HYDROTHERMAL OPTIMAL POWER FLOW USING CONTINUATION METHOD

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Abstract The problem of optimal economic operation of hydrothermal electric power systems is solved using powerful continuation method. While in conventional approach, fixed generation voltages are used to avoid convergence problems, in the proposed algorithm, they are treated as variables so that better solutions can be obtained. The algorithm is tested for a typical 5-bus and 17-bus New Zealand networks. Its capabilities and promising results are assessed.

Key Words Hydrothermal Optimal Power Flow, Continuation Method, Optimization

چکیده در این مقاله، برای حل مسأله پخش بار بهینه در شبکه‌های قدرت دارای نیروگاههای آبی و حرارتی، استفاده از روش تداومی پیشنهاد شده است. درحالیکه در روشهای معمول برای حل مسأله فوق، به منظور پیش‌گیری از واگرا شدن معادلات، ولتاژ گرههای تولید ثابت فرض می‌شوند، در روش پیشنهادی امکان متغیر گرفتن این ولتاژها و دستیابی به پاسخ بهتر وجود دارد. الگوریتم در مورد شبکه نمونه ۵ شینه و شبکه ۱۷ شینه نیوزیلند آزمایش شده که قابلیتهای روش و نتایج رضایت بخش آن، ارائه شده است.

INTRODUCTION

Optimal Power Flow (OPF) in electric power systems seeks the power flow solution which optimizes a performance function such as fuel costs, while simultaneously enforcing the loading limits imposed by the system equipment ratings and various constraints. In a mixed hydrothermal power system, the solution is obtained while utmost use of free (or almost no cost) hydropower is guaranteed.

Hydro Thermal Optimal Power Flow (HTOPF) has received attention in power system literature [1-6]. Due to a large number of time intervals, solution convergence is more difficult in comparison with all-thermal power systems [5]. That is why in published literature, HTOPF problem is solved, normally by considering fixed generation voltages. In this case, even if the optimization problem is properly and fully formulated, a suboptimal solution is obtained.

To overcome the difficulty, a new algorithm is proposed in this paper. HTOPF is solved using powerful continuation method (CM) [7,8]. This method has already been applied to some engineering problems [9]. In 1994, the method was used to solve all-thermal OPF problem [7]. The method is extended in this work to account for hydro-thermal system.

The structure of the paper is as follows. First the formulation of the problem is described. Then, continuation method is briefly reviewed. The proposed algorithm is subsequently discussed. Test results on a small 5-bus typical power system and a real 17-bus power system of New Zealand [10] are demonstrated. Some concluding remarks are finally provided.

PROBLEM FORMULATION

A hydro-thermal electric power system consisting of N buses with N_s thermal and N_h hydro stations (totally N_g stations) is considered. The period of the study may be, say, 24 hours with N_t solution intervals, each of which is T^t hours long. The objective function to be minimized is the fuel cost of the thermal units during the period as follows:

$$J = \sum_{t=1}^{N_t} \sum_{i=1}^{N_s} F_i(P^t_{si}) T^t \qquad (1)$$

where $F_i(P^t_{si})$ is the fuel cost of unit i as:

$$F_i\left(P^t_{si}\right) = \alpha_i + \beta_i P^t_{si} + \gamma_i \left(P^t_{si}\right)^2$$
$$i = 1,\dots,N_s \qquad (2)$$

The network is modeled completely with its power flow equations as follows:

$$P^t_i + P^t_{di} - P^t_{gi} = 0$$
$$t = 1,\dots,N_t, \quad i = 1,\dots,N \qquad (3)$$

$$Q^t_i + Q^t_{di} - Q^t_{gi} = 0$$
$$t = 1,\dots,N_t, \quad i = 1,\dots,N \qquad (4)$$

where P_i (Q_i), P_{di} (Q_{di}) and P_{gi} (Q_{gi}) are injected, demand and generation active (reactive) powers, respectively. For load buses, P_{gi} and Q_{gi} are set to zero.

For a hydro unit, hydrogeneration (P_{hi}) and water flow through turbine (q_i) are related by:

$$q^t_i = d_i + j_i P^t_{hi}$$
$$t = 1,\dots,N_t, \quad i = 1,\dots,N_h \qquad (5)$$

The inequality constraints to be satisfied include:

-Total water accessible during the period:

$$\sum_{t=1}^{N_t} q^t_i T^t \le b_i \qquad i = 1,\dots,N_h \qquad (6)$$

where b_i is the upper bound for the water accessible to unit i.

