



## Probabilistic Assessment of Bending Strength of Statically Indeterminate Reinforced Concrete Beams

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### ABSTRACT

This paper presents a reliability analysis of a two-span reinforced concrete beam, taking into account of random variations in cross-sectional dimensions, area and position of reinforcement for sagging and hogging bending moments, material strengths, loads and model uncertainties. In addition, the limit state functions for the statically indeterminate beam were derived; considering the static equilibrium requirement after the moments were redistributed as well as the codified allowable limit for the adjusted moment at each beam section. A large number of Monte Carlo simulations were performed in which the basic variables were modeled with normal, lognormal and Gumbel distributions. When the elastic moment distribution was used in evaluating the beam reliability, the two-span beam behaved as a series system with three critical nodes located at the interior support and midspan sections. The probability that the system had at least one overloaded node was greater than the failure probability of an individual node. However, considering moment redistribution made it possible to reduce the amount of reinforcement whilst maintaining the reliability of the beam. When the reinforcement area was reduced by 26% at the support section or 14% at the midspan sections, the failure probability was predicted to be  $6.90 \times 10^{-5}$ , which is deemed acceptable for a 50 year reference period.

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## 1. INTRODUCTION

Building structures are to satisfy requirements including safety of the structures against collapse, limitations on damage, deflection, vibration or other criteria. According to current structural codes, the design of reinforced concrete beams as well as other structural members is normally based on partial safety factors and characteristic values of action and resistance effects [1]. The code-based method is a refined version of the deterministic approach and can be classified as semi-probabilistic. On the other hand, a full probabilistic approach would not use partial factors of safety but directly consider inherent uncertainties in the loading, material properties and other random variables relevant to the structure behavior and safety.

Reliability is the ability of a structure to satisfy the specified requirements at any time during its design life

[2]. Each requirement can be considered as a limit state. Let  $R$  and  $E$  be the resistance and load effect respectively. The failure probability  $P_f$  of a structure can be written as  $P_f = P(R < E)$  or  $P_f = P(R/E < 1)$  or in general:

$$P_f = P(Z(R, E) < 0) \quad (1)$$

where  $Z$  is the limit state function and  $P_f$  is the probability of limit state violation [3]. The measurement of reliability can be identified with the survival probability  $P_s = (1 - P_f)$  or the reliability index  $\beta$  which is the ratio of the mean value of  $Z$  to its standard deviation if  $Z$  is normally distributed. For other distribution of  $Z$ ,  $\beta$  is just a conventional measure of the reliability and using  $P_f$  or  $P_s$  would be more meaningful. For structural members of residential and office buildings assessed at the ultimate limit state, the recommended target reliability index is 3.8 corresponding to a target failure probability of  $7.23 \times 10^{-5}$  for a reference period of 50 years [4]. Examples of

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application of reliability analysis in building structures include prediction of flexural behavior of beams subjected to pitting corrosion [5], evaluation of shear strength of deep beams with and without web reinforcement [6], torsional design of reinforced concrete beams strengthened with CFRP laminate [7]. The list also includes assessment of existing reinforced concrete beams when strengthened with additional reinforcing bars [8], evaluation of bearing capacity of slabs considering compressive membrane action [9], structural fire safety assessment of slabs exposed to fire [10], reliability analysis of seismic hazard [11], seismic assessment of buildings with soft-story and torsional irregularities [12]. As a contribution to the trend of reliability-based design, the present paper discusses a probabilistic procedure for evaluation of flexural strength of a statically indeterminate reinforced concrete beam considering the effect of moment redistribution.

Whilst a beam is normally designed based on its elastic moment envelope, moment redistribution allows the transfer of moments from critical sections where plastic hinges have formed to underutilized sections. Experimental studies showed that moment redistribution in reinforced concrete beams could occur not only at the ultimate limit state but also at the serviceability limit state [13]. A good capacity for plastic rotation and moment redistribution was also observed in high-strength concrete beams with low tensile reinforcement ratios [14]. The neutral axis depth of the beam was found to effect the redistribution ratio [15]. The plastic zones in the beam after yielding could behave like rotational springs [16]. The practical approach to allow for moment redistribution is using codified moment redistribution factor to adjust the bending moment diagram obtained from a linear elastic analysis without explicit verification of the rotation capacity. Alternatively, a plastic analysis is performed to determine the rotational demand and capacity of the hinges from first principles [17]. Some nonlinear failure analysis models have been proposed such as a stress resultant beam element with embedded discontinuity in rotations [18], a damaged-plasticity model for the concrete [19], and a fictitious crack model based on nonlinear fracture mechanics [20]. To ensure rotation capacity at the section of plastic hinges, contemporary design codes specify the allowable redistribution ratio as a function of the ratio of neutral axis depth to the section effective depth [21, 22]. The redistribution limit can also be based on the net tensile strain of the reinforcement [23].

