DIGITAL DISTANCE RELAYING

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Abstract Increasing interest is being shown in the use of microprocessor for protection, switching and data acquisition required in modern high voltage substations. One of the most difficult functions to fulfill is that of transmission line distance protection employing samples of the voltage and current waveform taken from high voltage transducer equipment at the usual relaying point. This paper examines and compares different digital algorithms suitable for distance measurement, with particular reference to their accuracy and speed of calculation.

چکیده با پیشرفت تکنولوژی کامپیوتر و کاهش ارزش آن علاقه روزافزونی به استفاده از میکروپروسسور جهت حفاظت ، صدور فرمان قطع و وصل کلیدها و جمع‌آوری اطلاعات در پستهای فشار قوی بوجود آمده است . یکی از مشکل‌ترین کارهائی که باید توسط میکروپروسسور در پست انجام گیرد حفاظت دیستانس خط انتقال انرژی با نمونه‌برداری از موجهای ولتاژ و جریان است . دراین مقاله روشهای مختلفی که برای محاسبه دیجیتالی امپدانس خط میتواند بکار رود با درنظرگرفتن سرعت و دقت آنها مورد بررسی و مقایسه قرار گرفته است .

INTRODUCTION

Several methods can be used for the digital calculation of transmission line fault impedance using a microprocessor on-line. All methods rely on sampling and storing voltage and current waveforms at a single measurement or relaying point. For measurement purposes, the most onerous condition is when a fault occurs at a voltage maximum on one phase, because the consequent discharge of potential energy through the line inductance causes harmonics to be generated in the current and voltage. In addition, noise and non-linearity of the transducers result in further harmonics against which any algoithm used to calculate fault impedance should be immune. This paper examines and compares different algorithms suitable for fault impedance measurement, with particular reference to their accuracy and speed of calculation. These algorithms have been chosen because of their ability to deal with low order harmonics.

FAULT MEASUREMENT ALGORITHMS

There two main methods proposed for digital distance relaying, namely:

(i) Determination of fundamental frequency phasors $\overline{V}$ and $\overline{I}$ at the relaying point so that the impedance "seen" can be calculated from

$$\overline{Z} = \frac{\overline{V}}{\overline{I}} = R + jx \qquad (1)$$

This method was proposed by Slemon et.al. [12], and developed by Mann and Morrison [3], Gilbert and Shovlin [7], Rockefeller [4] and others.

(ii) Instantaneous values of v and i obtained by sampling without corruption from harmonics or noise can be inserted into an equation of the form

$$V = Ri + L\frac{di}{dt} \qquad (2)$$

and solved for R and L using two successive V and i values.

PEAK DETERMINATION METHOD

Establishing Z of equation (1) by predicting the peak values V_p and I_p from samples was one of the early methods investigated by Mann and Morrison [3] and Rockefeller and Udren [4]. The method required that the d.c. offset of fault waveforms be eliminated by using a mimic impedance as a secondary burden comprising the average source impedance and 90% of the line impedance. In addition, Rockefeller and Udren proposed and implemented a digital relay using first and second derivatives of voltage and current to determine impedance from

$$Z = \frac{v^2 + (\frac{V^{\prime\prime}}{\omega})^2}{i^2 + (\frac{i^{\prime\prime}}{\omega})^2} \qquad (3)$$

To find V' and i', the difference expressions using sampled values can be used, typically:

$$V'_K = \frac{1}{2T}(V_{K+1} - V_{K-1}) \qquad (4)$$

or

$$V'_K = \frac{1}{T}(V_K - V_{K-1}) \qquad (5)$$

Where T is the time between successive samples V_K and V_{K-1}. Appendix 1 shows how these expressions can be looked upon as digital filters with voltage samples as inputs and voltage derivatives as outputs. More elaborate difference expressions could be used but it can be shown that no great benefit results and increased computational burdens are incurred.

The frequency spectrum of equation (4) is given by

$$H_1(f) = 50\,N \left| \sin\frac{\pi f}{25N} \right| \qquad (6)$$

where N is the number of samples per cycle. The maximum output from this filter which will be useful should occur at fundamental frequency, i.e. 50 Hz, giving the condition

$$\frac{\pi.50}{25N} = \pi/2, \frac{3\pi}{2}, 5\pi/2, \ldots \text{ or } N = 4 \text{ s/c for}$$

practical sampling.
This is, therefore, the optimum sampling rate for this filter. The spectrums for 4 and 16 s/c are shown for comparison in Figure 1 where it can be seen that for sampling rates higher than the optimum, many components larger than 50 Hz are accentuated.

