RCD RULES AND POWER SYSTEMS OBSERVABILITY

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Abstract Power system state estimation is a process to find the bus voltage magnitudes and phase angles at every bus based on a given measurement set. The state estimation convergency is related to the sufficiency of the measurement set. Observability analysis actually tests this kind of problem and guarantees the state estimation accuracy. A new and useful algorithm is proposed and applied in this paper for power system observability. Through the use of this practical method, one can test the observability of power systems for a specific measurement set. In this new method, which we call Row-Column-Diagonal (RCD) method, the electric power network is transformed to a useful graphical network named pseudo-network and then observability study is applied. This new method is implemented on some example networks which are already tested.

Key Words State Estimation, Observability, Power Systems, Pseudo-Network, RCD Rules

چکیده تخمین حالت سیستمهای قدرت فرآیندی است که براساس اندازه گیریهای انجام یافته از سیستم اقدام به تخمین مقادیر دامنه و زاویه فاز ولتاژ شینها مینماید. همگرایی عملیات تخمین حالت بستگی به کافی بودن تعداد اندازه گیریها دارد. تحلیل رویت پذیری، مناسب بودن مجموعه اندازه گیریها را بررسی نموده و همگرایی عملیات تخمین حالت را تضمین می کند. دراین مقاله الگوریتم جدیدی برای تست رویت پذیری سیستمهای قدرت پیشنهاد شده است. با استفاده از این روش، براحتی می توان رویت پذیری سیستمهای قدرت را براساس مجموعه اندازه گیریها انجام داد. این روش RCD نامیده می شود و درآن سیستم قدرت به گراف دیگری بنام شبه شبکه تبدیل شده و سپس مطالعات رویت پذیری در روی این شبکه اعمال می گردد. این روش جدید برروی برخی از شبکه های قدرت که قبلاً بررسی شده اند اعمال می شود.

INTRODUCTION

Observability study plays an important role in state estimation calculations. The state estimation is a basic analysis for power systems control, which determines the optimal states of the networks with filtering the noisy measurements [1]. The correction and convergence of the state estimation is fully related to the types, number, and location of the measurement set. Therefore, attempt should be made to use algorithms capable of searching amongst various measurement sets and choosing the best option.

The power system will be observable if the variables of the state estimation process (magnitude and angles of the buses voltage) are calculated based on the network measurements. In a power system the types of the possible measurements are as follows:

1. Active and reactive bus injection powers.
2. Active and reactive line flow powers.
3. Bus voltage magnitudes.
4. Generator buses frequency.

The measurements observe the whole non-redundant states of the network of which the values do not reject one another.

Power system observability is classified into two categories: topological observability and numerical observability. The topological observability is the general definition of the observability and is completed by numerical observability which is related to the convergence of the state estimation process steps.

The power system is topologically observable if the whole states could be estimated based on the measurement set. In other words, in topological observability the gain matrix must have the minimum rank equal to the network states (magnitudes and angles of the buses voltage without slack bus voltage angle). The gain matrix is the product of the transpose of the observer matrix (in the observer model which defines the relations of the measurements and states), inverse of the observer convariance matrix and the observer matrix. The power system is numerically observable if the state estimation process is converged based on the measurement set and initial assumptions. In other words, the network is numerically observable if the gain matrix have the minimum rank of the network states in each estimation step. If a network is not topologically observable, then it is required that the observability of the network be tested through numerical observability method.

Each observability case is divided into active power-voltage angle (P-δ) and reactive power-voltage magnitude (V-Q) observability. In the P-δ observability, the existence of estimation of the whole bus voltage angles is tested using the active power measurements. The existence of estimation of the whole bus voltage magnitudes is also checked by the Q-V observability analysis based on the reactive power and bus voltage magnitude measurements.