In this paper, a two-span concrete beam is first reinforced in accordance with Eurocode 2. The limit state functions for flexural strength of the beam are then developed considering the codified limits for the redistribution ratio as well as the requirement for static equilibrium after the moments were redistributed, following the lower bound approach of plastic theory [1].

The effectiveness of moment redistribution in maintaining the beam reliability when the provided steel areas are less than the elastic-moment-based steel areas is examined via a large number of Monte Carlo simulations.

## 2. METHODS

### 2. 1. Ultimate Bending Strength and Moment Redistribution to Eurocode 2

The design moment capacity at the ultimate limit state of a singly reinforced rectangular beam,  $M_{Rd}$ , assuming that the reinforcement has yielded, can be obtained from the following expressions:

$$M_{Rd} = A_s f_{yd} (d - 0.5x) \quad (2)$$

$$x b f_{cd} = A_s f_{yd} \quad (3)$$

where  $A_s$  and  $f_{yd}$  are the area and design yield strength of the tension reinforcement,  $d$  and  $b$  are the effective depth and width of the section respectively;  $f_{cd}$  is the design compressive strength of concrete and  $x$  corresponds to the depth of the equivalent rectangular concrete stress block. The design strengths are taken as  $f_{cd} = 0.85 f_{ck} / \gamma_c$  and  $f_{yd} = f_{yk} / \gamma_s$  in which  $f_{ck}$  and  $f_{yk}$  are the characteristic compressive strength of concrete and yield strength of reinforcement,  $\gamma_c = 1.5$  and  $\gamma_s = 1.15$  are the partial factors of safety for the concrete and reinforcement, respectively [1].

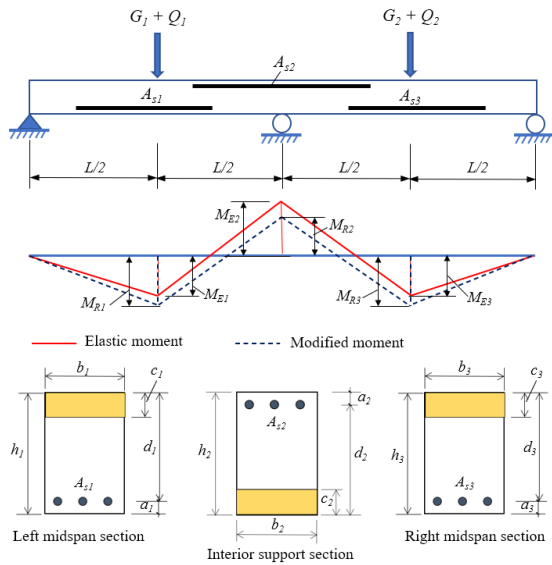
In case moment redistribution is implemented, the redistribution ratio  $\delta$ , which is the ratio of the modified moment to elastic moment at a section, for concrete with  $f_{ck}$  less than or equal to 50 MPa, should satisfy:

$$\delta \geq 0.44 + 1.25c/d \quad (4)$$

where  $c = x/0.8$  is the depth of the neutral axis of the section. It is also recommended that the bending moment capacity at any section should not be less than 70% of the elastic moment, i.e.  $\delta \geq 0.7$ . Static equilibrium must be maintained after redistribution of moments. Do Carmo and Lopes tested 10 two-span beams up to failure [14]. The recommendations of Eurocode 2 were found to be within safety limits and very similar to the experimental results for both normal-strength and high-strength concrete beams. The test also showed that the recommendations of ACI 318 were conservative for high-strength concrete beams whilst the Canadian code prediction for high values of  $c/d$  might be unsafe.

### 2. 2. Case Study Beam and Random Variables for Reliability Analysis

Figure 1 depicts a two-span beam subjected to the concentrated permanent loads  $G_1$ ,  $G_2$  with the same characteristic value of  $G_k$  and imposed load  $Q_1$ ,  $Q_2$  with the same characteristic value of  $Q_k$ . The beam self-weight was already included in the permanent load. The beam has a span length of 8 meters and is part of a floor system for general office use. The characteristic



**Figure 1.** Bending moment diagram and cross sections details of investigated beam

compression strength of the concrete is  $f_{ck} = 25$  MPa and yield strength of the steel reinforcement is  $f_{yk} = 500$  MPa. The cross sections have a nominal overall depth of  $h = 600$  mm, width  $b = 300$  mm and reinforcement axis distance of  $a = 60$  mm. The maximum moments derived from an elastic analysis are  $M_{E1}$ ,  $M_{E2}$  and  $M_{E3}$ . The areas of tension reinforcement provided at the critical sections with maximum moments are  $A_{s1}$  and  $A_{s3}$  at midspans and  $A_{s2}$  over the interior support of the beam. The moments of resistance of the sections associated with the steel areas  $A_{s1}$ ,  $A_{s2}$  and  $A_{s3}$  are  $M_{R1}$ ,  $M_{R2}$  and  $M_{R3}$ , respectively.

