APPLICATION OF A ONE-DIMENSIONAL COMPUTER MODEL TO FLOOD ROUTING IN NARROW RIVERS

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Abstract This paper deals with the development of a computer model for flood routing in narrow rivers. Equations describing the propagation of a flood wave in a channel-flood plain system are presented and solved using an implicit finite difference scheme. Particular emphasis has been given to the treatment of the friction term in the governing equation of motion.

Key Words Computer Model, Finite Difference Scheme, Flood Routeing, Narrow Rivers

چکیده در این مقاله جزئیات مربوط به یک الگوی کامپیوتری جدید که می تواند برای ردگیری سیلاب در رودخانه های کم عرض مورد استفاده قرار گیرد، توضیح داده شده است. معادلات مربوط به انتشار امواج سیلاب در کانال وسیل گیر رودخانه ها ارائه شده و به طریق تفاوتهای محدود حل شده اند. اصطکاک ته رودخانه و کف سیل گیر بطور دقیق مطالعه شده ویک راه حل جدید برای اعمال نیروی اصطکاک در معادلات پیشنهاد شده است. انتشار سیل در رودخانه «تی ز» انگلستان بعنوان مثال، شرح داده شده است.

INTRODUCTION

Computer models of flood events in natural water systems are based on the formulation and solution of appropriate hydrodynamical equations of continuity and motion. The general shallow water equations describing the propagation of a long wave in a narrow channel are well known in hydrodynamics. However, before the direct application of the general equations for real cases, the special flow conditions in any water system should be considered. In narrow rivers, the bed conveys the flow of water predominantly in one direction and the motion is effectively one-dimensional. In addition, the relatively low cost of the one-dimensional flood routing model in terms of gathering necessary hydrographic data and computer simulations has made this approach the most commonly adopted technique. However, as the flood wave moves along the course of a narrow river at places where the land morphology allows, water spills over the river banks and floods the adjacent plain. Such flow is not strictly unidirectional and the use of a purely one-dimensional model for its simulation mutst involve a great deal of care. In the present work we describe a one-dimensional food routing model which is based on a definition of the roughness coefficient, in the friction term, as a composite factor.

In addition to the representation of the resistance to flow within the main river channel, this composite factor also incorporates the firction caused by spilling over the channel banks and motion of water on the flood plain. We present prototype outputs generated by the computer model for naturally occurring and synthetic floods in a narrow river. Comparison of these results with the observed data shows that our method effectively copes with various situations.

MATHEMATICAL MODEL

The model is based on the mathematical integration of the following hydrodynamical equations of continuity and motion:

$$\frac{\partial Q}{\partial x} + B\frac{\partial h}{\partial t} = q_m \tag{1}$$

$$\frac{\partial h}{\partial x} = -\frac{1}{gA}\frac{\partial Q}{\partial t} - \frac{1}{gA}\left(\frac{\partial}{\partial x}\frac{Q^2}{A}\right) - \frac{|Q|Qn^2}{A^2H^{4/3}}$$

$$-\frac{0.75H}{2000.0 + 1.5S}\frac{\partial S}{\partial x} \tag{2}$$

where in S.I. units

Q= discharge (=AU)

h= Water surface elevation with respect to datum

B= breadth of water surface A= cross-sectional area of flow channel H= hydraulic radius of flow channel	hydrographic data

n= Manning's friction coefficient

S= water salinity

q_m= tributary discharge

g= acceleration due to gravity

x,t= space and time variables respectively

U= depth mean velocity

Equations 1 and 2 describe the propagation of a flood wave in a tidal river [1]. The friction term in Equation 2 is written partly in terms of the magnitude of discharge to ensure the representation of the resistance always in the direction opposed to the flow. Water surface elevation h(stage) is given with respect to a datum in order to include the river's depth and its bed slope.

Numerical Scheme

The system of non-linear hyperbolic partial differential equation of 1 and 2 is too complex for any analytical solution. Numerical solutions based on finite difference and finite element methods are developed by various investigators, for example see [2] and [3]. The advantage of the finite element method over the finite difference schemes in handling geometrically-complicated flow domains is not relevant in a one-dimensional analysis. Furthermore, since there is no mathematical justification for adopting a complicated finite element approach [4], we have used a four-point implicit finite difference scheme as is shown in Figure 1.