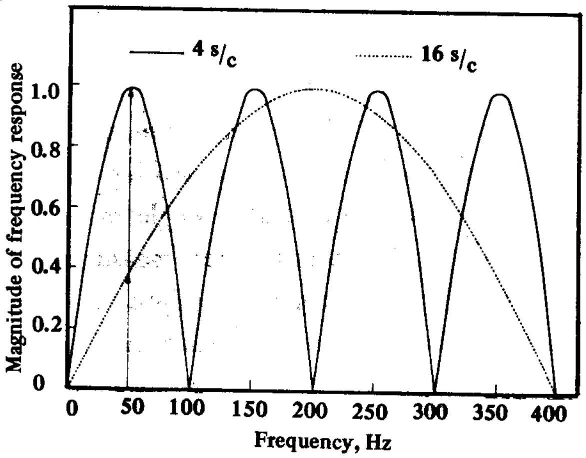

Figure 1. Spectrum of $H_1(f)$ for 2 sampling rates.

A similar argument can be applied to the difference equations and their corresponding filter equivalent frequency response for determination of second derivatives V'' and i''. It can be shown that, unless 4 s/c is used, unwanted components above 50 Hz are amplified, as would be expected from any method relying on differentiation for fault determination. To overcome this problem Rockefeller and Udren [4] used analogue filters with low cut-off frequencies having long time delays and a sampling rate of 12 s/c. Methods which could combine acceptable sampling rates with simple analogue filters are much to be desired and will be explored in the next section.

FOURIER METHODS

These methods depend upon establishing the fundamental components of V and I by eliminating harmonics which are integer-multiples of the fundamental. The elemination of these harmonics will be shown by the use of a finite impulse response digital filter approach.

Assume the fundamental components of voltage and current waveform are:

$$v = V_p \sin(\omega t + \lambda + \delta) = A_V \sin\omega t + B_V \cos\omega t \tag{7}$$

$$i = I_p \sin(\omega t + \delta) = A_i \sin\omega t + B_i \cos\omega t \tag{8}$$

In complex form: $\overline{V} = A_V + j B_V$

$\overline{I} = A_i + jB_i$

Hence from equation (1)

$$\overline{Z} = \frac{\overline{V}}{\overline{I}} = R + jx = \frac{A_V A_i + B_V B_i}{A^2 + B^2} + j\,\frac{B_V A_i - A_V B_i}{A_i^2 + B_i^2}$$

or

$$R = \frac{A_V A_i + B_V Bi}{A_i^2 + B_i^2} \tag{9}$$

and

$$X = \frac{B_V A_i - A_V B_i}{A_i^2 + B_i^2} \tag{10}$$

Slemon et.al. [12] suggested calculation of the modulus and phase angle of transmission line impedance as:

$$Z = \sqrt{A_V^2 + B_V^2} \quad \sqrt{A_i^2 + B_i^2} \tag{11}$$

and

$$\lambda = \tan^{-1}\frac{B_V}{A_V} - \tan^{-1}\frac{B_i}{A_i} \tag{12}$$

It is very difficult and time consuming to calculate Z from equation [11] by microprocessor and the calculation of arc tangents is lengthy by exact means. An alternative is to use a coarse look-up table. The method of calculation of R and X from equations [9] and [10] avoids these difficulties and makes possible the production of an ideal characteristic.

