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Price and Service Competition between Two Leader-follower Retailer-Stackelberg Supply Chains

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In this paper, two leader-follower supply chains consisting of one manufacturer and one retailer are considered. In-chain competition is addressed besides the chain-to-chain competition in which the retailer is the leader and the manufacturer is the follower. The competition elements are price and service, which are investigated in three different scenarios: decentralized leader-decentralized follower, integrated leader-decentralized follower, and decentralized leader-integrated follower. Using the backward induction, we start the solving process from the follower supply chain and derive the follower's best response function. Then the leader strategies are examined after the substitution of the follower's best response function in leader profit function. Finally, we analyzed the effects of the price competition intensity and service investment coefficient of both chains on the equilibrium values in all three scenarios. The results show that increasing the price competition intensity will decrease the profit of the leading supply chain. In contrast, small values of price competition intensity are beneficial for the follower supply chain. Moreover, the service investment coefficient of both supply chains has a direct impact on follower optimal values and an inverse impact on the leader ones.

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NOMENCLATURE

c_m	the unit production cost, which is assumed equal for both manufacturers	γ_s, γ_p	The intensity of competition between two retailers to the retailer's service and price, $\gamma_s, \gamma_p \geq 0$
α_i	the potential market demand for supply chain *i*	s_i	the service level of retailer *i*
p_i	the retail price of supply chain *i*	π^{ri}	The profit function of retailer *i*
w_i	the wholesale price of supply chain *i*	π^{mi}	The profit function of manufacturer *i*
η_i	the coefficient of service investment efficiency for retailer *i*.	π^{Ti}	Total profit of supply chain *i*
β_s, β_p	the sensitivity of retailer's market demand to the retailer's service and retailer's price, $\beta_s, \beta_p \geq 0$		

1. INTRODUCTION

Pricing is one of the critical components of the success of an organization and one of the most significant parts of business behaviors. Therefore, competing companies are involved in price competition to attract more customers and earn a fair market share. Along with the price, the service level is also considered a crucial factor affecting customer purchasing decisions. In recent years, competition between supply chains has been considered not only on price but also on the service level. This is because services provided to the customers play an important role in attracting customers and the acquired share of the market.

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In a competitive market, there are supply chains with different power structures. In most businesses, there is a chain that has more leadership power in the market because of its superior brand, position, or higher quality of products which has a competitive advantage to reach more customers. Such structures are modeled as a Stackelberg game, in which the chain with more power acts as a leader and the others as followers. In addition, unlike chain-to-chain competition in which two or more distinct supply chains compete with each other. There is the in-chain competition which is among the members of a supply chain.

There are some novelty and contributions which make this article different from other research. First, we examine an industry composed of two competing supply chains in which the retailers play a leading role in each chain. A case that has received less attention while being used in many retail industries. This paper is the first to examine both chain-to-chain and in-chain competitions over service and price between two leader-follower retailer-led supply chains. In addition to the leading retailers in both chains, a supply chain has more power in the market, and the competition is discussed from two perspectives of price and service level. Furthermore, the effect of integration or decentralization of each supply chain is investigated through different scenarios.

The remainder of the paper is organized as follows. In section 2, a brief description of the relevant literature is presented. Next, the model along with the assumptions and notations are provided in section 3. Section 4 is devoted to analyzing the model in which three different scenarios are addressed to examine the various structures of the model. Section 5 investigates the effect the main parameters on the equilibrium solutions in all three scenarios through numerical analysis. Eventually, in section 6, the study is concluded, and the main findings and some future research direction are described.

2. LITERATURE REVIEW

Price and service competition have attracted many researchers in recent decades. Most studies considered the price as the only factor of competition [1-4]. For example, Mahmoudi and Tofigh [5] considered five competitive firms using the game-theoretic approach in a dynamic competitive market. Amin-Naser and Azari-Khojasteh [6] examined two supply chains with risk-averse retailers and uncertain demand (one leader and one follower) competing on price. Mahmoodi [7] addressed the simultaneous pricing and replenishment policy in a duopoly environment with a unique Nash equilibrium. Sadjadi and Alirezaee [8] examined how different pricing strategies and cooperative advertising have influenced a two-echelon supply chain. Khanlarzade, Zegordi and Nakhai Kamalabadi [9] considered the price contest of two multi-echelon supply chains under two different scenarios. In the first scenario, there was a Nash game between both supply chains, and in the second one, there was an imbalance power structure between supply chains. Lou et al. [10] assessed a supply chain consisting of two levels, including a manufacturer and a retailer. Finally, Widodo and Januardi [11] considered a dual-channel supply chain and obtained the Nash equilibrium solution. The above studies and the references therein show that price competition attracts much attention in the literature. However, none of the above papers considered the service as a factor of competition.

Another stream of relevant literature is one focused on service competition [12-16]. Jamshidi et al. [17] studied the impact of manufacturers and retailers' service level on customers' demand. They applied a game-theoretic approach in a supply chain consist of one manufacturer and a common retailer. Wu et al. [18] investigated optimal service decisions of two supply chains with a leading manufacturer under horizontal Stackelberg structure. In another study, Wu et al. [19] addressed the impact of competition on optimal decisions in a network of two supply chains. They applied the game theory deciding on green or non-green production under different competitive situations.

Several studies considered the competition from the perspective of both price and service level [20-23]. Xiao and Yang [24] studied an uncertain market that includes two supply chains competing on both the price and service level. At the same time, the manufacturer is risk-neutral, and the retailer is risk-averse. They found that when the retailers are more sensitive to risk, the optimal values equivalent to price and service level would be less. Chen et al. [25] investigated the problem of optimal price, service level, and quality decisions in a supply chain under different structures. Using the backward induction and a two-stage optimization game, they formulated the integrated and decentralized models.

Based on literature review, price and service competitions exist among researchers. In supply chain literature, vertical and horizontal competition between the industry members and the competition between supply chains is one of the growing areas in supply chain management. However, the leader-follower structure has received less attention from previous studies. Meanwhile, there has always been a supply chain with more leadership power in the actual competitive market that influences the decision of other supply chains.

Furthermore, in most studies, a manufacturer-Stackelberg structure is assumed, and none of the previous studies has addressed price and service competition for two leader-follower supply chains with the retailer-Stackelberg system. However, the retailer-Stackelberg system is widely used in retail industries like Walmart and Kmart. In this supply chains, retailer plays the leading role in determining the wholesale price of the manufacturer. Therefore, considering two competing leader-follower supply chains, we examine the interaction between manufacturer and retailer in each chain and the competition between the supply chains. This paper covers the shortcomings in this area and extends the literature by formulating the leader-follower supply chains under the retailer-Stackelberg structure. Moreover, both price and service competition exists between market members.

The most related papers to our study are Amin-Naser and Azari-Khojasteh [6] and Xiao and Yang [24]. Amin-Naser and Azari-Khojasteh [6] investigated the price competition between two leader-follower supply chains. However, they did not consider the service competition, and they did not investigate the various centralization scenarios for the supply chains in their study. In contrast, we considered price and service competition in this paper and analyzed different centralization scenarios for the supply chains. On the other hand, Xiao and Yang [24] examined price and service competition. However, they did consider neither the leader-follower supply chains nor the retailer-led structure for the supply chains. However, we addressed price and service competition for two leader-follower supply chains with a retailer-led structure.