After division of flow domain into different segments according to this scheme at a point P a function f= f(x,t) and its temporal and spatial derivations are given by:

$$f(x,t) = \frac{\theta}{2}(f_{i+1}^{j+1} + f_i^{j+1}) + \frac{1-\theta}{2}(f_{i+1}^{j} + f_i^{j}) \tag{3}$$

$$\frac{\partial f(x,t)}{\partial t} = \frac{f_{i+1}^{j+1} - f_{i+1}^{j} + f_i^{j+1} - f_i^{j}}{2\Delta t} \tag{4}$$

$$\frac{\partial f(x,t)}{\partial x} = \frac{\theta(f_{i+1}^{j+1} - f_i^{j+1}) + (1-\theta)(f_{i+1}^{j} - f_i^{j})}{(\Delta x)_m} \tag{5}$$

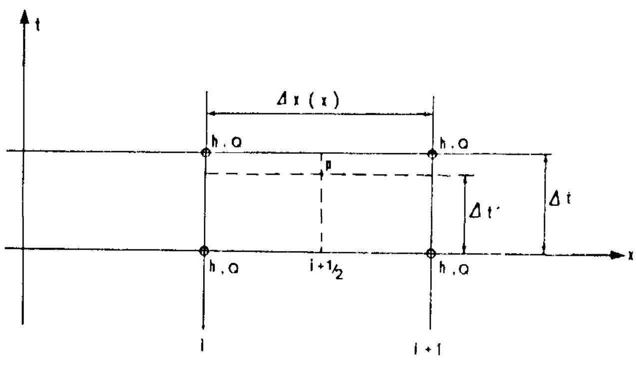

Figure 1. Computational scheme

where time weighting factor θ= (Δt'/Δt) [5]. Equations 3 to 5 represent a dissipating scheme and theoretical stability analysis of linearized hydrodynamical models based on this technique show that for $0.5 \leq \theta \leq 1.0$ it is unconditionally stable [6]. Application of the four-point implicit scheme to the governing Equations of 1 and 2 results in the following working equations:

$$Q^{j+1}_{i+1} - Q^{j+1}_{i} + G_1 (h^{j+1}_{i+1} + h^{j+1}_{i}) - G_2 = 0.0 \qquad (6)$$

$$h^{j+1}_{i+1} - h^{j+1}_{i} + G_3 (Q^{j+1}_{i+1} + Q^{j+1}_{i}) - G_4 = 0.0 \qquad (7)$$

These algebraic equations relate the prime unknowns of h and Q at the boundaries of every segment. Coefficients G_1, G_2, G_3 and G_4 are relations in terms of space and time intervals, hydrographic data, roughness, salinity and the values of the prime unknowns at the old and new time levels. The exact forms of these relationships adopted in the present model are:

$$G_1 = \frac{(\Delta x)_m \, \theta \, (B^{j+1}_{i+1} + B^{j+1}_{i}) + (\Delta x)_m \, (1-\theta) \, (B^{j}_{i+1} + B^{j}_{i})}{4\,\theta \Delta t} \qquad (8)$$

$$G_2 = G_1 \, (h^{j}_{i+1} + h^{j}_{i}) - \frac{1-\theta}{\theta} (Q^{j}_{i+1} - Q^{j}_{i}) \qquad (9)$$

$$G_3 = \frac{(\Delta x)_m}{g\,\theta \Delta t\,[\theta\,(A^{j+1}_{i+1} + A^{j+1}_{i}) + (1-\theta)\,(A^{j}_{i+1} + A^{j}_{i})]}$$

$$+ \frac{4\,[\theta\,(Q^{j+1}_{i+1} - Q^{j+1}_{i}) + (1-\theta)\,(Q^{j}_{i+1} - Q^{j}_{i})]}{g\,[\theta\,(A^{j+1}_{i+1} + A^{j+1}_{i}) + (1-\theta)\,(A^{j}_{i+1} + A^{j}_{i})]^2}$$

$$- \frac{2\,\theta\,[\theta\,(A^{j+1}_{i+1} - A^{j+1}_{i}) + (1-\theta)\,(A^{j}_{i+1} - A^{j}_{i})]\,(Q^{j+1}_{i+1} + Q^{j+1}_{i})}{g\,[\theta\,(A^{j+1}_{i+1} + A^{j+1}_{i}) + (1-\theta)\,(A^{j}_{i+1} + A^{j}_{i})]^3}$$