Table 1 presents the statistical properties of basic random variables  $X$  adopted by the reliability analysis, which include dimensions of cross sections, areas and positions of reinforcements, strengths of materials, loads and model uncertainties. The statistical data of the variables are obtained from real buildings in European countries and reported in the European publication EUR 29410 [24]. The normal distribution is suitable for symmetric random variables with low variation (coefficient of variation less than 0.3) such as the dead load and geometrical dimensions of cross-sections. The lognormal distribution with lower limit at zero is recommended for representation of mechanical properties of materials whose logarithms are normally distributed. The Gumbel distribution, which has a simple exponential shape, can be used to represent the distribution of extreme values of random variables such as the live load and wind pressure. In Table 1, the subscripts 1, 2, 3 included in the parameters  $h$ ,  $b$ ,  $A_s$ ,  $a$ ,  $f_s$ ,  $f_c$  correspond to the midspan section of the left span, the interior support section, and the midspan section of the right span, respectively. The live loads  $Q_1$ ,  $Q_2$  were modeled by a Gumbel distribution with an average of

**TABLE 1.** Statistical properties of random variables

Symbol, $X$	Mean, $\mu_X$	Standard deviation, $\sigma_X$	Distribution
$b_1, b_2, b_3$	$b$	10 mm	Normal
$A_{s1}, A_{s2}, A_{s3}$	$A_{s1}, A_{s2}, A_{s3}$	$0.02\mu_X$	Normal
$a_1, a_2, a_3$	$a$	10 mm	Normal
$f_{y1}, f_{y2}, f_{y3}$	$f_{yk} + 2\sigma_X$	$0.053\mu_X$	Lognormal
$f_{c1}, f_{c2}, f_{c3}$	$f_{ck} + 2\sigma_X$	$0.121\mu_X$	Lognormal
$G_1, G_2$	$G_k$	$0.1\mu_X$	Normal
$Q_1, Q_2$	$0.6Q_k$	$0.35\mu_X$	Gumbel
$\theta_R$	1	0.1	Normal
$\theta_E$	1	0.1	Normal

$0.6Q_k$  and coefficient of variation of 0.35 as recommended by the EUR 29410 for general offices with a 50 year reference period. The model uncertainties factors  $\theta_E$  and  $\theta_R$  take account of imprecision and incompleteness of the relevant theoretical models for load and resistance effects [24].

### 2. 3. Limit State Functions

**2. 3. 1. Without Moment Redistribution** Since redistribution of moment is not considered, the bending capacity of each critical section of the beam must be checked against the elastic moment at that section. The limit state functions  $Z_1(X)$ ,  $Z_2(X)$  and  $Z_3(X)$  for flexural strength of individual cross sections (left midspan, interior support, right midspan) are given by Equations (5)-(7) where the expressions for moment capacity  $M_{R1}$ ,  $M_{R2}$  and  $M_{R3}$  were derived from Equations (2)-(3). The expressions of maximum moments  $M_{E1}$ ,  $M_{E2}$  and  $M_{E3}$  at the midspan and support sections were obtained from a conventional elastic structural analysis of the two-span beam [25].

$$Z_1(X) = M_{R1} - M_{E1} = \theta_R A_{s1} f_{y1} \left( d_1 - \frac{0.5 A_{s1} f_{y1}}{0.85 b_1 f_{c1}} \right) - \theta_E \{0.2031(G_1 + Q_1) - 0.0469(G_2 + Q_2)\}L \quad (5)$$

$$Z_2(X) = M_{R2} - M_{E2} = \theta_R A_{s2} f_{y2} \left( d_2 - \frac{0.5 A_{s2} f_{y2}}{0.85 b_2 f_{c2}} \right) - \theta_E (G_1 + Q_1 + G_2 + Q_2)0.09375L \quad (6)$$

$$Z_3(X) = M_{R3} - M_{E3} = \theta_R A_{s3} f_{y3} \left( d_3 - \frac{0.5 A_{s3} f_{y3}}{0.85 b_3 f_{c3}} \right) - \theta_E \{0.2031(G_2 + Q_2) - 0.0469(G_1 + Q_1)\}L \quad (7)$$