3. MODEL DESCRIPTION

We consider a market of two rival chains, where each of them contains a manufacturer (she) and a retailer (he) selling substitutable products to a common market. There is no cross-selling between the members of the supply chains, which means that each manufacturer offers her product only to the retailer in her supply chain, and at the same way, the retailer offers it to the end customers. The leader-follower relationship is considered not only among the supply chains but also between the members of a supply chain. We assume that Supply chain 1 has the role of leader and Supply chain 2 has the role of follower. In addition, the retailer is known as the leader and the manufacturer as the follower inside each supply chain. The decision variables of each retailer are the service level and the retail price, while the decision variable of each manufacturer is her wholesale price. Each agent aims to set his/her decision variable(s) to maximize his/her profit considering the strategy of the other agents. It is presumed that both chains have similar conditions in production cost and demand sensitivity to their price and service level. However, one of them has a bigger potential market size. These assumptions are rational because an incumbent may have a higher market share in a business environment compared to a new entrant.

The demand function of retailer i is a linear function of retail price and service level of both retailers as follows:

$$d_i = \alpha_i - \beta_p p_i + \gamma_p\left(p_j - p_i\right) + \beta_s s_i - \gamma_s\left(s_j - s_i\right) \tag{1}$$

Using $\beta'_s = \beta_s + \gamma_s$ and $\beta'_p = \beta_p + \gamma_p$, the demand function is better to be expressed as follows:

$$d_i = \alpha_i - \beta'_p p_i + \gamma_p p_j + \beta'_s s_i - \gamma_s s_j \tag{2}$$

This demand function is common in the literature; for example, Xiao and Yang [24] , Tsay and Agrawal [20], and many other researchers have employed similar demand functions in their papers. Furthermore, as mentioned in the model of Xiao and Yang [24], the retailers in both chains will provide customers' services. Moreover, retailer *i's* cost of providing service level s_i is $\frac{1}{2}\eta_i \cdot s_i^2$.

Therefore, the profit functions of the retailers and manufacturers could be written as follows:

$$\begin{aligned}\pi^{ri} &= \left(p_i - w_i\right)\left(\alpha_i - \beta'_p p_i + \gamma_p p_j + \beta'_s s_i - \gamma_s s_j\right) \\ &- \frac{1}{2}\eta_i s_i^2; i = 1,2;\ j = 3 - i\end{aligned} \tag{3}$$

$$\begin{aligned}\pi^{mi} &= \left(w_i - c_m\right)\left(\alpha_i - \beta'_p p_i + \gamma_p p_j + \beta'_s s_i - \gamma_s s_j\right) \\ i &= 1,2;\ j = 3 - i\end{aligned} \tag{4}$$

The total profit of a supply chain will be

$$\pi^{Ti} = \pi^{mi} + \pi^{ri} \quad ; i = 1, 2; \tag{5}$$

The assumption that the retail price is bigger than the wholesale price in both chains can be defined by $p_i = w_i + v_i$. Therefore, after determining the wholesale price, the retailer could decide on his retail price by setting v_i.

In this study, the following three scenarios are analyzed.

Scenario 1: Both leader and follower supply chains are decentralized. Thus, each agent aims to maximize his/her profit, setting his/her decision variable(s) independently without any cooperation.

Scenario 2: The leading supply chain is integrated, and the follower is decentralized. The manufacturer and retailer of the leader chain cooperate and set their decision variables to maximize the total profit. However, the members of the follower supply chain do not cooperate and make their decisions independently.

Scenario 3: The follower supply chain is integrated while the leader is decentralized.

4. MODEL ANALYSIS

In this section, the equilibrium solution is provided for the scenarios mentioned above. Due to the leader-follower relation of supply chains, each scenario is analyzed employing the concepts of the Stackelberg game in which the leader chooses his/her strategy, and then the follower determines his/her strategy given the leader's decision. Thus, the leader can predict the follower's next move. Effectively, the backward induction technique is employed to obtain the optimal values.

4. 1. Decentralized Leader-decentralized Follower

In the first scenario, the members of both leader and follower supply chains make their price and service level decisions independently. We use the backward induction technique based on which the solving process would be:

Stage 1: Considering the leader supply chain's decision variables as constant, we determine the decision variables of the follower supply chain in terms of the leader's variables. Furthermore, concerning the leader-follower relationship inside of the follower supply chain, the following sub-stages are used to determine its decision variables.

Sub Stage 1.1: The wholesale price of the follower chain's manufacturer is determined as a subordinate of the leader chain's decision variables and the follower chain's retailer decision variables.

Sub Stage 1.2: Considering the wholesale price of the follower chain's manufacturer, the retail price and service level of the follower chain's retailer are obtained as functions of the leader chain's decision variables.

Stage 2: Considering the equations obtained for the follower supply chain's decision variables as the response function, we determine the leader supply chain's decision variables. The following sub-stages are used to determine its decision variables regarding the leader-follower relationship within the leader supply chain.

Stage 2.1: The wholesale price of the leader chain's manufacturer is defined as a function of the leader chain's retailer decision variables.

Stage 2.2: Regarding the equations obtained as the best responses, the decision variables of the leader chain's retailer are determined;

Stage 3: Eventually, in a back-substitution process, all of the variables are determined.

Therefore, the solving process starts from the manufacturer of the follower chain whose profit function is

$$\pi^{m2}=(w_2-c_m)\left(\alpha_2-\beta'_p(w_2+v_2)+\gamma_p p_1+\beta'_s s_2-\gamma_s s_1\right) \quad (6)$$

Since $\partial\pi^{m2}/\partial w_2<0$, the best response of Manufacturer 2 could be obtained from the first-order condition. Accordingly, we have

$$w_2=\frac{1}{2}(c_m-v_2)+\frac{\left(\alpha_2+\gamma_p p_1+s_2\beta'_s-\gamma_s s_1\right)}{2\beta'_p} \quad (7)$$

The profit function of Retailer 2 is:

$$\pi^{r2}=(p_2-w_2)\left(\alpha_2-p_2\beta'_p+\gamma_p p_1+\beta'_s s_2-\gamma_s s_1\right)-\frac{1}{2}\eta_2 s_2^2 \quad (8)$$

Retailer 2 anticipates the best response of Manufacturer 2; therefore, his profit function is obtained by substitution of Equation (7) in Equation (8) as below.

$$\pi^{r2}=v_2\left(\frac{\alpha_2+\gamma_p p_1+\beta'_s s_2-\gamma_s s_1-(c_m+v_2)\beta'_p}{2}\right)-\frac{1}{2}\eta_2 s_2^2 \quad (9)$$

LEMMA 1. In the first scenario, for given values of Chain 1's decision variables, the profit function of Retailer 2 is concave, if and only if, $0\le 1/\eta_2\le 4.\beta'_p/\beta'^2_s$.

All proofs are presented in Appendix A.