$$- \frac{4\,(1-\theta)\,[\theta\,(A^{j+1}_{i+1} - A^{j+1}_{i}) + (1-\theta)\,(A^{j}_{i+1} - A^{j}_{i})]\,(Q^{j}_{i+1} + Q^{j}_{i})}{g\,[\theta\,(A^{j+1}_{i+1} + A^{j+1}_{i}) + (1-\theta)\,(A^{j}_{i+1} + A^{j}_{i})]^3}$$

$$+ \frac{(n^{j}_{i+1} + n^{j}_{i})^2 \,(\Delta x)_m \,|\,\theta (Q^{j+1}_{i+1} + Q^{j+1}_{i}) + (1-\theta)\,(Q^{j}_{i+1} + Q^{j}_{i})\,|}{4[\theta\,(A^{j+1}_{i+1} + A^{j+1}_{i}) + (1-\theta)\,(A^{j}_{i+1} + A^{j}_{i})]^2 \left[\frac{\theta}{2}\,(H^{j+1}_{i+1} + H^{j+1}_{i}) + \frac{1-\theta}{2}\,(H^{j}_{i+1} + H \right.}$$

(10)

$$G_4 = - \frac{1-\theta}{\theta} (h^{j}_{i+1} - h^{j}_{i}) + \frac{(\Delta x)_m \,(Q^{j}_{i+1} + Q^{j}_{i})}{g\,\theta \Delta t\,[\theta (A^{j+1}_{i+1} + A^{j+1}_{i}) + (1-\theta)(A^{j}_{i+1} + A^{j}_{i})]}$$

$$- \frac{4[\theta (Q^{j+1}_{i+1} - Q^{j+1}_{i}) + (1-\theta)(Q^{j}_{i+1} - Q^{j}_{i})]\,(1-\theta)(Q^{j}_{i+1} + Q^{j}_{i})}{g\,\theta[\theta (A^{j+1}_{i+1} + A^{j+1}_{i}) + (1-\theta)(A^{j}_{i+1} + A^{j}_{i})]^2}$$

$$+ \frac{2\,(1-\theta)^2\,(Q^{j}_{i+1} + Q^{j}_{i})^2\,[\theta (A^{j+1}_{i+1} - A^{j+1}_{i}) + (1-\theta)(A^{j}_{i+1} - A^{j}_{i})]}{g\,\theta[\theta (A^{j+1}_{i+1} + A^{j+1}_{i}) + (1-\theta)(A^{j}_{i+1} + A^{j}_{i})]^3} -$$

$$\frac{(\Delta x)_m\,(1-\theta)\,|\,\theta (Q^{j+1}_{i+1} + Q^{j+1}_{i}) + (1-\theta)(Q^{j}_{i+1} + Q^{j}_{i})\,|\,(n^{j}_{i+1} + n^{j}_{i})^2\,(Q^{j}_{i+1} + Q^{j}_{i})}{4\theta[\theta (A^{j+1}_{i+1} + A^{j+1}_{i}) + (1-\theta)(A^{j}_{i+1} + A^{j}_{i})]^2 \left[\frac{\theta}{2}(H^{j+1}_{i+1} + H^{j+1}_{i}) + \frac{1-\theta}{2}(H^{j}_{i+1} + H^{j}_{i})\right]^{4}}$$

$$- \frac{1.5\left[\frac{\theta}{2}(H^{j+1}_{i+1} + H^{j+1}_{i}) + \frac{1-\theta}{2}(H^{j}_{i+1} + H^{j}_{i})\right](S^{j}_{i+1} - S^{j}_{i})}{\theta\left[4000.0 + 1.5\,(S^{j}_{i+1} - S^{j}_{i})\right]}$$

(11)

The presence of the new time level values of the prime unknowns (i.e. h^{j+1} and Q^{j+1}) and the hydrographic data (i.e. A^{j+1}, B^{j+1} and H^{j+1}) in Equations 8 - 11 reflect the non-linearity of the mathematical model.

Solution Algorithm

The developed solution algorithm has been described elsewhere [7] and is only summarised here. The flow domain is divided into segments of variable length. Boundaries of the segments in practical cases are selected to coincide with locations where the hydrographic data are collected and the water levels and discharges are recorded.

Starting from $t_1 = 0$ and assuming $Q^1 = 0$ and $h^1 =$ bed level everywhere except the upstream and downstream boundaries i.e. cold start, a set of non-linear algebraic equations is constructed using the working Equations of 6 and 7 for all segments. The prescription of the boundary conditions makes the derived set of equations determinate. The boundary conditions for flood routing in a tidal river systems are usually the recorded flood hydrograph at the upstream end and the observed tidal stage at the downstream end. The set of equations is then solved using a band-solver routine. the non-linearity is treated by iterations using successive substitutions over each time step. The entire algorithm is iterated over three tidal cycles to make sure that the obtained solution has completely converged and the effects of the numerical noise caused by the cold start is damped out.

