

## APPLICATION OF BOUNDARY ELEMENT METHOD TO 3-D SUBMERGED STRUCTURES WITH OPEN ENDS

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**Abstract** This paper presents a three dimensional application of direct Boundary-Element Method (BEM) for computing interaction of sinusoidal waves with a large submerged open bottom structure near the floor with finite depth. The wave diffraction problem is formulated within the framework of linearized potential theory and solved numerically with direct BEM. A computer program based on BEM is developed to calculate the wave-exciting hydrodynamic forces. Comparisons of the results with those obtained by several previous investigators reveal a good agreement. In this study, we are primarily interested in the wave forces on a shell defined by an open-end surface. The formulation of this problem is similar to that for a solid, except that the solution is contained in a singular integral equation. This solution is extended to a cylindrical open bottom structure and the wave forces are compared to those for the corresponding seated structures. This method, however, can be applied for any 3-D geometrical objects with or without open ends.

**Key Words** BEM, Scattering, Green's Function, Boundary Integral, Boundary Conditions, Submerged

چکیده در این مقاله، فرمولاسیون عددی مسائل سه بعدی امواج خطی در حالت کلی ارائه شده و همچنین اثر برخورد موج سینوسی سطحی به سازه حجیم در زیر دریا (مخزن نفت ته باز) و فرمولاسیون مساله شرایط حدی تفرق امواج حول یک جسم سه بعدی و در عمق محدود مورد بررسی قرار گرفته است. مسائل پراکندگی امواج در چهارچوب تئوری پتانسیل خطی مورد بحث می باشد. دامنه صحت کاربرد این روش با توجه به فرضیات مساله یعنی استفاده از تئوری امواج خطی و نیز صرفنظر کردن از اثرات حاصل از لزجت است. نهایتاً مساله به روش المانهای مرزی حل شده و نیروهای هیدرودینامیکی محاسبه گردیده است.

### INTRODUCTION

This research was prompted by an increase in offshore activities by the oil industry in recent years and the use of this type of structures storing oil near to production fields, prior to export.

These structures are subject to surface wave forces in both horizontal and vertical directions. In many instances, these structures must be held in place by piling in addition to their own weight to withstand the wave loads.

Frequently a piled structure is scoured out at the

bottom, leaving its underside exposed to the wave action. Indeed for a shell-like structure the entire inside will be open to the wave action.

In this paper we are interested in the study of a large cylindrical shape with open ends and subjected to regular incident waves. Mathematically, it is a boundary value problem governed by the three-dimensional Laplace equation with fixed boundary conditions on the free surface. Problems of this type are generally solved by Green's function method [1-2]. In this paper, an alternative method is described. The weighted residual

principle known as the boundary element method is used to determine the velocity potential all along the domain boundary. With this knowledge, the potentials inside the fluid domain are calculated.

In this study an approach using a direct boundary-element method is developed which has more physical relation to the wave-body interaction problem. Like the finite element method, the direct boundary-element method also divides the external surface of a domain into a series of elements over which the functions under consideration can progress in different ways. This capability is important as the boundary integral equation formulations were generally restricted to constant source strength over the elements and the sources were assumed to be concentrated at a series of points on the external boundary surface.

### BOUNDARY VALUE FORMULATION AND BOUNDARY CONDITIONS

We consider a right-handed Cartesian coordinate system with the origin on the free surface in which the z axis is vertically upward as shown in Figure 1.

The basic flow is assumed to be oscillatory, inviscid, incompressible and irrotational, which has a large Reynolds number. The incident wave is chosen as the linear wave and the direction of propagation of the incident wave makes an angle  $\alpha$  with the positive x-axis. The xy-plane coincides with the free surface when the fluid is at rest. It is also assumed that objects in the fluid domain of the sea bottom reside in a limited domain, and depth h is constant every where outside. For free floating

objects and mooring systems, the average forward velocity is zero, and for problems concerning interaction of waves with objects, the linearized formulations are used to represent their dynamic behavior. The assumption of potential wave theory leads us to a Laplace equation.

