RESEARCH NOTE

ON THE OPTIMUM DIRECTIVITY OF UNIFORMLY SPACED BROADSIDE ARRAYS OF PARALLEL HALF-WAVE DIPOLES

J. Rashed-Mohassel

Department of Electrical and Computer Engineering
Faculty of Engineering
Tehran University

Abstract The nominal directivity for uniformly spaced broadside parallel half-wave dipoles associated with a uniform excitation is evaluated. The amplitude distribution for an optimized directivity is then obtained for different numbers of elements with the separations between the dipoles as a variable. The optimum and nominal directivities are compared for different spacings of the elements. While these directivities are different for small separations, they are practically the same for increased spacings of the array elements.

Key Words Antennas, Linear Arrays, Antenna Arrays, Arrays of Dipoles

چکیده راستاوری نامی برای آرایه ای از دوقطبی های نیم موج موازی با تابش جانبی (Broadside) و با تحریک جریان یکسان محاسبه شده و سپس توزیع دامنه تحریک برای بهینه سازی راستاوری با تعداد عناصر مختلف بد ست آمده است. دراین محاسبه فواصل بین دوقطبی های نیم موج به عنوان متغیر انتخاب گردیده است. راستاوری های نامی و بهینه برای فواصل مختلف عناصر آرایه مقایسه شده است. درحالیکه این راستاوری ها برای فاصله های کم عناصر با هم متفاوتند، با افزایش فاصلهٔ عناصر آرایه عملاً با هم یکسان می گردند.

INTRODUCTION

Optimization of the directivity in linear arrays has been the subject of many investigations and different schemes of optimization have been considered in the past [1-3]. A generalized treatment of the optimization problem is reported in a concise manner [4-6]. The problem of maximizing the directivity of a linear array with equally spaced isotropic elements was first studied in 1946 [7]. The maximum directivity of some linear arrays with half-wave dipole elements from the viewpoint of self and mutual resistances of the elements has also been addressed and few calculations mentioned [8]. The optimization of fundamental sources such as uniformly spaced isotropic elements and infinitesimal dipoles has been thoroughly considered and the corresponding curves are reported [9]. However, these fundamental sources are not used as practical elements in application. As for the half-wave dipoles, the optimum and nominal directivities of a broadside array of collinear dipoles have been considered [10] and the amplitude distribution for an optimum directivity is obtained. In this communication linear arrays of parallel half-wave dipoles are considered and the nominal directivities of the corresponding uniformly spaced arrays are obtained. The directivity is presented as several sets of curves with the spacings of the elements as a variable. The optimum directivity for the broadside case is then obtained and the amplitude distribution is presented for different numbers of elements and various spacings.

GENERAL FORMULATION

Figure 1 shows the geometry of the parallel half-wave dipoles located in the Cartesian coordinate system. The array axis is assumed along the z axis and "d" is the separation between the dipole elements.

The array factor of a linear broadside array is well known and given by

$$\sum_{n=1}^{N} a_n e^{j(n-1)kd\cos\theta}$$

where, N is the number of elements and k is the propagation constant. The element pattern function in the (θ, φ) direction, $f(\theta, \varphi)$ in the given coordinate system is

$$f(\theta,\varphi)=\frac{\cos\left(\frac{\pi}{2}\cos\theta_x\right)}{\sin\theta_x} \tag{1}$$

in which θ_x is the angle between the (θ, φ) direction and the x axis. The nominal directivity for the broadside radiation, $D_n(\theta, \varphi)$ is therefore given by

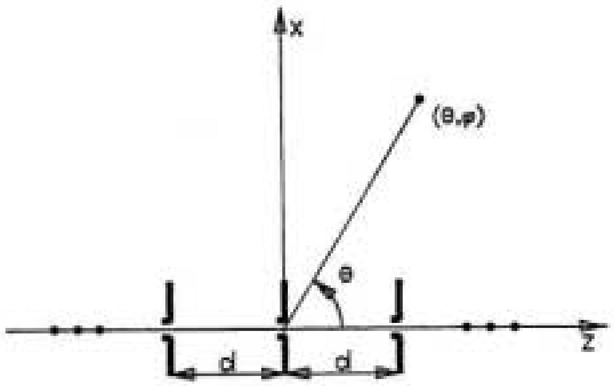

Figure 1. The geometry of parallel half-wave dipoles.