APPLICATION OF THE MODEL TO FLOOD ROUTING IN THE RIVER TEES (U.K.)

The river Tees in the North East, U.K., flows in a general easterly direction, from the Pennines to the North Sea, see Figure 2. We have applied our developed model to simulate flood conditions in the upper parts of this river, between Low Moor and Yarm a distance of 12 km; Figure 2. The first step was the calibration and validation of the computational model using a number of past flood and tidal events of known magnitude. It is apparent that the river-bed roughness coefficient, Manning's "n" in the firction term of the equation of motion is the central factor in the calibration and validation stage of the model development. The situation is especially complicated since in the upper parts of the river, beyond the limit of the tidal reach at Low Worsall, extensive flooding is caused by fluvial floods. In the present investigation the roughness coefficient was taken as consisting of two parts, the first representing the main channel friction and the second representing the flood plain friction. When the water surface was confined to the main channel then the friction coefficient was taken as n_c but when the water surface covered the flood plain the coefficient n_f was introduced. The composite friction n was taken as:

$$n(x,h) = \frac{B_c(x,h)}{B_T(x,h)} . n_c(x,h) + \frac{B_T(x,h) - B_c(x,h)}{B_T(x,h)} . n_f(x,h) \tag{12}$$

where $B_c =$ channel width,

$B_T =$ channel width + flood plain width

After the initial selection of n_c and n_f values for each cross-section at any stage, the latter were altered by a process of optimization in order to achieve the best possible agreement between the computed and observed water levels. This optimization process is somewhat constrained since there are limiting values for the roughness coefficient at all cross-sections in the river. The optimum roughness factor for river sections n_c, was found to vary between 0.016 to 0.029, and for the flood plain n_f, varied from 0.02 to 0.030. These values fall within an acceptable range for a typical British river-estuary system. The observed and computed water levels for the calibration event, spring tide of 23rd November 1988 at Tees Dock together with a typical hundred year flood, 23rd March 1968

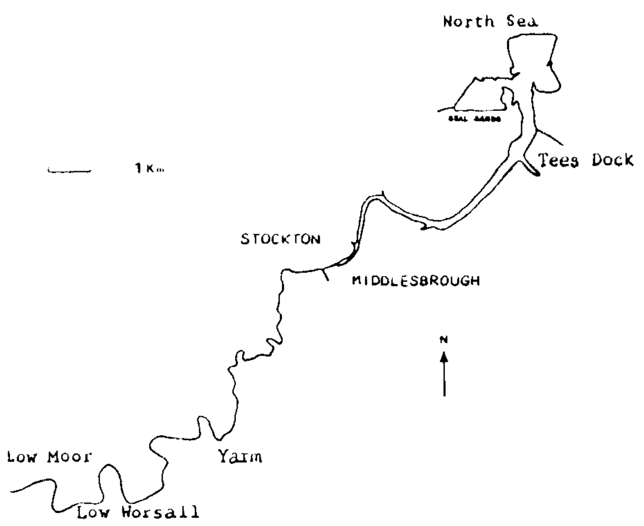

Figure 2. River Tees

hydrograph, at Low Moor, is shown in Figure 3. The level of agreement between the two sets of results suggests that the developed friction factor distributed along the river is acceptable. This is confirmed by the study of the result of the verification event shown in Figure 4. Following the verification, the model was used to predict the extent of the flooding in the upper parts of the Tees river. A series of simulations based on the synthetic 10, 40 and 100 year flood hydrographs at Low Moor routed for 12 km in the upper Tees is shown in Figure 5.