We assume that the condition of Lemma 1 holds; therefore, the first-order condition of Equation (9) gives the equivalent values of v_2 and s_2.

$$v_2=2\eta_2\left(\alpha_2+\gamma_p p_1-\gamma_s s_1-\beta'_p c_m\right)\Big/\left(4\eta_2\beta'_p-\beta'^2_s\right) \quad (10)$$

$$s_2=\beta'_s\left(\alpha_2+\gamma_p p_1-\gamma_s s_1-c_m\beta'_p\right)\Big/\left(4\eta_2\beta'_p-\beta'^2_s\right) \quad (11)$$

Based on leader-follower relation, Manufacturer 2 can anticipate the best response of Manufacturer 1 and Retailer 1. Therefore, the profit function of Manufacturer 1 is obtained by substitution of Equations (7), (10), and (11) in Equation (4).

$$\pi^{m1}=(w_1-c_m)\left(\begin{array}{l}\alpha_1-(v_1+w_1)\beta'_p+\gamma_p\left(\begin{array}{l}\dfrac{\left(\alpha_2+\gamma_p(v_1+w_1)-\gamma_s s_1\right)}{2\beta'_p}\\+\left(\dfrac{\alpha_2+\gamma_p(v_1+w_1)-\gamma_s s_1-\beta'_p c_m}{D}\right)R+\dfrac{1}{2}c_m\end{array}\right)\\+\beta'_s s_1-\gamma_s\left(\dfrac{\beta'_s}{D}\left(\alpha_2+\gamma_p(v_1+w_1)-\gamma_s s_1-\beta'_s c_m\right)\right)\end{array}\right) \quad (12)$$

where for simplification, we define the following notations;

$$D=4\eta_2\beta'_p-\beta'^2_s,\ A=\frac{\gamma_p^2}{2\beta'_p},\ B=\frac{\gamma_s\gamma_p\beta'_s}{D},\ E=\frac{\gamma_s\beta'_s}{D}$$

$$C=-\frac{\gamma_s\gamma_p}{2\beta'_p},\ R=\eta_2+\frac{\beta'^2_s}{2\beta'_p},\ H=\beta_p'-A+B-R\frac{\gamma_p^2}{D}$$

It is easy to show that $\partial^2\pi^{m1}/\partial w_1^2\le 0$, therefore, the best response of Manufacturer 1 is calculated by the first-order condition.

$$\frac{\partial\pi^{m1}}{\partial w_1}=0\Rightarrow w_1=\frac{1}{2}(c_m-v_1)+\left(C+E\gamma_s+\beta'_s-\frac{RB}{\beta'_s}\right)s_1\Big/2H+G \quad (13)$$

where

$$G=\left(\alpha_1+\left(\frac{A}{\gamma_p}+R\frac{\gamma_p}{D}-E\right)\alpha_2+\left(-R\left(\frac{\gamma_p\beta'_p}{D}\right)+\frac{\gamma_p}{2}+\frac{B\beta'_p}{\gamma_p}\right)c_m\right)/(2H)$$

Next, considering the best responses of Retailer 2 and Manufacturers 1 and 2, the profit function of Retailer 1 can be expressed as follows.

$$\pi^{r1}=(v_1)\left(\begin{array}{l}\alpha_1-\left(\frac{1}{2}(c_m+v_1)+Ks_1+G\right)\beta'_p\\ +\gamma_p\left(\begin{array}{l}\frac{1}{2}c_m+\frac{\alpha_2+\gamma_p\left(\frac{1}{2}(c_m+v_1)+Ks_1+G\right)-\gamma_s s_1}{2\beta'_p}\\ +\left(\frac{\alpha_2+\gamma_p\left(\frac{1}{2}(c_m+v_1)+Ks_1+G\right)-\gamma_s s_1-\beta'_p c_m}{D}\right)R\end{array}\right)\\ +\beta'_s s_1-\gamma_s\left(\frac{\beta'_s}{D}\left(\alpha_2+\gamma_p\left(\frac{1}{2}(c_m+v_1)+Ks_1+G\right)-\gamma_s s_1-c_m\beta'_p\right)\right)\end{array}\right)-\frac{1}{2}\eta_1 s_1^2 \quad (14)$$

where $K=\left(C+E\gamma_s-\frac{RB}{\beta'_s}+\beta'_s\right)\Big/2H$.

LEMMA 2. In the first scenario, the profit function of Retailer1 is concave, if and only if,

$$0\le\frac{1}{\eta_1}\le 4\left(\beta'_p+B-A-\frac{R\gamma_p^2}{D}\right)\Big/\left(C+E\gamma_s+\beta'_s-\frac{\gamma_p^2 R}{D}\right)^2$$

and $\dfrac{3\gamma_p}{\gamma_s\beta'_s+\beta'_p}\le\dfrac{1}{\eta_2}\le\dfrac{4\beta'_p}{\beta'^2_s}$

Regarding the conditions of Lemma 2 is correct, v_1 and s_1 could be obtained from the first-order conditions.

$$\frac{\partial\pi^{r1}}{\partial v_1}=0 \;\; and \;\; \frac{\partial\pi^{r1}}{\partial s_1}=0\Rightarrow$$

$$v_1^*=\left(G-\frac{c_m}{2}\right)\eta_1\Big/\eta_1-K^2H \quad (15)$$

$$s_1^*=\left(\frac{KH}{\eta_1}\right)\left(G-\frac{c_m}{2}\right)\eta_1\Big/\eta_1-K^2H \quad (16)$$

Now, all of the other decision variables could be obtained using the back-substitution process, which their equations are presented as follows.

$$w_1^*=\frac{1}{2}(c_m-O)+\frac{HK^2}{\eta_1}O+G \quad (17)$$

$$v_2^*=\frac{2\eta_2 X}{D} \quad (18)$$

$$s_2^*=\frac{\beta'_s X}{D} \quad (19)$$

$$w_2^*=c_m+\left(\frac{1}{2\beta'_p}+\frac{\beta'^2_s}{2D\beta'_p}-\frac{\eta_2}{D}\right)X \quad (20)$$

where in addition to the previous notations, we define the following notations to simplify the equations.

$$O=\frac{\left(G-\frac{c_m}{2}\right)\eta_1}{\eta_1-K^2H},\; X=\alpha_2+\gamma_p\left(\frac{1}{2}(c_m+O)+\frac{HK^2}{\eta_1}O+G\right)-\gamma_s\left(\left(\frac{KH}{\eta_1}\right)O\right)-\beta'_p c_m$$

4. 2. Integrated Leader-decentralized Follower

As stated in previous sections, the leader chain is integrated in the second scenario, which means that the manufacturer and retailer decide together on pricing and determine the service level based on profit maximization. However, the manufacturer and retailer act as two distinct agents in the follower chain to maximize their profit independently. Therefore, the decision variables in the leader supply chain are retail price and service level, while in the follower supply chain, they are wholesale price, retail price, and service level.

The solving sequence is the same as that of the first scenario, however, in stage 2, there are no sub-stages, and the leader chain decision variables are obtained in one step. Furthermore, the profit functions of the manufacturers and the retailers are similar to the functions expressed in the first scenario. Moreover, the total profit of the integrated chain is represented as below.

$$\begin{array}{l}\pi^{T1}=(p_1-c_m)\left(\alpha_1-\beta'_p p_1+\gamma_p p_2+\beta'_s s_1-\gamma_s s_2\right)-\frac{1}{2}\eta_1 s_1^2=\\ (w_1+v_1-c_m)\left(\alpha_1-\beta'_p(w_1+v_1)+\gamma_p(w_2+v_2)+\beta'_s s_1-\gamma_s s_2\right)-\frac{1}{2}\eta_1 s_1^2\end{array} \quad (21)$$

