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Modeling Traffic Signal Control System at an Isolated Intersection using Queuing Systems

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A B S T R A C T

As the population grows in cities worldwide, the number of vehicles present on the roadways also increases, resulting in slow-moving and congested traffic. Therefore, a widespread problem in large cities concerns the traffic in the streets. Traffic signals are one of the most powerful tools available to city authorities for urban traffic control. Their proper installation can improve both traffic flow and the safety of all road users. Extensive research has been conducted to reduce the impacts of long car queues, based mainly on traffic signal timing optimization. This paper estimates the average waiting time at an isolated intersection and optimizes the timing of the green and red phases using an analysis of queueing systems. The control system is assumed to be the fixed-time type, and the Poisson process is considered for the arrivals. The proposed model is applied to real traffic data at a two-phase intersection in Bojnurd, Iran. It needs to be noted that the current situation at the intersection under study reduces average waiting time only for one side, but the analytic model can reduce average waiting time for the whole intersection. Moreover, simulation experiments are carried out, the results of which verify the capabilities of the proposed methodology in traffic signal control applications.

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NOMENCLATURE

Sets and Indices		N_i	Intersection capacity
i	Movement phase index ($i = 1,2$)	w_i	Average waiting time in phase i
Parameters		W	Average waiting time at the intersection
λ_i	Arrival rate	**Decision Variables**	
μ_i	Service rate	A_1	Green light time length for phase 1
C	Yellow light time length	B_1	Red light time length for phase 1
n_{i1}	Number of vehicles in phase i	A_2	Red light time length for phase 2
n_{i2}	Signal state for phase i	B_2	Green light time length for phase 2
n_{i3}	The current state of the green or red light in phase i.		

1. INTRODUCTION

Traffic congestion is a serious problem in urban areas, where transportation demand exceeds road capacity. A frequent aspect is the induced air pollution with negative effects on health and living environments and the global economy due to the wasted time. Mere construction of new roads might not provide the best solution to congestion problems due to the enormous financial requirements and complex network effects. Instead, there is huge potential to improve the conditions through efficient traffic management and optimization of transportation networks. There are two major aspects to analyze and optimize urban transportation networks: traffic assignment, which is an important tool for forecasting traffic flow over the urban transportation network, and traffic signal timing, used to improve the services. Traffic signals are often controlled as fixed-time

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or real-time. Each of these strategies can be subdivided into isolated-intersection (controlling a single intersection and disregarding others) and coordinated-intersection (considering more than one intersection). The latter can be further subdivided into arterial and grid-network. When several closer intersections on an arterial are independent of signal control, the upstream vehicles are likely to meet the red light at the intersection downstream. The isolated control method applied among intersections unavoidably causes frequent stops. The main characteristic of arterial control is that the same cycle is established, and there are several semaphores with relative phase differences. Arterial control is appropriate for intersections with relatively short distances and heavy traffic flow. Grid-network control is an extension of arterial control, which adopts coordinated control to several sets of semaphores on a vast area over the road network.

In the fixed-time mode, signal timing is scheduled in advance for a specified period. The fixed-time signal control uses preset time intervals repeated every time the signal cycles, regardless of traffic volume changes. Some fixed-time systems use different preset time intervals for the morning or evening rush hours and other busy times. The fixed-time control system is the simplest type, and a great deal of research has been conducted to assess the performance of intersections with this control system. One of the first studies on signal timing with the fixed-time control system is the work by Webster [1], where an analytic model was presented to set the duration of the green signal and the fixed cycle length. The objective is to minimize vehicle delay at an isolated intersection, and the arrivals are assumed to have the Poisson distribution. After that, many papers were focused on fixed-time signal control. For example, Miller [2] and Newell [3] proposed approximation approaches to calculating the residual queue length at the end of the green phase in a fixed-time control system. For the first time, Heidemann [4] presented a relationship between the distribution function of queue length and delay with Poisson-distributed arrivals, an isolated intersection, and a fixed-time control system. Hu et al. [5] proposed an $M/D^X/1$ queueing model with server vacations for a fixed-time control system, which could be considered somewhat as a generalization of Heidemann's work for cases with several lanes in each street at the intersection. Chanloha et al. [6] compared the performance of the Q-learning framework to that of the M/M/1 and D/D/1 models to signal timing at an isolated intersection with a fixed cycle length. The total delay for each queuing model and, accordingly, the optimal green time were obtained on that basis, and the results were then compared to those of the Q-learning algorithm. The results indicate that the Q-learning algorithm can significantly improve network throughput and total delay with respect to those in queueing models. Van Den et al. [7] compared a queueing model for fixed-time signal timing with a batch-service queueing model and presented new equations for average vehicle delay. Habibi et al. [8] presented two algorithms to reduce traffic density and delay. Akçelik and Rouphail [9] studied a queueing model with batch arrivals to optimize the number of vacations and average queue length at an isolated intersection with a fixed-time control system. Pacheco et al. [10] analyzed an M/D/1 queue with vacations to estimate queue length variance and delay at an intersection with a fixed-time control system. Yang and Shi [11] proposed a queueing model with batch services and batch arrivals, where the objective is to minimize the average queue length on a multilane road. Ghasemi and Rasekhi [12] proposed an approach for predicting traffic signals using game theory and neural networks with swarm particle optimization. Boon et al. [13] derived the queue length distribution for a fixed-time control system, avoiding the computational challenges that previous studies had faced to solve several characteristic equations. Sumi and Ranga [14] proposed a new intelligent traffic management system (TMS), an approach for smart cities to control traffic lights and ease ambulance movement in cities. Amini and Shahi [15] and Faghri [16] investigated the influence of geometric and control features on the quality of traffic services.