$$D_n(\theta,\varphi)=\frac{N^2}{\frac{1}{4\pi}\int_0^{2\pi}\int_0^{\pi}\frac{\cos^2\left(\frac{\pi}{2}\sin\theta\cos\varphi\right)}{1-\sin^2\theta\cos^2\varphi}\sum_{m=1}^{N}\sum_{n=1}^{N}e^{jrD\cos\theta}\sin\theta d\theta d\varphi}$$

$$=\frac{N^2}{\sum_m\sum_n\beta_{mn}} \tag{2}$$

where, $r = m-n$ and $D= kd$. The β_{mn} in (2) is defined by

$$\beta_{mn}=\frac{1}{4\pi}\int_0^{2\pi}\int_0^{\pi}\frac{\cos^2\left(\frac{\pi}{2}\sin\theta\cos\varphi\right)}{1-\sin^2\theta\cos^2\varphi}e^{jrD\cos\theta}\sin\theta d\theta d\varphi \tag{3}$$

Decomposition of the exponential term in the integrand of Equation 3 into cosine and sine functions will reduce β_{mn} to

$$\beta_{mn}=\int_0^{2\pi}F(\theta)\cos(rD\cos\theta)\,d\theta \tag{4}$$

where,

$$F(\theta)=\frac{1}{8\pi}\int_0^{2\pi}\frac{\cos(\pi\sin\theta\cos\varphi)}{1-\sin^2\theta\cos^2\varphi}d\varphi \tag{5}$$

In contrast to the case of collinear half-wave dipoles [10], the double integral involved in β_{mn} term is not separable. In an attempt to find a closed form expression for β_{mn}, we substitute $z=\exp(j\varphi)$ and Equation 5 is transformed to

$$F(\theta)=\frac{1}{2\pi j}\oint_c\frac{z\cos\left[\frac{\pi}{2}\sin\theta\left(\frac{1+z^2}{z}\right)\right]}{4z^2-\sin^2\theta(1+z^2)^2}dz \tag{6}$$

where C is the unit circle. The integrand has four poles at: $\pm\tan\theta/2$ and $\pm\cot\theta/2$ with an essential singularity at z=0. It is straightforward to show that for $0<\theta<\pi/2$ the first pair of poles, i.e., $\pm \tan\theta/2$ is within the unit circle and for $\pi/2<\theta<\pi$ the second pair is interior to that circle and in both cases the residues of these poles cancel each other and therefore $F(\theta)$ is reduced to

$$F(\theta)=\mathrm{Res}\left\{\frac{z\cos[\frac{\pi}{2}\sin\theta\,(\frac{1+z^2}{z})]}{4z^2-\sin^2\theta\,(1+z^2)^2}\right\}_{z=0}$$

where the right hand side is the residue at the essential singularity and can be obtained by expanding $F(\theta)$ in a Laurent series. An investigation shows that this residue yields a series involving modified Bessel functions of the first kind which can not be expressed in a closed form. In view of the above discussion, the β_{mn}, in its most simple form is given by

$$\beta_{mn}=\frac{1}{8\pi}\int_0^{2\pi}\int_0^{\pi}\left[\frac{\cos(\pi\sin\theta\cos\varphi)}{1-\sin^2\theta\cos^2\varphi}\right]\cos(rD\cos\theta)\sin\theta\,d\theta\,d\varphi \tag{7}$$

Although the procedure in obtaining β_{mn} for collinear half-wave dipoles [10] is similar to the case of parallel dipoles, the integration should be evaluated numerically in the latter. In view of Equation 3 and using $\beta_{mn}=\beta_{nm}=P_r(D)=P_{-r}(D)$, the nomial directivity, D_n, is reduced to

$$D_n=\frac{N^2}{NP_0+2\sum_{r=1}^{N-1}(N-r)P_r(D)} \tag{8}$$

with $P_r(D)$ given by Equation 7 which should be evaluated numerically. For isotropic sources and infinitesimal dipoles, this expression is given by simple trigonometric functions [9]. In the case of collinear half-wave dipoles, β_{mn} can be explicitly given in a closed form [10]. For an endfire array in the $\theta=0$ direction, the array factor is slightly modified and with parallel half-wave dipoles as elements of an endfire array, β_{mn} follows from a similar procedure and is reduced to

$$\beta_{mn}=e^{-jrD}P_r(D) \tag{9}$$

and the corresponding nominal directivity after some manipulations reduces to

$$D_{ne}=\frac{N^2}{NP_0+2\sum_{r=1}^{N-1}(N-r)P_r(D)\cos rD} \tag{10}$$

where $P_r(D)=\beta_{mn}$ is defined as before.