The follower chain's profit function is similar to the previous case; therefore, the best response of the follower chain is the same as the first scenario. As was discussed in the first scenario, the retail price and service level are as follows:

$$\begin{array}{l}p_2=\frac{1}{2}c_m+\frac{\alpha_2+\gamma_p(w_1+v_1)-\gamma_s s_1}{2\beta'_p}\\ +\left(\frac{\left(\alpha_2+\gamma_p(w_1+v_1)-\gamma_s s_1-\beta'_p c_m\right)}{D}\right)R\end{array} \quad (22)$$

$$v_2=\frac{2\eta_2\left(\alpha_2+\gamma_p p_1-\gamma_s s_1-\beta'_s c_m\right)}{D} \quad (23)$$

$$s_2=\frac{\beta'_s\left(\alpha_2+\gamma_p p_1-\gamma_s s_1-\beta'_p c_m\right)}{D} \quad (24)$$

$$w_2=\frac{1}{2}(c_m-v_2)+\frac{\alpha_2+\gamma_p p_1+\beta'_s s_2-\gamma_s s_1}{2\beta'_p} \quad (25)$$

Consequently, with substitution of the above best responses, the leader chain profit will be

$$\pi^{T1}=(v_1+w_1-c_m)\left(\begin{array}{l}\alpha_1-\beta'_p(w_1+v_1)\\ +\gamma_p\left(\begin{array}{l}\frac{1}{2}c_m+\frac{\alpha_2+\gamma_p(w_1+v_1)-\gamma_s s_1}{2\beta'_p}\\ +\left(\frac{\alpha_2+\gamma_p(w_1+v_1)-\gamma_s s_1-\beta'_p c_m}{D}\right)R\end{array}\right)\\ +\beta'_s s_1-\gamma_s\left(\frac{\beta'_s\left(\alpha_2+\gamma_p(w_1+v_1)-\gamma_s s_1-\beta'_p c_m\right)}{D}\right)\end{array}\right)-\frac{1}{2}\eta_1 s_1^2 \quad (26)$$

LEMMA 3. In the second scenario, the total profit function of the integrated chain is concave, if and only if, $0 \le \frac{1}{\eta_1} \le 2\left(\beta'_p + B - A - \frac{\gamma_p^2 R}{D}\right) \Big/ \left(C + E\gamma_s + \beta'_s - \frac{BR}{\beta'_s}\right)^2$ and $\frac{3\gamma_p}{\gamma_s \beta'_s + \beta'_p} \le \frac{1}{\eta_2} \le \frac{4\beta'_p}{\beta'^2_s}$

Assuming the conditions of Lemma 3 are held, the equilibrium equations for w_1, v_1 and s_1 are obtained from the first-order conditions of Eq. (26). Then, the variables of Chain 2 are obtained using the back-substitution process. After all, the equilibrium equations in the second scenario are obtained as follows:

$$s_1^* = M \tag{27}$$

$$p_1^* = \frac{c_m}{2} + KM + G \tag{28}$$

$$v_2^* = \frac{2\eta_2 X'}{D} \tag{29}$$

$$s_2^* = \frac{\beta'_s X'}{D} \tag{30}$$

$$w_2^* = c_m - \frac{\eta_2 X'}{D} + \frac{X'}{2\beta'_p}\left(\frac{\beta'^2_s}{D} + 1\right) \tag{31}$$

$$p_2^* = c_m + \frac{X'}{2\beta'_p} + \frac{X' R}{D} \tag{32}$$

where

$M = \dfrac{2KH\left(-\frac{c_m}{2} + G\right)}{\eta_1 - 2K^2 H}$ and

$$X' = \alpha_2 + \gamma_p\left(\frac{c_m}{2} + K\left(\frac{2KH\left(-\frac{c_m}{2}+G\right)}{\eta_1 - 2K^2H}\right) + G\right) - \gamma_s\left(\frac{2KH\left(-\frac{c_m}{2}+G\right)}{\eta_1 - 2K^2H}\right) - \beta'_p c_m$$

4. 3. Decentralized Leader- Integrated Follower

In the third scenario, in contrast to the second scenario, the leader chain is decentralized, and the follower is integrated, which means that the retailer and manufacturer make their maximization decisions on price and service level cooperatively.

The solving sequence is similar to that of the first scenario; however, in stage 1, there are no sub-stages, and the follower chain decision variables are obtained in one step. Furthermore, the profit functions of the manufacturers and the retailers are similar to the functions expressed in the first scenario. Moreover, the total profit of the integrated follower chain is stated as:

$$\begin{aligned}\pi^{T2} &= \left(p_2 - c_m - c_r\right)\left(\alpha_2 - \beta'_p p_2 + \gamma_p p_1 + \beta'_s s_2 - \gamma_s s_1\right) - \frac{1}{2}\eta_2 s_2^2 = \\ &\left(w_2 + v_2 - c_m\right)\left(\alpha_2 - \left(w_2 + v_2\right)\beta'_p + \gamma_p p_1 + \beta'_s s_2 - \gamma_s s_1\right) - \frac{1}{2}\eta_2 s_2^2\end{aligned} \tag{33}$$

LEMMA 4. In the third scenario, for given values of Chain 1's decision variables, the total profit function of chain 2 is concave, if and only if $0 \le \frac{1}{\eta_2} \le \frac{2\beta'_p}{\beta'^2_s}$.

It is assumed that the condition of Lemma 4 is held; therefore, the best response of Chain 2 is obtained from the first-order condition:

$\frac{\partial \pi^{T2}}{\partial v_2} = 0,\ \frac{\partial \pi^{T2}}{\partial s_2} = 0, \text{and} \frac{\partial \pi^{T2}}{\partial w_2} = 0$; which together gives

$$s_2 = \frac{\beta'_s\left(\alpha_2 + \gamma_p p_1 - \gamma_s s_1 - \beta'_p c_m\right)}{2\eta_2 \beta'_p - \beta'^2_s} \tag{34}$$

$$p_2 = \frac{\beta'_s}{2\beta'_p}\left(\frac{\beta'_s\left(\alpha_2 + \gamma_p p_1 - \gamma_s s_1 - \beta'_p c_m\right)}{2\eta_2\beta'_p - \beta'^2_s}\right) + \frac{\alpha_2 + \gamma_p p_1 - \gamma_s s_1 + \beta'_p c_m}{2\beta'_p} \tag{35}$$

Manufacturer 2 could anticipate the above best responses; consequently, her profit function converts to

$$\pi^{m1} = \left(w_1 - c_m\right)\left(\begin{aligned}&\alpha_1 - \beta'_p\left(w_1 + v_1\right) \\ &+ \gamma_p\left(\begin{aligned}&\frac{\beta'^2_s}{2\beta'_p}\left(\frac{\alpha_2 + \gamma_p\left(w_1+v_1\right) - \gamma_s s_1 - \beta'_p c_m}{2\eta_2\beta'_p - \beta'^2_s}\right) \\ &+ \frac{\alpha_2 + \gamma_p\left(w_1+v_1\right) - \gamma_s s_1 + \beta'_p c_m}{2\beta'_p}\end{aligned}\right) \\ &+ \beta'_s s_1 - \gamma_s\left(\frac{\beta'_s\left(\alpha_2 + \gamma_p\left(w_1+v_1\right) - \gamma_s s_1 - \beta'_p c_m\right)}{2\eta_2\beta'_p - \beta'^2_s}\right)\end{aligned}\right) \tag{36}$$

Since $\frac{\partial^2 \pi^{m1}}{\partial w_1^2} < 0$, the best response of Manufacturer 1 is obtained from the first-order condition.

$$\frac{\partial \pi^{m1}}{\partial w_1} = 0 \Rightarrow w_1 = \frac{1}{2}\left(c_m - v_1\right) + \frac{\left(CQ + C + \beta'_s + \frac{I\gamma_s}{\gamma_p}\right)}{2H'} s_1 + G' \tag{37}$$

In which the following notations are used to simplify the presentation.

