



## Supervised and Unsupervised Clustering Based Dimensionality Reduction of Hyperspectral Data

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### ABSTRACT

Nowadays, hyperspectral images (HIs) are widely used for land cover land use (LCLU) mapping. Hyperspectral sensors collect spectral data in numerous adjacent spectral bands, which are usually redundant. Hyperspectral data processing comes with important challenges such as huge processing time, difficulties in transfer, and storage. In this study, two supervised and unsupervised dimensionality reduction methods are proposed for hyperspectral feature extraction based on the band clustering technique. In the first method, the unsupervised method, after the unsupervised band clustering stage with some statistical attributes, the principal component transform is used in each cluster, and the first PC component is considered an extracted feature. In the second method, the supervised method, bands are clustered based on training samples mean vectors of each class, and the weighted mean operator is used for feature extraction in each cluster. The experiment is conducted on the classification of real famous HI named Indian Pines. Comparing the obtained results and some other state of art methods proved the proposed method's efficiency.

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## 1. INTRODUCTION

Hyperspectral sensors have high discrimination capabilities of ground surface materials due to recording hundreds of spectral portions of the electromagnetic wave. Classification is one of the most important methods for information extraction from HIs. Hughes phenomenon is the main problem in the supervised classification of HIs that degrades the classification accuracies. This phenomenon expresses that by increasing the number of features above some threshold, classification accuracy usually decreases [1, 2].

Dimensionality reduction is a common way to tackle this problem. Generally, feature selection and feature extraction are two main groups of dimensionality reduction methods. Feature selection methods try to find a lower-dimensional subset of the original feature so that essential discriminative information is preserved, while feature extraction methods try to find a transformation to map the features in lower-dimensional space. The main difference between these two methods lies in reduced

features' physical meaning, which is only preserved in feature selection methods. In this study, we focus on feature extraction methods.

Numerous studies are available in the literature for feature extraction from HIs. Principal components analysis (PCA) and minimum noise fraction (MNF) are two widely used unsupervised methods that map the original features in lower dimensional space so that the first few reduced features contain the most information [3, 4]. Another version of PCA named segmented principal component analysis (SPCA) is proposed, which enhanced the original PCA version in the computational time aspect [5, 6]. Also, the nonlinear version of PCA named Kernel Principal Component (KPCA) is used in several studies dimensionality reduction of hyperspectral data [7, 8]. The wavelet-based dimensionality reduction method is another unsupervised method [9, 10]. In this method, high frequency and low-frequency components of the spectral signature curve (SSC) are separated, and the smoother version of SSC is used as the reduced features. The rational function curve fitting method is

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recently used for hyperspectral feature extraction [11, 12]. In this method, a specific rational function approximation is developed to fit each pixel's SSC, and coefficients of the numerator and denominator are considered the newly extracted features.

Another widely used method that uses labeled samples of classes is linear discriminant analysis (LDA) [13, 14]. This method tries to maximize the within-class similarity and between-class separability. A generalized version of LDA that uses the kernel function is proposed by Baudat and Anouar [15]. LDA has three important drawbacks; first, this method works well only if the distributions of classes are normal-like. Second, this method can only extract (L-1) features (L is the number of classes), and third, the singularity of the within-class scatter matrix often occurs in the face with hyperspectral data [16]. Another supervised method named decision boundary feature extraction (DBFE) is a method that is extracted discriminately feature from the decision boundary between classes [17]. Nonparametric weighted feature extraction (NWFE) is another important feature extraction method for hyperspectral data [18]. The main idea of NWFE is to put different weights on every sample, compute the weighted means, and then define nonparametric between-class, and within-class scatter matrices. As a drawback, this method takes enormous time for the data reduction process. The kernel version of NWFE named kernel NWFE (KNWFE) is proposed to extend the NWFE for a nonlinear situation [16].

Recently, clustering-based methods are used for feature extraction [19, 20]. A supervised method named PSBS has been proposed based on k-means clustering of bands for band selection of HIs [21]. In a detailed paper, previous authors proposed the PSFE method based on fuzzy c-means clustering and feature weighting by class membership values [22]. Clustering-based feature extraction (CBFE) is proposed by Imani and Ghassemian [23]. This method considers a vector of mean values of training samples of all classes in each band and then a clustering algorithm such as k-means, groups these vectors in some clusters, and subsequently, the mean of bands whose associated vectors are located in a cluster is considered as an extracted feature. Usually, clustering-based methods work well even in small sample size situations (SSS situation) and do not have the other problems of some conventional supervised feature extraction methods such as enormous computation times, limitation of L-1 features (in LDA), and problems in estimating covariance matrix. An unsupervised version of band selection based on clustering of some bands statistics such as variance, mean absolute deviation (MAD), and standard deviation are for endmember extraction and classification of HIs [24-28].