If A_V, B_V, A_i and B_i are calculated for each phase, the impedance of the transmission line seen from a single relaying point can be determined. Now, using Fourier analysis, the coefficients can be calculated as:

$$A_V = \frac{1}{\pi}\int_{\gamma}^{\gamma+2\pi} v \sin\omega t \, d(\omega t)$$
$$B_V = \frac{1}{\pi}\int_{\gamma}^{\gamma+2\pi} v \cos\omega t \, d(\omega t) \tag{13}$$

$$A_i = \frac{1}{\pi}\int_{\gamma}^{\gamma+2\pi} i \sin\omega t \, d(\omega t)$$
$$B_i = \frac{1}{\pi}\int_{\gamma}^{\gamma+2\pi} i \cos\omega t \, d(\omega t) \tag{14}$$

where γ is an arbitrary angle form which to start the calculation. By using the trapezoidal rule (which requires less manipulation than Simpson's rule, but produces about the same accuracy) the coefficients can be calculated from N s/c as follows:

$$A_{VK} = \frac{1}{N}\left[V_{K-N}\sin\gamma + 2V_{K-N-1}\sin(\gamma + \frac{2\pi}{N}) + \ldots + 2V_{K-1}\sin(\gamma + \frac{N-1}{N}2\pi) + V_K \sin(\gamma + 2\pi)\right] \tag{15}$$

and

$$B_{VK} = \frac{1}{N} [V_{K-N} \cos\gamma + 2V_{K-N-1}$$

$$\cos(\gamma + \frac{2\pi}{N}) + ... + 2V_{K-1}\cos(\gamma + \frac{N-1}{N} 2\pi) +$$

$$V_K \cos(\gamma + 2\pi)] \qquad (16)$$

where V_K, V_{K-1}, ... are evenly spaced samples of voltage. Similar equations can be written for A_i and B_i.

1. Immunity of Fourier Method to Noise and High Frequency Components

Equations (15) and (16) are two finite impulse response digital filters whose frequency spectrums are shown in Appendix 2 to be respectively

$$\left|H_A(f)\right| = \frac{2 \sin \frac{\pi f}{50}}{N(\cos \frac{\pi f}{25N} - \cos \frac{2\pi}{N}} \times$$

$$(\sin^2 \frac{2\pi}{N} \cos^2\gamma + \sin^2\gamma \sin^2 \frac{\pi f}{25N})^{1/2} \qquad (17)$$

$$\left|H_B(f)\right| = \frac{2 \sin \frac{\pi f}{50}}{N(\cos \frac{\pi f}{25N} - \cos \frac{2\pi}{N})} \times$$

$$\sin^2 \frac{2\pi}{N} \sin^2\gamma + \cos^2\gamma \sin^2 \frac{\pi f}{25N})^{1/2} \qquad (18)$$

where γ is the angle after zero that the first sample is taken. By varying γ between extremes of 0 and $\pi/2$ it is possible to obtain a variety of characteristics dependent upon the sampling rate N. Figure 2 shows typical spectrums with 16 s/c and γ=0, $\pi/4$ and $\pi/2$.

Further study shows that by increasing the sampling rate, more unwanted components can be filtered out and that the portion of the characteristic between zero and 100 Hz is approximately independent of the sampling rate. Thus, if components above 100 Hz are filtered out by analogue means, the accuracy of the Fourier method would be independent of the sampling rate. However an analogue filter with 100 Hz cut-off would have a time delay of about ½ cycle which would be unacceptable for fast fault determination. If at least three successive calculations must be done with new samples to identify a fault, then a sampling rate of at least 8 s/c must be selected. This rate is therefore chosen for further study.

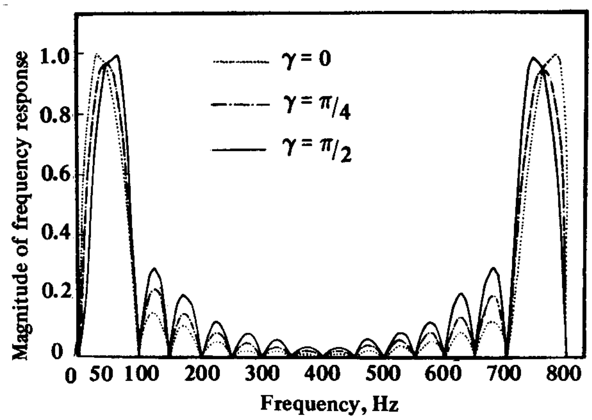

Figure 2. Spectrum of Fourier Method with 16 s/c

2. Fourier Spectrum With 8 s/c and A Butterworth Analogue Filter

The spectrum of equation (17) with 8 s/c and a double pole Butterworth analogue filter having a 150 Hz cut-off frequency is shown in Figure 3. This combination was chosen as the best after comparison with spectrums obtained from various analogue filters from 60 Hz single pole up to 200 Hz three pole. A 150 Hz double pole filter produces a time delay of about 1/8th cycle which should be acceptable in a practical scheme. Increased sampling rates (e.g. 12, 16, 20) could improve the characteristic but not significantly enough to compensate for increased hardware complexity.

