

**PREDICTION OF TEMPERATURE PROFILE OF  
A BURIED GAS PIPELINE THROUGH UTILIZATION OF  
CORRESPONDING STATES PRINCIPLE**

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**Abstract** A new analytical equation for prediction of temperature profile of a buried gas pipeline is developed. Utility of this equation is illustrated by its application to corresponding states principle. The resulting equation is tested through prediction of the actual Schorre data. It is shown that the new equation can predict temperature profile more accurately than the others without using any chart of tables. This equation can also be used for prediction of gas mixture temperature profile flowing in a buried pipeline.

**چکیده**

در این پژوهش معادله جدیدی برای نیمرخ دمای خطوط لوله گاز مدفون در زمین بدست آمده است. این معادله با بکار گرفتن اصول Corresponding States - حالات متناظر ارائه شده و نتایج حاصل از آن با نتایج تجربی Schorre مقایسه گردیده است. در اینجا نشان داده شده است که این معادله نیمرخ دما را با دقت بیشتری نسبت به معادلات دیگر در این زمینه بدست میدهد. در صورت بکار گرفتن این معادله برای تعیین نیمرخ دما بر خلاف روشهای دیگر، نیازی به استفاده از منحنی و جدول خواص نیست. مزیت دیگر این معادله بر معادلات دیگر این است که میتوان آنرا برای مخلوط گازهایی جاری در خطوط لوله نیز مورد استفاده قرار داد.

## INTRODUCTION

There are a few analytical equations which are used to predict temperature profile in a buried gas pipeline. Schorre Equation [1] is one which has been used since 1954. Forrest [2] interpreted the Schorre equation and then Coulter [3] solved the energy equation by using compressibility factor  $z$ .

While there has been extensive activity in the development of an analytical and more accurate equation, there has been little attention given to the fact that the Joule-Thomson coefficient and heat capacity are not constant and these are functions of temperature as well as pressure. In the present study we introduce a new concept for development of an analytical equation through the corresponding states principle. This concept is based on the fact that  $\mu$  and  $C_p$  are not considered constant.

## Derivation of Equation

Differential formulation of the first law of thermodynamics has been considered for a control volume segment  $dx$  of a pipeline which is

$$A(dq_x/dx) + \rho v A(dh/dx) + q_r \rho = 0 \quad (1)$$

In derivation of Equation 1, kinetic and potential energies have been neglected. The first and second term in the above equation represent the axial conduction and the axial enthalpy flow, respectively. Because Peclet number is the ratio between the axial enthalpy flow and the axial conduction, one can only consider the axial enthalpy flow when Peclet no. is very large (greater than 100) [4]. In gas pipelines and in our case study Peclet no. is usually of the order of 10,000 and the axial conduction term has

been neglected.  $q_r$  is the radial heat conduction flux. Heat conduction can be considered radially lumped when Biot number is less than 0.1 and therefore the following relation is used for  $q_r$ :

$$q_r = U(T - T_g) \quad (2)$$

where the overall heat transfer coefficient  $U$  is chosen to be [5]:

$$U = 2k/D(1/\ln(4z/D)) \quad (3)$$

The enthalpy of gas is a function of temperature and pressure and is defined as follows:

$$dh = C_p dT - \mu C_p dP \quad (4)$$

By substituting Equations 3 and 4 into equation (1) one can obtain:

$$dT/dx + \xi(T - T_g) = \mu dp/dx \quad (5)$$

where  $\xi = \pi DU / (\rho v A C_p)$

Equation 5 can be reduced to a dimensionless form as the following:

$$d\theta_r/dx^* + \xi^* \theta_r = \mu P_c / T_c dP/dx^* \quad (6)$$

where  $\xi^* = \xi D$ ,  $\theta_r = (T - T_g) / T_c$  and  $x^* = x/D$

$\mu$ , the Joule-Thomson coefficient, is obtained from the following Relation [6]:

$$C_p P_c / T_c = T_r d(BP_c / RT_c) / dT_r - (BP_c / RT_c) \quad (7)$$

$$\text{where } BP_c / RT_c = (0.1445 + 0.073\omega) - (0.33 - 0.46\omega)T_r^{-1} - (0.1385 + 0.5\omega)T_r^{-2} - (0.012 + 0.097\omega)T_r^{-3} - 0.0073\omega T_r^{-8} \quad (8)$$

by substituting Equation 8 into Equation 7 and upon linearization of the resulting equation one obtains:

$$\mu C_p P_c / RT_c = (1.278 + 2.01\omega)T_r^{-1} / (0.338 + 1.302\omega) \quad (9)$$

Heat capacity is also considered as a function of  $T$  and  $P$  [6]:

$$C_p = C_{p0}(T) + \Delta C_p(P, T)$$

where  $C_{p0}(T) = A_1 + A_2 T + A_3 T^2 + A_4 T^3$  and

$$\Delta C_p(P, T) = P_r R [0.66 - 0.92\omega] T_r^{-2} + (0.83 + 3\omega) T_r^{-3} + (0.145 + 1.1\omega) T_r^{-4} + 0.526\omega T_r^{-9}$$