This paper offers a new topological method call RCD method here that has a simple algorithm and fast convergence calculation operations. In this approach, the electric network structure is transformed to a pseudo- network construction, and the relation of buses and lines are represented by the special shapes on a lower triangular matrix. The RCD method is applied on the power test networks which are presented in the references [2-10]. The results are compared with other methods in the references. For simplicity of operations, only the P-δ observability is checked. The results show that the RCD method is faster than other most frequently used approaches under unequal conditions, with a single processor and with low clock rate of a PC-80486 computer. One of the important advantages of the RCD method is the simplicity of the method which does not require the network bus numbering sequences. Neither does the method require heavy searching calculations and definitions.

PREVIOUS APPROACHES

In recent years, the power systems observability has been the subject of various investigations some of which are listed in the references. The power systems observability methods can be categorized into two classes of topologically based [2-5] and numerically (or floating-point calculations) based [6-10].

Krumpholtz, Clements and Davis [2] have used a graph-theoretic algorithm for topological observability. According to their works, the observability of the power systems depends on at least one spanning tree of full rank finding. The algorithm provided has a complex implementation since all trees of the network must be tested to find an observable tree.

Quintana, Simoes-Costa and Mandel [3] developed a modified searching method to find an observable spanning tree which could also be formulated as a Matroid intersections algorithm.

Bargiela, Irving and Sterling [4] proposed an algorithm based on observability graph theory [2] to find an observable spanning tree for observable cases and the maximum observable forest for unobservable cases. The algorithm is also computationally complex.

Nucera and Gilles [5] used the concept of augmented sequences to determine a maximal forest of full rank utilizing the graph theory concept [2]. It is obvious that, the above and other non-numeric methods, or topological observability algorithms, originally ultimate the graph theory approach presented in reference [2].

However, the topological (combinatorial or symbolic) methods do not analyze the measurement of Jacobian or gain matrix and hence have difficulties for observability detection in the situation of singular gain matrix. The numerical observability methods, on the other hand, are advantageous in observability determination, through they require heavy calculations.

Monticelli and Wu [6] proposed a method for numerical observability using triangular factorization of the gain matrix. The triangular factorization is derived for state estimation process using normal equation approach.

A method based on symbolic reduction of the measurement of Jacobian matrix was developed by Slutsker and Scudder [7]. This method is quite simple and fast, but observability tests may give incorrect results for each non-zero element which is located in the Jacobian matrix.

Wu, et. al. proposed another floating-point observability method using Hachtel's method [8]. In state estimation analysis using Hachtel's approach, a solvable matrix equation is made instead of the gain matrix with equality constraints. Based on this process, they extended the factorization method to observability analysis.

Through the use of the approach presented by Slutsker and Scudder [7], Chen developed an integer algorithm for determining the observability with linearized state estimation model [9]. In this method, only the integer values of the non-zero measurement elements are located in the Jacobian matrix. However, the method leads to incorrect results for certain cases.

Falcao and Arias [10] introduced approach which utilizes the Echelon form of the linearized measurements model. This method is a generalized form of matrix triangular factorization. The results obtained are congruent with the results of the cases mentioned in reference [6].

In most topological methods, the critical or spanning tree foundations is almost to the same extent problematic. The numerical methods find the gain matrix rank and use the triangular matrix form to reduce calculations. The observability analysis has three objectives as follows:

1. Testing the network observability using given measurement set.

2. Determining the observable and unobservable islands in unobservable cases.

3. Adding the minimal pseudo-measurements to the measurement set of the unobservable network to make it observable.

Unfortunately, most methods do not provide answers to all observability problems; most of them only determine the network observability test. Therefore, these methods require another algorithm to solve other observability requirements.

FUNDAMENTALS OF RCD METHOD

In this paper, we propose an answer to the questions of the power systems observability in topological form, which is based on the fundamentals of the method described in [11]. The rules of this method are applied to the transformed power network, named as pseudo-network. In pseudo-network, the network lines and buses are located in the rows, columns and diagonals of a lower triangular construction. In the pseudo-network configuration, the buses and lines are shown with rectangular and circular shapes. If the elements include an injection or flow power measurements, then the related shape is filled. Therefore, the proposed rules are related to rows, columns and diagonals elements. Hence, they are called RCD rules.