### 2. 3. 2. With Moment Redistribution

When moment redistribution is considered, it is not compulsory to reinforce each beam section based on the elastic moment. The beam can be designed on the basis of the lower bound approach (or “safe” or “static” method) which is allowed by Eurocode 2. The modified moments can now be taken as the moments of resistance  $M_{R1}$ ,  $M_{R2}$

and  $M_{R3}$  of the sections as shown in Figure 1. The yield condition is hence not violated anywhere. In order to maintain static equilibrium after the moments were redistributed as required by the lower bound method [1], the limit state functions  $Z_4(X)$  and  $Z_5(X)$  are written as Equations (8) and (9) for the left and right spans. The total bending resistance is  $(M_{R1} + 0.5M_{R2})$  for the whole left span and  $(M_{R3} + 0.5M_{R2})$  for the whole right span. The total static applied moment of the left and right spans are  $(G_1 + Q_1)L/4$  and  $(G_2 + Q_2)L/4$  respectively when the concentrated loads are applied at midspan of the beam.

$$Z_4(X) = \theta_R \left\{ A_{s1} f_{y1} \left( d_1 - \frac{0.5A_{s1} f_{y1}}{0.85b_1 f_{c1}} \right) + 0.5A_{s2} f_{y2} \left( d_2 - \frac{0.5A_{s2} f_{y2}}{0.85b_2 f_{c2}} \right) \right\} - \theta_E (G_1 + Q_1)L/4 \quad (8)$$

$$Z_5(X) = \theta_R \left\{ A_{s3} f_{y3} \left( d_3 - \frac{0.5A_{s3} f_{y3}}{0.85b_3 f_{c3}} \right) + 0.5A_{s2} f_{y2} \left( d_2 - \frac{0.5A_{s2} f_{y2}}{0.85b_2 f_{c2}} \right) \right\} - \theta_E (G_2 + Q_2)L/4 \quad (9)$$

In addition, the limit state functions  $Z_6(X)$ ,  $Z_7(X)$  and  $Z_8(X)$  of Equations (10)-(12) are derived to reflect the allowable limits for the redistribution ratio given by Equation (4). These functions relate the adjusted moment ratios  $M_{R1}/M_{E1}$ ,  $M_{R2}/M_{E2}$  and  $M_{R3}/M_{E3}$  to the ratio of the depth of the compression zone to the effective depth of the sections  $c_1/d_1$ ,  $c_2/d_2$  and  $c_3/d_3$ . Eurocode 2 suggests that using the codified redistribution ratios with linear elastic analysis is possible without explicit verification of the rotation capacity in continuous beams [1].

$$Z_6(X) = \frac{M_{R1}}{M_{E1}} - \max \left\{ \left( 0.44 + \frac{1.25f_{y1}A_{s1}}{0.68f_{c1}b_1d_1} \right); 0.7 \right\} \quad (10)$$

$$Z_7(X) = \frac{M_{R2}}{M_{E2}} - \max \left\{ \left( 0.44 + \frac{1.25f_{y2}A_{s2}}{0.68f_{c2}b_2d_2} \right); 0.7 \right\} \quad (11)$$

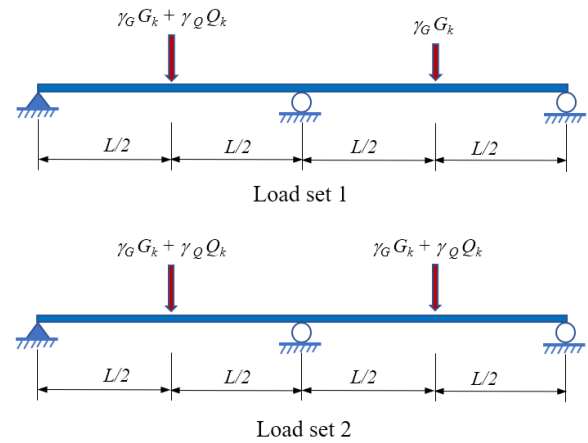
$$Z_8(X) = \frac{M_{R3}}{M_{E3}} - \max \left\{ \left( 0.44 + \frac{1.25f_{y3}A_{s3}}{0.68f_{c3}b_3d_3} \right); 0.7 \right\} \quad (12)$$

### 2. 3. 3. Monte Carlo Simulation and Strength Evaluation

A large number of Monte Carlo simulations were performed to evaluate the ultimate flexural strength of individual cross sections as well as the whole beam. Firstly, independent random values of the variables with statistical properties given by Table 1 were generated using MATLAB built-in random number generators [26]. The limit state functions, or safety margins, of individual cross sections, Equations (5)-(7), and the whole beam, Equations (8)-(12), were then calculated. The random simulation process was repeated for 10 million times during which the number of failure events with negative safety margins were counted. Table 2 presents the criteria for a failure event relating to the strengths of an individual cross section and of the whole beam. The ratio of the number of failure events to the total number of trials defines the failure probability  $P_f$ .