By linearizing kinematics and dynamic free surface boundary conditions, the result is a linear wave system. For the cases of monochromatic waves of frequency  $\omega$ , the free surface displacement  $\zeta$ , the velocity vector  $V$  and the potential  $\Phi$  can be expressed as [3]

$$V = R_e \{ \bar{u}(x, y, z) e^{-i\omega t} \} \quad (1)$$

$$\zeta = R_e \{ \eta(x, y) e^{-i\omega t} \} \quad (2)$$

$$\phi = R_e \{ \phi(x, y, z) e^{-i\omega t} \} \quad (3)$$

The structure is considered as a large rigid body oscillating sinusoidally and the amplitudes of the structural motions are assumed to be small. For small amplitude water waves, the fluid velocity may be represented as the gradient of a scalar potential function. Under potential theory, the total velocity potential is obtained as a sum of the incident and a scattered potential and the motion may be described by a velocity potential  $\Phi$  which satisfies the Laplace equation.

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} + \frac{\partial^2 \phi}{\partial z^2} = 0 \quad (4)$$

The wave diffraction of an incident wave represented by its potential  $\phi_i$  on a general three-dimensional body can also be represented by the diffracted potential  $\phi_s$ .

### BOUNDARY CONDITIONS

For small amplitude waves, the following boundary conditions are imposed :

**(i) At the Free Surface** The linearized kinematics and dynamic free surface boundary conditions can be written as:

$$\eta = \frac{i}{\omega} \frac{\partial \phi}{\partial z} \quad z = 0 \quad (5)$$

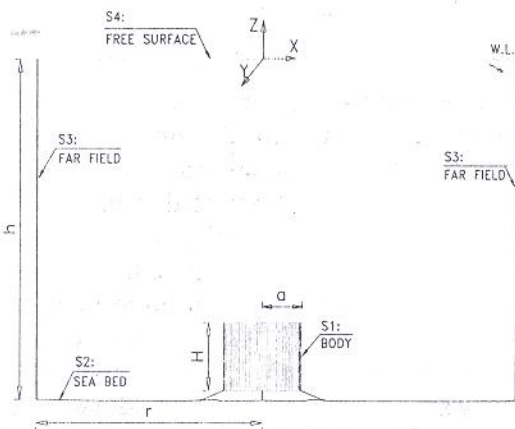


Figure 1. Definition sketch of mathematical model.



$$\eta = \frac{i\omega}{g}\phi \quad z = 0 \quad (6)$$

The normal velocity of the fluid at the free surface is equal to the velocity of the surface itself. In the linear theory, this condition reduces to

$$\frac{\partial\phi}{\partial z} - v\phi = 0 \quad \text{on CF} \quad (7)$$

(ii) Along the rigid stationary sea bed,  $z = h(x, y)$ , the normal velocity vanishes, i.e.,

$$\frac{\partial\phi}{\partial z} - y\phi = 0 \quad \text{on CB} \quad (8)$$

(iii) On the Surface of Submerged Object the normal velocity is prescribed, i.e.

$$\frac{\partial\phi}{\partial z} = y_n = 0 \quad \text{on Cb} \quad (9)$$

$$\frac{\partial\phi_s}{\partial n}(x, y, z) = -\frac{\partial\phi_i}{\partial n}(x, y, z) \quad \text{on Cb} \quad (10)$$

Where  $\frac{\partial}{\partial n} = \mathbf{n} \cdot \nabla$ , and  $\mathbf{n}$  is unit normal vector,

(iv) **Far-field Boundary Conditions** In this problem, the fluid domain is not bounded in the horizontal directions, the water depth is assumed to be uniform far away from objects. The radiation condition, which requires the disturbance generated by the presence of the object to be outgoing waves, is needed at infinity. We need to impose a radiation boundary condition when  $r \rightarrow \infty$  to make the solution unique. This condition is introduced by the Sommerfield boundary condition at infinity and can be expressed as:

$$\text{Lim}\sqrt{kr}\left[\frac{\partial\phi}{\partial r} - ik\phi\right] = 0 \quad (11)$$

In which  $r$  is the radial ordinate and  $k$  is the wave number. The problem is linear, and the potential  $\phi$  can be represented as the sum of an incident and a diffracted wave potential

$$\phi = \phi_i + \phi_s \quad (12)$$

The expression of the velocity potential in the

form of Equation 12, involving a separation into undisturbed incident wave and scattered wave components, constitutes the basis of diffraction theory. The incident wave potential satisfies Equations 4,7-11 and a spatial periodicity; it is specified in complex form as

$$\phi_i(x, y, z) = -\frac{iga_0}{2\omega} \frac{\cosh[k(z+h)]}{\cosh(kh)} e^{ik(x\cos\alpha + y\sin\alpha)} \quad (13)$$