Another type of traffic signal control system is the real-time control system, in which the traffic signal timing is carried out simultaneously with the inspection of traffic status using automatic cameras, and the green and red times in each cycle depend on the intersection traffic status. The real-time traffic signal control system can be of two sub-types: actuated and adaptive traffic signal control. Unlike fixed-time control, actuated control constantly adjusts the timing of the green light and, in some cases, the order of the phases. These settings are based on the traffic demand criteria recorded by the detectors at the intersections. This control method usually reduces delay and increases capacity and can be safer than fixed-time control, but it is very expensive to implement and requires advanced training for proper execution. In an adaptive control system, the traffic signal time constantly varies by the changes in vehicle arrival patterns at the intersection. This traffic information is collected by the detectors at the intersection and then evaluated. Finally, correction is made to the signal timing, where the traffic signal times are updated.

However, not many papers have considered real-time traffic signal timing control systems. Zhang and Wang [17] investigated an actuated traffic signal control system to minimize mean vacation and mean queue length and maximize vehicle throughput. Jiao et al. [18] presented a multi-objective model to minimize average delay, minimize the average number of stops, maximize traffic capacity for an inductive traffic control system, and optimize cycle time and time-varying green time using the Particle Swarm Optimization (PSO) algorithm.

Mirchandani and Zou [19] proposed an M/G/1 queueing model for an adaptive traffic signal control system. The objective is to minimize the mean queue length and average waiting time for vehicles. In that research, an intersection with two straight movement phases is considered and turns taken to the right, and left is ignored. An adaptive control system is also considered. When the signal shows the green light at one side of the intersection, the system will not turn it into red until the queue on that street is entirely empty. Recently, Chedjou and Kyamakya [20] have reviewed the studies on traffic signal control systems, investigating various strategies and their strengths and drawbacks, along with the challenges involved in their applications.

Table 1 summarizes some articles in this field that are categorized based on the intersection type, decision variables, and control system type. Despite the extensive research on traffic signal timing, some problems still require further inspection. Since the residual queue at the end of the green time is hard to obtain precisely, most studies have used approximate equations to calculate its values in fixed-time control systems and failed to provide strict equations for the objective functions. This paper proposes an analytic approach to obtain average waiting time at an isolated intersection for the fixed-time control system using queueing system analysis. For this purpose, an appropriate definition of queuing system state is first provided, and the corresponding equilibrium equations are then extracted and solved, from which the limiting probabilities are derived. Finally, the performance criterion (average waiting time) for the queueing system is obtained, and the corresponding equation is considered as the objective function of the mathematical model.

TABLE 1. Literature review summary

	Intersection Type		**Decision Variables**				**Control System Type**		
	Isolated	**Arterial**	**Cycle Time**	**Green Time**	**Phases sequences**	**Red Time**	**Fixed Time**	**Actuated**	**Adoptive**
Webster [1]	*			*			*		
Chanloha [6]	*			*			*		
Akçelik [9]	*			*			*		
Pacheco [10]	*			*			*		
Yang [11]	*			*			*		
Zhang and Wang [17]	*	*		*			*		
Jiao et al. [18]	*		*	*				*	
Mirchandani and Zou [19] Chedjou and Kyamakya [20]	*		*	*					*
Ceylan and Bell [21]		*	*	*			*		
Wunderlich et al. [22] Wismans et al. [23]	*						*	*	
Ghavami et al. [24]	*			*	*		*	*	*
Ren et al. [25]	*			*			*		
Zhou and Cai [26]	*			*			*		
Dujardin et al [27]	*			*		*			*
Wu and Wang [28]	*			*			*		
Peñabaena et al. [29]		*	*	*			*		
Anusha et al. [30]	*		*	*			*		
Olszewski [31]	*		*	*			*		
Shirvani and Maleki [32]	*			*				*	
Lim et al. [33]	*			*			*		
Chin et al. [34]		*	*	*	*		*		
Current research	*		*	*			*		

The rest of the paper is organized as follows. Section 2 provides the problem definition and model assumptions. Section 3 presents equilibrium equations that are used to generate the mathematical model. Section 4 describes the results. Finally, section 5 summarizes the implications.

2. PROBLEM DEFINITION

Let us consider an isolated intersection with two straight movement phases at each of the two intersecting streets (phase 1 for north-to-south movements and phase 2 for east-to-west movements), where one line exists for vehicles to pass along in each phase. The arrivals in each phase are considered as a Poisson process with a rate of λ_i $(i = 1, 2)$, and the service is assumed to be an exponential process with a rate of μ_i $(i = 1, 2)$. Fixed-time timing is assumed for the traffic signal. In this strategy, the timing plan of traffic signals is adjusted to a preset time according to prior data. The decision variables are the green time and the red time. For analysis of the queueing system applied to the intersection, the green time (red time) for phase 1 is first assumed to have Erlang distribution with degrees of freedom g_1 (r_1) and rate A_1 (B_1). If the degrees of freedom tend to infinity in the limiting state, a constant will be obtained. Therefore, if the degrees of freedom for the green time (red time), *i.e.*, g_1 (r_1), tend to infinity, the green time (red time) will have a constant value with parameter A_1 (B_1), where A_1 (B_1) is the decision variable. The same analysis can be provided for phase 2; that is, we can assume in phase 2 that the red time (green time) has Erlang distribution with degrees of freedom g_2 (r_2) and rate A_2 (B_2). Again, if the degrees of freedom tend to infinity, a constant value with parameter A_2 (B_2) will be obtained for the red time (green time). The yellow time is assumed in the analysis to be constant for each phase and have Erlang distribution with degrees of freedom l and rate C, like the green and red times. Furthermore, if the degrees of freedom tend to infinity, a constant value with parameter C will be obtained for the yellow time. Unlike the green and red times, however, it is not a decision variable.