Equations 9 and 10 have been substituted into Equation 6 and upon integration of the resulting equation, the following expression has been derived for prediction of temperature profile.

$$T_2 / (T_2 - T_1) \ln[(T_1 - T_2) / (T_0 - T_2)]$$

$$T_1 / (T_2 - T_1) \ln(T - T_1) / (T_0 - T_1)$$

$$= \pi Da / (m^0 b) \ln[a / (a + bx)]$$

where

$$T_1 = -C_1 / 2 + T_g / 2 - [(T_g - c_1)^2 / 4 + c_2]^{0.5}$$

$$T_2 = -c_1 / 2 + T_g / 2 + [(T_g - c_1)^2 / 4 + c_2]^{0.5}$$

$$c_1 = (0.34 + 1.30\omega) RT_c / P_c \cdot (dp/dx) m^0 / (\pi DU)$$

$$c_2 = (1.28 + 2.01\omega) RT_c^2 / P_c \cdot (dp/dx) m^0 / (\pi DU)$$

$$a = a_1 + a_2 P$$

$$b = a_2 dp/dx$$

$$a_1 = C_{p0}(T)$$

$$a_2 = \Delta C_p(P, T) / p$$

The above equation can be used either to obtain the temperature profile in a pipeline or for a specified gas temperature, to find the length of the pipeline. In derivation of the

equation, virial equation of state has been used to derive a relation for the Joule-Thomson coefficient. Therefore, this equation can also be used for gas mixture flowing in a pipeline. To do this one has to choose an appropriate mixing rule and apply it to second virial coefficient.

Equation 11 is a non-linear equation with respect to  $T$  and linear with respect to  $x$ . To find  $T$  at a specified length of a pipe a method for solving a non-linear equation can be used.

### Calculations and Results

The new analytical solution can predict temperature profiles more precisely than other equations. Figures 1 and 2 show that for the same conditions, the analytical equation presented here gives more accurate results. In addition, since the equation developed in this work is based on the theorem of corresponding states, it can be extended to the mixtures in a straight forward manner by use of the pseudocritical approach. The temperature profile equation developed in this work can be used along with one of the gas flow rate equations such as the Panhandle or Weymouth equations to predict pressure drop in a gas pipeline accurately.

### NOTATION

$A$	pipe cross section
$A_1, A_2, A_3, A_4$	Ideal gas heat capacity coefficient
$B$	Second virial coefficient
$C_p$	Heat capacity
$D$	Pipe diameter
$h$	Enthalpy of fluid
$k$	Soil thermal conductivity
$m^{\circ}$	Molar flow rate
$\rho$	Pipe perimeter
$P_c$	Reduced pressure
$q_x$	Axial heat conduction flux
$q_r$	Radial heat conduction flux
$R$	Universal gas constant
$T$	Gas temperature
$T_0$	Initial gas temperature
$T_c$	Critical temperature
$T_r$	Reduced temperature
$T_g$	Ground temperature
$\frac{\dot{q}}{T}$	Average temperature
	Overall heat transfer coefficient
$v$	Velocity of fluid
$x$	Pipe length
$dp/dx$	Average pressure drop
$\rho$	Density of fluid
$\mu$	Joule-Thomson coefficient
$\omega$	Acentric factor

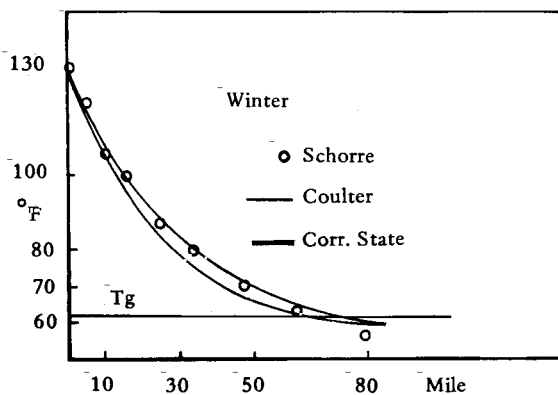


Figure 1. Temperature profile in a gas pipeline considering winter ground temperature

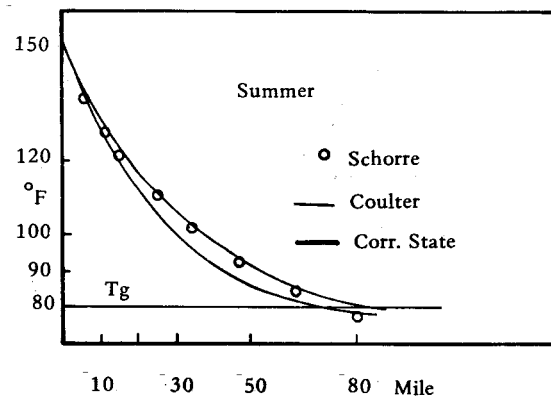


Figure 2. Temperature profile in a gas pipeline considering summer ground temperature

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