RESEARCH NOTE

AN ALGORITHM TO COMPUTE THE COMPLEXITY OF A STATIC PRODUCTION PLANNING

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Abstract Complexity is one of the most important issues of any production planning. The increase in complexity of production planning can cause inconsistency between a production plan and an actual outcome. The complexity generally can be divided in two categories, the static complexity and the dynamic complexity, which can be computed using the ant ropy formula. The formula considers the probability of a system in different scenarios in which it can happen and based on the formula it computes the complexity of the system. However, the method is not able to make a difference between the complexities of different scenarios such as busy, idle, setup, etc. This paper presents a new algorithm to compute the complexity of a static production planning. Our method ranks the importance of the complexity for each scenario and then computes the complexity of the overall system.

Key Words Complexity, Entropy Formula, Linear Programming, Production Planning

چکیده پیچیدگی یکی از مهمترین مشکلات هر مسئله برنامه ریزی تولید است. افزایش پیچیدگی باعث ناهمگونی بین برنامه تولید و خروجی عملی آن می‌گردد. پیچیدگی معمولا به دو گروه ایستا و پویا تقسیم بندی گـردیده و به کمک فرمول انتروپی محاسبه می‌شود. فرمول مزبور بر اساس حالتهای اتفاق افتاده در سیستم و در نظـر گرفتـن احـتمال وقـوع آنهـا مـیزان پیچیدگـی را محاسـبه می‌کند. با این وجود این فرمول قادر به تفکیک سناریوهای مختلف مانند فعال، بیکار و یا تنظیم نمی‌باشد. این مقاله یک الگوریتم جدید برای محاسبه پیچیدگی سیستم با در نظر گرفتن حالت های مختلف سیستم ارایه می‌دهد. در روش جدید سناریوهای مختلف مورد رتبه بندی قرار گرفته و سپس پیچیدگی الگوریتم مورد محاسبه قرار می‌گیرد.

1. INTRODUCTION

Flexible Manufacturing System (FMS) is one of the most important requirements of a world class manufacturing systems. FMS has many advantages such as customer satisfaction, better competition, etc. The lack of a good FMS can create an inefficient decision, impractical programs, high inventories that could result an unproductive production planning [3,6,8,9]. On the other hand, increasing more flexibility itself can lead to a more complex production planning and this could result to low quality in product scheduling and lack of a good degree of reliability [1,7,10]. Therefore, a FMS plan is good as long as it can handle a high degree of complexity. Although different types of complexities have already been discussed but there is not a unique and standard way to define the complexity. Frizelle and Woodcook [4] are among few people who introduce a mathematical model to compute the complexity of a system. In their implementation, complexity is computed based on three basic assumptions. The first assumption is that each subsystem is a process of input or output. The second one tells us that as the complexity of a process increases, the system has less reliability and finally it is very likely for a process with high complexity to become a bottleneck. Their model considers that the complexity is also a reflection of the processes with high setup times. Calinescv et al. [2] explain complexity as static and dynamic. The static complexity is defined as the following:

$$H_{static} = -\sum_{i=1}^{M} \sum_{j=1}^{N_j} P_{ij} \ln P_{ij}, \tag{1}$$

where M, N_j represent the number of resources and scenarios, respectively and P_{ij} denotes the possibility of resource i in scenario j. In this paper, we use a normalized form of (1) as follows:

$$H_{static} = -\frac{1}{\ln M} \sum_{i=1}^{M} \sum_{j=1}^{N_j} P_{ij} \ln P_{ij}. \tag{2}$$

Next section, we present a new algorithm to compute the complexity of a system. The implementation of the new algorithm is also discussed using some practical examples.

2. A NEW ALGORITHM TO COMPUTE THE COMPLEXITY OF A STATIC SYSTEM

As we explained in previous section, the complexity of a system depends on the number of sections (e.g. setup, busy, idle) and their likelihoods. According to (1), static complexity depends on the variance of the likelihoods. In other word, higher complexity represents higher variance with the likelihoods [4]. For example, consider a system with three subsystems 1, 2, 3 and with the same likelihoods of their presentation in system. Now consider the same system with three different likelihoods of 0.95, .025 and .025, respectively. This indicates that an increase to the complexity of a system is a direct result of big variances among all subsystems. However (1) does not explain how important each component can participate in system's complexity. For example, consider a system with only one subsystem. In this case, when the system is idle, it represents low priority whereas when it encounters with a busy status it can represents higher complexity regardless of the likelihood. In other example, let's look at a system with two components called busy and idle. Now, consider two different cases, 1 and 2. For case one, the busy and idle occur with the likelihoods of 0.9 and 0.1, respectively. For case two, the busy and idle occur with opposite possibilities of 0.1 and 0.9, respectively. It is clear that case one represents higher complexity than case two. However, applying (1) yields unique results for both cases. Therefore, we need to make some additional assumption in (1) in order to show the effects of the system's status. This paper presents a new algorithm that incorporates this assumption using a linear programming model. In our algorithm, we need to change the likelihoods as the following form:

$$H_{static} = -\sum_{i=1}^{M} \sum_{j=1}^{N_j} \alpha_{ij} \ln \alpha_{ij}, \tag{3}$$

where α_{ij} is the modified P_{ij} which is computed as the solution of the following linear programming model,

$$\max \sum_{t=1}^{N} \frac{1}{t^2} \alpha_t \tag{4}$$

$$\text{s.t } \alpha_t \geq \gamma_{ft} P_f, \sum_{t=1}^{N} \alpha_t = 1, 0 < \alpha_t < 1 \tag{5}$$

where f,t = 1 … N. In (4), t = {1, 2, …, N} represents different scenarios in terms of their effects on increasing the complexity. In other word, $t = 1$ represents the minimum complexity while $t = N$ denotes the maximum complexity. Let P_f represent the likelihood of a particular resource in case f and γ_{ft} is a portion of P_f when system is in case t. Obviously, an expert can easily set Pf. We now start two simple cases in order to show the effects of the implementation of our LP formulation.

2.1. Example 1

Consider a production system where there are only two situations of busy and idle. When the system is busy it has maximum complexity and obviously when it is idle, the system has minimum complexity. Let P_B and P_I be the likelihoods of busy and idle, respectively. Therefore, we can expect the maximum complexity for $P_I = \varepsilon$ and $P_B = 1$. Also we anticipate the minimum complexity when $P_I = 1$ and. $P_B = \varepsilon$ According to (1) the maximum complexity occurs when $P_I \approx P_B$ and the minimum complexity happens when $(P_I, P_B) = (1, \varepsilon)$ or

$(P_I, P_B) = (\varepsilon,1)$. For instance, consider the following likelihoods for the two cases:

- Case 1 $P_I = \varepsilon, P_S = 1$. In this case we consider the following for γ_{1t}: $\gamma_{1I} = \frac{1}{2}, \gamma_{1B} = \frac{1}{2}$.
- Case 2 $P_I = 1, P_S = \varepsilon$. In this case we consider the following for γ_{1t}: $\gamma_{1I} = 1, \gamma_{1B} = \varepsilon$.

Let t = {I,B}, then we write the following LP model:

$$\max \quad \alpha_I + \frac{1}{4}\alpha_B$$
$$\text{s.t.} \quad \alpha_B \ge 0.5\alpha_B,\ \alpha_I \ge P_I,\ \alpha_I \ge 0.5P_B, \qquad (6)$$
$$\alpha_I + \alpha_B = 1, 0 < \alpha_I, \alpha_B \le 1$$

Table 1 demonstrates the complexity of the system under different conditions. The results indicate that the normalized complexity computed based on the proposed LP formulation depends entirely on the status of the system. For example, when $P_B = 0.90$ and $P_I = 0.10$ then the system is met with 99.27 percent of complexity. Conversely, for $P_B = 0.10$ and $P_I = 0.90$ the complexity is only 28.6 percent. We now consider a system with three different subsystems.

2.2. Example 2 Consider a more complicated system that contains three situations of Setup, Busy and Idle. Let P_s, P_B and P_I represent the likelihoods corresponding to Setup, Busy and Idle, respectively. We consider the maximum complexity when $(P_I, P_B, P_I) = (\varepsilon,\varepsilon,1)$ the minimum complexity where $(P_I, P_B, P_I) = (1,\varepsilon,1)$ and finally the medium complexity when $(P_I, P_B, P_I) = (1,1,\varepsilon,)$. For instance, consider the following likelihood for the three cases,

- Case 1 $P_S = 1$. In this case we consider the following for γ_{1t}:

  $\gamma_{1I} = \frac{1}{3}, \gamma_{1B} = \frac{1}{3}, \gamma_{1S} = \frac{1}{3}$,

- Case 2 $P_I = 1$. In this case we consider the following for γ_{1t}: $\gamma_{2I} = 1,\ \gamma_{2B} = 0,\ \gamma_{2S} = 0$,

- Case 3 $P_B = 1$. In this case we consider the following for γ_{1t}:

  $\gamma_{3I} = 0.84,\ \gamma_{3B} = 0.08,\ \gamma_{3S} = 0.08$.