Where  $\alpha_0$  is the wave height and  $\alpha$  its angle of incidence. The wave number satisfies the dispersion relation

$$K \tanh(kh) = v = \frac{\omega^2}{g} \quad (14)$$

because all the equations of the problem are linear. The potential  $\phi_s$  also satisfies Equations 4, 7 - 11, as well as the radiation condition (11). The body surface boundary condition (9) provides a link between  $\phi_s$  and  $\phi_i$  in the form

$$\frac{\partial\phi_s}{\partial n} = -\frac{\partial\phi_i}{\partial n} = 0 \quad (15)$$

Equations 4, 7 - 10 applied to  $\phi_s$ , together with 11 and 15, define the problem in terms of  $\phi_s$ . The interaction of the wave and the submerged object is thus posed as a problem in potential theory, which can be solved by Green's function method. Green's function,  $G$ , is chosen as a singular potential which satisfies the same boundary conditions as the reflected potential at the free surface. There are two different schemes of solution of integral equations within the BEM. The first one employs the so-called free space Green's function,  $G = 1/r$ , where  $r$  is the distance between the source and field points. Since no boundary condition is directly taken into account, this requires discretization of all boundaries of the fluid region (seabed, body surface, free surface and radiation boundaries). The main advantages of the above scheme are the simplicity of the fundamental solution and the possibility of incorporating a variable depth in the vicinity of the body.

The second and more common scheme of solution uses a particular fundamental solution,  $G = 1/r + \bar{G}$ , which directly satisfies the seabed, free surface and radiation boundary conditions.



Thus, only the body surface needs to be discretized. The function  $\overline{G}$  is a regular one satisfying  $\nabla^2 \overline{G} = 0$  throughout the fluid region. Advantages of this scheme include a much simpler data input and a much smaller number of unknowns. The main disadvantage is the fact that the fundamental solution has a very complicated form [4-10].

Solution of the above- defined problem through the BEM can be obtained by using both the direct and indirect formulations.

### BOUNDARY ELEMENT FORMULATION

There are many numerical methods such as finite element variational formulations, Green's function method and boundary element method known, which deal with the hydrodynamic analysis of arbitrary shaped large bodies in the presence of regular waves. In fact these methods can also be used in studying the interaction effects between the incident waves and rigid body.

The boundary element method is well suited to problems in which the limits of the domain are infinite or difficult to define, in that the solution is applied to the boundary rather than the domain. There are two types of boundary element methods, the "direct" and "indirect". The indirect boundary element method is the "source" method. In this method the unknowns are not the physical variables of the problem. On the other hand, the "direct" boundary element method not only allows solving the problem in terms of physical variables but also serves as the first step towards a better understanding of the technique and its relationship to other approximate methods. In particular mixed finite-element methods and the boundary element technique is powerful. It is convenient in applications such as fluid-shell interaction problems. This combination should be achieved satisfying full compatibility and equilibrium at the interfaces between fluid and solid, which requires using the same types of elements for both solutions.

The direct boundary element method used to solve boundary value problems can be formulated rigorously, using either an approach based on Green's theorem or a particular case of the

weighted residual. With the advantages of being more powerful and the ability to relate boundary solutions to other more classical engineering techniques, the residual method is preferred.

The system of equations can now be rewritten in a weighted residual form, to minimize errors when using an approximate method of solution such as boundary elements.

$$\xi = \nabla^2 \phi_s \text{ on } \Omega \quad (16)$$

$$\xi = \frac{\partial \phi_s}{\partial n} + \frac{\partial \phi_i}{\partial n} \text{ on CB} \quad (17)$$

$$\xi = \frac{\partial \phi_s}{\partial n} \text{ on Cb} \quad (18)$$

$$\frac{\partial}{\partial n} = n \cdot \nabla \text{ on CR} \quad (19)$$

$$\xi = \frac{\partial \phi_s}{\partial n} - \frac{\omega^2}{g} \phi_s \text{ on CF} \quad (20)$$

Let us consider an arbitrary function  $u^*$  and its derivative  $q^* = du^*/dn$ .

Later on we will associate these functions with the virtual increment type of function used with the fundamental solutions of boundary elements. In this case the function  $u^*$  can be associated with a full space Green's function such that :

$$\Delta^2 u^* + \delta^i = 0 \quad (21)$$

where  $\delta^i$  is a Dirac delta function. From fundamental solutions the form of  $u^*$  is

$$u^* = \frac{1}{4\pi r} \quad (22)$$

where  $r = \sqrt{(x - x_i)^2 + (y - y_i)^2 + (z - z_i)^2}$ ,  $(x, y, z)$  and  $(x_i, y_i, z_i)$  are the coordinates of the field and source points respectively.