This study introduced two unsupervised and supervised methods for dimensionality reduction of hyperspectral data based on bands clustering. As a result

of the literature, when PCA transforms modified so that transformation is carried out by avoiding the low correlations between highly correlated blocks, PCA's efficiency improved [29]. This study's first proposed method is the unsupervised method named unsupervised clustering based principal components analysis (UCPCA). In this method, in the first stage, bands are clustered based on some statistical attributes. We expect adjacent and non-adjacent correlated bands to concentrate in the same cluster. In the second stage in each cluster, we carried out the PCA transform, and the first principal component (PC) is considered reduced features in each cluster. Unsupervised band clustering is the main novelty of this method, making UCPCA, like the original PCA, an unsupervised method. In the second proposed method, the supervised method named weighted mean clustering-based feature extraction (WMCFE), bands are clustered based on the mean values of training samples in each class, and the weighted mean of bands is based on MAD considered as an extracted feature in each cluster. The major differences between UCPCA and WMCFE can be summarized as follows:

1. Each cluster of bands in the UCPCA is formed based on some statistical attributes of bands in an unsupervised manner without using any training samples, but bands clustering space in the WMCFE is formed based on the mean value of training samples in each class.
2. The UCPCA method used PCA transform for extracting the informative feature of each cluster of bands, but the WMCFE used a weighted mean for feature extraction in each cluster. Based on this explanation UCPCA method is grouped in the information-based feature extraction method, but WMCFE is the statistical feature extraction method.

In the next part, the methodology of the study that consists of two proposed methods is presented. In the second part, after introducing the data set results and related analysis of experiments are obtained. Finally, in the last part conclusion is presented.

## 2. METHODOLOGY

**2. 1. UCPCA** Unsupervised clustering-based principal components analysis (UCPCA) is an unsupervised method the same as PCA. This method has two stages. In the first stage, hyperspectral bands are grouped by K-means clustering based on some statistical attributes. Each band of HI can be represented by three statistical measures that are shown in Table 1.

In the above formulas,  $B_i$  is the value of  $i^{\text{th}}$  pixel,  $M$ , and  $N$  are image dimensions,  $mean(b)$  is the mean of all pixel values in band  $b$ . In other words, each band can be shown as a point in 3-dimensional statistical space, and the K-means clustering algorithm tries to group each

**TABLE 1.** Statistical attributes

Statistical attribute	Formula
Variance (Var)	$SM1_b = \frac{1}{M.N} \sum_{i=1}^{M.N} (B_i - mean(b))^2$
Mean	$SM2_b = \frac{\sum_{i=1}^{M.N} B_i}{M.N}$
Kurtosis	$SM3_b = \frac{\frac{1}{M.N} \sum_{i=1}^{M.N} (B_i - mean(b))^4}{(\frac{1}{M.N} \sum_{i=1}^{M.N} (B_i - mean(b))^2)^2}$

band of HI so that the bands with similar information lie in the same cluster. K-means is one of the simplest unsupervised learning algorithms and consisted of four stages [30]:

1) Place K points into the attribute space represented by the statistics of bands. K is the number of desired reduced features.

2) Assign each band to the group that has the closest centroid based on some distance metrics such as square Euclidean

3) recalculate the positions of the K centroids by taking the mean of all bands assigned to that centroid's cluster

4) Repeat steps 2 and 3 until stopping criteria met (such as no centroid move, the sum of the distances is minimized, or user-defined maximum number of iterations)

Because of the algorithm's sensitivity to the initial randomly selected cluster centers and the probability of local optimum situation of results, the k-means algorithm may run multiple times.

In the second stage of the proposed algorithm, when the clusters formed, principal components analysis is applied in each cluster (for more information about the principal component analysis, referred to literature [31]), and the first PC of each cluster that contains the most information is considered as a new extracted feature.

## 2. 2. Weighted Mean Clustering-based Feature Extraction

Weighted mean clustering-based feature extraction (WMCFE) is a supervised method based on the bands' K-means clustering and weighed mean operator. This method has three stages. In the first stage, based on training samples, we calculate the mean vector of training samples in each class for each band of HI. This vector is as follows:

$$b_i = [m_{i1} \quad m_{i2} \quad m_{i3} \quad \dots \quad m_{ic}]^T \quad (1)$$

In the above vector  $b_i$  is  $i^{\text{th}}$  band of HI and  $m_{ic}$  is mean of class c in  $i^{\text{th}}$  band. In other words, if we want to describe this method as the same as the UCPCA method, in the WMCFE method, the means of classes are attributes of each band. In the second stage, k-means clustering is introduced to group the bands the same as the UCPCA. The number of clusters in k-means

clustering is user-defined and is equal to the desired numbers of reduced extracted features. In the last stage, the weighted mean of bands in each cluster is considered as extracted features. In this study, the weighted mean operator in each cluster is formulated as follows:

$$\text{weighted mean}(\mathbf{B}) = \frac{\sum_{i=1}^k w_i b_i}{\sum_{i=1}^k w_i} \quad (2)$$

