

Figure 5. Band-gap zone versus filling factor.

5. CONCLUSIONS

In this paper, two methods for calculating the TEM propagation constant of a periodic structure are compared. It is possible to apply these two methods to the calculation of TM and TE propagation modes. These methods can be generalized to 2-D photonic crystals.

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