**TABLE 2.** Failure criteria for individual cross sections and whole beam

Checked item	Failure criteria
Left midspan section, without moment redistribution	$Z_1(X) < 0$
Interior support section, without moment redistribution	$Z_2(X) < 0$
Right midspan section, without moment redistribution	$Z_3(X) < 0$
Whole beam, without moment redistribution	$(Z_1(X) < 0)$ or $(Z_2(X) < 0)$ or $(Z_3(X) < 0)$
Whole beam, with moment redistribution	$(Z_4(X) < 0)$ or $(Z_5(X) < 0)$ or $(Z_6(X) < 0)$ or $(Z_7(X) < 0)$ or $(Z_8(X) < 0)$



**Figure 2.** Load arrangements for design bending moment

## 3. RESULTS AND DISCUSSIONS

**3. 1. Design Reinforcement to Eurocode 2** The required reinforcement areas were first determined using the Eurocode 2 conventional approach. For this symmetric beam, the elastic moment envelope can be obtained by considering the load arrangements shown in Figure 2; with the load set 1 aiming at getting the maximum sagging moment at the left midspan section and the load set 2 aiming at finding the maximum hogging moment at the interior support section.

At the ultimate limit state, the partial safety factors are  $\gamma_G = 1.35$  and  $\gamma_Q = 1.5$  for the permanent and imposed loads respectively. The maximum design moments were  $M_{Ed2} = 317$  kNm at the interior support and  $M_{Ed1} = 298$  kNm at the midspan section. The corresponding reinforcement areas computed using Equations (2)-(3) were  $A_{s2} = 1591$  mm<sup>2</sup> at the interior support and  $A_{s1} = A_{s3} = 1476$  mm<sup>2</sup> at the midspan sections.

### 3. 2. Reliability of Individual Cross Sections versus Whole Beam

The reinforcement areas based on the

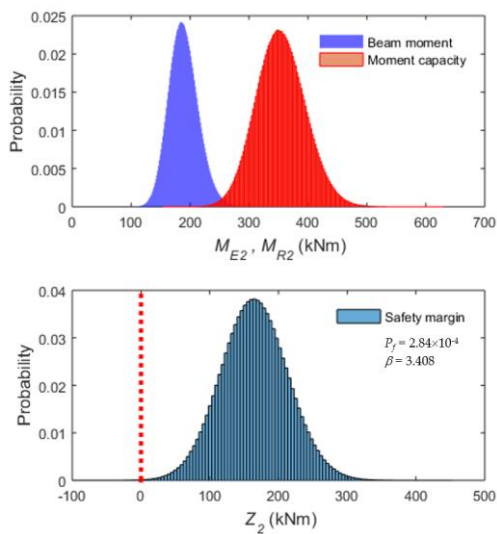
elastic moment envelope were considered as the mean values of the reinforcement areas used in the Monte Carlo simulations. Table 3 presents the failure probability  $P_f$  and reliability index  $\beta$  for individual sections and the whole beams with and without moment redistribution (MR), obtained from  $10^7$  simulations.

The  $\beta$  values of both the midspan and support sections were higher than the recommended target value of 3.8. The  $P_f$  values for the individual sections and the whole beam either with or without moment redistribution were all well below the target failure probability of  $7.23 \times 10^{-5}$ , indicating that the traditional design approach was conservative. Without moment redistribution, the probability that the beam had at least one overloaded section was higher than the failure probability of any individual cross section. By contrast, considering moment redistribution resulted in the failure probability of the beam being significantly lower than the elastic-moment-based failure probability (Table 3).

Further Monte Carlo simulations were performed in which the mean reinforcement areas were reduced. Figure 3 shows the probability density functions (PDFs) of the elastic-based-moment and safety margin for the interior support section when its steel reinforcement area

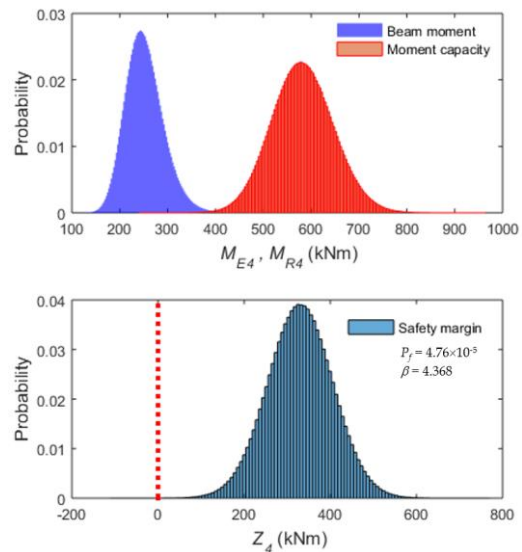
**TABLE 3.** Reliability of beam with Eurocode-2-based reinforcement areas