-Upper and lower bounds on active power generations:

$$P^{min}_{gi} \le P^t_{gi} \le P^{max}_{gi} \qquad i = 1,\dots,N_g \quad (7)$$

-Upper and lower bounds on reactive power generations:

$$Q^{min}_{gi} \le Q^t_{gi} \le Q^{max}_{gi} \qquad i = 1,\dots,N_g \quad (8)$$

-Upper and lower bounds on reservoir volume:

$$v^{min}_i \le v^t_i \le v^{max}_i \qquad i = 1,\dots,N_h \quad (9)$$

-Upper and lower bounds on bus voltage magnitudes:

$$V^{min}_i \le V^t_i \le V^{max}_i \qquad i = 1,\dots,N \quad (10)$$

The optimization problem, thus formed, will be solved using continuation method. The theoretical background is presented next.

CONTINUATION METHOD

Basic Principles Newton method is a powerful technique for solving nonlinear equations. The method, however, may fail to perform satisfactorily as shown in Figure 1.

The aim is to find the solution of:

$$f(X) = 0 \qquad (11)$$

As seen in the Figure 1, initial choice of A_1, A_2 and A_3 would not result in convergence to solution O_1, although all of them are chosen on left-hand side of the solution. The choice of A_4 will result in solution convergence.

In continuation method, a new function is formed as follows:

$$F(X,\varepsilon) = f(X) - (1-\varepsilon) f(X_0) = 0 \qquad (12)$$

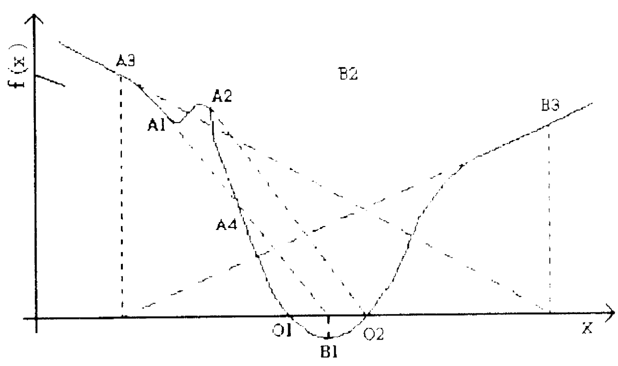

Figure 1. The performance of N-R in zero finding

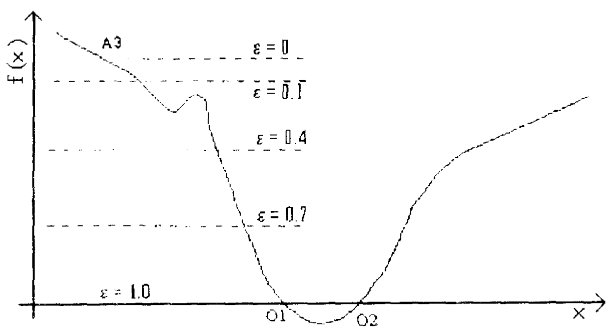

Figure 2. CM in zero finding.

so that for parameter $\varepsilon=0$, X_0 is the known and initial solution. The final solution is for the case where $\varepsilon=1$. As shown in Figure 2, ε is initially taken to be zero for the initial choice of A_3. Its value is gradually increased to 1. In this case, the algorithm converges to solution O_1 although Newton method does not. In fact, in continuation method, the solution to $f(X) = A$ is found for different values of A during the trajectory. These solutions are either directly subsequently employed in the problem solution process or indirectly employed as appropriate initial solution points.

For more details on continuation method, see Reference [8].

Parametric Optimization The continuation method may be effectively employed in optimization problems as follows.

The optimization problem already described may be stated as:

$$\min_X J(X) \tag{13}$$

subject to:

$$g(X) = 0 \qquad (\lambda) \tag{14}$$

$$h(X) \le 0 \qquad (\mu) \tag{15}$$

where λ and μ are Lagrange multipliers for equality and inequality constraints, respectively.

Let X_0 and λ_0 be arbitrary initial guesses for X and λ, respectively. To use continuation

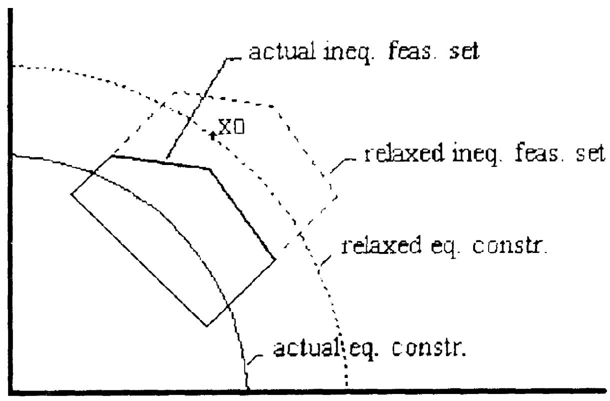

Figure 3. Modification of feasible set in the CM.