In the above,  $b_i$  is the  $i^{\text{th}}$  band in a cluster,  $w_i$  is its corresponding weight and  $k$  is the number of bands in each cluster. Weights in this study are defined by MAD statistical measure:

$$\frac{1}{M.N} \sum_{i=1}^{M.N} |B_i - mean(b)| \quad (3)$$

Bands with higher values of MAD get larger weights in each cluster because these bands contain more information, and so have more contribution to the extracted feature of each cluster [27].

## 3. EXPERIMENTAL RESULTS

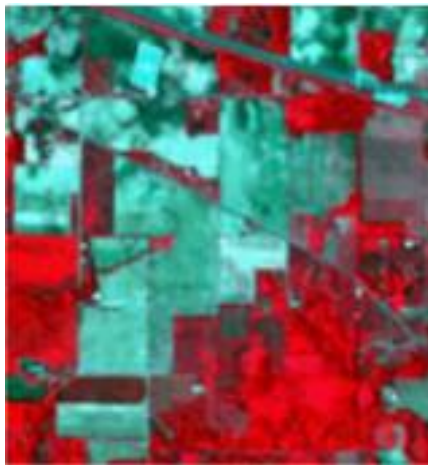
**3. 1. Data Set** The airborne visible/ Infrared Imaging Spectrometer (AVIRIS) sensor acquired the Indian Pines scene from a mixed forest/agricultural from the Indian Pines Site in the USA. This data set consists of  $145 \times 145$  pixels and 224 spectral bands. This data set's spatial resolution is 20 m, and band wavelengths are from 400 to 2500 nm with 10 nm resolution. After removing water absorption and noisy bands, the remaining 200 bands are used in this study. This scene contains 16 different land covers. Figure 1 illustrates the false-color composition of the original image and the ground-truth map .

After discarding four classes named alfalfa, grass-pasture-mowed, oats, and stone-steel-towers with a small number of samples in the ground truth map, the other 12 classes are used in this study.

## 3. 2. Results

For evaluating proposed dimensionality reduction methods, a conventional maximum likelihood classifier is used to classify extracted features. The classification results for proposed methods are compared with four other dimensionality reduction methods named PCA, CBF and NWFE. In this study, average and overall accuracies are two approaches used accuracy metrics [32].

Experiments are done in a few training-size situations with 60 training samples per class randomly chosen from the ground truth map. This training set is used for training classifiers and feature extraction. Half of these training samples are used in supervised feature extraction methods. The obtained overall and average accuracies values in the mentioned experiment for 60 training sample sizes are shown in Figures 2(a) and 2(b).



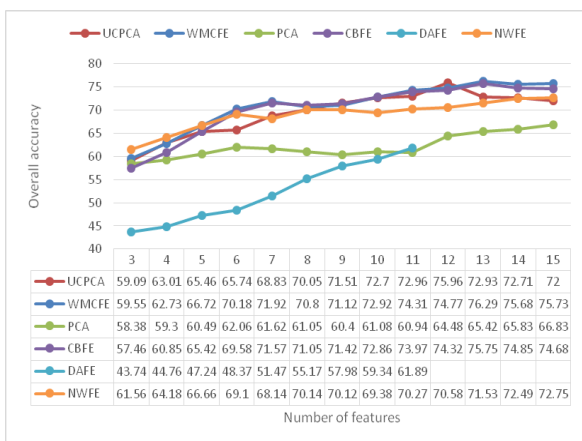
(a)



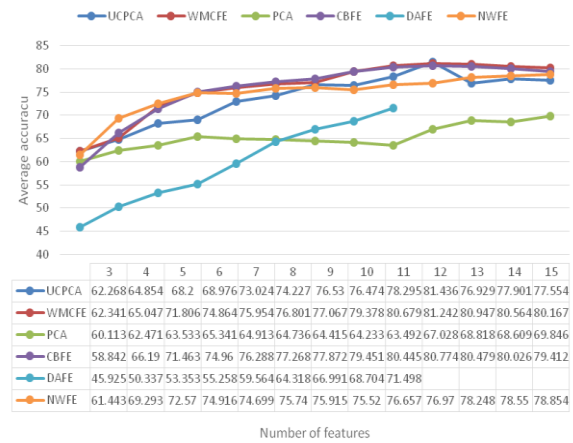
(b)

**Figure 1.** Indian pines data set - a) color composite image - b) ground-truth map

The highest ML achieved classification overall accuracies of extracted features from different methods



(a)