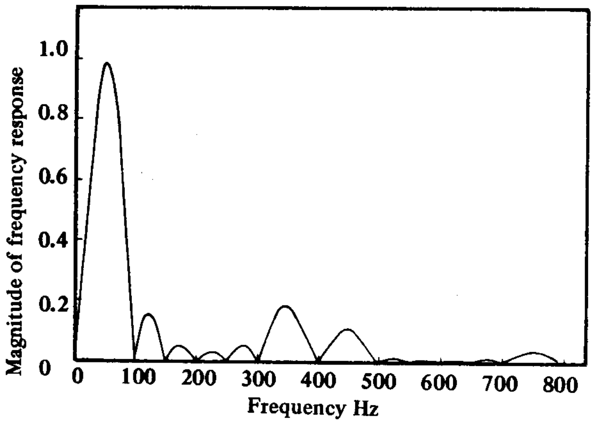

Figure 3. Overall spectrum of Fourier method with 8 s/c and 2-pole Butterworth analogue filte with 150 Hz cut-off.

For 8 s/c, A_{VK} and B_{VK} of equations (15) and (16) can be calculated as

$$A_{VK} = \frac{1}{8}\,[2(V_{K-6} - V_{K-2}) + \sqrt{2}\,(V_{K-7} + V_{K-5} - V_{K-3} - V_{K-1})] \tag{19}$$

$$B_{VK} = \frac{1}{8}\,[(V_{K-8} - 2V_{K-4} + V_K) + \sqrt{2}(V_{K-7} - V_{K-5} - V_{K-3} + V_{K-1})] \tag{20}$$

In each equation only one multiplication is necessary. But even this can be avoided by approximating $\sqrt{2}$ with 1.5 and writing equations (19) and (20) as:

$$A_{VK} = 2\,(V_{K-6} - V_{K-2}) + 1.5\,(V_{K-7} + V_{K-5} - V_{K-3} - V_{K-1}) \tag{21}$$

$$B_{VK} = (V_{K-8} - 2V_{K-4} + V_K) + 1.5\,(V_{K-7} - V_{K-5} - V_{K-3} - V_{K-1}) \tag{22}$$

The coefficient $\frac{1}{8}$ has now been omitted because it will be cancelled out in the impedance calculation. These two equations can only be calculated by addition and shifting operations. The frequency response of the method is:

$$|H_A(f)| = \frac{1}{2} \sin\frac{\pi f}{100}\,(1 + \sqrt{2}\cos\frac{\pi f}{200})$$

and with 1.5 instead of $\sqrt{2}$ it becomes:

$$|H_A(f)| = \frac{1}{2} \sin\frac{\pi f}{100}\,(1 + 1.5\cos\frac{\pi f}{200})$$

Figure 4 shows these two spectrums; the maximum difference between the two is less than 4% and for practical purposes they can be considered identical.

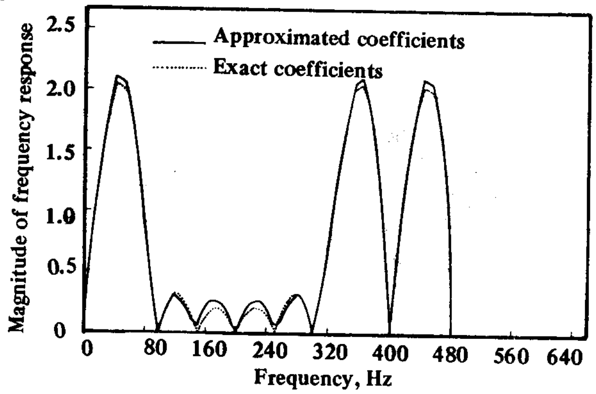

Figure 4. Spectrum of Fourier method with 8 s/c

The accuracy of the Fourier method with any sampling rate is impaired by its inability to remove the exponential d.c. offset effectively. A constant d.c. offset can be removed completely, but a decaying one contains low order frequency components and so its effects cannot be completely eliminated but only attenuated. Therefore it is concluded that Fourier methods are most applicable to transmission systems with high X/R ratios in which very little offset decay occurs in the first cycle following a fault. Alternatively, methods based on equation (2) could provide an improved response and accuracy. These mehtods will be studied next.