The idea of the RCD method originates from the basic electrical laws of KVL and KCL. These laws explain the relations between the lines (with number n_l) in some network loops or connected to some network buses (with number n_b). Therefore, using the KVL and KCL laws, the relations of buses and lines are written in terms of power equations. The number of KVL and KCL equations are equal to the number of network loops (n_l-n_b+1) and buses without slack bus (n_b-1). The results of the RCD rules application must be similar to those of the linear equation set solution which is obtained from the nodes and loops relations. This is done by appropriate rules sequences operation.

For example, suppose the RCD method is applied to a full connected 3-bus network. Figures 1 and 2 show this network and the related pseudo-network, respectively.

Choosing the earth as the reference bus, we write the KCL, KVL, OHM and power relations as follows:

definition: $Y_{i.j} = Y_{j.i}$ $\quad I_{i.j} = -I_{j.i}$

$V_{i.j} = V_i\text{-}V_j$ $\quad V_{i.j} = -V_{j.i}$

KCL:
$$I_1 = I_{1.2} + I_{1.3}$$
$$I_2 = I_{2.1} + I_{2.3} \qquad (1)$$
$$I_3 = J_{3.1} + I_{3.2}$$
$$(I_1 + I_2 + I_3 = 0)$$

KVL: $$V_{1.2} + V_{2.3} + V_{3.1} = 0 \qquad (2)$$

OHM: $$Y_{1.2}\, V_{1.2} = I_{1.2}$$
$$Y_{2.3}\, V_{2.3} = I_{2.3} \qquad (3)$$
$$Y_{1.3}\, V_{1.3} = I_{1.3}$$

POWER: $$S_i = V_i I_i^* \qquad (4)$$
$$S_i^* = P_i - jQ_i = V_i^* I_i \qquad (5)$$

Notice that two equations from KCL Equations 1 (n_b - 1 equations) and one KVL Equation 2 (n_l-n_b+1 equations) are used to calculate the network currents and voltages. If the injected parameters of the buses are known and one of the current or voltage relations is eliminated (using OHM equations), then there are three unknown voltages or currents in the 3-bus network. In other words, if we choose the bus currents or voltages, then there are n_b unknown parameters and network equations. If the line currents or voltages are chosen, then there are n_l unknown parameters and also network equations. Further, the KCL equations can be written in the form of load flow equations using OHM and powers relations. In the load flow form, the variables are chosen as the bus voltage magnitudes and angles in a complex form.

$$P_1\text{-}jQ_1 = V_1^* (-Y_{1.1} V_1 - Y_{1.2} V_2 - Y_{1.3} V_3)$$
$$P_2\text{-}jQ_2 = V_2^* (-Y_{2.2} V_2 - Y_{2.1} V_1 - Y_{2.3} V_3)$$
$$P_3\text{-}jQ_3 = V_3^* (-Y_{3.3} V_3 - Y_{3.1} V_1 - Y_{3.2} V_2)$$

$$Y_{i.i} = -\sum_{\substack{j=1 \\ j \neq i}}^{j=n_b} Y_{i.j} \qquad i = 1, \ldots, n_b$$

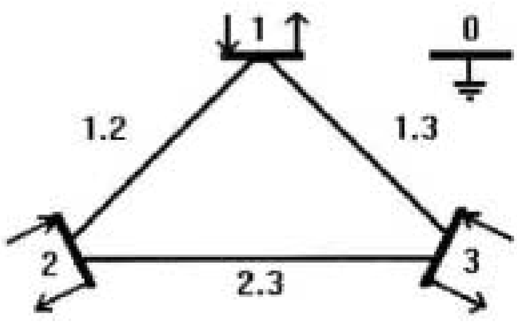

Figure 1. 3-bus power network.