The weighted residual statement for the problem can be written as:

$$\int_{\Omega} \nabla^2 \phi_s u^* d\Omega = \int_{CB=S_1} \left( \frac{\partial \phi_s}{\partial n} + \frac{\partial \phi_i}{\partial n} \right) u^* ds + \int \frac{\partial \phi_s}{\partial n} u^* ds + \int_{CR=S_3} \left[ \frac{\partial \phi_s}{\partial n} - ik\phi_s \right] u^* ds + \int_{CF=S_4} \left[ \frac{\partial \phi_s}{\partial n} - \frac{\omega^2}{g} \phi_s \right] u^* ds \quad (23)$$

By using Green's second identity the left hand side of the above formulation can be simplified as

follows:

$$\begin{aligned} \text{Left hand side} &= \int_{\Omega} (\nabla^2 \phi_s) u^* d\Omega \pm \int_{\Omega} (\nabla^2 u^*) \phi_s d\Omega \\ &= \int_{\Omega} \left\{ u^* (\nabla^2 \phi_s) - \phi_s (\nabla^2 u^*) \right\} d\Omega + \int_{\Omega} \phi_s (\nabla^2 u^*) d\Omega \\ &= \int_s \left( u^* \frac{\partial \phi_s}{\partial n} - \phi_s \frac{\partial u^*}{\partial n} \right) ds + \int_{\Omega} \phi_s (-\delta^i) d\Omega \quad (24) \end{aligned}$$

Hereford Equation 23 can be rearranged as follows

$$-\phi_s - \int_s \phi_s \frac{\partial u_i^*}{\partial n} ds = \int_{s_1} \frac{\partial \phi_s}{\partial n} u_i^* ds - \int_{s_2} ik \phi_s u_i^* ds - \int_{s_3} \frac{\omega^2}{g} \phi_s u_i^* ds \quad (25)$$

By expanding the second term on the right hand side of Equation 25, one obtains:

$$\begin{aligned} \phi_s - \int_{s_1+s_2} \phi_s \frac{\partial u_i^*}{\partial n} ds + \int_{s_3} \left[ \frac{\partial u_i^*}{\partial n} - ik u_i^* \right] ds + \int_{s_4} \left[ \frac{\partial u_i^*}{\partial n} - \frac{\omega^2}{g} u_i^* \right] \\ = - \int_{s_1} \frac{\partial \phi_s}{\partial n} u_i^* ds \quad (26) \end{aligned}$$

Note that s is the total boundary, i.e.  $s = s_1 + s_2 + s_3 + s_4$  Equation 26 implies an integral relationship between the potential at the field point "i" inside the domain  $\Omega$  and its values on the boundary "s" of the domain. When this point is moved to the boundary s, the integrals involved in this equation become singular at "i"

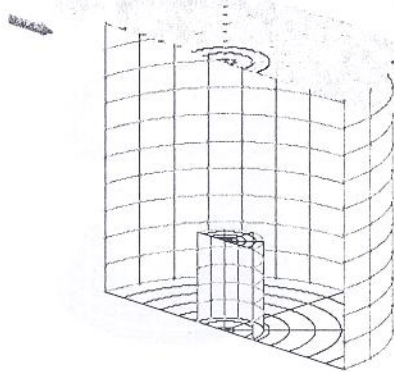


Figure 2. Mesh generation and discretized surfaces.

and must be evaluated in the Cauchy principle value sense [3].

Thus, for any point "i" on the boundary surface "s", the boundary integral formulation for a point on the boundary can be written:

$$\begin{aligned} C_i \phi_s^i + \int_{s_1+s_2} \phi_s \frac{\partial u_i^*}{\partial n} ds + \int_{s_3} \left[ \frac{\partial u_i^*}{\partial n} - ik u_i^* \right] ds + \int_{s_4} \left[ \frac{\partial u_i^*}{\partial n} \right] ds \\ = - \int_{s_1} \frac{\partial \phi_s}{\partial n} u_i^* ds \quad (27) \end{aligned}$$