$N = 2\beta'_p\left(2\eta_2\beta'_p - \beta'^2_s\right), Q = \frac{\beta'^2_s}{2\eta_2\beta'_p - \beta'^2_s}, I = \frac{\gamma_s\gamma_p\beta'_s}{2\eta_2\beta'_p - \beta'^2_s},$

$H' = \beta'_p - AQ - A + I,\ G' = \left(\alpha_1 + \left(\frac{A}{\gamma_p} + \frac{AQ}{\gamma_p} - \frac{I}{\gamma_p}\right)\alpha_2 + \left(-\frac{\gamma_p Q}{2} + \frac{\gamma_p}{2} + \frac{I\beta'_p}{\gamma_p}\right)c_m\right) \Big/ 2H'$

Furthermore, by substitution of the above best responses, the profit function of Retailer 1 would be:

$$\pi^{r1} = v_1 \left(\begin{array}{l} \alpha_1 - \beta'_p \left(\frac{1}{2}(c_m + v_1) + N' s_1 + G' \right) \\ + \gamma_p \left(\left(\frac{\beta'^2_s \left(\alpha_2 + \gamma_p \left(\frac{1}{2}(c_m + v_1) + N' s_1 + G' \right) - \gamma_s s_1 - \beta'_p c_m \right)}{2\beta'_p \left(2\eta_2 \beta'_p - \beta'^2_s \right)} \right) + \frac{\alpha_2 + \gamma_p \left(\frac{1}{2}(c_m + v_1) + N' s_1 + G' \right) - \gamma_s s_1 + \beta'_p c_m}{2\beta'_p} \right) \\ + \beta'_s s_1 - \gamma_s \left(\frac{\beta'_s \left(\alpha_2 + \gamma_p \left(\frac{1}{2}(c_m + v_1) + N' s_1 + G' \right) - \gamma_s s_1 - \beta'_p c_m \right)}{2\eta_2 \beta'_p - \beta'^2_s} \right) \end{array} \right) - \frac{1}{2}\eta_1 s_1^2 \tag{38}$$

where $N' = \left(CQ + C + \beta'_s + \frac{I\gamma_s}{\gamma_p} \right) \Big/ 2H'$

LEMMA 5. In the third scenario, the profit function of Retailer 1 is concave, if and only if, $0 \le \frac{1}{\eta_2} \le \frac{2\beta'^2_p - \gamma_p^2}{\beta'^2_s \beta'_p - \beta'_p \gamma_s \gamma_p}$ and $0 \le \frac{1}{\eta_1} \le \frac{1}{2H' N'^2}$.

Suppose that the conditions of Lemma 5 are held; the equilibrium equations are obtained by setting the gradient to zero.

$$\partial \pi^{r1} / \partial s_1 = 0 \;\; and \;\; \partial \pi^{r1} / \partial v_1 = 0 \Rightarrow$$

$$s_1{*} = N' H' M' / \left(\eta_1 - N'^2 H' \right) \tag{39}$$

where to simplify the writing, we define the following abbreviation.

$$X'' = \left(\eta_1 - 4N'^2 H' \right) c_m \Big/ \left(4\left(\eta_1 - N'^2 H' \right) \right) + 3\eta_1 G' \Big/ \left(2\left(\eta_1 - N'^2 H' \right) \right), \;\; M' = G' - \frac{c_m}{2}$$

Eventually, using the back-substitution process, the equilibrium equation of other previous decision variables would be as follows.

$$v_1^* = M' \eta_1 \Big/ \left(\eta_1 - N'^2 H' \right) \tag{40}$$

$$w_1^* = X'' + \left(2\eta_1 \Big/ \left(4\left(\eta_1 - N'^2 H' \right) \right) \right) \left(c_m - 2G' \right) \tag{41}$$

$$p_1^* = X'' \tag{42}$$

$$p_2^* = (Q+1) \left(\left(\frac{\gamma_p}{2\beta'_p} \right) X'' - \frac{\gamma_s}{2\beta'_p} \left(\frac{N' H' M'}{\eta_1 - N'^2 H'} \right) \right) + \frac{Q\left(\alpha_2 - \beta'_p c_m \right) + \alpha_2}{2\beta'_p} + \frac{c_m}{2} \tag{43}$$

$$s_2^* = \frac{I}{\gamma_s} X'' - \frac{I}{\gamma_p} \left(\frac{N' H' M'}{\eta_1 - N'^2 H'} \right) + \frac{\beta'_s \left(\alpha_2 - \beta'_p c_m \right)}{2\eta_2 \beta'_p - \beta'^2_s} \tag{44}$$

5. NUMERICAL ANALYSIS

This section is devoted to numerically investigate the impact of main parameters on the equilibrium points and the profit of players. The base example is considered as follows.

$$\beta_p = 0.1, \gamma_p = 0.1, \beta_s = 0.1, \gamma_s = 0.1, \; \alpha_1 = 12, \alpha_2 = 10, c_m = 10, \eta_1 = \eta_2 = 0.3$$

We are interested in assessing the influence of price competition intensity and service investment efficiency on the optimal values. Therefore, we vary their related parameters in each scenario to explore their impacts and find some managerial insights.

5. 1. Effects of Price Competition Intensity

Since γ_p represents price competition intensity, the base example is analyzed for a various amount of γ_p, the results of which are reported in Table 1 for the first scenario. The analysis is done for second and third scenarios as well; however, their results are not reported due to the similarity. In all three scenarios, the market with more considerable price competition intensity has a smaller wholesale price, retail price, and service level in both supply chains. By increasing the intensity of competition, the retailers in all three scenarios try to decrease the retail price to attract more customers and gain more market share. In addition, the leader supply chain always has higher retail and wholesale prices than the follower supply chain.

Thus, for small values of γ_p, a part of market demand is attracted to the follower supply chain because of the lower retail price. Figure 1 shows the behavior of the supply chains and the industry's profits in terms of the price competition intensity. The profit functions of the follower chain's manufacturer and retailer are first increasing and then decreasing to the price competition intensity. Therefore, the total profit of the follower supply chain has similar behavior. Consequently, the existence of a low degree of price competition is better for the follower chain compared to having no competition. Furthermore, the increase of price competition intensity results in the decrease of leader chain's manufacturer and retailer profits and the total profit of the supply chain.

Amin-Naser and Azari-Khojasteh [6] observed that when the competition intensity increases, the total profit of the leading supply chain decreases, while the total profit of the follower supply chain increases. Their result is partially consistent with our result. That is, for the small values of competition intensity, the results are the same. However, for a higher value of competition intensity, we observe that the total profit of both supply chains is decreasing with competition intensity. The difference is due to the difference between the considered demand functions. Amin-Naser and Azari-Khojasteh [6] assumed the classical version of the linear demand function. At the same time, we considered a modified version of the linear demand function, which draws the competition behavior better than the classical model.