3. MATHEMATICAL MODEL

To obtain the equilibrium equations, the state of the system for each phase is defined as n_{i1}, n_{i2}, n_{i3}, where n_{i1} represents the number of vehicles in phase i $(0 \le n_{i1} \le N_i)$. Since it is not possible to solve the equilibrium equations for cases where system capacity is infinite, the intersection capacity is assumed to be a finite value N_i for $i = 1, 2$, providing a good approximation for the case of infinite capacity.

It should be noted that n_{i2} denotes the signal state for phase i, where $n_{i2} = 0$ represents the red signal, $n_{i2} = 1$ indicates the green light, and $n_{i2} = 2$ shows the yellow light. Moreover, n_{i3} represents the current state of the green or red light in phase i. In other words, when the traffic signal shows green light ($n_{12} = 1$) in phase 1, n_{13} increases stepwise from 1 to g_1, where $n_{13} = 1$ indicates the beginning of the green light, and $n_{13} = g_1$ denotes the end for phase 1. Once $n_{13} = g_1$, the signal for phase 1 shows yellow light ($n_{12} = 2$), and $n_{13} = 1$ again, increasing stepwise to 1. $n_{13} = l$ implies that the signal light in phase 1 should be changed from yellow to red and that in phase 2 should be changed from red to green. Therefore, $n_{12} = 0$ (phase 1 signal light is red), and $n_{22} = 1$ (phase 2 signal light is green). Then, n_{13} increases stepwise from 1 to r_1, and phase 1 signal light turns green when it reaches r_1. A similar analysis can be provided for phase 2. As a result, the bounds for n_{13} can be given as follows.

$$\text{Phase 1:} \begin{cases} 1 \le n_{13} \le r_1 \ , & n_{12} = 0 \\ 1 \le n_{13} \le g_1 \ , & n_{12} = 1 \\ 1 \le n_{13} \le l \ , & n_{12} = 2 \end{cases}$$

$$\text{Phase 2:} \begin{cases} 1 \le n_{23} \le g_2 \ , & n_{22} = 0 \\ 1 \le n_{23} \le r_2 \ , & n_{22} = 1 \\ 1 \le n_{23} \le l \ , & n_{22} = 2 \end{cases}$$

The system state is illustrated in the following diagrams for better understanding.

State (0, 1, 1): In this state (Figure 1), in phase 1, there is no vehicle, the light is green, and the first stage of the green time is dominant. The degrees of freedom for the green time is g_1, and n_{13} increases stepwise from 1 to g_1, when the light turns yellow. At that moment, $n_{13} = g_1$ indicates the end of the green time, $n_{12} = 2$, and $n_{13} = 1$. The latter denotes the beginning of the yellow time.

The equilibrium equation for state (0, 1, 1) is as follows.

$$\left(\lambda_1 + \frac{g_1}{A_1}\right)\pi_{(0,1,1)} = \frac{r_1}{B_1}\pi_{(0,0,r)} + \mu_1\pi_{(1,1,1)} \quad (1)$$

State $(\boldsymbol{n_{11}}, \mathbf{1}, \boldsymbol{n_{13}})$: In this state (Figure 2) in phase 1, there are n_1 vehicles. The signal indicates the green light because $n_2 = 1$, and the green signal is in mode n_3.

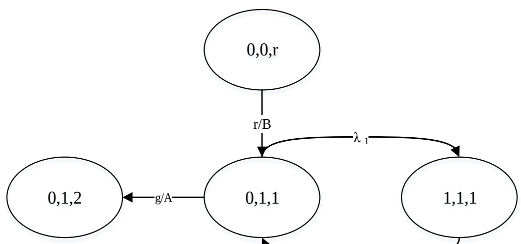

Figure 1. Diagram for state (0, 1, 1)

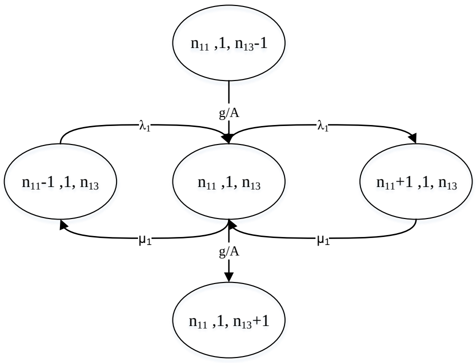

Figure 2. Diagram for state $(n_{11}, 1, n_{13})$

The equilibrium equation for state $(n_{11}, 1, n_{13})$ is as follows.

$$\left(\lambda_1 + \frac{g_1}{A_1} + \mu_1\right)\pi_{(n_{11},1,n_{13})} = \frac{g_1}{A_1}\pi_{(n_{11},1,n_{13}-1)} + \lambda_1\pi_{(n_{11}-1,1,n_{13})} + \mu_1\pi_{(n_{11}+1,1,n_{13})} \quad ,1 \le n_{11} \le N_1 \ ,2 \le n_{13} \le g \tag{2}$$

State (0, 0, 1): In this state (Figure 3) in phase 1, there is no vehicle, the signal shows red light since $n_2 = 0$, and the first stage of the red light is dominant. In this state and others where $n_2 = 0$, there is no service because the light is red.