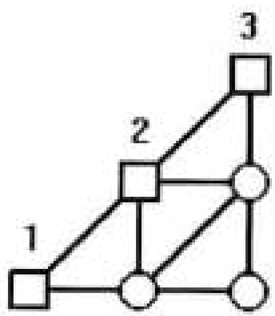

Figure 2. 3-bus power pseudo-network.

The above load flow equations could be solved for unknown complex bus voltages $(V_i = V_i \angle \delta_i)$ by iterative analytical methods. This is possible if the active and reactive injection powers are known. Otherwise, there will be a state estimation problem, and a number of measurements must be done for the estimator.

In the RCD method, the bus injection powers are analyzed using the injection and flow powers and voltage magnitudes measurements. Then, the power system observability is defined, and voltage magnitudes and angles are calculated. The RCD rules relate the KCL and KVL equations which are transformed to the complex power variables. Therefore, the RCD method works on the changed KCL and KVL equations which are indicative of the existence of the network powers. Rejecting the complex power representation, the RCD equations are as follows:

$$\begin{aligned} &1+2+3=0 \\ &1=1.2+1.3 \\ &2=2.1+2.3 \\ &3=3.1+3.2 \\ &1.2+2.3+3.1=0 \end{aligned} \qquad (6)$$

In the RCD equations, if there is a measured bus or line, it is shown by underlined numeral (e.g. $\underline{3}$) in the equation set. The unvalued elements which are calculated by RCD rules are marked by over head bar (e.g. $\overline{1.2}$). After applying the measurement elements to RCD equations, the RCD rules analyze the equations solution on the pseudo-network. Notice that in the RCD equations, only the number of $n_l\,(n_l=(n_b-1)_{KCL}+(n_l-n_b+1)_{KVL})$ equations are used which are equal to the sum of KCL and KVL equations numbers.

THE RCD RULES

The RCD method tranforms the electric power network to a lower triangular matrix form, which is named pseudo-network. In this representation, the network buses are located at the major diagonal with shapes of square. The network lines are at the minor diagonals with circular shapes. If there is a bus injection or a line flow power measurement, the related shape will be filled in the pseudo-network. To complete the pseudo-network, the parallel lines are drawn with minor diagonals which contain a number of connected network lines.

In the RCD method, observability testing is done by calculating the values of unmeasured rows, columns and diagonals elements using the values of the measured elements. In the power network, it is like the solution of the equation set to find the unknown buses and lines or state estimation process. In the static state estimation, the states of the network are estimated on the basis of the measurement model. The nonlinear and linearized measurement or observer models are as follows:

$$z = h(x) + v \qquad (7)$$

$$\Delta z = H.\,\Delta x + v \qquad (8)$$

In Equation 7, the measurement vector z contains the known measured values, the state vector $\boldsymbol{x}$ includes the unknown bus voltage angles and magnitudes, and v is the measurement noise vector. The nonlinear relation between z and $\boldsymbol{x}$ is characterized by nonlinear equation $\boldsymbol{h(x)}$ which is linearized as $\boldsymbol{H} = \partial \boldsymbol{h(x)}/\partial \boldsymbol{x}$ in Equation 8. In a load flow process, z is a deterministic vector, and $\boldsymbol{H} = \boldsymbol{J}$ is Jacobian matrix in which the state vector $\boldsymbol{x}$ is calculated by iteration methods. However, in the state estimation, z is random and noisy vector, and $\boldsymbol{x}$ is estimated by optimizing an object function, such as least squares of calculated and measured values.

$$\Delta \hat{x} = (H^T R^{-1} H)^{-1} H^T R^{-1} \Delta z \qquad (9)$$

R is the covariance matrix of the network measurements. Therefore, the state estimation convergency is directly related to adequacy of the measurement vector z and observation matrix $\boldsymbol{H}$. Ideally, if there exists a complete measurement vector with all buses injection powers or lines flow powers, a successful process of the state estimation can be expected.