MCINNES & MORRISON METHOD (5)

This method, based on equation (2), has been quoted in a number of papers and extensively tested in the development laboratory. It would appear to have the potential of fast fault detection provided high order harmonics and noise can be reliably and quickly removed. To achieve this, equation (2) is usually integrated over two successive time periods t_k to (t_k+KT) and t_{k+1} to $(t_{k+1}+KT)$ where K is an arbitrary integer constant whose optimum value will be determined. The following pair of equations result:

$$\int_{t_k}^{t_k+KT} v\,dt = R\int_{t_k}^{t_k+KT} i\,dt + L\int_{t_k}^{t_k+KT} di \quad (25)$$

$$\int_{t_{k+1}}^{t_{k+1}+KT} v\,dt = R\int_{t_{k+1}}^{t_{k+1}+KT} i\,dt + L\int_{t_{k+1}}^{t_{k+1}+KT} di \quad (26)$$

Which can be written as:

$$SV_k = R.\ SI_k + L\ .\ DI_k \quad (27)$$

$$SV_{k+1} = R\ .\ SI_{k+1} + L\ .\ DI_{k+1} \quad (28)$$

where SV_k, SI_k, DI_k ... represent the integration terms. From the pair of equation (27) and (28), R and L can be calculated as:

$$R = \frac{SV_{k+1}\ .\ DI_k - SV_k\ .\ DI_{k+1}}{SI_{k+1}\ .\ DI_k - SI_k\ .\ DI_{k+1}} \quad (29)$$

and

$$L = \frac{SV_k\ .\ SI_{k+1} - SV_{k+1}\ .\ SI_k}{SI_{k+1}\ .\ DI_k - SI_k\ \ DI_{k+1}} \quad (30)$$

Using the trapezoidal integration rule and N s/c, SV_k, and DI_k can be written as (similar expression apply for SV_{k+1}, SI_{k+1} and DI_{k+1}):

$$2/_T\ .\ SV_k = V_{k+k} + V_k + 2\sum_{n=1}^{k-1} V_{k+n} \quad (31a)$$

$$2/_T\ .\ SI_k = i_{k+k} + i_k + 2\sum_{n=1}^{k-1} i_{k+n}$$

$$DI_k = i_{k+k} - i_k$$

where V_k and i_k ... are instantaneous samples of voltage and current taken at equally spaced intervals of period T senconds.

Regarding these equations as three non-recursive digital filters with voltage and current samples as inputs and SV_k and DI_k as outputs produces spectrums of (see Appendix 3):

$$\left|H_{SV_k}(f)\right| = \left|H_{SI_k}(f)\right| = \frac{1}{N}\left|\sin\frac{K\pi f}{50N}\ .\ \cot\frac{\pi f}{50N}\right| \quad (32)$$

$$\left|H_{DI_k}(f)\right| = 2\left|\sin\frac{K\pi f}{50N}\right| \quad (33)$$

Equation (33) shows that the frequency response of the DI_k filter is dependent upon two parameters: N (sampling rate) and k representing the interval over which the integration is performed. To attenuate all unwanted components with respect to the 50 Hz fundamental requires that $\sin\frac{50k\pi}{50N} = 1$ or $k = \frac{N}{2}, \frac{3N}{2}, \frac{5N}{2}, \ldots$

This is a very interesting result since to attenuate all components with respect to 50 Hz, the interval of integration should be 1/2, 3/2, 5/2 cycles, etc.

Thus, for the fastest response, a half cycle integration period is required, independent of the sampling rate. Increase or decrease of this interval only accentuates the undesirable frequency components. Figure 5 shows the spectrum of H_{DI_k} (f) for any sampling rate N.