The equilibrium equation for state (0, 0, 1) is as follows.

$$\left(\lambda_1 + \frac{r_1}{B_1}\right)\pi_{(0,0,1)} = \frac{l}{C}\pi_{(0,2,l)} \tag{3}$$

The equilibrium equation for state $(n_{11}, 0, n_{13})$ is as follows (Figure 4).

$$\left(\lambda_1 + \frac{r_1}{B_1}\right)\pi_{(n_{11},0,n_{13})} = \frac{r_1}{B_1}\pi_{(n_{11},0,n_{13}-1)} + \lambda_1\pi_{(n_{11}-1,0,n_{13})}, 1 \le n_{11} \le N_1 \ ,2 \le n_{13} \le r \tag{4}$$

According to the above definitions, the equilibrium equations for phase 1 are as follows.

$$\left(\lambda_1 + \frac{r_1}{B_1}\right)\pi_{(0,0,1)} = \frac{l}{C}\pi_{(0,2,l)} \tag{5}$$

$$\left(\lambda_1 + \frac{r_1}{B_1}\right)\pi_{(0,0,n_{13})} = \frac{r_1}{B_1}\pi_{(0,0,n_{13}-1)} \quad ,2 \le n_{13} \le r_1 \tag{6}$$

$$\left(\lambda_1 + \frac{r_1}{B_1}\right)\pi_{(n_{11},0,1)} = \frac{l}{C}\pi_{(n_{11},2,l)} + \lambda_1\pi_{(n_{11}-1,0,1)} \ ,1 \le n_{11} \le N_1 \tag{7}$$

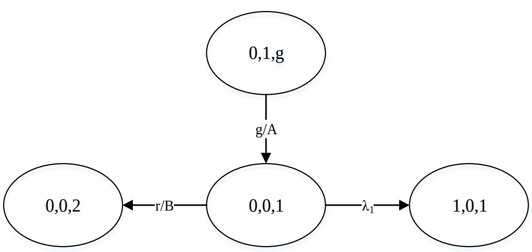

Figure 3. Diagram for state (0, 0, 1)

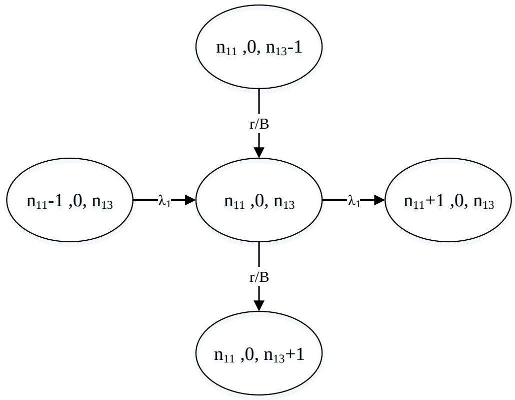

Figure 4. Diagram for state $(n_{11}, 0, n_{13})$

$$\left(\lambda_1 + \frac{r_1}{B_1}\right)\pi_{(n_{11},0,n_{13})} = \frac{r_1}{B_1}\pi_{(n_{11},0,n_{13}-1)} + \lambda_1\pi_{(n_{11}-1,0,n_{13})} \ ,1 \le n_{11} \le N_1 \ ,2 \le n_{13} \le r_1 \tag{8}$$

$$\left(\lambda_1 + \frac{g_1}{A_1}\right)\pi_{(0,1,1)} = \frac{r_1}{B_1}\pi_{(0,0,r_1)} + \mu_1\pi_{(1,1,1)} \tag{9}$$

$$\left(\lambda_1 + \frac{g_1}{A_1}\right)\pi_{(0,1,n_{13})} = \frac{g_1}{A_1}\pi_{(0,1,n_{13}-1)} + \mu_1\pi_{(1,1,n_{13})} \ ,2 \le n_{13} \le g_1 \tag{10}$$

$$\left(\lambda_1 + \frac{g_1}{A_1} + \mu_1\right)\pi_{(n_{11},1,1)} = \frac{r_1}{B_1}\pi_{(n_{11},0,r_1)} + \lambda_1\pi_{(n_{11}-1,1,1)} + \mu_1\pi_{(n_{11}+1,1,1)} \quad ,1 \le n_{11} \le N_1 \tag{11}$$

$$\left(\lambda_1 + \frac{g_1}{A_1} + \mu_1\right)\pi_{(n_{11},1,n_{13})} = \frac{g_1}{A_1}\pi_{(n_{11},1,n_{13}-1)} + \lambda_1\pi_{(n_{11}-1,1,n_{13})} + \mu_1\pi_{(n_{11}+1,1,n_{13})}, 1 \le n_{11} \le N_1 \ ,2 \le n_{13} \le g_1 \tag{12}$$

$$\left(\lambda_1 + \frac{l}{C}\right)\pi_{(n_{11},2,n_{13})} = \frac{g_1}{A_1}\pi_{(n_{11},1,g_1)} + \lambda_1\pi_{(n_{11}-1,2,n_{13})} + \frac{l}{C}\pi_{(n_{11},2,n_{13}-1)}, 0 \le n_{11} \le N_1 \ ,2 \le n_{13} \le l \tag{13}$$

$$\sum_{n_{13}=1}^{r_1}\sum_{n_{11}=0}^{N_1}\pi_{(n_{11},0,n_{13})} + \sum_{n_{13}=1}^{g_1}\sum_{n_{11}=0}^{N_1}\pi_{(n_{11},1,n_{13})} + \sum_{n_{13}=1}^{l}\sum_{n_{11}=0}^{N_1}\pi_{(n_{11},2,n_{13})} = 1 \tag{14}$$

The equilibrium equations for phase 2 can be presented along the same lines.

For example, consider state $(n_{21}, 2, n_{23})$, with n_1 vehicles in phase 2 (Figure 5). The signal indicates the yellow light because $n_2 = 2$, where the yellow signal is in mode n_3, and there is no service because the light is yellow.