The RCD rules check the solution possibility of unknown buses and lines by other measured elements. If all network buses have measured or calculated values, then all network lines will be valued and power system will be observable. Therefore, the RCD rules are applied to pseudo-network to test the existence of the buses injection power values. The pseudo-network structure varies with the variation of bus numbers. Therefore, the RCD rules must be independent of bus numbering. In the pseudo-network, we identify the various consisting elements and their interconnections as a set of relationships. The general RCD rules are explained as follows [11]:

1. RC-bus Rule: Each bus value in the main diagonal is equal to the sum of the line values in the related row and column or *RC-bus* equation. If only one of the elements in each *RC-bus* relation does not have a measured or calculated value, it will be calculated by this rule.

2. D-bus Rule: The sum of bus values in the main diagonal (*D-bus*) is equal to zero. Hence, if only one bus does not carry a known value, it will be calculated. Once the D-bus rule is executed, an observable network is attained.

3. RC-line Rule: Each *RC-line* equation includes a set of related lines. Based on this rule, each unknown line will be calculated if all other elements in *RC-line* equation have known values. If the beginning and end of a sequential set of lines is placed on a *D-line* equation and include common line located on another *D-line*, these will be a combined *RC-line* equation.

4. D-line Rule: A selected group of unknown lines will be computed if it has the necessary observability condition. If it is assumed that a group of unknown lines is calculated, then it is required that all the elements of their related buses must be computed using other rules. These groups are defined by the rows and columns for which all their elements are unknown.

If a network is unobservable, the minimum number of unknown lines in *D-line* equations defines the number of the pseudo-measurements.

The main advantage of these RCD rules is its simple logic which does not require extra and complex definitions. The RCD method searches the pseudo-network through its effective rules without numerical computations. Hence, a powerful computer language, such as MATLAB, is chosen to implement the proposed algorithm with its inherent search mechanism. The RCD rules must be applied to RC-bus, D-bus, RC-line and D-line rules, respectively, to obtain correct results.

The RCD method solves the three observability problems easily. After applying the RCD rules, if any bus does not have the measured or calculated value, the power system is unobservable. In this case, some of the unknown lines also divide the network to some observable and unobservable islands. The unvalued buses and lines are candidates as pseudo-measurements to make the network observable.

RESULTS

In this section, we apply the RCD method to the 6,9,13 and 14-bus example networks which are described in the references. These networks are special cases and proper to check the accuracy of the method. The results are exactly the same as the results given in the references. They show that the RCD method is powerful. The networks and their pseudo-networks are shown in Figures 3-7. For simplicity of the operations, only the observability case of P-δ is only

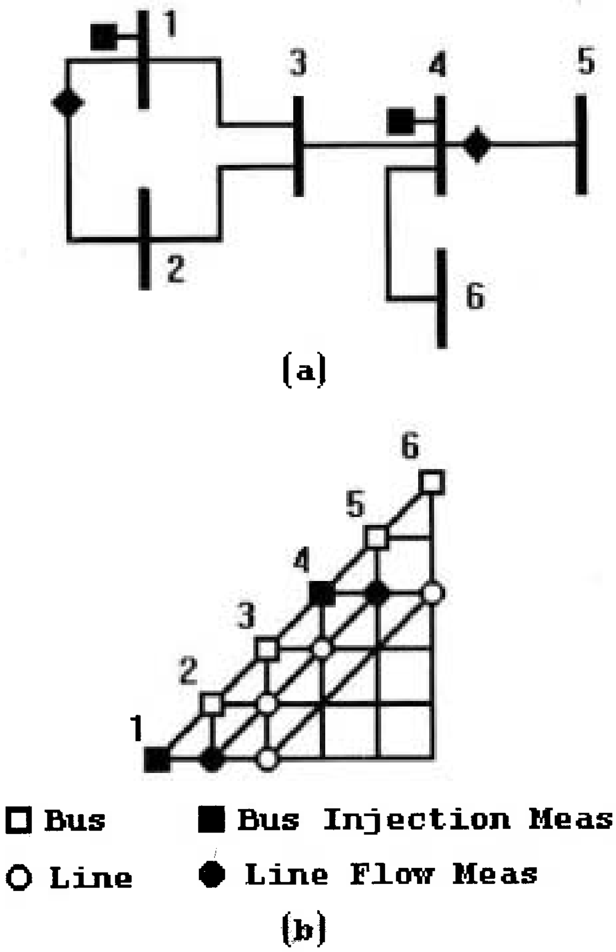

Figure 3. (**a**) The 6-bus power network (**b**) and its pseudo-network.