Individual cross sections				Whole beam	
Midspan		Interior support		Without MR	With MR
$P_f$	$\beta$	$P_f$	$\beta$	$P_f$	$P_f$
$2.39 \times 10^{-5}$	4.401	$0.46 \times 10^{-5}$	4.320	$5.02 \times 10^{-5}$	$1.20 \times 10^{-5}$

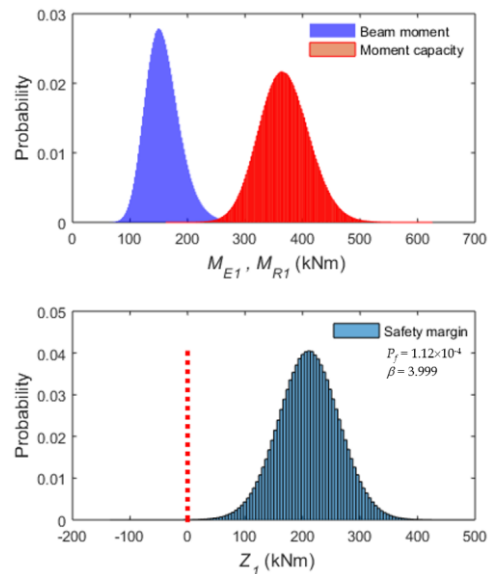


**Figure 3.** PDF of moment capacity of cross section at support with 20% reduction in  $A_{s2}$

was reduced by 20%. The elastic-moment-based  $P_f$  of the interior support section was found to be  $2.84 \times 10^{-4}$  which is nearly four times higher the recommended target failure probability. However, the redistributed-moment-based  $P_f$  of the corresponding beam was  $4.76 \times 10^{-5}$  which is below the target failure probability (Figure 4). In the event that the reinforcement amount of each midspan section was reduced by 10%, the elastic-moment-based  $P_f$  of the midspan sections was  $1.12 \times 10^{-4}$  (Figure 5) whilst the beam with moment redistribution was still robust with a  $P_f$  value of  $4.16 \times 10^{-5}$  (Figure 6).



**Figure 4.** PDF of moment capacity of whole beam with 20% reduction in  $A_{s2}$



**Figure 5.** PDF of moment capacity of cross section at midspan with 10% reduction in  $A_{s1}$  and  $A_{s3}$

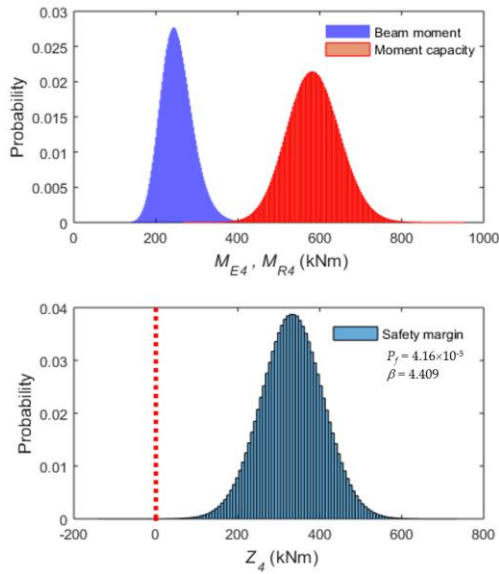


Figure 6. PDF of moment capacity of whole beam with 10% reduction in  $A_{s1}$  and  $A_{s3}$

Let  $A_{s,prov}$  be the steel areas provided for the beam sections and  $A_s$  be the steel areas resulted from the elastic moment envelope. Considering the  $A_{s,prov}$  to  $A_s$  ratio varying from 0.7 to 1, the random simulations were performed again with  $A_{s,prov}$  assigned to the mean values of the reinforcement areas. Figures 7 and 8 compare the elastic-moment-based  $P_f$  and  $\beta$  values of the sections with the modified-moment-based  $P_f$  and  $\beta$  values of the beam, for various  $A_{s,prov}/A_s$  values.