The equilibrium equation for state $(n_{21}, 2, n_{23})$ would be as follows.

$$\left(\lambda_2 + \frac{l}{C}\right)\pi_{(n_{21},2,n_{23})} = \frac{r_2}{B_2}\pi_{(n_{21},1,g_2)} + \lambda_2\pi_{(n_{21}-1,2,n_{23})} + \frac{l}{C}\pi_{(n_{21},2,n_{23}-1)} \ ,0 \le n_{21} \le N_1 \quad 2 \le n_{23} \le l \tag{15}$$

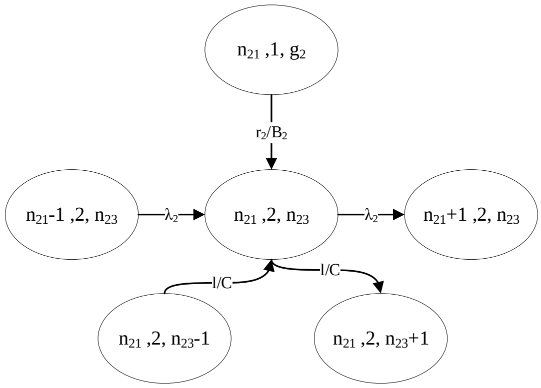

Figure 5. Diagram for state $(n_{21}, 2, n_{23})$

State $(n_{21}, 1, n_{23})$: In this state (Figure 6) in phase 2, there are n_1 vehicles. The signal indicates the green light because $n_2 = 1$, and the green signal is in mode n_3.

The equilibrium equation for state $(n_{21}, 1, n_{23})$ is as follows:

$$\left(\lambda_2 + \frac{r_2}{B_2} + \mu_2\right)\pi_{(n_{21},1,n_{23})} = \frac{r_2}{B_2}\pi_{(n_{21},1,n_{23}-1)} + \lambda_2\pi_{(n_{21}-1,1,n_{23})} + \mu_2\pi_{(n_{21}+1,1,n_{23})} \quad ,1 \le n_{21} \le N_2 \ ,2 \le n_{23} \le r_2 \tag{16}$$

The objective function of the model is to minimize the average waiting time at the intersection, considered as a weighted sum of the average waiting times in all the phases, as follows.

$$\min W = \frac{\lambda_1 w_1 + \lambda_2 w_2}{\lambda_1 + \lambda_2} \tag{17}$$

$$\lambda_1 w_1 = \sum_{n_{11}=0}^{N_1} n_{11}\pi_{(n_{11},n_{12},n_{13})}\ , \quad for \begin{cases} 1 \le n_{13} \le r_1 \ , & n_{12} = 0 \\ 1 \le n_{13} \le g_1 \ , & n_{12} = 1 \\ 1 \le n_{13} \le l \ , & n_{12} = 2 \end{cases} \tag{18}$$

$$\lambda_2 w_2 = \sum_{n_{21}=0}^{N_2} n_{21}\pi_{(n_{21},n_{22},n_{23})}\ , \quad for \begin{cases} 1 \le n_{23} \le g_2 \ , & n_{22} = 0 \\ 1 \le n_{23} \le r_2 \ , & n_{22} = 1 \\ 1 \le n_{23} \le l \ , & n_{22} = 2 \end{cases} \tag{19}$$

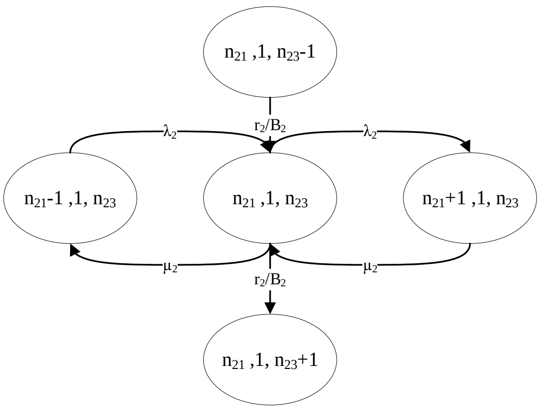

Figure 6. Diagram for state $(n_{21}, 1, n_{23})$

The set of constraints consists of all the equilibrium equations, each pertaining to a phase.
subject to:

$$\left(\lambda_1 + \frac{r_1}{B_1}\right)\pi_{(0,0,1)} = \frac{l}{C}\pi_{(0,2,l)} \tag{20}$$

$$\left(\lambda_1 + \frac{r_1}{B_1}\right)\pi_{(0,0,n_{13})} = \frac{r_1}{B_1}\pi_{(0,0,n_{13}-1)} \quad 2 \le n_{13} \le r_1 \tag{21}$$

$$\left(\lambda_1 + \frac{r_1}{B_1}\right)\pi_{(n_{11},0,1)} = \frac{l}{C}\pi_{(n_{11},2,l)} + \lambda_1\pi_{(n_{11}-1,0,1)}, 1 \le n_{11} \le N_1 \tag{22}$$

$$\left(\lambda_1 + \frac{r_1}{B_1}\right)\pi_{(n_{11},0,n_{13})} = \frac{r_1}{B_1}\pi_{(n_{11},0,n_{13}-1)} + \lambda_1\pi_{(n_{11}-1,0,n_{13})} \ ,1 \le n_{11} \le N_1 \ ,2 \le n_{13} \le r_1 \tag{23}$$

$$\left(\lambda_1 + \frac{g_1}{A_1}\right)\pi_{(0,1,1)} = \frac{r_1}{B_1}\pi_{(0,0,r_1)} + \mu_1\pi_{(1,1,1)} \tag{24}$$