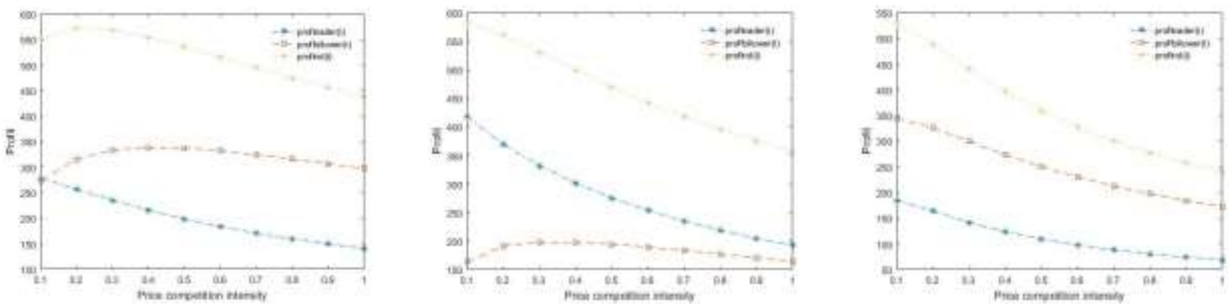
5. 2. Effects of Service Investment Coefficient To investigate the relationship between equilibrium solutions and the service investment coefficient of the leading supply chain, we change its quantity in the base example and report the results for the first scenario in Table 2 (the second and third scenarios are similar). As can be seen, increasing the service investment coefficient (decreasing the service investment efficiency) will decrease the leader chain's service level.

Decreasing the service investment efficiency means that investing in the service levels does not positively influence the profit of the supply chain as it should. Thus, the leader supply chain decreases the service level, and therefore, the retail price will be decreased. On the other hand, the follower supply chain increases its service level results in gaining more customers. As a result, the retail price of the follower supply chain will be enhanced, which directly impacts the chain's whole profit. In addition, the profit functions of the follower chain's manufacturer and retailer are increasing, and those of the leader chain are decreasing. Also, the industry's total profit is decreasing because of the higher effect of the leading supply chain. The lower the service investment coefficient of one retailer, the lower the service level and retail price of his rival will be. This result is consistent with the result of Xiao and Yang [24], in which the authors considered two competing supply chains with manufacturer-Stackelberg structure.

5. 3. Comparison of Three Scenarios and Managerial Implications In all three scenarios, decentralization for both supply chains leads to having more retail prices in the leader and the follower supply chains. In the first scenario in which both chains are decentralized, the manufacturers and retailers try to increase their profit independently by increasing the retail price and wholesale price in both supply chains. In other words, centralization will decrease the retail price either in the follower or in the leading supply chain.

TABLE 1. Effect of varying γ_p in Scenario 1

γ_p	p_1^*	s_1^*	p_2^*	s_2^*	w_1^*	w_2^*	π^{m1}	π^{r1}	π^{t1}	π^{m2}	π^{r2}	π^{t2}	$\pi^{t1}+\pi^{t2}$
0.1	84.57	13.67	77.90	15.09	34.85	32.63	101.95	175.86	277.81	102.46	170.78	273.25	551.06
0.2	74.13	10.68	68.38	12.97	31.37	29.46	91.41	165.68	257.10	113.62	201.99	315.61	572.72
0.3	66.88	9.04	61.46	11.43	28.96	27.15	82.52	152.76	235.28	117.70	215.79	333.50	568.79
0.4	61.24	7.93	56.13	10.25	27.08	25.37	75.02	140.61	215.63	118.22	220.69	338.92	554.55
0.5	56.66	7.09	51.85	9.30	25.55	23.95	68.68	129.82	198.51	116.80	220.62	337.42	535.93
0.6	52.86	6.42	48.33	8.51	24.28	22.77	63.29	120.38	183.67	114.30	217.72	332.03	515.71
0.7	49.64	5.88	45.37	7.86	23.31	21.79	58.65	112.11	170.77	111.25	213.23	324.48	495.25
0.8	46.88	5.43	42.85	7.30	22.29	20.95	54.64	104.85	159.50	107.92	207.85	315.77	475.28
0.9	44.48	5.05	40.66	6.81	21.49	20.22	51.13	98.44	149.57	104.50	201.04	306.55	456.13
1	42.37	4.72	38.76	6.39	20.79	19.58	48.04	92.74	140.79	101.10	196.07	297.17	437.96



Scenario 1 Scenario 2 Scenario 3

Figure 1. Profit versus the price competition intensity

TABLE 2. Effect of varying η_1 in Scenario 1

η_1	η_2	p_1^*		p_2^*	s_2^*	w_1^*	w_2^*	π^{m1}	π^{r1}	π^{t1}	π^{m2}	π^{r2}	π^{t2}	$\pi^{t1}+\pi^{t2}$
0.1	0.3	119.47	60.22	72.66	13.92	46.49	30.88	219.73	258.18	477.91	87.27	145.46	232.73	710.65
0.2	0.3	91.03	22.28	76.93	14.87	37.01	32.31	120.37	191.09	311.47	99.56	165.94	265.50	576.98
0.3	0.3	84.57	13.67	77.90	15.09	34.85	32.63	101.95	175.86	277.81	102.46	170.78	273.25	551.06
0.4	0.3	81.71	9.86	78.33	15.18	33.90	32.77	94.28	169.12	263.41	103.76	172.94	276.71	540.12
0.5	0.3	80.10	7.71	78.57	15.23	33.36	32.85	90.09	165.32	255.41	104.50	174.17	278.67	534.09
0.6	0.3	79.06	6.33	78.73	15.27	33.02	32.91	87.45	162.88	250.33	104.97	174.96	279.93	530.27
0.7	0.3	78.34	5.37	78.83	15.29	32.78	32.94	85.63	161.18	246.81	105.30	175.51	280.11	527.63
0.8	0.3	77.81	4.66	78.91	15.31	32.60	32.97	84.31	159.93	244.24	105.55	175.91	281.46	525.71
0.9	0.3	77.40	4.11	78.97	15.32	32.46	32.99	83.30	158.96	242.27	105.73	176.22	281.96	524.24
1	0.3	77.08	3.68	79.02	15.33	32.36	33.00	82.50	158.20	240.71	105.86	176.47	282.36	523.08

On the contrary, centralization raises the service level. In the second scenario (integrated leader), the service level of the leading supply chain is more than the other two scenarios. Besides, in the third scenario (integrated follower), the service level of the follower supply chain is higher. Clearly, in the integrated structure, both the manufacturer and the retailer tend to maximize the total profit of the supply chain cooperatively. Subsequently, by decreasing the retail price and increasing the service level, more market share will be gain.

Comparing the results obtained from increasing the price competition intensity indicates that only for the small amount of price competition intensity, the centralization is the most sensible choice for the follower supply chain in the third scenario. In other words, for $\gamma_p \geq 0.3$, the total profit of the follower supply chain in the first scenario (decentralized leader and follower supply chains) has a higher total profit. Accordingly, the industry's total profit is the most in the first scenario, and only for $\gamma_p = 0.1$, the industry's total profit is the most in the second scenario. At the same time, changing the service investment coefficient of both the leader and follower supply chains, the integrated supply chain has the most profit among the three scenarios. It means that in the second scenario, the total profit of the leading supply chain is the most of the three scenarios, and in the third scenario, the follower supply chain has the same situation. Since the leader supply chain has a higher market share and more power, it has a more impressive impact on the market than the follower supply chain. Hence, the total profit of the industry is the most in the second scenario, and obviously, the centralization of the leading supply chain is the best choice for the total profit of the industry.