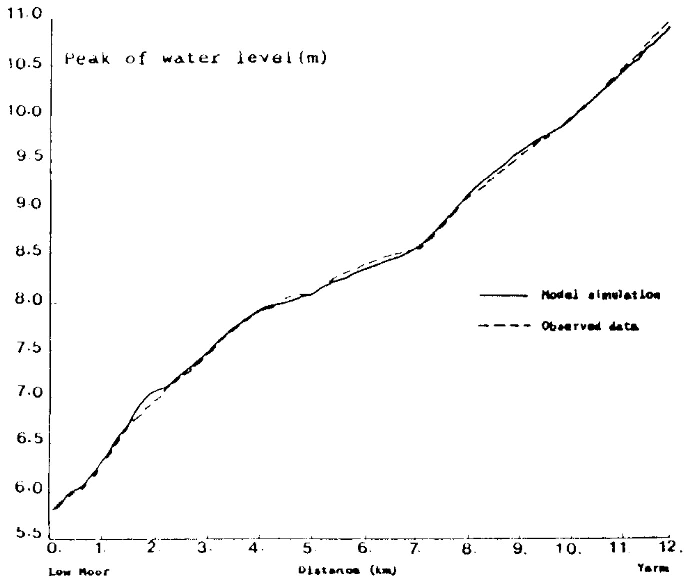

Figure 3. Calibration event (spring tide co-incidental with 100 year flood)

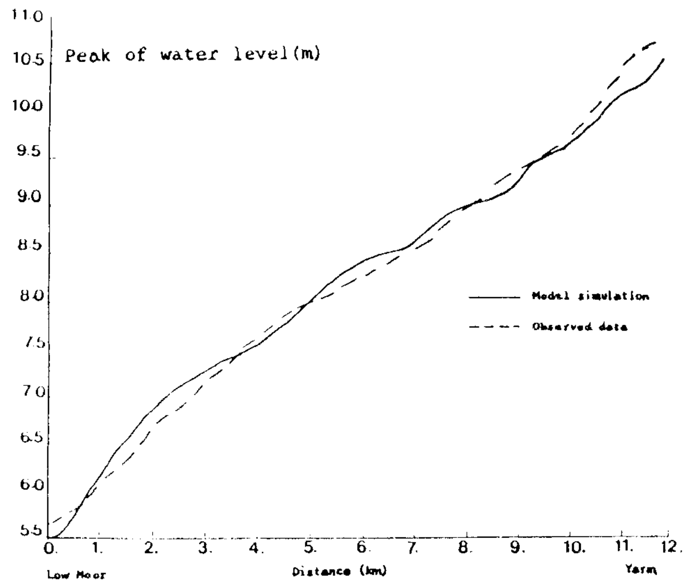

Figure 4. Verification event (spring tide co-indidental with 1963 flood)

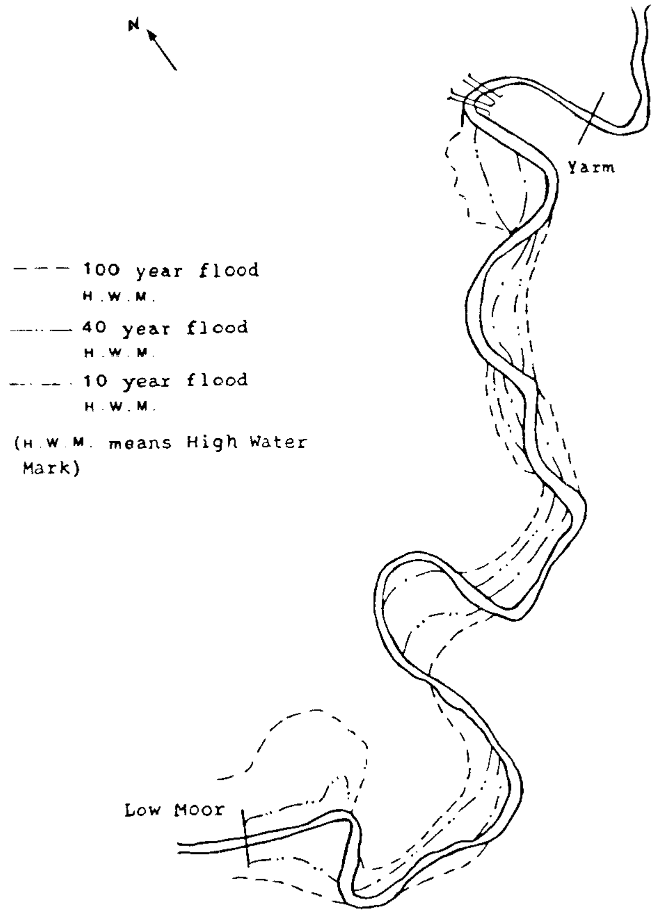

Figure 5. Flood routing in River Tees

CONCLUSION

The investigation has shown that the developed numerical model can be used to predict maximum levels of future flood events in a one-dimentional river-estuary system. The successful application of the model to any particular system largely depends on the careful selection of the roughness coefficient distribution during the model calibration.

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