$$\left(\lambda_1 + \frac{g_1}{A_1}\right)\pi_{(0,1,n_{13})} = \frac{g_1}{A_1}\pi_{(0,1,n_{13}-1)} + \mu_1\pi_{(1,1,n_{13})} \ , \ 2 \le n_{13} \le g_1 \tag{25}$$

$$\left(\lambda_1 + \frac{g_1}{A_1} + \mu_1\right)\pi_{(n_{11},1,1)} = \frac{r_1}{B_1}\pi_{(n_{11},0,r_1)} + \lambda_1\pi_{(n_{11}-1,1,1)} + \mu_1\pi_{(n_{11}+1,1,1)} \ ,1 \le n_{11} \le N_1 \tag{26}$$

$$\left(\lambda_1 + \frac{g_1}{A_1} + \mu_1\right)\pi_{(n_{11},1,n_{13})} = \frac{g_1}{A_1}\pi_{(n_{11},1,n_{13}-1)} + \lambda_1\pi_{(n_{11}-1,1,n_{13})} + \mu_1\pi_{(n_{11}+1,1,n_{13})} \ ,1 \le n_{11} \le N_1 \ ,2 \le n_{13} \le g_1 \tag{27}$$

$$\left(\lambda_1 + \frac{l}{C}\right)\pi_{(n_{11},2,n_{13})} = \frac{g_1}{A_1}\pi_{(n_{11},1,g_1)} + \lambda_1\pi_{(n_{11}-1,2,n_{13})} + \frac{l}{C}\pi_{(n_{11},2,n_{13}-1)} \ ,0 \le n_{11} \le N_1 \ ,2 \le n_{13} \le l \tag{28}$$

$$\sum_{n_{13}=1}^{r_1}\sum_{n_{11}=0}^{N_1}\pi_{(n_{11},0,n_{13})} + \sum_{n_{13}=1}^{g_1}\sum_{n_{11}=0}^{N_1}\pi_{(n_{11},1,n_{13})} + \sum_{n_{13}=1}^{l}\sum_{n_{11}=0}^{N_1}\pi_{(n_{11},2,n_{13})} = 1 \tag{29}$$

$$\left(\lambda_2 + \frac{g_2}{A_2}\right)\pi_{(0,0,1)} = \frac{l}{C}\pi_{(0,2,l)} \tag{30}$$

$$\left(\lambda_2 + \frac{g_2}{A_2}\right)\pi_{(0,0,n_{23})} = \frac{g_2}{A_2}\pi_{(0,0,n_{23}-1)}, 2 \le n_{23} \le g_2 \tag{31}$$

$$\left(\lambda_2 + \frac{g_2}{A_2}\right)\pi_{(n_{21},0,1)} = \frac{l}{C}\pi_{(n_{21},2,l)} + \lambda_2\pi_{(n_{21}-1,0,1)} \ ,1 \le n_{21} \le N_2 \tag{32}$$

$$\left(\lambda_2 + \frac{g_2}{A_2}\right)\pi_{(n_{21},0,n_{23})} = \frac{g_2}{A_2}\pi_{(n_{21},0,n_{23}-1)} + \lambda_2\pi_{(n_{21}-1,0,n_{23})} \ ,1 \le n_{21} \le N_2 \ ,2 \le n_{23} \le g_2 \tag{33}$$

$$\left(\lambda_2 + \frac{r_2}{B_2}\right)\pi_{(0,1,1)} = \frac{g_2}{A_2}\pi_{(0,0,g_2)} + \mu_2\pi_{(1,1,1)} \tag{34}$$

$$\left(\lambda_2 + \frac{r_2}{B_2}\right)\pi_{(0,1,n_{23})} = \frac{r_2}{B_2}\pi_{(0,1,n_{23}-1)} + \mu_2\pi_{(1,1,n_{23})} \ ,2 \le n_{23} \le r_2 \tag{35}$$

$$\left(\lambda_2 + \frac{r_2}{B_2} + \mu_2\right)\pi_{(n_{21},1,1)} = \frac{g_2}{A_2}\pi_{(n_{21},0,g_2)} + \lambda_2\pi_{(n_{21}-1,1,1)} + \mu_2\pi_{(n_{21}+1,1,1)} \quad, 1 \le n_{21} \le N_2 \tag{36}$$

$$\left(\lambda_2 + \frac{r_2}{B_2} + \mu_2\right)\pi_{(n_{21},1,n_{23})} = \frac{r_2}{B_2}\pi_{(n_{21},1,n_{23}-1)} + \lambda_2\pi_{(n_{21}-1,1,n_{23})} + \mu_2\pi_{(n_{21}+1,1,n_{23})} \quad, 1 \le n_{21} \le N_2 \ , 2 \le n_{23} \le r_2 \tag{37}$$

$$\left(\lambda_2 + \frac{l}{C}\right)\pi_{(n_{21},2,n_{23})} = \frac{r_2}{B_2}\pi_{(n_{21},1,g_2)} + \lambda_2\pi_{(n_{21}-1,2,n_{23})} + \frac{l}{C}\pi_{(n_{21},2,n_{23}-1)} \quad, 0 \le n_{21} \le N_1 \quad 2 \le n_{23} \le l \tag{38}$$

$$\sum_{n_{23}=1}^{g_2}\sum_{n_{21}=0}^{N_2}\pi_{(n_{21},0,n_{23})} + \sum_{n_{23}=1}^{r_2}\sum_{n_{21}=0}^{N_2}\pi_{(n_{21},1,n_{23})} + \sum_{n_{23}=1}^{l}\sum_{n_{21}=0}^{N_2}\pi_{(n_{21},2,n_{23})} = 1 \tag{39}$$

$$0 \le \pi_{(n_{i1},n_{i2},n_{i3})} \le 1 \quad, \ i = 1,2 \tag{40}$$

$$A_{low} \le A \le A_{high} \tag{41}$$

$$B_{low} \le B \le B_{high} \tag{42}$$

Equations (18)-(27) show the equilibrium equations for phase 1, and Equations (28)-(37) show those for phase 2. The latter two determine the upper and lower bounds for the green and red times. If the green time in phase 1 is very long, the waiting time in phase 2 will increase substantially due to the long red time and vice versa. Therefore, the green and red times are required not to be longer than a specific amount.