If equation (32) is considered in a similar way, K=N/2 again produces an optimum but now the spectrum H_{SV_k} (f) depends upon N as shown for N=4, 8 and 16 s/c in Fig.6

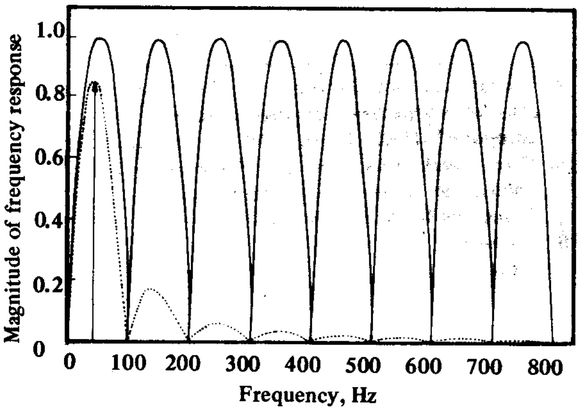

Figure 5. Spectrum of H_{DI_k}(f) for any sampling rate. Dotted line shows the effect of 2-pole Butterworth analogue filter with 60 Hz cut-off.

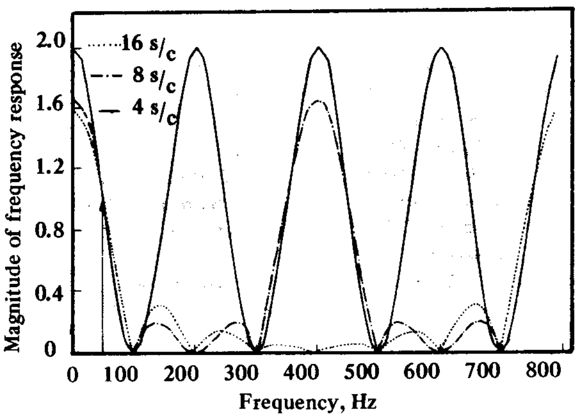

Figure 6. Spectrum of H_{SV_k}, various sampling rates. No analogue filter.

From the above argument, it can be deduced that the most vulnerable spectrum is that of H_{DI_k} (f) with respect to harmonic and non-harmonic components above 50 Hz. This spectrum (Figure 5) is similar to that of a Fourier method (section 5.1) with 4 s/c for which a double pole Butterworth analogue filter with 60 Hz cut-off would have been necessary to reduce effectively components above 50 Hz as shown by the dotted line in Figure 5. With this analogue filter, the spectrums of SV_k and SI_k with any sampling rate greater than 4 s/c would be acceptable but from a numerical accuracy viewpoint at least 8 s/c should be used. Tests have shown (14) that with 8 s/c and a 60 Hz double pole Butterworth filter, the McInnes and Morrison method enables distant faults to be detected reliably in less than one fundamental cycle. This is generally faster than the Fourier method which needs samples of one complete cycle for the correct determination of distant faults.

DISCUSSION AND CONCLUSIONS

Digital protection is now receiving considerable attention as a means of using microprocessor technology applied to the relaying task. It has often been quoted as a way of producing faster co-ordinated protection than can be obtained by using analogue relays of either the electromagnetic or solid state type. However, a careful analysis of relaying waveforms, using sampled data theory and filter design methods, has tended to show that there are more fundamental limitations to speed of response due to line and system characteristics than there is to the hardware implementation in the digital relay itself. Kohlas [17] was one of the first to recognise this in a highly mathematical paper several years ago and this has since been explored by Johns et.al., [1] AEP [18] and others in subsequently published material. In this paper it has been explored the best that can be done with signal conditioning, assuming realistic rates of sampling up to 24 s/c. If higher rates are economically feasible, then the problem of detecting faults at one relaying point using fundamental components must be re-examined. One possibility is to measure the higher harmonics to identify fault types as has been proposed by Ogden et.al., [19] and Vitins [20]. Incremental values of voltage and current as implemented by ASEA [25] could also be used in digital relaying, but a re-examination of filtering is necessary, depending on the system protected.

From the analysis given here, it can be concluded that Foruier techniques provide a sound way of establishing fundamental components of voltage and current waveforms from which impedance can be calculated. References 3, 6, 8 and 15 give examples of the speed of fault detection possible, mainly under laboratory conditions, and show that even for distant faults,one cycle operation can be assured. Faster detection of distant faults is feasible by the McInnes and Morrison [5] method of section 5, provided care is taken to eliminate unwanted harmonics and noise with a suitably designed analogue filter. Examples of this method are given by the author in reference 14 and show fault detection in about $\frac{1}{4}$ cycle. Initially, this method is recommended for digital relaying with 8 s/c and a double-pole Butterworth filter. With more powerful microprocessors allowing fast multiplication and adequate storage, the method could be developed to include shunt capacitance effects in the protected line or it could even be applied to distance protection of long cables.