OPTIMUM DIRECTIVITY

Let a_i be the current amplitude of the *i*th element of the broadside array. The directivity, which is to be optimized is given by

$$D_0=\frac{\sum_i\sum_j a_i a_j}{\sum_{i=1}^{N}\sum_{j=1}^{N}\beta_{ij}\,a_i\,a_j} \tag{11}$$

For a maximized directivity, D_0, we have

$$\frac{\partial D_0}{\partial a_p}=0 \qquad p=1,2,\ldots,N \tag{12}$$

which is transformed to the matrix form [9];

$$[\beta]\,[a]=[k] \tag{13}$$

where, β_{ij} in (13) are the elements of the N×N matrix and k is a constant independent of P. substituting a_i's from Equation 13 in Equation 11, the optimum directivity, D_0, reduces to [9, 10];

$$D_0 = \frac{\sum_{i=1}^{N} a_i}{k} \qquad (14)$$

The relative amplitudes and the optimum directivity can be obtained from Equations 13 and 14 respectively. Since the relative amplitudes are of interest, k is an arbitrary constant and can have any value.

NUMERICAL RESULTS

A numerical evaluation of the $P_r(D) = (\beta_{mn})$ function given by Equation 7 and substituting in Equation 8 yields the nominal directivity, D_n for the broadside parallel half-wave dipoles. To find the optimum directivity, the current distribution, $[a] = [a_1, a_2, \ldots, a_N]^T$ can be obtained from Equation 13 and the optimum directivity, D_0, then follows from Equation 14. In these calculations, the separation d, is varied up to 2λ. Figures 2 and 3 show the nominal and optimum directivities, for N equals to 3,4,5,6, respectively. A comparison between the optimum directivities of short dipoles [9] and half-wave dipoles is shown in Figure 4. These curves show an increase in the overall directivity of an array with half-wave dipole elements. Since the directivity of a single half-wave dipole is slightly higher than that of a short dipole, this increase in the optimum directivity is predictable. The current distribution for optimizing the directivity of several arrays was obtained. The relative current distribution for an array with 4 and 6 half-wave dipole elements is given in Figures 5 and 6 respectively. In these curves, the excitation amplitudes of the dipoles are normalized to the amplitude of the first element and the relative amplitudes are plotted against the separation between the elements. An investigation of the current distribution shows that although the amplitudes vary significantly from

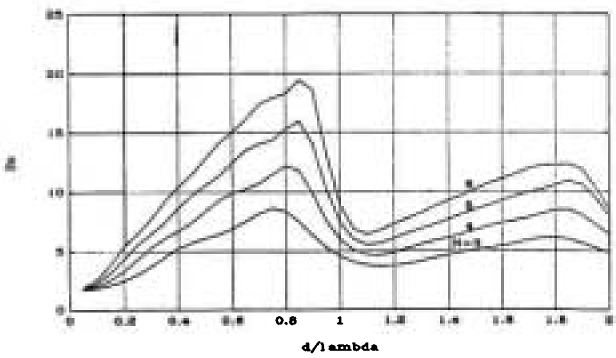

Figure 2. Nominal directivity of parallel half-wave dipoles, N=3 to 6.

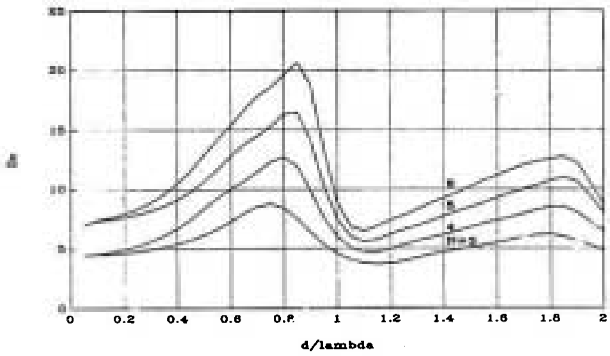

Figure 3. Optimum directivity of parallel half-wave dipoles, N=3 to 6.

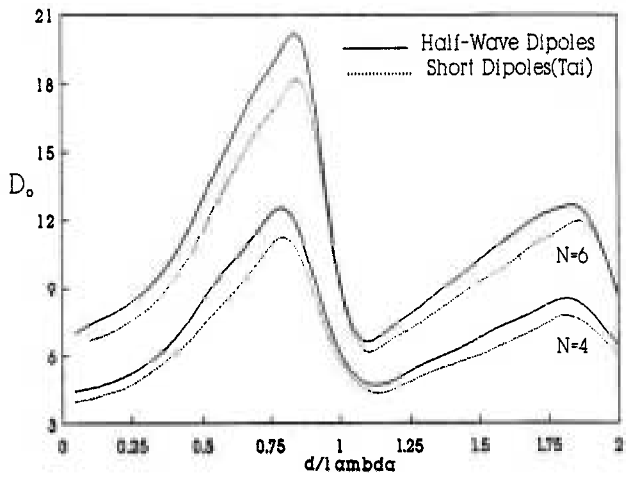

Figure 4. Comparison of the optimum directivities of parallel short dipoles and half-wave dipoles.