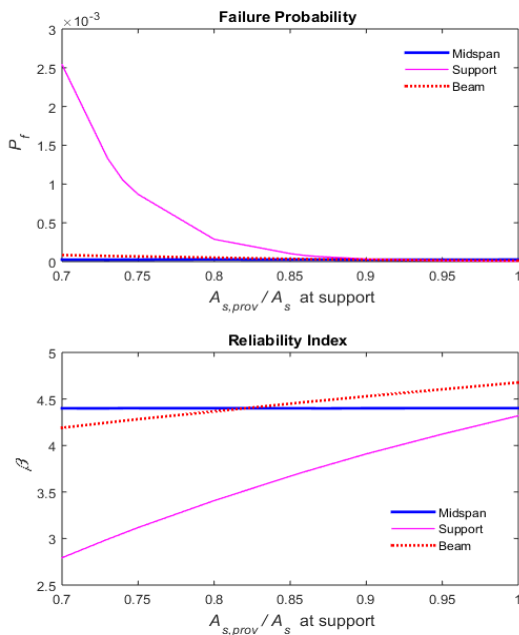


Figure 7. Reliability of individual cross sections and whole beam with reduction in  $A_{s2}$

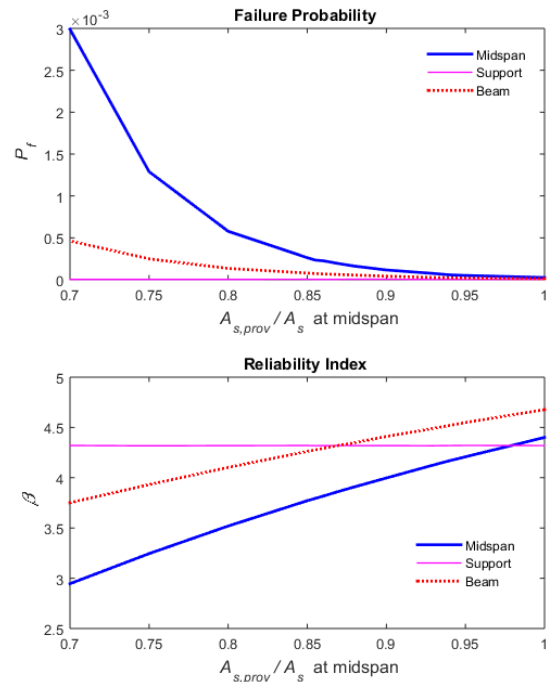


Figure 8. Reliability of individual cross sections and whole beam with reduction in  $A_{s1}$  and  $A_{s3}$

### 3. 3. Reinforcement Modification and Beam Performance

Figure 9 allows identification of the failure probability and reliability index of the beam with moment redistribution considered when the

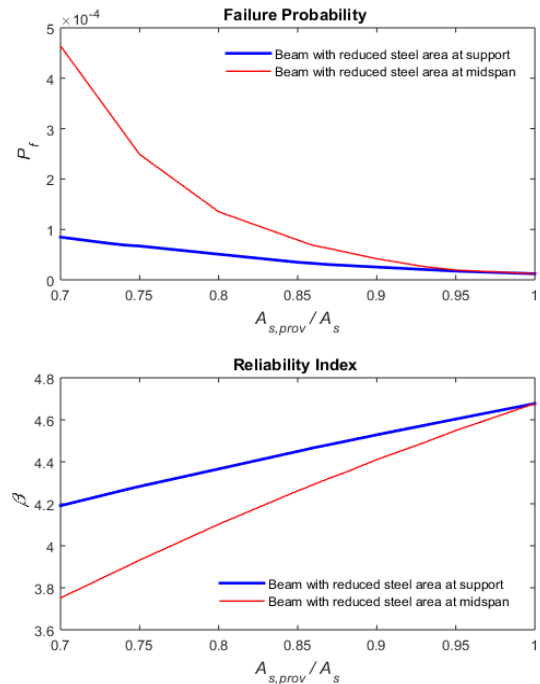


Figure 9. Performance of beam with reduced reinforcement areas



reinforcement areas at either the support or midspan sections were modified. For instance, using  $A_{s,pro}/A_s = 0.9$  at the interior support section would provide  $P_f = 2.53 \times 10^{-5}$  and  $\beta = 4.529$  for the beam. Alternatively, taking  $A_{s,pro}/A_s = 0.9$  at the midspan sections would result in  $P_f = 4.18 \times 10^{-5}$  and  $\beta = 4.410$  for the beam. For a further steel reduction with  $A_{s,pro}/A_s$  equal to 0.74 at the support section or 0.86 at the midspan sections, the beam would still be acceptable with  $P_f = 6.90 \times 10^{-5}$  and  $\beta = 4.280$ . The advantage of moment redistribution in statically indeterminate reinforced concrete beams was also confirmed by experimental data presented in the relevant literature. Testing 33 two-span beams, Scott and Whittle [13] observed that 23 specimens which failed in flexure essentially all achieved the designed 30% moment modification by the end of the test. The redistribution ratio limit of 0.7 specified by Eurocode 2 was hence guaranteed. The formation of plastic hinges was well recognized by the test. Another experimental work on two-span beams performed by Ehsani et al. [27] revealed a reduction of 18.5% in the measured moment compared with the elastic moment at the ultimate load, indicating a redistribution ratio of 0.815.