method, new formulation should be established. The optimization problem may be restated as [7]:

$$\min_X J(X,\varepsilon) \tag{16}$$

subject to:

$$g(X,\varepsilon) = g(X)-(1-\varepsilon)g(X_0) = 0 \qquad (\lambda) \tag{17}$$

$$h(X,\varepsilon) = h(X)-(1-\varepsilon)\Delta h \le 0 \qquad (\mu) \tag{18}$$

where:

$$J(X,\varepsilon) = J(X) - (1-\varepsilon)J_0^T X + 1/2(1-\varepsilon)\, W \|X - X_0\|^2 \tag{19}$$

$$J_0 = \frac{\partial J}{\partial X}(X_0) + \frac{\partial g^T}{\partial X}(X_0)\lambda_0 \tag{20}$$

$$\Delta h_i = 0 \qquad \text{if} \qquad h_i(X_0) < 0 \tag{21}$$

$$\Delta h_i > h_i(X_0) \qquad \text{if} \qquad h_i(X_0) \le 0 \tag{22}$$

and where ε is a parameter which should assume values from zero to 1, so that at 1, the final solution is obtained.

The optimization problem is so arranged that:

-Arbitrary choice of X_0 would result in the optimal solution at $\varepsilon=0$. It not only satisfies (17) and (18), but also satisfies the first and the second order optimality conditions (see next section). Figure 3 shows how the feasible set is modified by relaxation of (14) and (15) to (17) and (18). W is a parameter so selected that at

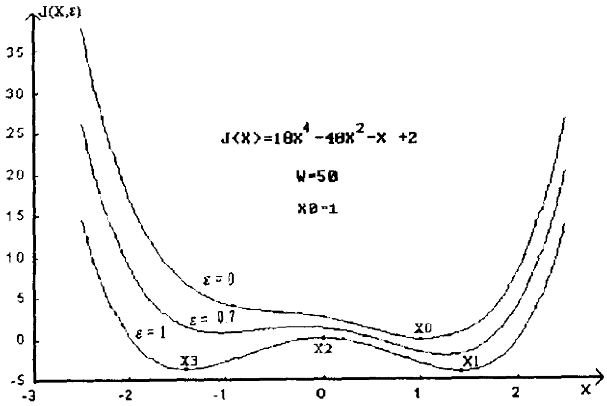

Figure 4. Effect of W on meeting SOOC.

initial trajectory point ($\varepsilon=0$), the second order optimality condition (SOOC) is met for X_0. Figure 4 shows a sample objective function for different values of ε. The effect of W on meeting the SOOC is demonstrated for $\varepsilon=0$ and $X=X_0$.

-The parameter variation process progressively returns the loads back to their original values at $\varepsilon=1$ so that the problem formulated by (16) to (22) returns back to the original problem defined by (13) to (15). As a result, the solution at $\varepsilon=1$ is the minimum of the actual objective function.

Optimality Conditions The parametric Lagrangian is given by:

$$l(X\lambda,\mu,\varepsilon) = J(X,\varepsilon) + \lambda^T{}_g(X,\varepsilon) + \mu^T{}_A h_A(X,\varepsilon) \tag{23}$$

where index A corresponds to the index set of active inequalities for some value of ε. If I represents the corresponding set of inactive inequalities, we have:

$$h_A(X,\varepsilon) = 0 \tag{24}$$

$$h_I(X,\varepsilon) < 0 \tag{25}$$

According to Kuhn-Tucker conditions given by:

$$\mu_i h_i(X,\varepsilon) = 0 \tag{26}$$

$$\mu_i \geq 0 \tag{27}$$

practical conditions in the algorithm may be restated as:

$$h_i(X,\varepsilon) < 0 \tag{28}$$

$$\mu_A \geq 0 \tag{29}$$

For X^* to be the optimum solution of (16), two optimality conditions should be met. The first order optimality conditions are described as:

$$\nabla l = 0 \tag{30}$$

or

$$\frac{\partial l}{\partial X} = \frac{\partial J(X,\varepsilon)}{\partial X} + \frac{\partial g^T}{\partial X}(X,\varepsilon)\lambda + \frac{\partial h_A^T}{\partial X}(X,\varepsilon)\mu_A = 0 \tag{31}$$

$$\frac{\partial l}{\partial \lambda} = g(X,\varepsilon) = 0 \tag{32}$$

$$\frac{\partial l}{\partial \mu_A} = h_A(X,\varepsilon) = 0 \tag{33}$$

To satisfy the second order optimality condition, the Hessian matrix should be at least semi-positive definite for any value of ε. A sufficient large value of W coefficient in (19) can satisfy this condition at and near $\varepsilon=0$ since it appears only as diagonal elements in Hessian matrix (see Test Results).