tested. Therefore, the measurements in each network are active bus injection and line flow powers.

The network in Figure 3(a) has KCL and KVL equations which can be transferred to power relations. These equations are changed to RCD relations in the pseudo-network in Figure 3(b). First the RC and D elements are defined.

Each element which is measured in power network or filled in the pseudo-network is shown in the RCD equations with an under line. If an element is calculated by RCD rules then it is shown by over head bar.

• Measurement Set:

Bus Power Injection Meas.= {$\underline{1}$, $\underline{4}$}

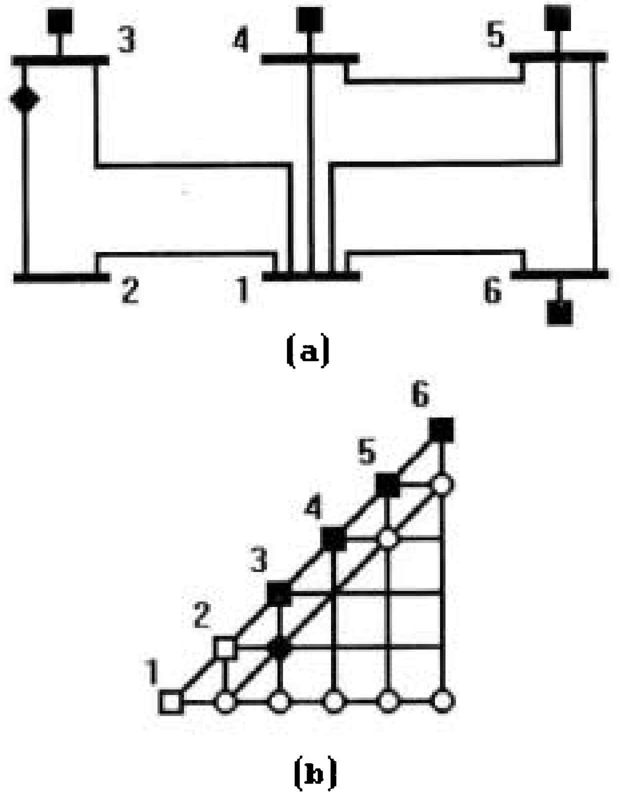

Figure 4. (**a**) Another 6-bus power network (**b**) and its pseudo-network.

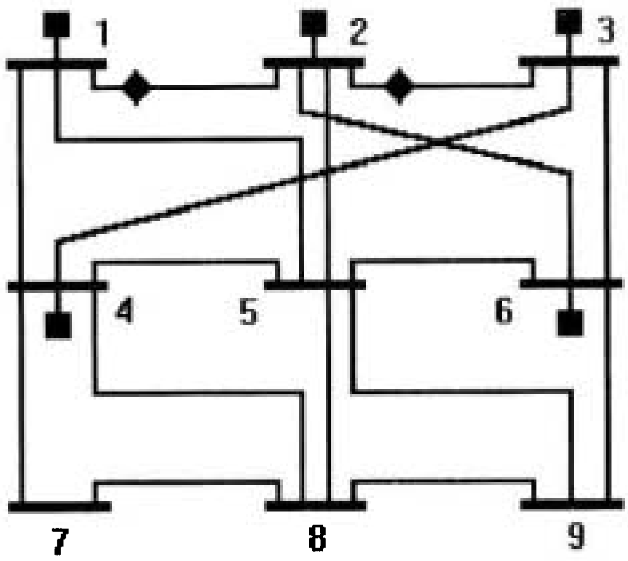

Figure 5. The 9-bus power network.