Some managerial implications of our findings could be elaborated on as follows. In a market with high price competition intensity, when the leader supply chain is decentralized, the follower supply chain should choose the decentralized structure to maximize its profit. In contrast, centralization is more profitable for the follower supply chain in a market with low price competition intensity. Furthermore, when the leader supply chain is integrated, the best situation occurred for the entire industry when a low price competition intensity is introduced. Moreover, it is better for the entire industry in a market with a decentralized leader that the follower chain also has a decentralized structure. In addition, a market with an integrated follower is preferred from the costumers' point of view since both wholesale and retail prices have the least amount. In this case, customers who buy the product from the follower supply chain also have the advantage of receiving a higher service level.

6. CONCLUSION

This study considered the competition of two leader-follower supply chains where they compete on retail price and service level. Each supply chain consisted of a leading retailer and a follower manufacturer. Three scenarios with different structures were investigated to examine the impact of the competition intensity and the investment efficiency coefficient, on the optimal values of retail and wholesale prices, service levels, and profit functions of supply chain members.

The numerical analysis showed that if the price competition intensity increases, the retail price, the wholesale price, and the service level of both chains would decrease in all three scenarios. Furthermore, the profits of the leader chain and its members generally decrease by increasing the competition intensity. Moreover, in the follower supply chain, the manufacturer profits, the retailer profits, and the total profit in the first and second scenarios are first increasing and then decreasing as the price competition intensity increases.

However, this is not the case for the follower supply chain when a small competition intensity is introduced. In this scenario, the follower's retailer and manufacturer act the same as the leader ones, and by increasing the price competition intensity, the profit functions of the manufacturers and the retailers in both supply chains decrease. More specifically, the existence of limited price competition is beneficial for the follower supply chain.
By increasing the investment efficiency coefficient of both chains, a rise happened in the follower chain's retail price, wholesale price, and service level, and a drop happened in leader's ones in all three scenarios. Therefore, having more investment efficiency coefficient both in the leader and follower supply chains is not profitable for the leading supply chain and will decrease its total profit while it leads to an increase in the follower's total profit.

Some directions less noted by researchers can be applied as a basis for future research. For example, one may consider a stochastic demand model that is more realistic. However, it is more complex than the current model. Another direction is to consider inventory decisions along with the price and service decisions.

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Appendix A: Proof of all lemmas

Proof of lemma 1.

The Hessian matrix of the follower chain's retailer is

$$H_2=\begin{bmatrix}\dfrac{\partial^2\pi^{r2}}{\partial v_2{}^2} & \dfrac{\partial^2\pi^{r2}}{\partial v_2\partial s_2}\\ \dfrac{\partial\pi^{r2}}{\partial s_2\partial v_2} & \dfrac{\partial\pi^{r2}}{\partial s_2{}^2}\end{bmatrix}=\begin{bmatrix}-\beta'_p & \dfrac{\beta'_s}{2}\\ \dfrac{\beta'_s}{2} & -\eta_2\end{bmatrix}$$

π^{r2} is concave on retail price and service level if and only if H_2 is negative semi-definite. Therefore, the following two conditions must be held:

$$1)\ -\beta'_p\le 0$$

$$2)\ \begin{vmatrix}-\beta'_p & \dfrac{\beta'_s}{2}\\ \dfrac{\beta'_s}{2} & -\eta_2\end{vmatrix}\ge 0\rightarrow M_1=\eta_2\beta'_p-\left(\frac{\beta'_s}{2}\right)^2\ge 0$$

The first condition is correct by model assumptions, and the second condition is satisfied for the domain Lemma:

$$0\le\frac{1}{\eta_2}\le\frac{4\beta'_p}{\beta'_s{}^2}\ \square$$

Proof of lemma 2.

From the first condition of the leader's Hessian matrix, we know that:

$$-\beta'_p+A-B+\frac{\gamma_p^2.R}{D}\le 0\rightarrow\frac{\gamma_p}{D}\left(\gamma_s\beta'_s-\gamma_p\eta_2\right)\ge\frac{\gamma_p^2}{2\beta'_p}\left(1+\frac{\beta'^2_s}{D}\right)-\beta'_p$$

Simplifying the equations above leads to:

$$\gamma_s\beta'_s+\beta'_p\ge 3\gamma_p\eta_2$$

According to the second condition of the follower's retailer hessian matrix:

$$0\le\frac{1}{\eta_2}\le\frac{4\beta'_p}{\beta'^2_s}\qquad\text{A-1}$$

Also, from the first condition of the leader's Hessian matrix:

$$\frac{1}{3\gamma_p\eta_2}\ge\frac{1}{\gamma_s\beta'_s+\beta'_p}\Leftrightarrow\frac{1}{\eta_2}\ge\frac{3\gamma_p}{\gamma_s\beta'_s+\beta'_p}\qquad\text{A-2}$$

Therefore, according to A-1 and A-2, we can define an interval for the follower's investment efficiency at service level:

$$\frac{3\gamma_p}{\gamma_s\beta'_s+\beta'_p}\le\frac{1}{\eta_2}\le\frac{4\beta'_p}{\beta'^2_s}$$

We can change the second condition of the leader's retailer hessian matrix as follows:

$$H\eta_1\ge\left(2KH-KH\right)^2\Leftrightarrow 0\le\frac{1}{\eta_1}\le\frac{1}{K^2H}$$

Proof of lemma 3.

The leader's Hessian matrix is:

$$H_1=\begin{bmatrix}\dfrac{\partial\pi^{T1}}{\partial v_1^2} & \dfrac{\partial\pi^{T1}}{\partial v_1\partial s_1}\\ \dfrac{\partial\pi^{T1}}{\partial s_1\partial v_1} & \dfrac{\partial\pi^{T1}}{\partial s_1^2}\end{bmatrix}=\begin{bmatrix}2\left(-\beta'_p+A-B+\dfrac{\gamma_p^2R}{D}\right) & C+E\gamma_s+\beta'_s-\dfrac{BR}{\beta'_s}\\ C+E\gamma_s+\beta'_s-\dfrac{BR}{\beta'_s} & -\eta_1\end{bmatrix}$$

If the Hessian matrix of the leader's chain is negative definite, the total profit function of the leader's chain will be concave with respect to price and service level. The determinant of the matrix is:

$$M_1=2\eta_1\left(\beta'_p-A+B-\frac{\gamma_p{}^2R}{D}\right)-\left(C+E\gamma_s+\beta'_s-\frac{BR}{\beta'_s}\right)^2$$

$$1)\left(-\beta'_p+A-B+\frac{\gamma_p{}^2R}{D}\right)\le 0$$

$$2)\begin{vmatrix}2\left(-\beta'_p+A-B+\dfrac{\gamma_p{}^2R}{D}\right) & C+E\gamma_s+\beta'_s-\dfrac{BR}{\beta'_s}\\ C+E\gamma_s+\beta'_s-\dfrac{BR}{\beta'_s} & -\eta_1\end{vmatrix}\ge 0\rightarrow M_1\ge 0$$

The first condition of the leader's chain in the second scenario is the first condition of the leader's chain in the first scenario. In other words, the follower's investment efficiency at the service level must be:

$$\frac{3\gamma_p}{\gamma_s\beta'_s+\beta'_p}\le\frac{1}{\eta_2}\le\frac{4\beta'_p}{\beta'^2_s}$$

The second condition of optimality can be identified as:

$$2\eta_1H\ge K^2\Leftrightarrow\eta_1\ge\frac{\left(C+E\gamma_s+\beta'_s-\dfrac{BR}{\beta'_s}\right)^2}{2\left(\beta'_p+B-A-\dfrac{R\gamma_p^2}{D}\right)}\Leftrightarrow$$

$$0\le\frac{1}{\eta_1}\le\frac{2\left(\beta'_p+B-A-\dfrac{\gamma_p^2R}{D}\right)}{\left(C+E\gamma_s+\beta'_s-\dfrac{BR}{\beta'_s}\right)^2}$$

Proof of lemma 4.