PROPOSED ALGORITHM

The hydrothermal optimal power flow problem may be solved based on the theory developed. For X as shown below:

$$X = \{V^t_i, \delta^t_i, P^t_{gk}, Q^t_{gk}\} \qquad i=1,\dots,N, \quad j=2,\dots,N, \quad k=1,\dots,N_g, \quad t=1,\dots,N_t \tag{34}$$

The flowchart of proposed algorithm is shown in Figure 5. The steps may be briefly described as follows:

1. Select initial values for X_0 and λ_0, set $\varepsilon=0$

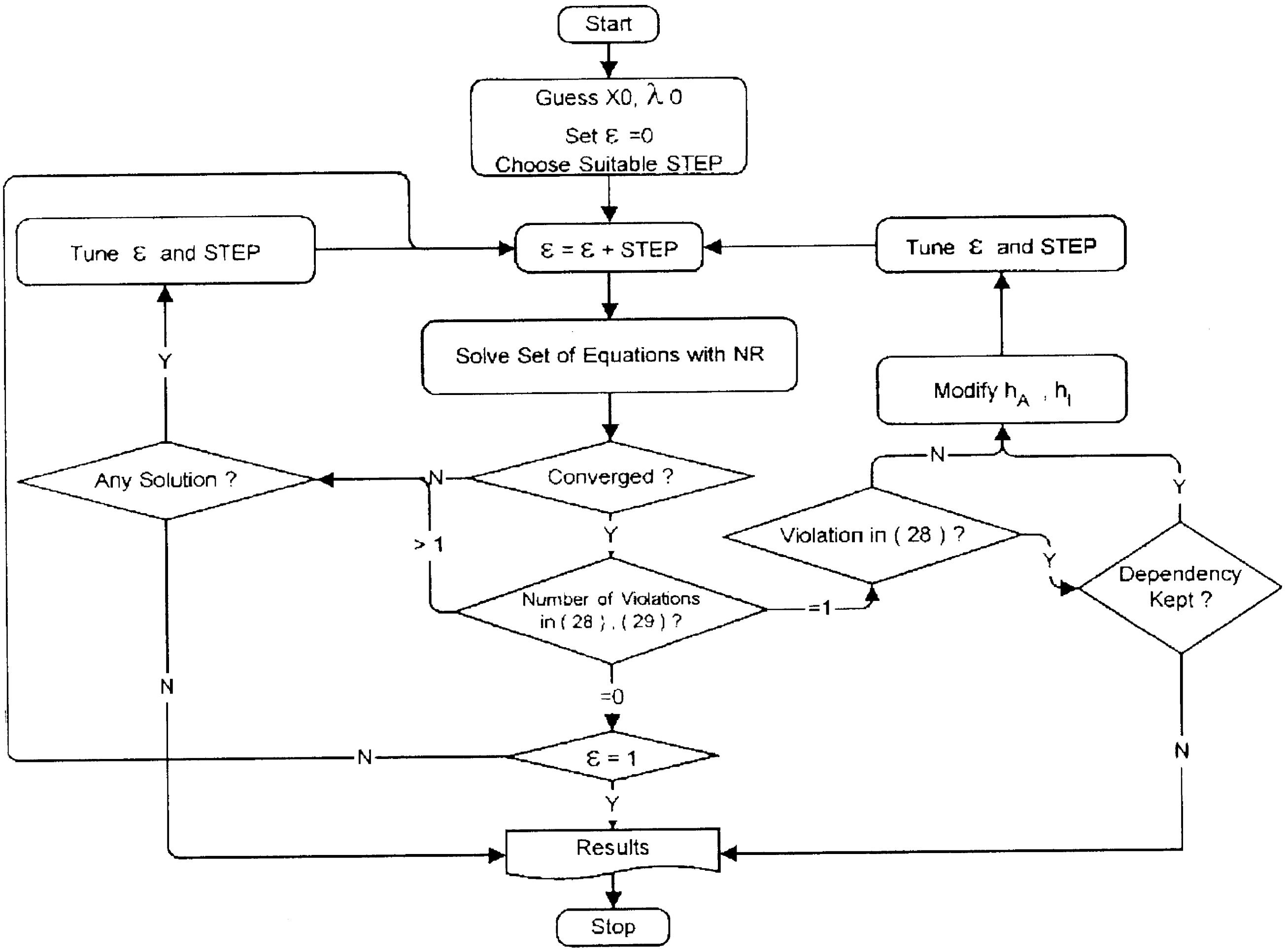

Figure 5. Flow chart of continuation method.

and choose an appropriate value for STEP in $\varepsilon=\varepsilon$ + STEP (A value of 0.1 for STEP is satisfactory. In fact any value between 0 and 1 results in convergence but with different number of iterations).

2. $\varepsilon=\varepsilon$ + STEP

3. Solve (31) to (33), using Newton Raphson. For the case of no convergence (detected by $||\Delta X||$ not to be less than a specified tolerance), go to step 8.

4. Check (28) and (29):
   - For no violation, go to step 5.
   - For single violation of (29), go to step 6.
   - For single violation of (28), go to step 7.
   - For multiple violations, go to step 8.