where  $C_i = 1/2$  for smooth boundaries and for a sharp corner its value is proportional to the interior angle.  $C_i = 1$  if the point "i" is in  $\Omega$  and  $C_i = 0$  if "i" is outside the domain  $\Omega$  and boundary "s". The no dimensional form of the above formulation is given as follows:

$$\begin{aligned} \hat{\phi} &= \frac{\phi}{gA} & , d\hat{s} &= k^2 ds \\ \hat{n} &= kn & , \hat{n}_j &= k_j n \\ \hat{u}^* &= \frac{u^*}{k} = \frac{1}{4\pi kr} = \frac{1}{4\pi \hat{r}} & , \hat{A} &= kA \quad (28) \end{aligned}$$

$$\begin{aligned} C_i \hat{\phi}_s^i + \int_{\hat{s}_1+\hat{s}_2} \hat{\phi}_s \frac{\partial \hat{u}_i^*}{\partial \hat{n}} d\hat{s} + \int_{\hat{s}_3} \left[ \frac{\partial \hat{u}_i^*}{\partial \hat{n}} - ik \hat{u}_i^* \right] d\hat{s} + \\ \int_{\hat{s}_4} \left[ \frac{\partial \hat{u}_i^*}{\partial \hat{n}} - \tanh(\hat{h}) \hat{u}_i^* \right] d\hat{s} = - \int_{\hat{s}_1} \frac{\partial \hat{\phi}_s}{\partial \hat{n}} \hat{u}_i^* d\hat{s} \quad (29) \end{aligned}$$

The boundary "s" can now be discretized into a series of N elements as shown in Figure 2, the types of elements are the most important in the analysis, as we will see, but for simplicity let us consider that the elements are constant  $\Phi$  to be stored at the center of the element. The potential  $\Phi$  and let the values of within each element be defined as:

$$\hat{\Phi}_s^i = [N_i][\Phi_e^i] \quad (30)$$

Where  $[N_i]$  is the interpolation function vector and  $[\Phi_e^i]$  is a vector defining the potential at the nodes of elements  $s_e$ , the boundary integral Equation



29 now becomes :

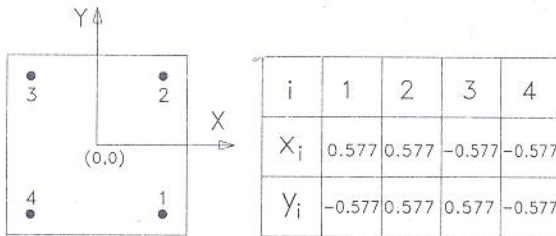
$$C_i \hat{\Phi}_s + \sum_{k=1}^{n_1+n_2} \left\{ \int_{s_k} N_L \frac{\partial \hat{u}_i^*}{\partial \hat{n}} ds \right\} \hat{\Phi}_s + \sum_{k=1}^{n_3} \left\{ \int_{s_k} N_L \left[ \frac{\partial \hat{u}_i^*}{\partial \hat{n}} - i \hat{u}_i^* \right] ds \right\} \hat{\Phi}_s + \sum_{k=1}^{n_4} \left\{ \int_{s_k} N_L \left[ \frac{\partial \hat{u}_i^*}{\partial \hat{n}} - \tanh(\hat{h}) \hat{u}_i^* \right] ds \right\} \hat{\Phi}_s = - \sum_{k=1}^{n_4} \left\{ \int_{s_k} N_L \frac{\partial \hat{\Phi}_l}{\partial \hat{n}} \hat{u}_i^* ds \right\} \quad (31)$$

We will now study how to use integrals over the elements. Two types of integrals can be distinguished:

$$H_{ij} = \int_{s_j} \left[ \frac{\partial u_i^*}{\partial n} \right] ds \quad G_{ij} = \int_{s_j} [u^*] ds \quad (32)$$

These integrals represent influence functions between element "i" at which the fundamental solution is applied and any other element "j" under consideration. When  $i \neq j$ , we can usually apply a standard four points Gauss integration rule as follow.