The follower's hessian matrix of profit function is:

$$H_2=\begin{bmatrix}\dfrac{\partial\pi^{T2}}{\partial v_2{}^2} & \dfrac{\partial\pi^{T2}}{\partial v_2\partial s_2}\\ \dfrac{\partial\pi^{T2}}{\partial s_2\partial v_2} & \dfrac{\partial\pi^{T2}}{\partial s_2{}^2}\end{bmatrix}=\begin{bmatrix}-2\beta'_p & \beta'_s\\ \beta'_s & -\eta_2\end{bmatrix}$$

The two conditions of the Hessian matrix are:

$$1)-2\beta'_p \le 0$$

$$2)\begin{vmatrix} -2\beta'_p & \beta'_s \\ \beta'_s & -\eta_2 \end{vmatrix} \ge 0 \rightarrow$$

$$M_2 = 2\beta'_p \eta_2 - \beta'^{\,2}_s \ge 0$$

β'_p is positive; therefore, the first condition is satisfied.

The second condition is satisfied if and only if the follower's retailer investment efficiency at service level will be $0 \le \frac{1}{\eta_2} \le \frac{2\beta'_p}{\beta'^2_s}$.

Proof of lemma 5.

The hessian matrix of leader's retailer profit function is:

$$H_1 = \begin{bmatrix} \frac{\partial \pi^{r1}}{\partial v_1^2} & \frac{\partial \pi^{r1}}{\partial v_1 \partial s_1} \\ \frac{\partial \pi^{r1}}{\partial s_1 \partial v_1} & \frac{\partial \pi^{r1}}{\partial s_1^2} \end{bmatrix} =$$

$$\begin{bmatrix} -\beta'_p + AQ + A - I & \left(-\beta'_p + AQ + A - I\right)N' + \left(CQ + C + \beta'_s + \frac{I\gamma_s}{\gamma_p}\right) \\ \left(-\beta'_p + AQ + A - I\right)N' + \left(CQ + C + \beta'_s + \frac{I\gamma_s}{\gamma_p}\right) & -\eta_1 \end{bmatrix}$$

Moreover, the two conditions of the Hessian matrix are:

$$1)-\left(\beta'_p - AQ - A + I\right) \le 0$$

$$2)\begin{vmatrix} \left(-\beta'_p + AQ + A - I\right) & \left(-\beta'_p + AQ + A - I\right)N' + \left(CQ + C + \beta'_s + \frac{I\gamma_s}{\gamma_p}\right) \\ \left(-\beta'_p + AQ + A - I\right)N' + \left(CQ + C + \beta'_s + \frac{I\gamma_s}{\gamma_p}\right) & -\eta_1 \end{vmatrix} \ge 0 \rightarrow$$

$$M_1 = \eta_1\left(\beta'_p - AQ - A + I\right) - \left(\left(-\beta'_p + AQ + A - I\right)N' + \left(CQ + C + \beta'_s + \frac{I\gamma_s}{\gamma_p}\right)\right)^2 \ge 0$$

The first condition is:

$$\beta'_p - AQ - A + I \ge 0 \Leftrightarrow$$

$$\frac{\beta'_p\left(2\eta_2\beta'_p - \beta'^{\,2}_s\right) + \gamma_s\gamma_p\beta'_s}{2\eta_2\beta'_p - \beta'^{\,2}_s} \ge \frac{\eta_2\gamma_p^2}{\left(2\eta_2\beta'_p - \beta'^{\,2}_s\right)} \Leftrightarrow$$

$$\eta_2 \ge \frac{\beta'^{\,2}_s\,\beta'_p - \beta'_s\,\gamma_s\gamma_p}{2\beta'^2_p - \gamma_p^2}$$

The first condition of the Hessian matrix of leader's retailer is satisfied if and only if the follower's retailer investment efficiency at service level will be:

$$0 \le \frac{1}{\eta_2} \le \frac{2\beta'^2_p - \gamma_p^2}{\beta'^{\,2}_s\,\beta'_p - \beta'_s\,\gamma_s\gamma_p}$$

By considering $\frac{CQ + C + \beta'_s + \frac{I\gamma_s}{\gamma_p}}{2H'} = N'$, the second condition will be: $0 \le \frac{1}{\eta_1} \le \frac{1}{H'N'^2}$.

Persian Abstract

چکیده

رقابت یکی از فاکتورهای اساسی در تعیین استراتژی زنجیره های تامین است. به علاوه، با توجه به وجود عدم تعادل در سهم بازار، در اکثر موارد یک زنجیره دارای قدرت کنترل بیشتر در بازار بوده و نقش رهبر را در بازار رقابتی بازی می کند. در این تحقیق، دو زنجیره تامین رهبر–پیرو متشکل از یک تولیدکننده و یک خرده فروش مورد بررسی قرار می گیرند. علاوه بر رقابت بین زنجیره ای، رقابت درون زنجیره ای نیز مورد مطالعه قرار می گیرد که در آن خرده فروش نقش رهبر و تولیدکننده نقش پیرو را بازی می‌کند. دو عامل رقابتی قیمت و سطح سرویس در سه سناریوی مختلف مورد بررسی قرار می گیرند: رهبر غیرمتمرکز–پیرو غیرمتمرکز، رهبر متمرکز–پیرو غیرمتمرکز و رهبر غیرمتمرکز–پیرو متمرکز. با استفاده از رویکرد استنتاج بازگشتی، روند حل مسئله از زنجیره پیرو آغاز شده و توابع بهترین پاسخ زنجیره پیرو بدست می‌آیند. سپس استراتژی زنجیره رهبر پس از جایگذاری توابع بهترین پاسخ زنجیره پیرو در توابع سود زنجیره رهبر، تعیین می‌گردند. در نهایت، تاثیر شدت رقابت در قیمت و ضریب سرمایه‌گذاری در سطح سرویس هر دو زنجیره تامین را بر روی مقادیر بهینه در هر سه سناریو مورد تجزیه و تحلیل قرار می‌گیرد. نتایج بدست آمده نشان می دهد که افزایش شدت رقابت در قیمت موجب کاهش سود زنجیره رهبر می‌شود. در مقابل، مقادیر کم شدت رقابت در قیمت سودآوری بیشتری برای زنجیره پیرو در پی خواهد داشت. به علاوه، ضریب سرمایه‌گذاری در سطح سرویس در هر دو زنجیره رهبر و پیرو، تاثیری مستقیم بر روی مقادیر بهینه زنجیره پیرو و تاثیری معکوس بر روی زنجیره رهبر خواهد داشت. همچنین، سناریوی اول (رهبر غیرمتمرکز–پیرو غیرمتمرکز) بیشترین مقدار قیمت عمده فروشی و خرده فروشی را در بین هر سه سناریو دارد، در حالی که زنجیره متمرکز دارای بیشترین مقدار سطح سرویس در بین دو زنجیره خواهد بود.