# OPTIMUM AGGREGATE INVENTORY IN THE LOT SCHEDULING PROBLEM WITH NON-ZERO SET UP TIME


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**Abstract** In this paper, the minimization of the maximum aggregate inventory of all products for the common cycle time approach to the scheduling problem of a multi-products single machine system is considered. In the case of non-zero set up time and limited resources, a simple and easy to apply rule has been obtained for this optimization problem. Using this rule, one can obtain the optimal production sequence among n! possible schedules, just by comparison of values of the maximum aggregate inventory of only n possible schedules.



**چکیده** در این مقاله کمینه‌سازی حداکثر موجودی جمعی تمام محصولات برای روش دور مشترک در مسأله برنامه‌ریزی یک سیستم تک ماشینی چند-محصولی در نظر گرفته شده است. برای حالت زمان آماده‌سازی غیر صفر و منبع محدود، یک قاعده ساده که کاربرد آن آسان است برای این مسأله بهینه‌سازی بدست آمده است. با استفاده از این قاعده، می‌توان توالی تولید بهینه را، از بین !n برنامه ممکن، فقط با مقایسه حداکثر موجودی جمعی n برنامه بدست آورد.


## INTRODUCTION

Consider the problem of obtaining a low cost schedule for a production system in which a number of products is manufactured in a fixed sequence repeatedly from cycle to cycle. For any given problem, optimum manufacturing frequencies for individual products and cycle time can be easily determined but the problem arises when we try to obtain a feasible schedule. If it is possible to obtain a feasible schedule without altering the optimum manufacturing frequencies or cycle time for individual products then this is the optimum production schedule. In practice such a happy coincidence of events rarely occurs and it becomes essential to alter some of the values of the manufacturing frequencies of individual products and/or the optimum cycle time in order to obtain a feasible schedule which, in turn, is improved upon. A description of different methods which have been given for tackling this scheduling problem can be found in references 2-4 and 8-12. In one of the approaches each product will be produced once during each cycle time [6, 7]. In this approach, (the common cycle time approach, in obtaining the optimal cycle time which minimizes the annual variable cost, it is implicitly assumed that unlimited resources are available for allocation to aggregate inventory of all products. But in most real life situations the available resources are limited and reducing the maximum aggregate inventory for the given cycle time, as well as the optimal cycle time, is desirable.

In this paper, for the common cycle time approach and in the case of non-zero set up time and limited resources, a rule has been obtained for the scheduling problem. By using this rule, we can obtain the optimal schedule, among the n! possible schedules, by comparison of values of the maximum aggregate inventory of only possible schedules.

## THE MATHEMATICAL MODEL

The following notations are used in this paper:

n = Number of products.

T = Length of the cycle time.

D = Aggregate demand rate of all products in terms of limited resource.

For the j - th product:

$S_{(j)}$ = Set up time

$T_{(j)}$ = Length of time, in each cycle time T, during which only the product j is produced.

$D_{(j)}$ = Demand rate in terms of units of the limited resource..

$P_{(j)}$ = Production rate of machine in terms of units of the limited resources.

The following assumptions are made:

1. Shortage are not permitted.
2. The total increase of aggregate inventory during the production of product j is greater than the aggregate demand during the set up time of product j, that is,

$$[P(j) - D] * T(j) > D * S(j)$$

3. In each cycle time T, once the production starts, then all n products will be produced in lots and except for the necessary set up time, there is no idle time between the production runs of products in the cycle.

Let $L_{(j,t)}$ be the inventory position of product j at time t and Z the maximum aggregate inventory, then:

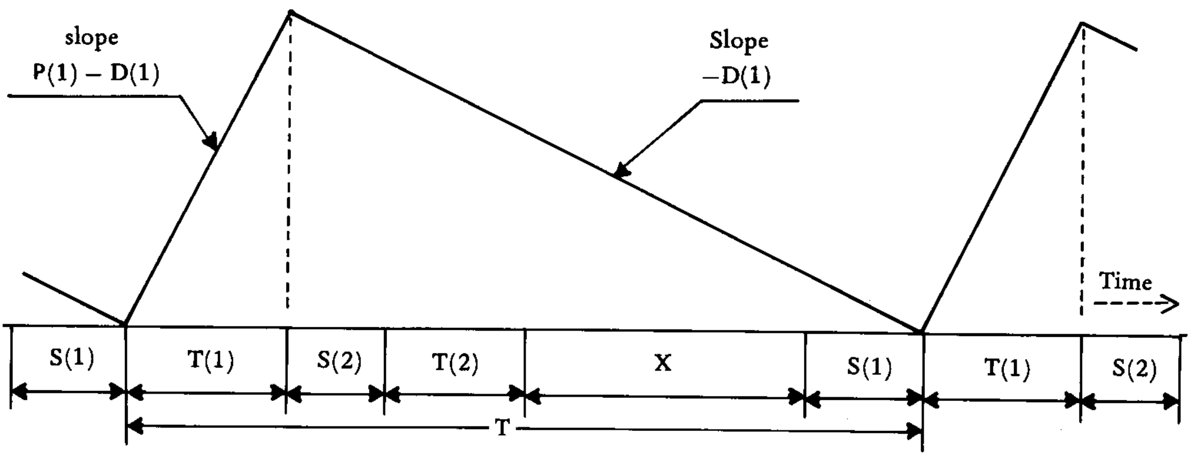


*Figure 1(a). Inventory position of product 1*

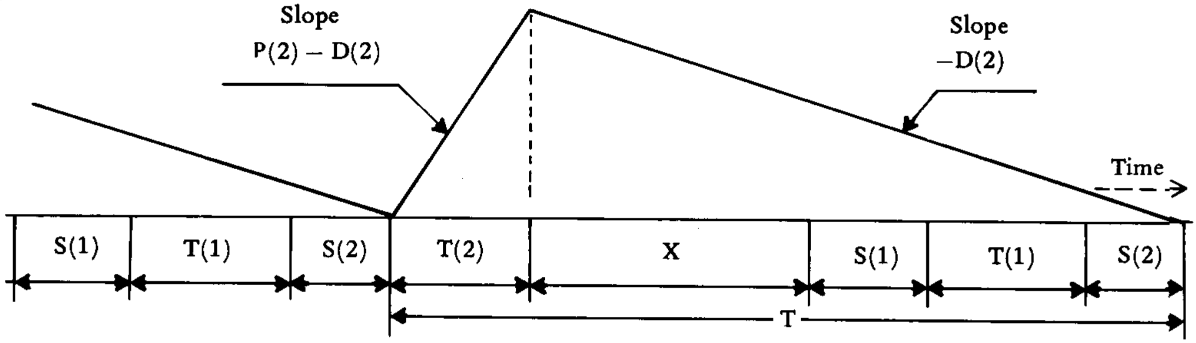


*Figure 1(b). Inventory position of product 2.*

$$\max_{0<t<T} \sum_{j=1}^{n} I(j, t)$$

The value of Z depends on the order of the production of products. To see this, consider the case of 2 products, (Figure 1 (a,b)).

In each cycle time, T, the inventory of product j will accumulate at a rate $[P_{(j)} - D_{(j)}]$ during the period $T_{(j)}$ and it will decrease at a rate $D_{(j)}$ during the time interval $T - T_{(j)}$, as shown in Figure 1 (a, b).

Because of assumption 2, $[P(2) - D]*T(2) > D*S(2)$, the aggregate inventory for the given order in Figure 1(a, b) will reach its maximum, Z, at the end of production of product 2, and it is equal to:

$$Z1 = [P(1) - D(1)]*T(1) - D(1)*[S(2) + T(2)] + [P(2) - D(2)]*T(1). \quad (1)$$

But if we change the order, then in a similar way, the aggregate inventory will reach its maximum Z at the end of production of product 1, and is equal to

$$Z2 = [P(2) - D(2)]*T(2) - D(2)*[S(1) + T(1)] + [P(1) - D(1)]*T(1). \quad (2)$$

It is clear that, in general, Z1 and Z2 are not equal.

to find Z for the case of n products consider an arbitrary order of products, i1, i2, ..., in (ij represents the product that has the j-th position in the given order). In each inventory cycle time T, after the start of production of product i1 and before the end of production of product in, the aggregate inventory increases at the rate $[P_{(ij)} - D]$ during $T_{(ij)}$, $j = 1, 2, \ldots n$, and decreases at the rate D during $S_{(ij)}$, $j = , 2, 3, \ldots n$. Because of assumption 2, $[P_{(ij)} - D]^{*}T_{(ij)} > D^{*}S_{(ij)}$, the aggregate inventory reaches its maximum Z at the end of production of product in. Then the aggregate inventory decreases at the rate D during both the idle time X and the set up time S(i1), (Figure 2) and it reachesits minimum z at the end of S(i1), that is, at the start of production of product i1 (at the start of the next cycle). Thus,

$$Z = z + \sum_{j=1}^{n} [P(ij) - D]*T(ij) - D\sum_{j=2}^{n} S(ij).$$

$$Z = z + \sum_{j=1}^{n} [P(ij) - D]*T(ij) - D\sum_{j=1}^{n} S(ij) + D*S(il)$$

It is clear that for the given cycle time T

$$\sum_{i=1}^{n} [P(ij) - D]*T(ij)$$

is independent of the order of production.