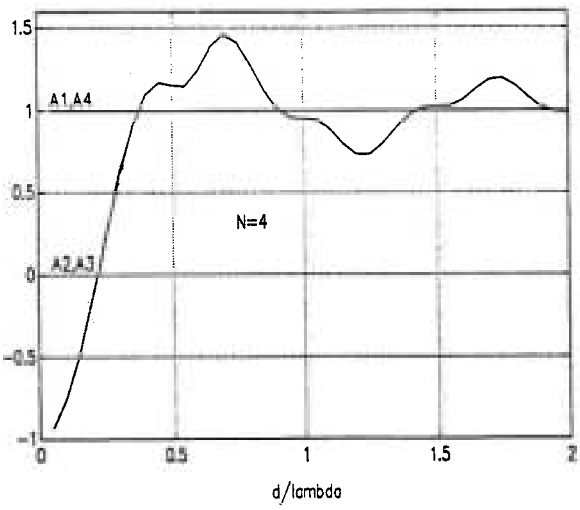

Figure 5. Amplitude of excitation for an array with 4 elements.

element to element, the optimum directivity itself exceeds, only marginally, the nominal directivity associated with the uniform excitation particularly for higher separations. The same situation is reported for short parallel dipoles [9]. Figure 7 compares the nominal and optimum directivities for N=4. For separations greater than ≈0.8λ, which also results in longer arrays, a uniform excitation is therefore sufficient.

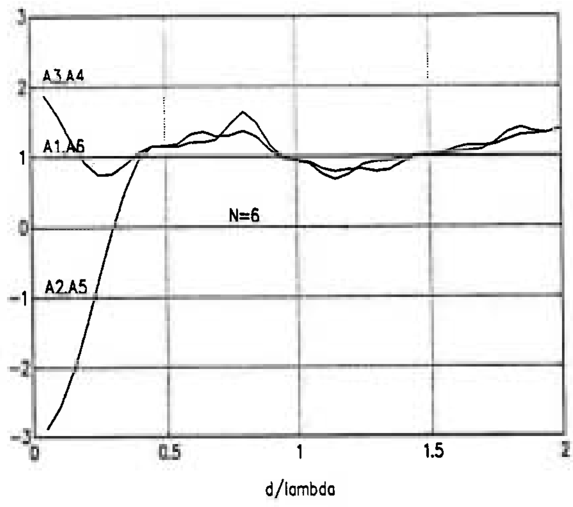

Figure 6. Amplitude of excitation for an array with 6 elements.

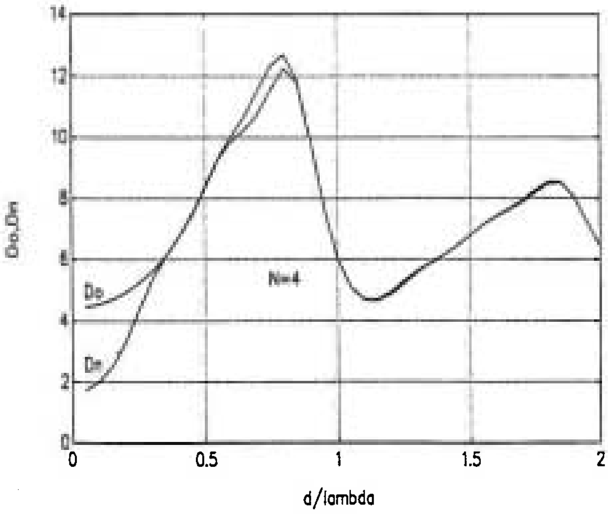

Figure 7. Comparison of nominal and optimum directivities for parallel half-wave dipoles, N=4.

CONCLUSIONS

The nominal directivity of a broadside array of parallel half-wave dipoles associated with a uniform excitation is presented in this work. The amplitude distribution for an optimized directivity is then obtained. It was observed that except for a separation d, below roughly 0.8λ the optimum and nominal directivities are practically the same. Similar results applies for short parallel dipoles. For d≤0.8λ, where a marked difference in D_0 and D_n is observed, the current distribution significantly differs from element to element. Because of the proximity of the dipoles and the presence of mutual coupling, in the supergain region and for very small separations, the results are not accurate [3] and hence not shown in the figures.

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