Applying (4) to example yields,

$$\max \quad \alpha_t + \frac{1}{4}\alpha_s + \frac{1}{9}\alpha_{s,} \qquad (6)$$

subject to

$$\alpha_I \ge \frac{1}{3}P_I, \alpha_s \ge \frac{1}{3}P_{S,} \alpha_B \ge \frac{1}{3}P_S \qquad (7)$$

and

$$\alpha_s \ge P_I, \alpha_I \ge 0.84P_B, \alpha_I \ge \frac{1}{3}P_S \qquad (8)$$

TABLE 1. The Complexity of Example 1.

P_B	P_I	α_B	α_I	H_{static}
1	ε	0.5	0.5	100
ε	1	0	1	0
0.5	0.5	0.25	0.75	81
0.75	0.25	0.375	0.625	95.3
0.1	0.9	0.05	0.95	28.6
0.9	0.1	0.45	0.55	99.27
0.3	0.7	0.15	0.85	60.8

TABLE 2. The Complexity of Example 2.

P_I	P_B	P_S	α_I	α_B	α_S	H_{static}
1	ε	ε	1	0	0	0
ε	ε	1	1/3	1/3	1/3	100
ε	1	ε	0.84	0.08	0.08	50
0.2	0.3	0.5	0.668	0.166	0.166	87.5
0.9	0.08	0.02	0.9868	0.0066	0.0066	7.93
0.5	0.45	0.05	0.928	0.0036	0.036	30.8
0.3	0.65	0.05	0.896	0.052	0.0052	40.6
0.2	0.75	0.05	0.88	0.06	0.06	45
ε	0.95	0.05	0.848	0.076	0.076	53
ε	0.05	0.95	0.368	0.316	0.316	99.75

and

$$\alpha_I + \alpha_B + \alpha_S = 1, 0 < \alpha_I, \alpha_B, \alpha_S \leq 1. \quad (9)$$

Table 2 summarizes the results of the complexities of the system under different conditions, which are similar to the results of the example (1).

For instance, when $P_I = \varepsilon$, $P_B = 0.05$ and $P_S = 0.95$, we get $H_{static} = 99.75$ and conversely, when $P_I = 0.9, P_B = 0.08$ and $P_S = 0.02$ we have $H_{static} = 0.07$. These results are highly desirable and can realistically reflect the complexity of the system. Therefore, the new algorithm provides better results for the complexity of the system under different conditions.

3. CONCLUSIONS

We have presented a new algorithm to use the implementation of linear programming in order to compute the complexity of any particular system. We have explained that traditional methods cannot include the effects of the subsystems under different cases on overall complexity. The new algorithm presented in this paper is able to consider the effects of different cases of a system in terms of their priorities on overall complexity. Numerical results for two different practical examples have been presented in order to show the effectiveness of the proposed algorithm.

4. REFERENCES

1. 1.Bermejo, J., Calinescu, A. J., Efstanthiou, J. and Schirn, J., "Dealing with Uncertainty in Manufacturing, the Impact on Scheduling", ***Proceeding of the 32nd International Matador Conference***, (Ed.: Kochhar, A.), Macmillan Press, UK. (1977), 149-154.
2. Calinescv, A., Efstathio, J., Schirn, J. and Bermejo, J, "Applying and Assessing Two Methods for Measuring Complexity in Manufacturing", ***Journal of Operational Research Society***, Vol. 49, (1988), 133-151.
3. Efstashiou, J., Calinescu, A and Bermejo, J., "Modeling the Complexity of Production Planning and Control", ***Proceedings of the 2nd International Conference on Production Planning and Control in Metals Industry***, Institute of Materials, London, (1996), 60-66.
4. Frizelle, G. and Woodcook, E., "Measuring Complexity As an Aid to Developing Operational Strategy", ***International Journal of Operations and Production Management***, Vol. 25, No. 15, (1994), 26-39.
5. Frizelle, G., "An Entropic Measurement of Complexity in Manufacturing Operations", ***Research Report, Department of Engineering, University of Cambridge***, UK. (1996).
6. Lewis, F., Huang, F., Pastravanu, O. and Gurel, A., "Control System Design for Flexible Manufacturing Systems", In: Raouf A. and Daya M. (Eds.), Flexible Manufacturing Systems; Recent Developments, Elsevier Science, Amsterdam, New York, (1995).
7. McKay, K., Safayeni, F. and Buzacott, J., "Common Sense Realities of Planning and Scheduling in Printed Circuit Board Production", ***International Journal of Production Resource***, Vol. 33, No. 6. (1995), 1585-1603.
8. Neely, A., Gregory, M. and Platts, A., "Performance Measurement System Design", ***International Journal of Operational and Production Management***, Vol. 15, No. 4, (1995), 80-116.
9. Slack, N., "Operations Management", Pitman Publishing: London, UK, (1995).
10. Stoop, P. and Wiers, V., "The Complexity of Scheduling in Practice", ***International Journal of Production Management***, Vol. 16, No. 10, (1996), 37-53.