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APPENDICES

1. Frequency Spectrum of Peak Determination Algorithm (Section 3)

Equations (4) and (5) can be regarded as digital filters with voltage samples as inputs and voltage derivatives as outputs. Application of the Z-transform gives

$H_1(Z) = \frac{1}{2T}(1 - Z^{-2})$ for equation (4)

and

$H_2(Z) = \frac{1}{T}(1 - Z^{-1})$ for equation (5)

Replacing Z with $e^{j\omega T}$ gives the frequency spectrum as

$$\left|H_1(f)\right| = 50N \left|\sin \frac{\pi f}{50N}\right| \quad \text{and}$$

$$\left|H_2(f)\right| = 100N \left|\sin \frac{\pi f}{50N}\right|$$

2. Frequency Spectrum of Fourier Method Algorithm (Section 4.1)

The Z-transform of both sides of equation (15) is

$$Z(A_{VK}) = \frac{1}{N}\left[Z^{-N}\sin\gamma + 2Z^{-N+1}\sin\left(\gamma + \frac{2\pi}{N}\right) + \dots + 2Z^{-1}\sin\left(\gamma + 2\pi\frac{2\pi}{N}\right) + \sin(\gamma + 2\pi)\; Z(V_K)\right] \quad (39)$$

The transfer function of the filter is therefore given by

$$H_A(z) = \frac{Z(A_{VK})}{Z(V_K)}$$

Using the two relations:

$$\sum_{k=1}^{n-1} P^k \sin(kx) = \frac{P\sin x - P^n \sin(nx) + P^{n+1}\sin(n-1)x}{1 - 2P\cos x + P^2}$$

and

$$\sum_{k=0}^{n-1} P^k \cos(kx) = \frac{1 - P\cos x - P^n\cos(nx) + P^{n+1}\cos(n-1)x}{1 - 2P\cos x + P^2}$$

it is possible to simplify the term in square brackets of equation (39) to give

$$H_A(Z) = \frac{1}{N}\,\frac{(1-Z^{-N})(Z^2\sin\gamma - 2Z\sin\frac{2\pi}{N}\cos\gamma - \sin\gamma)}{1 - 2Z\cos\frac{2\pi}{N} + Z^2}$$

Putting $Z = e^{j2\pi fT}$ and $T = \frac{0.02}{N}$ S then

$$\left|H_A(f)\right| = \frac{2\sin\frac{\pi f}{50}}{N(\cos\frac{\pi f}{25N} - \cos\frac{2\pi}{N})} \times$$

$$(\sin^2\frac{2\pi}{N}\cos^2\gamma + \sin^2\gamma\sin^2\frac{\pi f}{25N})^{½}$$

Similarly

$$\left|H_B(f)\right| = \frac{2\sin\frac{\pi f}{50}}{N(\cos\frac{\pi f}{25N} \quad \cos\frac{2\pi}{N}} \times$$

$$(\sin^2\frac{2\pi}{N}\sin^2\gamma + \cos^2\gamma\sin^2\frac{\pi f}{25N})^{½}$$

3. Frequency Spectrum of Mcinnes and Morrison Method

The Z-transform of DI_k (equation (31c)) is:

$$Z(DI_k) = (1 - Z^{-k})\, Z(i_k)$$

So:

$$H_{DI_k}(z) = (1 - Z^{-k})$$

In terms of frequency f this gives

$$H_{DI_k}(f) = 2\sin \frac{K\pi f}{50N}\, e^{j\left(\frac{1}{2} - \frac{Kf}{50N}\right)\pi}$$

So:

$$\left| H_{DI_k}(f) \right| = 2\sin \frac{K\pi f}{50N}$$

Similarly, the transfer function of SV_k is

$$H_{SV_k} = \frac{T}{2} \times \frac{(1 + Z^{-1})(Z^{-k} - 1)}{Z^{-1} - 1}$$

to give the spectrum

$$\left| H_{SV_k}(f) \right| = \frac{1}{N} \left(\sin \frac{K\pi f}{50N} \cot \frac{\pi f}{50N}\right)$$