Number of features

(b)

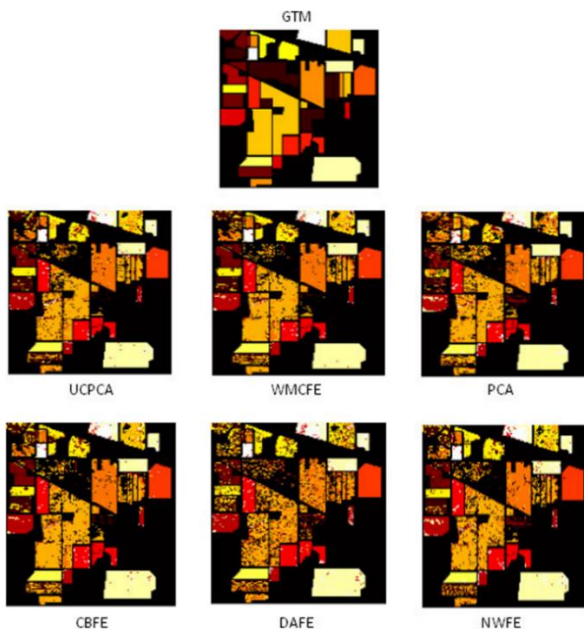
**Figure 2.** Classification accuracies for different feature extraction methods in different number of features- a) overall accuracy - b) average accuracy

that are shown in Table 2 demonstrate that both proposed methods (UCPCA and WMCFE) perform better than the other feature extraction methods and at last WMCFE is superior. The main reason for the superiority of the UCPCA against the PCA is the bands clustering technique, which leads to the more homogenous band groups, and as a result, improves the performance of the PCA. In the comparison to CBFE, WMCFE uses the weighted mean operator that informative band has more contribution to the extracted feature of each cluster. Compared to some classical methods such as the DAFE and the NWFE, whose performance is dependent on a large number of the training samples (for accurate parameter estimation of models), the proposed method (UCPCA and WMCFE) works well even when few training samples are available.

The ground truth map of the Indian data set and the classified maps of the methods in the situation of Table 2 are shown in Figure 3. Based on Figure 3, one can understand that the proposed methods produced more smooth classification results.

**TABLE 2.** highest ML classification overall accuracies

	Methods					
	UCPCA	WMCFE	PCA	CBFE	DAFE	NWFE
<b>OA</b>	<b>75.96</b>	<b>76.29</b>	66.83	75.75	61.89	72.75
<b># of features</b>	12	13	15	13	11	15



**Figure 3.** Ground truth map (GTM) and classified maps of different feature extraction methods

#### 4. CONCLUSION

In this study, two unsupervised and supervised feature extraction methods based on bands clustering have been introduced for dimensionality reduction of hyperspectral data. The unsupervised method, named UCPCA, is based on unsupervised bands clustering and principal components analysis feature extraction. The supervised method, named WMCFE, is based on supervised bands clustering based on samples mean of classes and weighed mean operator based on the MAD measure. The experimental results showed better performance of proposed methods compared with some conventional feature extraction methods such as PCA, CBFE and NWFE.

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### Persian Abstract

#### چکیده

در سال‌های اخیر از تصاویر ابرطیفی به طور گسترده‌ای برای تهیه نقشه از کاربری اراضی تحت پوشش زمین استفاده می‌شود. سنسورهای ابرطیفی باندهای طیفی را در نواحی مجاور متعددی که معمولاً اضافی هستند جمع‌آوری می‌کنند، از این رو پردازش داده‌های طیفی فوق با چالش‌های مهمی مانند زمان پردازش زیاد، مشکلات در انتقال و ذخیره‌سازی داده رویرو است. در این مطالعه، دو روش کاهش ابعاد تحت نظارت و بدون نظارت برای استخراج ویژگی‌های طیفی بر اساس روش خوشه‌بندی باند پیشنهاد شده است. در روش اول، که روش بدون نظارت است، پس از مرحله خوشه‌بندی بدون نظارت باندها با استفاده از مشخصات آماری، در هر خوشه، از تبدیل مولفه اصلی استفاده می‌شود و اولین مولفه آن در هر خوشه به عنوان ویژگی استخراج شده در نظر گرفته می‌شود. در روش دوم که روش نظارت شده است، باندها بر اساس میانگین بردارهای نمونه‌های آموزشی هر کلاس خوشه‌بندی شده و از عملگر میانگین وزندار برای استخراج ویژگی در هر خوشه استفاده می‌شود. آزمایش‌های این مقاله بر روی تصویر ابرطیفی مشهور واقعی ایندین پابن انجام شده است. مقایسه بین نتایج بدست آمده و برخی دیگر از روشهای مرسوم بیانگر کارایی بیشتر روش پیشنهادی است.

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