$$\int_{-1}^1 \int_{-1}^1 F(x, y) dx dy = \sum_{i=1}^4 F(x_i, y_i)$$



With the above notations, Equation 31 can be written as :

$$\sum_{k=1}^{n_1+n_2} H_{ij} \hat{\Phi}_s + \sum_{k=1}^{n_3} [H_{ij} - iG_{ij}] \hat{\Phi}_s + \sum_{k=1}^{n_4} [H_{ij} - \tanh(\hat{h}) G_{ij}] \hat{\Phi}_s = - \sum_{k=1}^{n_4} \frac{\partial \hat{\Phi}_l}{\partial n} G_{ij} \quad (33)$$

where

$$H_{ij} = H_{ij} \quad \text{for } i \neq j$$

$$H_{ij} = H_{ij} + C_i \quad \text{for } i = j \quad (34)$$

Note that the total number of elements  $N = n_1 + n_2 + n_3 + n_4$  and that the subscript "i" refers to the element number. Applying this equation at all "i" boundary points, the following matrix form is produced:

$$AX = B \quad (35)$$

Where X contains all the unknowns, A is the coefficient matrix and the elements of B are known. The solution of Equation 33 will give the values of scattered potential for the diffraction problem. This is a complex system of equations, which has to be solved during the analysis. After the solution of (31), the dynamic pressure on the structure can be obtained by the linearized Bernoulli's equation. For a large object; the velocity-squared term is generally small compared to that of the linear term. However, in the case of a shell, it will not be small near the edges and may contribute significantly to the resultant forces when the wave length is large compared to the overall size of the shell. The pressure across the shell is given by: [11]

$$P = -\rho \frac{\partial \Phi}{\partial t} - \frac{\rho}{2} |\nabla \Phi|^2 \quad (36)$$

Where  $\rho$  and  $g$  are, respectively, the density of water and the acceleration of gravity. The velocity distribution and dynamic pressure across the shell are presented in Figures 3-7. The forces in the horizontal and vertical direction can now be

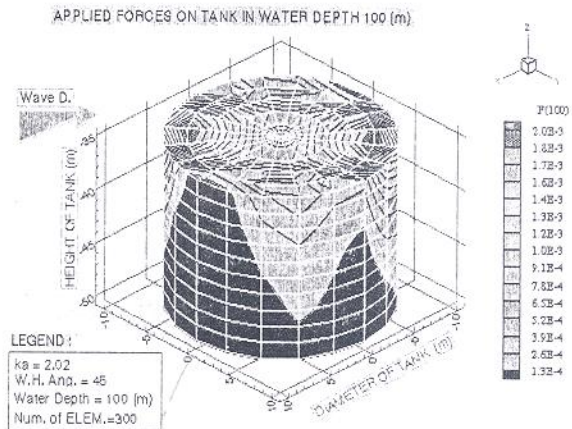


Figure 3. Applied forces on tank in water depth 100 (m).

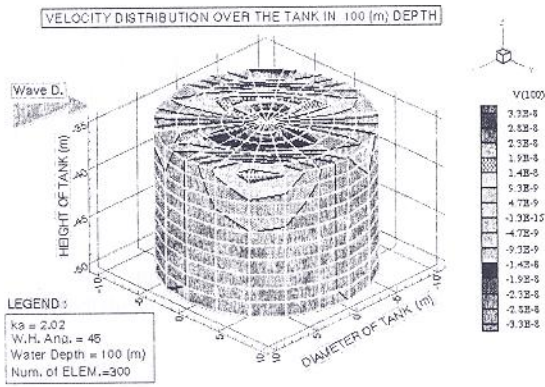


Figure 4. Velocity distribution over the tank in water depth 100 (m)

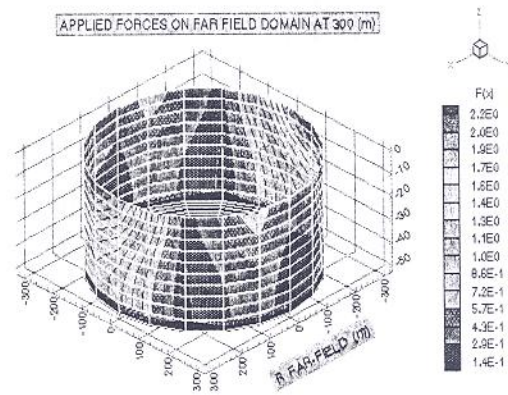


Figure 6 Applied forces on Far-Field domain at r = 300 (m) Radius.