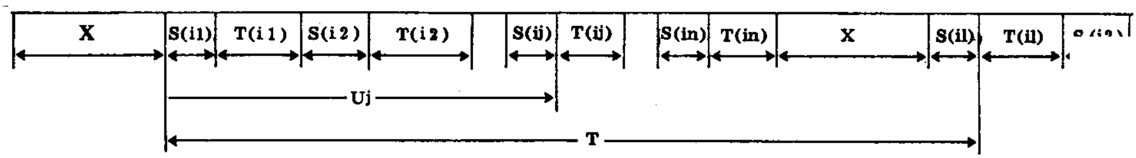


*Figure 2.*

Furthur,

$$D \quad \sum_{j=}^{n} \quad S(ij)$$

is a constant. Thus we can write

$$Z = z + C + D*S(il) \qquad (3)$$

where

$$C = \sum_{j=}^{n} [P(ij) - D]*T(ij) - D\sum_{j=1}^{n} S(ij).$$

Since no shortages are permitted, then $I_{(ij)}$, the inventory of product ij at the start of production of product il, is equal to demand for that product during Uj, the length of time from the start of production of il up to the start of production of product ij (Figure 2), that is,

$$Uj = \sum_{k=}^{j-} T(ik) + \sum_{k=2}^{j} S(ik). \qquad (4)$$

Thus for j  2, 3, ..  n

$$I(ij) = D(ij) \sum_{k=1}^{j-1} T(ik) + D(ij) \sum_{k=2}^{j} S(ik)$$

$$I(ij) = D(ij) \sum_{k=2}^{j} T(ik) + D(ij) \sum_{k=2}^{j} S(ik) + D(ij)*T(ij) - D(ij)*T(ij) \qquad (5)$$

Now z, the aggregate inventory at the start of product i1 is

$$z = \sum_{j=1}^{n}$$

and since I(i1) is zero then,

$$z = \sum_{j=2}^{n} I(ij). \qquad (6)$$

Let

$$Y(ik) = T(ik) + S(ik),$$

then from (5) and (6)

$$z = \sum_{j=2}^{n} D(ij) \sum_{k=2}^{j} Y(ik) + T(ij) \sum_{j=2}^{n} D(ij) - \sum_{j=2}^{n} D(ij)*T(ij) \qquad (7)$$

Now consider the set of sequences for which the product m has the first position and denote this set by Rm. Denote the corresponding z and Z for the requences of this set by zm and Zm, respectively. From(3), for the set Rm we have

$$Zm = zm + C - D*S(m),$$

and since for this set S(m) is constant, it is clear that minimization of Zm is equivalent to minimization of zm. Furthur, since in (7) for the set Rm

$$T(il) \sum_{j=2}^{n} D(ij) = T(m) \sum_{i=}^{n} D(j) - T(m)*D(m)$$

and

$$\sum_{j=2}^{n} D(ij)*T(ij) = \sum_{j=1}^{n} D(j)*T(j) - D(m)*T(m)$$

are both constant, then minimization of z(m) is equivalent to minimization of

$$Vm = \sum_{j=2}^{n} D(ij) \sum_{k=2}^{j} Y(ik).$$

But Vm is similar to total weighted flow time of (n - 1) jobs in the single machine problem [1], and minimization of Vm is equivalent to minimization of mean flow time in the single machine problem. Therefore, Vm will be minimized by WSPT sequencing

$$\frac{D(i2)}{Y(i2)} > \frac{D(i3)}{Y(i3)} \qquad > \frac{D(in)}{Y(in)}$$

This is a simple rule which gives the minimum of Zm. Let

$$f_m = \text{Min } Z_m$$

Then to find the optimal schedule, one needs only to compare n values of fm (m = 1, 2, ..., n). That is the minimization of Z, the maximum aggregate inventory, over n! possible schedules is reduced to comparison of only n values of fm.

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