Moreover, the desired reliability of the beam could still be maintained when the reinforcement areas at all three critical sections (midspan sections 1, 3 and support section 2) were reduced properly. For instance, Table 4 presents the failure probability  $P_f$ , survival probability  $P_s$  and reliability index  $\beta$  of the beam in response to some reinforcement modification scenarios. As can be seen, all the obtained  $P_f$  values were lower than the target failure probability of  $7.23 \times 10^{-5}$  and the predicted  $\beta$  values were higher than the target reliability index of 3.80 for a reference period of 50 years.

**TABLE 4.** Examples of successful reinforcement modifications options

$A_{s,pro}/A_s$ at section			Beam performance		
1	2	3	$P_f$	$P_f$	$\beta$
0.89	0.94	0.89	$6.74 \times 10^{-5}$	0.999933	4.283
0.91	0.90	0.91	$6.66 \times 10^{-5}$	0.999933	4.290
0.93	0.87	0.93	$6.68 \times 10^{-5}$	0.999933	4.284
0.95	0.83	0.95	$6.97 \times 10^{-5}$	0.999930	4.275
0.97	0.80	0.97	$6.90 \times 10^{-5}$	0.999931	4.282

#### 4. CONCLUSIONS

From the research that has been carried out, the effect of moment redistribution on the flexural strength reliability of the statically indeterminate reinforced concrete beam can be seen.

- When the elastic moment distribution was used, the two-span beam behaved as a series system with three critical nodes located at the interior support and midspan sections. The probability that the system had at least one overloaded node was proved to be greater than the failure probability of each node.
- Without using moment redistribution, a 9% reduction in the support reinforcement area or a 2% reduction in the midspans reinforcement area was found to place the elastic-based beam in a vulnerable state with a  $P_f$  of  $7.50 \times 10^{-5}$  which exceeded the target value.
- When moment redistribution was considered, it was possible to reduce the amount of reinforcement whilst maintaining the reliability of the beam. The beam can still be deemed acceptable with a  $P_f$  of  $6.90 \times 10^{-5}$  when the reinforcement area was reduced by 26% at the support section or 14% at the midspan sections.
- Reasonable adjustments of the reinforcements of the support section simultaneous with the midspan sections were also found to ensure the beam reliability yet save the steel reinforcement amount.

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### Persian Abstract

#### چکیده

این مقاله یک تحلیل قابلیت اطمینان یک تیر بتن مسلح دو دهانه را ارائه می‌کند، که تغییرات تصادفی در ابعاد مقطع، مساحت و موقعیت آرماتور برای ممان‌های خمشی فرورفتگی و گریز، مقاومت مواد، بارها و عدم قطعیت‌های مدل را در نظر می‌گیرد. علاوه بر این، توابع حالت حدی برای تیر از نظر استاتیکی نامعین، با در نظر گرفتن نیاز تعادل ایستایی پس از توزیع مجدد ممان‌ها و همچنین حد مجاز مدون برای گشتاور تنظیم شده در هر مقطع پرتو، استخراج شد. تعداد زیادی شبیه‌سازی مونت کارلو انجام شد که در آن متغیرهای پایه با توزیع‌های نرمال، لگ نرمال و گامبل مدل‌سازی شدند. هنگامی که توزیع گشتاور الاستیک در ارزیابی قابلیت اطمینان تیر مورد استفاده قرار گرفت، تیر دو دهانه به عنوان یک سیستم سری با سه گره بحرانی واقع در بخش پشتیبانی داخلی و بخش میانی رفتار کرد. احتمال اینکه سیستم حداقل یک گره اضافه بار داشته باشد بیشتر از احتمال شکست یک گره منفرد بود. با این حال، در نظر گرفتن توزیع مجدد لنگر، کاهش میزان آرماتور را در عین حفظ قابلیت اطمینان تیر ممکن کرد. زمانی که ناحیه آرماتور ۲۶ درصد در بخش پشتیبانی یا ۱۴ درصد در بخش‌های میانی کاهش یافت، احتمال شکست ۰/۰۰۰۰۶۹ پیش‌بینی شد که برای یک دوره مرجع ۵۰ ساله قابل قبول تلقی می‌شود.

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