5. If $\varepsilon=1$, the solution is reached, otherwise, go to step 2.

6. Omit the corresponding inequality from h_A and add to h_I. Then go to step 3.

7. Omit the corresponding inequality from h_I and add to h_A (if the new set of active constraints is independent); then go to step 3. In the case of dependency, go to step 9.

8. $\varepsilon=\varepsilon$-STEP; Reduce STEP and go to step2.

9. The problem has no solution.

The following points are worth mentioning:
- The operations involved in steps 4, 6, 7 and 8 are called Binary Search. Based on that, ε is so selected that proper solution is obtained.
- Reference [7] has shown that steps 6 and 7 would not create recycling.
- Steps 3, 4 and 8 can overcome convergence

problems provided the difficulty is due to a large STEP size.

- If the problem has no solution, it will be systematically detected in step 9. This is one of the main advantages of the algorithm.
- As full solution is obtained during each step of the trajectory; at the end, several side problems have been solved. These solutions may be effectively employed in rescheduling problems for other loading conditions (see section Basic Principles).

TEST RESULTS

Based on the model developed, a software package is developed using ^{+}C on a SUN compatible workstation. The algorithm is first tested on a small typical 5-bus network with two thermal units (buses 1 and 2) and one hydro unit (bus 3). The time period of the study (24 hours) is divided into 10 intervals. The data is provided in Appendix I.

Initially HTOPF problem is solved considering fixed generation voltages. The final solution is shown typically for two intervals in Table 1 with corresponding Lagrange multipliers. t_1 corresponds to the first and t_2 to the second time intervals. Negative values for Lagrange multipliers associated with bus 3 show that the final solution is not optimal. Table 2 shows that Newton method is unable to solve the equations even after 10 iterations.

The problem is subsequently solved using

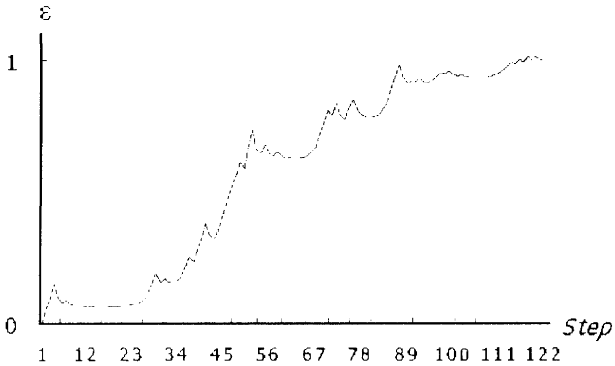

Figure 6. Growth of ε through trajectory (5-bus system).

TABLE 1. Effect of Fixed Voltages.

	V_1^{t1}	V_2^{t1}	V_3^{t1}	V_1^{t2}	V_2^{t2}	V_3^{t2}
Voltage Magnitude	1.05	1.05	1.05	1.05	1.05	1.05
Lagrange Multiplier	1.287	0.07	-0.857	0.551	0.735	-0.466

TABLE 2. Failure of NR.

iteration	V_1^{t1}	V_2^{t1}	V_3^{t1}	V_1^{t2}	V_2^{t2}	V_3^{t2}
1	1.0435	1.035	1.0	1.01	0.99	0.93
3	2.45	2.45	2.54	2.12	2.11	2.18
5	3.43	3.41	3.69	-3.06	-3.02	-3.05
10	20.4	19.6	21.3	-5.6	-10.4	0.04

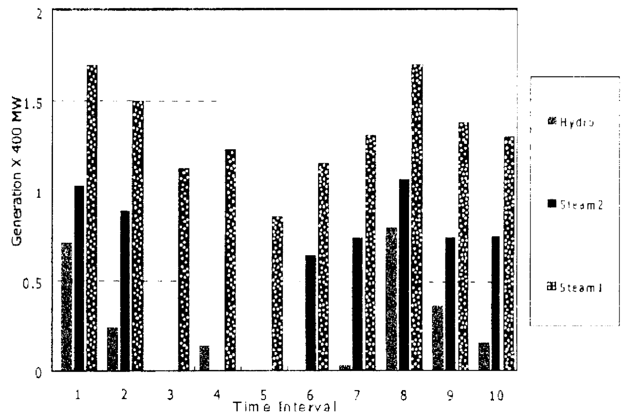

Figure 7. Final decision for generation (5-bus system).

continuation approach. Figure 6 shows how ε is varied during the solution process. The variations involved are due to multiple violations of (28) and (29) so that ε is properly reduced (see steps 4 and 8, previous section). Figure 7 shows the generations at the end of the trajectory. As expected, for heavy load periods, hydrogeneration is more pronounced.

Figure 8 shows reservoir volumes at the end of 10 time intervals. At the second and fourth intervals, minimum volumes are reached.