4. CASE STUDY

In this section, in order to evaluate the queuing model, we compare its numerical results to those of simulating a single intersection. The simulation model for a fixed-time control system is coded in MATLAB. To solve the queuing model and calculate the optimal value of average waiting time at the intersection, we consider different values for the green and red times and obtain the average value of waiting time for each case by solving the equilibrium equations through MATLAB. The process continues until the optimal value is obtained for the average waiting time.

As mentioned earlier, the average waiting time at the intersection is obtained for different green and red times, where various values of red time are considered for each value of green time from 25 to 40 seconds (Values below the lower bound or above the upper bound would result in a significant increase in the average waiting time at the intersection). The average waiting time for each case of green and red time is calculated, and the red time associated with the minimum case is finally recorded in Table 1. For example, for the case where the green time is 34 seconds, average waiting time is calculated for different values of red time, and the optimal case is obtained, that is the minimum value of average waiting time at the intersection for the case in which the red time is 35 seconds. The optimal red times for the other green times are also obtained similarly. Moreover, the numerical solution of the queuing model is compared to the simulation results, as mentioned earlier, for assessment of the performance of the queueing model. The simulation model is coded in MATLAB and implemented ten times for each green or red signal length, each for 10,000,000 seconds.

The arrival and departure rates for each phase are as follows. This information concerns a two-phase intersection (Figure 7) in the city of Bojnurd, Iran.

$\lambda_1 = 0.25$, $\mu_1 = 0.67$ $\quad \lambda_2 = 0.155$, $\mu_2 = 0.46$

The intersection capacity for each phase is assumed to be 50 cars (As stated earlier, this finite-capacity approach can provide a good approximation of the results for infinite capacity), and the degrees of freedom concerning Erlang distribution of the green, red, and yellow times are assumed to be 120. The yellow time is assumed to be 4 seconds for each phase (In the current state of the aforementioned intersection, the yellow time is 4 seconds).

Table 2 shows the green and red times and waiting time for phases 1 and 2 and the average waiting time at the intersection, obtained from the analytical model and simulation. The green and red times for phase 2 can be obtained easily from the results for phase 1, where the red time for phase 2 is obtained by adding the green and yellow times in phase 1. Moreover, the green time for phase 2 is obtained through subtraction of the yellow time for phase 2 from the red time for phase 1, Table 2.

$Green\ time_{phase\ 2} = Red\ time_{phase\ 1} - Yellow\ time_{phase\ 2}$

$Red\ time_{phase\ 2} = Green\ time_{phase\ 1} + Yellow\ time_{phase\ 1}$

Figure 8 compares the average waiting time obtained by the analytic model to that given by the simulation. As observed, there is an insignificant difference

Figure 7. Intersection under study

TABLE 2. Results of the model and simulation

Cycle	Phase 1 Green Time	Phase 1 Red Time	Phase 1 Waiting Time	Phase 2 Green Time	Phase 2 Red Time	Phase 2 Waiting Time	Average Waiting Time Model	Average Waiting Time Simulation
1	25	27	30.9	23	29	32.7	31.6	31.5
2	26	28	29.8	24	30	33.4	31.2	31.1
3	27	29	25.8	25	31	38.9	30.8	30.8
4	28	30	25.3	26	32	39.1	30.6	30.5
5	29	31	25.8	27	33	37.9	30.4	30.4
6	30	31	24.2	27	34	40.2	30.3	30.2
7	31	32	24.4	28	35	39.5	30.2	30.1
8	32	33	26.8	29	36	35.4	30.1	30.0
9	33	34	27.0	30	37	35.0	30.1	30.0
10	34	35	25.3	31	38	37.7	30.1	30.0
11	35	35	25.3	31	39	37.8	30.1	30.1
12	36	36	24.3	32	40	39.6	30.2	30.1
13	37	37	24.6	33	41	39.3	30.2	30.1
14	38	38	27.6	34	42	34.7	30.3	30.2
15	39	39	26.6	35	43	36.7	30.4	30.4
16	40	40	26.7	36	44	36.8	30.6	30.5

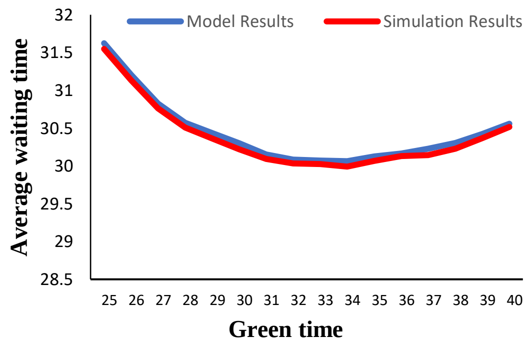

Figure 8. Average waiting time with respect to green time

between the results, and they are nearly matched. If the green time is longer or shorter than a certain amount, the average waiting time for the intersection will increase. This is because if the green time for phase 1 is long, the red time for phase 2 will increase, and so will the waiting time for phase 2 and the average waiting time for the whole intersection. Moreover, if the green time for phase 1 is short and that for phase 2 is long, the red time and the waiting time for phase 1 will increase. For this reason, the green and red times for each phase must be perfectly proportional to the arrival rate for that phase. At the intersection under study, the arrival rate for phase 1 is 1.6 times that for phase 2, and there should therefore not be much difference between the green times for the two phases. Figure 9 shows the waiting time for the two phases with respect to the cycle number. As can be seen, the waiting time for the former is always less than that for the latter because the arrival rate for phase 1 is greater than that for phase 2. Therefore, the green time for phase 1 is always greater than that for phase 2, leading to light traffic and a shorter average waiting time in phase 1 than in phase 2.