Line Power Flow Meas.= {$\underline{1.2}$, $\underline{4.5}$}

RC-bus 1: 1= $\underline{1.2}$ + 1.3 (10)

TABLE 3. Step of the RCD Rules Implementation for 9-bus Power Network in Figure 5.

Calculated Bus or Line	RCD Rule
group 1 1.4 3.4	D-line (fail)
group 2 2.6 3.6	D-line
$\overline{2.5}$	RC-bus 2
$\overline{3.4}$	RC-bus 3
$\overline{1.4}$	RC-line 5
$\overline{1.5}$	RC-bus 1
$\overline{4.5}$	RC-line 1
$\overline{5.6}$	RC-line 3
$\overline{6.9}$	RC-bus 6
Unobservable Network	—

TABLE 4. Steps of the RCD Rules Implementation for 13-bus Power Network in Figure 6.

Calculated Bus or Line	RCD Rule
$\overline{12.13}$	RC-bus 12
group 1 $\overline{3.4}$ $\overline{3.5}$ $\overline{4.6}$ $\overline{5.7}$ $\overline{6.8}$ $\overline{7.9}$	D-line
$\overline{8.10}$	RC-bus 8
$\overline{9.10}$	RC-bus 9
Unobservable Network	—

work is unobservable, and its islands are $\{\underline{1},\underline{2},\underline{3},\underline{4},\underline{5},\underline{6},\underline{7},\underline{8},\underline{9},10\}$ and $\{11,\underline{12},\underline{13}\}$. The 14-bus power network with injection measurement 5 is observable. Finally, the 14-bus power network without injection measurement 5 is unobservable, and using Table 6, the islands are $\{\underline{1},\underline{2},\underline{3},\underline{4},5,\overline{7},\overline{8},\underline{9}\}$, $\{\underline{6}\}$, $\{10\}$,

TABLE 5. Step of the RCD Rules Implementation for 14-bus with Injection Measurement 5.

Calculated Bus or Line	RCD Rule
$\overline{1.5}$	RC-bus 1
$\overline{7}$	RC-bus 7
$\overline{8}$	RC-bus 8
$\overline{2.5}$	RC-line 1
group 1 $\overline{2.3}$ $\overline{2.4}$ $\overline{3.4}$ $\overline{4.5}$ $\overline{5.6}$	D-line
group 2 $\overline{6.12}$ $\overline{6.13}$ $\overline{9.11}$	D-line
$\overline{6.10}$	RC-bus 6
$\overline{10.11}$	RC-bus 11
$\overline{10}$	RC-bus 10
$\overline{14}$	D-bus
Unobservable Network	—

TABLE 6. Steps of the RCD Rules Implementation for 14-bus without Injection Measurement 5.

Calculated Bus or Line	RCD Rule
$\overline{1.5}$	RC-bus 1
$\overline{7}$	RC-bus 7
$\overline{8}$	RC-bus 8
$\overline{2.5}$	RC-line 1
group 1 $\overline{2.3}$ $\overline{2.4}$ $\overline{3.4}$ $\overline{4.5}$	D-line
group 2 6.12 6.13 9.11	D-line (fail)
group 3 12.13	D-line (fail)
Unobservable Network	—

{11}, {12}, {13} and {14}.

CONCLUSIONS

Observability analysis is a basic problem in state estimation process. In this paper, the observability analysis based on the developed RCD method is proposed. The RCD method includes four rules which are applied in priority order to a transformed network, named pseudo-network. The RCD rules are capable of answering different kinds of observability questions. In this method, the power network will be observable if all buses have measured or calculated values. Otherwise, the power system is unobservable, and the observable, unobservable islands as well as pseudo-measurements are defined. The method is applied to 6,9,13, and 14-bus test networks.

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