Figure 9 demonstrates some of the voltages in some of the intervals during ε trajectory. As shown, in the 75th step, the 7th interval voltages are simultaneously violated. To prevent possible convergence problem, the voltage of bus 1 is decided to be fixed by binary search algorithm; other voltages remain within bounds. In the 87th step, the 4th interval voltages are

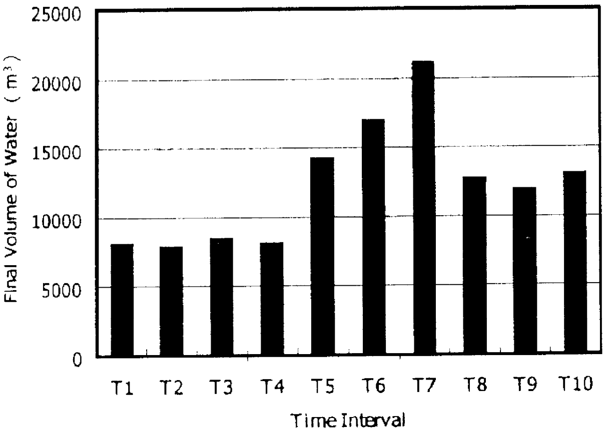

Figure 8. Reservoir volume at $\varepsilon = 1$ (5-bus system).

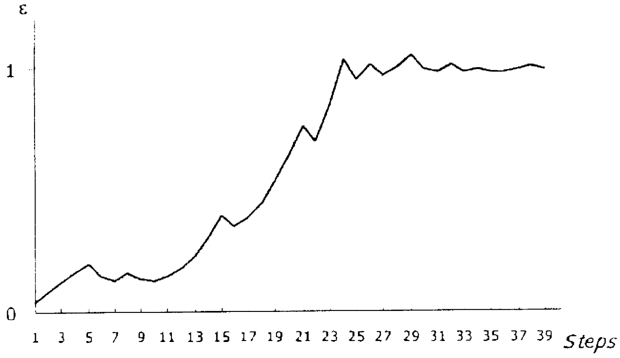

Figure 11. Growth of ε through trajectory (17-bus system).

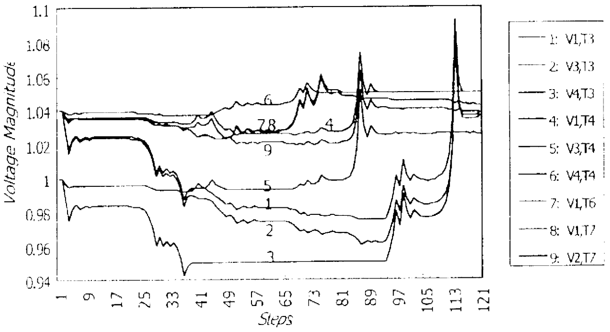

Figure 9. Behavior of some voltages during the trajectories.

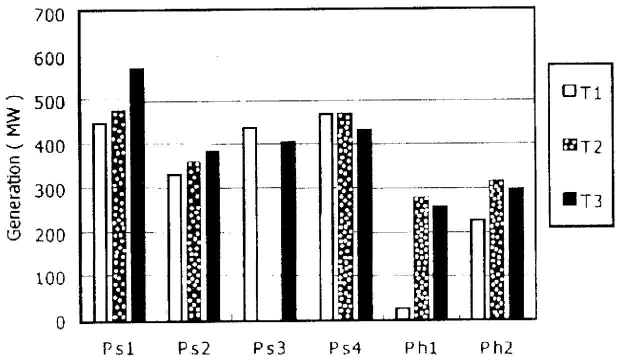

Figure 12. Final decision for generation (17-bus system).

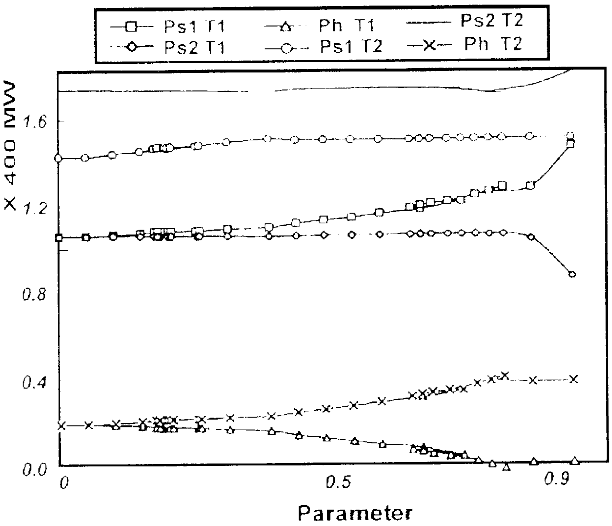

Figure 10. The case of no solution.