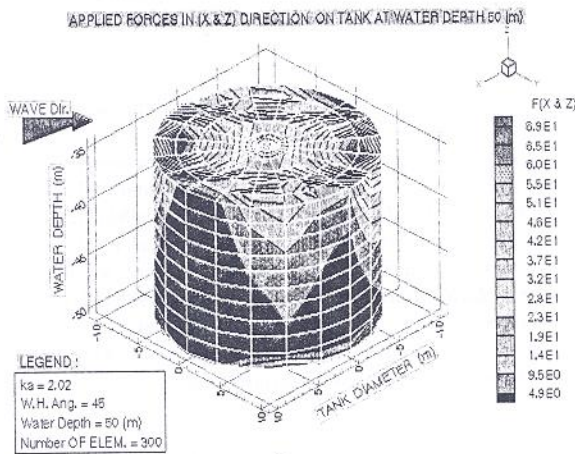


Figure 5. Applied forces in (X & Z) direction on tank in water depth 50 (m).

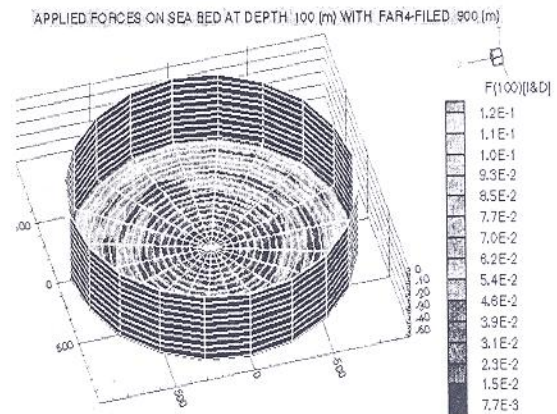


Figure 7. Applied forces on sea bed in water depth 100 (m) with r = 900 (m) Far-Field

obtained by integrating the pressure, i.e.

$$F = \int_s p n ds \quad (37)$$

Where,  $n = (n_x, n_z)$  is the normal vector into the body. Horizontal and vertical wave forces on a submerged storage tank with a radius "a" in a water depth of "5a" were computed. The nondimensionalized forces are shown in Figures 3-7. The results are presented in Figure 8 in comparison to the data obtained by exact solution. [7].

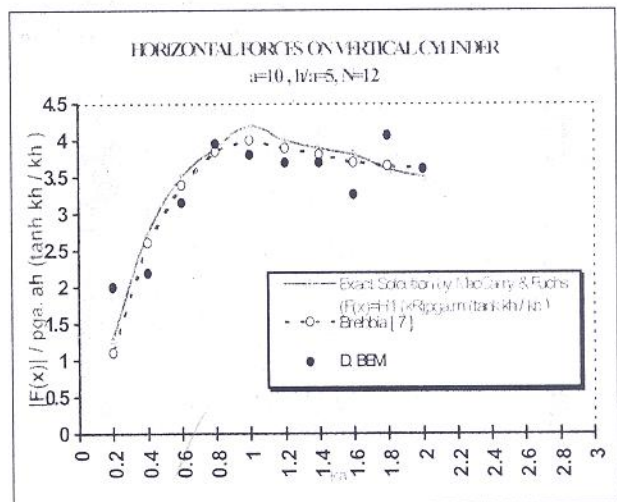


Figure 8. Horizontal forces on vertical cylinder



## CONCLUSIONS AND RECOMMENDATIONS

This paper presented the methodology for applying direct boundary elements to compute wave forces in submerged structures. The advantages of using boundary elements in comparison with the finite elements in fluid are clear. The case of the forces acting on submerged structures is presented in detail. An analysis of the wave interaction with a bottom mounted shell-like structure has been presented. Generally, the results of the analysis are in good agreement with those of other works. It is clearly demonstrated that at long wave periods a substantial reduction in the vertical wave force is achieved by having the structure open at the bottom. When the opening at the bottom is small, the pressure inside is uniform and can be approximated by the mean outside pressure at the bottom of the corresponding sealed structure. The vertical force on the open structure will differ from that on the sealed structure by this pressure times the base area, while the horizontal forces to a first approximation will be the same. Wave forces on a submerged cylinder of a radius "a" in a water depth of "5a" (i.e.  $h/a = 5$ ) were computed. The nondimensionalized forces, pressure and velocity are shown in Figures 3-7. It is easy to extend this formulation to the computation of added mass and damping for any arbitrarily shaped three-dimensional body. It would be of further interest to extend this method to study the wave resistance of a surface ship with forward speed. It should be pointed out that, in principle, any boundary geometries, no matter how complicated, can be handled by this method.

The boundary-element method is suitable for

solving problems that have a high ratio of domain volume to boundary surface area, especially for problems having infinite or semi-infinite domains.

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