Table 3 summarizes the optimal solution obtained by the analytic model, along with a comparison of the average waiting time at the intersection in the optimal state to that in the current state. In the current state, the green time for phase 1 substantially differs from that for phase 2, while the arrival rates for the two phases do not differ significantly ($\lambda_1 = 0.25$, $\lambda_2 = 0.155$). A long red time and a short green time in phase 2 lead to heavy traffic and a dramatic increase in the vehicle waiting time in that phase and, consequently, in overall average waiting time at the intersection. The red times and green times for the two phases are set by the analytic model so that the average waiting times for the two phases and that for the entire intersection increase. However, in the current state, the average waiting time is very short in phase 1 and extremely long in phase 2, meaning that the traffic is light in phase 1 but heavy in phase 2.

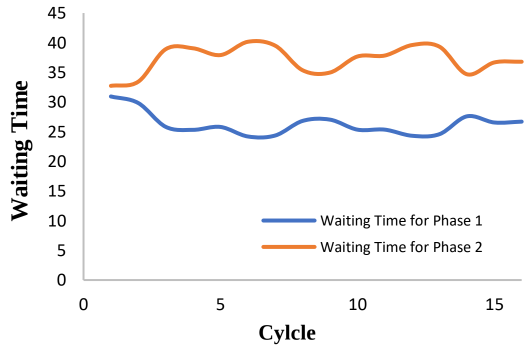

Figure 9. Waiting time for phases 1 & 2 with respect to cycle number

TABLE 3. A comparison of the optimal solution of the model and the current state

	Green Time Phase 1 (s)	Red Time Phase 1 (s)	Green Time Phase 2 (s)	Red Time Phase 2 (s)	Yellow Time (s)	Average Waiting Time (s)
Optimum result	34	35	31	38	4	30.1
Current state	50	30	26	54	4	99.2

From the Figures and tables above, the following results are implied.

1. The analytic model of intersection queue can significantly reduce the average waiting time at the isolated intersection. As indicated by the numerical results, the current conditions reduce average waiting time only in phase 1, but the analytic model sets the red times and green times for the two phases, so that average waiting time decreases in both phases, leading to relatively light traffic in them.
2. The analytic model can well estimate the average waiting time at the intersection, and its obtained results are very close to those given by the simulation of the fixed-time control system at the isolated intersection.

5. CONCLUSION

This paper has investigated the average waiting time obtained by a fixed-time signal control policy through analyses and simulations. We have analyzed a queuing model to estimate the average waiting time at a two-phase isolated intersection and calculated the optimal green and red times in a fixed-time traffic signal control system. The model has been solved for real traffic data collected from an isolated intersection in the city of Bojnurd, and the numerical results have been compared to those given by a simulation model. The comparison demonstrates that the analytical model approximates the simulation results very well. Moreover, it has been found that the analytical model obtained through analysis of the queuing model could substantially decrease the average waiting time at an isolated intersection. This paper has addressed the modeling of a fixed-time control system for a two-phase intersection. The method can be developed for adaptive or actuated control systems, and the results can be compared to those for the fixed-time control system. An assumption, and perhaps a drawback of the model, is that it considers a two-phase intersection, ignoring the turns taken to the left and right, which can greatly impact the average waiting time at the intersection. Furthermore, one can set green and red times for multiple intersections or a network instead of considering an isolated intersection, causing a decrease in the number of vehicle stops and average waiting time.

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Persian Abstract

چکیده

با افزایش جمعیت در شهرها، تعداد خودروهای موجود در جاده‌ها نیز افزایش می‌یابد که این منجر به کندی حرکت و تردد متراکم می‌شود. از جمله مشکلاتی که کم و بیش شهروندان شهرهای بزرگ با آن مواجه هستند، مسأله ترافیک در خیابان‌ها است. سیگنال‌های ترافیکی یکی از توانمندترین ابزارها برای کنترل ترافیک شهری هستند که نصب صحیح آنها می‌تواند هم جریان ترافیک و هم ایمنی کاربران را بهبود بخشد. برای کاهش اثر صف‌های طولانی خودروها در چهارراه‌ها تلاش زیادی صورت گرفته است. این تلاش‌ها عمدتاً در راستای بهینه‌سازی زمان‌بندی چراغ‌های راهنمایی بوده است. در این مقاله به کمک تحلیل سیستم صف، به محاسبه متوسط زمان انتظار در یک تقاطع منفرد و تنظیم زمان‌بندی طول چراغ سبز و قرمز پرداخته می‌شود. نوع سیستم کنترلی از نوع زمان ثابت و ورودی‌ها به تقاطع براساس فرآیند پواسان در نظر گرفته می‌شوند. پس از ارائه مدل ریاضی، داده‌های ترافیکی واقعی که از یک تقاطع دوفازه از شهر بجنورد تهیه شده است، برای حل مدل به کار گرفته می‌شوند و مقادیر بهینه طول چراغ سبز و قرمز بدست می‌آید. نتایج حاصل از شبیه‌سازی، قابلیت مدل پیشنهادی را در کنترل سیگنال‌های ترافیکی تایید می‌کند.