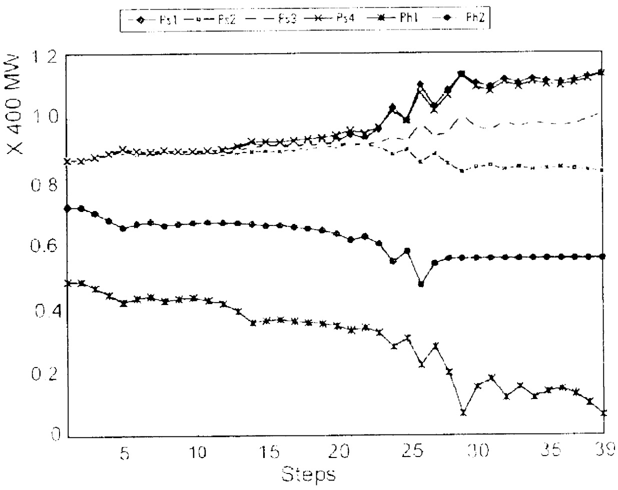

Figure 13. All units productions during 1st interval through trajectory.

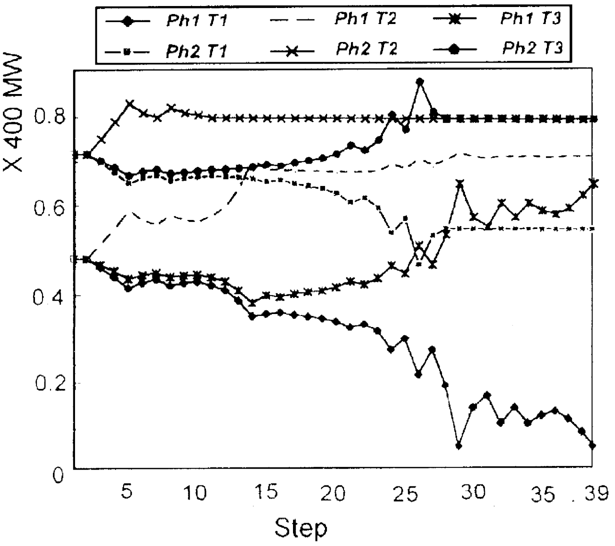

Figure 14. Hydro units productions through trajectory.

TABLE 3. Effect of W.

W	5	8	10	12	20	50
Number of Steps	50	43	40	41	42	57
Number of Breakpoints	11	9	9	10	11	11
Number of Violated* Constraints at ε=1	6	6	6	6	6	6

again violated. Following search, the voltage of bus 1 is fixed and others remain within bounds.

As stated earlier, if the problem has no solution it will be systematically detected. This is shown in Figure 10 where for a two-interval problem, total generation can not meet the required energy demand. That is why at $\varepsilon = 0.93$, the algorithm has stopped. So, other solutions such as unit recommitment, load shedding, etc. should be checked.

The choice of suitable values for W (Equation 19) is done through trial and error as shown in Table 3. It is evident that a value of 10 results in minimum number of steps.

To further show the capabilities of the algorithm, 17-bus New Zealand network with four thermal units and two hydro units is considered. The time period of study is 6 hours divided into 3 intervals. The data is provided in Appendix II.

Similar to Figures 6 and 7, the results for this system are shown in Figures 11 and 12. Also, in Figure 13, variations of all units' productions during the first interval are shown for the 39-step trajectory (see Figure 11). Figure 14, shows the variations of hydro units' productions during all three intervals.

CONCLUSIONS

Hydrothermal OPF problem was solved using continuation method. Besides its capabilities in solving problems which normal Newton method can not, better solutions are obtained as generation voltages are treated as variables (see Table 2). The authors are in the process of checking the algorithm for large-scale power systems.

APPENDIX I

Steam 1:

$F_1(P_1) = 561 + 5.92\,P_1 + .001\,P_1^2 \qquad 280^{MW} < P_1 < 800^{MW} \qquad -120^{MVAr} < Q_1 < 800^{MVAr}$

Steam 2:

$F_2(P_2) = 600 + 6.2\,P_2 + .001562\,P_2^2 \qquad 240^{MW} < P_2 < 720^{MW} \qquad -400^{MVAr} < Q_2 < 800^{MVAr}$

Hydro :

$q(P_h) = 320 + 12.3\,P_h \qquad 0 < P_h < 320^{MW} \qquad -80^{MVAr} < Q_h < 320^{MVAr}$

$V0 = 10000^{m3} \quad Vfinal = 10000^{m3} \quad Vmin = 8000^{m3} \quad Vmax = 22200^{m3}$

		T1	T2	T3	T4	T5	T6	T7	T8	T9	T10
Interval Length (hr)		2	1	1	1	5	2	4	3	3	2
Unit Commitment	Steam Unit 1	On	On	On	On	On	On	On	On	On	On
	Steam Unit 2	On	On	Off	Off	Off	On	On	On	On	On
Water Input (m^3/hr)	Hydro	1500	1500	1500	1500	1500	1450	1500	1500	1500	1450
Load (MW)	Node 1	320	240	88	80	60	140	208	260	288	204
	Node 2	280	200	100	120	104	148	200	304	176	252
	Node 3	280	220	96	100	56	128	120	280	212	168
	Node 4	280	240	100	100	60	160	124	252	100	80
	Node 5	200	140	60	140	60	140	180	280	220	180

APPENDIX II [10]

Steam Unit 1:

$F_1(P_1) = 561 + 5.92\,P_1 + .001\,P_1^2 \qquad 280^{MW} < P_1 < 600^{MW} \qquad -160^{MVAr} < Q_1 < 520^{MVAr}$

Steam Unit 2:

$F_2(P_2) = 600 + 6.2\,P_2 + .001562\,P_2^2 \qquad 240^{MW} < P_2 < 720^{MW} \qquad -200^{MVAr} < Q_2 < 600^{MVAr}$

Steam Unit 3:

$F_3(P_3) = 500 + 6.7\,P_3 + .0001\,P_3^2 \qquad 280^{MW} < P_3 < 800^{MW} \qquad -200^{MVAr} < Q_3 < 760^{MVAr}$

Steam Unit 4:

$F_4(P_4) = 600 + 5.8\,P_4 + .0012\,P_4^2 \qquad 120^{MW} < P_4 < 600^{MW} \qquad -200^{MVAr} < Q_4 < 600^{MVAr}$

Hydro Unit 1:

$q(P_{h1}) = 320 + 1.8\,P_{h1} \qquad 0 < P_{h1} < 720^{MW} \qquad -200^{MVAr} < Q_{h1} < 800^{MVAr}$

$V0_1 = 8000^{m3} \quad Vfinal_1 = 10000^{m3} Vmin_1 = 7800^{m3} \quad Vmax_1 = 22222^{m3}$

Hydro Unit 2:

$q(P_{h2}) = 320 + 2.37\,P_{h2} \qquad 0 < P_{h2} < 320^{MW} \qquad -80^{MVAr} < Q_{h2} < 320^{MVAr}$

$V0_1 = 12000^{m3} \quad Vfinal_1 = 12000^{m3} Vmin_2 = 7400^{m3} \quad Vmax_2 = 22200^{m3}$

		T1	T2	T3
Interval Length (hr)		2	2	2
Unit Commitment	Steam Unit 1	On	On	On
	Steam Unit 2	On	On	On
	Steam Unit 3	On	Off	On
	Steam Unit 4	On	On	On
Water Input (m^3/hr)	Hydro Unit 1	1000	1000	1000
	Hydro Unit 2	1000	1000	1000
Load (MW)		1868	1876	2424

REFERENCES

1. El-Hawary, M. E. and Ravindaranath, K. M., "Hydro Thermal Power Flow Scheduling Accounting for Head Variations", *IEEE Trans. on PWRS*, Vol. 7., No. 3, (1992), 1232-38.
2. Angeledis, G., "Hydrothermal Optimal Power Flow Based on Hydro Decomposition and Coordination", *Canadian Journal of Electrical and Computer Engineering*, Vol. 19, No. 2, (1994), 81-87.
3. Ibraham, E. S., "Dynamic Hydro-thermal Economic Dispatch with Water Availability Constraints and

Variable Water Head", ***Computer and Electrical Engineering***, Vol. 23, (1997), 15-23.

4. El-Hawary, M. E., Christensen, G. S., "Optimal Economic Operation of Electric Power Systems", Academic Press, New York, (1976).

5. Billinton, R. and Sachdeva, S. S., "Optimal Real and Reactive Power Operation in a Hydrothermal System", ***IEEE Trans. on PAS***, Vol. 91, (1972), 1405-1411.

6. Soares, S. and Ohishi, T., "Hydro Dominated Short-term Hydro-thermal Scheduling via Hybrid Simulation -Optimization Approach- A Case Study", ***IEE Proc.(Generation, Transmission and Distribution)***, Vol. 142, No. 6, (1995), 569-575.

7. Almeida, K. C., Galiana, F. D. and Soares, S., "A General Parametric Optimal Power Flow", ***IEEE Trans. on PWRS***, Vol. 9, No. 1, (1994), 540-547.

8. Garcia, C. B. and Zangwill, W. L., "Pathways to Solutions, Fixed Points and Equilibria", Prentice Hall, (1994).

9. Richter, S. L. and Decardo, R. A., " Continuation Method, Theory and Application", ***IEEE Trans. on AC***, Vol. 28, No. 6, (1983), 660-665.

10. Arrillaga, J. and Arnold, C. P., "Computer Analysis of Power Systems", John Wiley & Sons, (1990).