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Size-dependent Vibration Analysis of Stepped Nanobeams Based on Surface Elasticity Theory

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ABSTRACT

This paper investigates size-dependent vibrations of stepped nanobeams taken into account surface elasticity theory. To do this, the nanobeams are modeled as stepped beams and size-dependent governing vibration equations are derived considering compatibility conditions in stepped sections. Then, an analytical solution is developed to simulate natural frequencies and mode shapes of the nanobeam with various surface properties. Also, a backward procedure is proposed to verify the obtained results and calculate size-dependent effective surface modulus. The results indicate that surface effects and appropriate steps selection have noticeable impact on natural frequencies of non-uniform nanobeams. Also, the stepped modeling of the nanobeam became more important for longer and slender ones. Moreover, despite uniform nanobeams, the mode shapes of the non-uniform nanobeams are also extremely dependent on the surface effects.

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NOMENCLATURE

E	Young's modulus of the nanobeam	**Greek Symbols**	
E^s	Surface Young's modulus of the nanobeam	ρ	Mass density
EI_{eff}	Effective bending rigidity	ε	Strain components
L_i	Length of ith section	σ	Stress components
d^p	Particle diameter (μm)	σ^o	Residual normal stress
w	transverse deflection of the nanobeam	σ^s	Surface stress components

1. INTRODUCTION

In the recent years, remarkable developments in science and technology have afforded the ability to fabricate small size structures with micron to nano dimensions [1-2]. Among these novel structures, micro/nanobeams have recognized as important components of micro/nano systems such as micro/nano resonators, atomic force microscopes (AFM), etc [3-4]. Therefore, development of advanced micro/nano systems requires proper prediction of their mechanical behavior. Korayem et al. [5] presented dynamic modeling of an atomic force beam for micro/nano manipulation in which the beam was modeled as a lumped mass. However, lumped mass modeling cannot be approved as an accurate model for a continuous beam, especially in the nano-scale.

On the other hand, the capability of classical continuum theory to model micro/nano systems is strongly doubted through conducting experimental tests and molecular simulations. Accordingly, various analytical higher-order theories have been established to fix the problem. Bakhtiari-Nejad et al. [6] studied size-dependent free vibrations of piezoelectric nanobeams based on the nonlocal elasticity theory. Also, they [7] developed a general formulation of linear natural frequencies and corresponding quality factors for micro/nano composite beams having arbitrary laminated layers based on the nonlocal elasticity theory. Beni [8]

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employed couple stress theory to model and analyze an isotropic Euler-Bernoulli nano-beam. Nazemizadeh et al. [9] studied size-dependent nonlinear free vibration of a pizoelectic-laminated nanobeam considering the nonlocal elasticity theory. Also, they [10] investigated size effects on the nonlinear dynamic modeling and vibration analysis of a nanobeam at higher modes of vibration.

Furthermore, among the higher-order mechanics theories, the theory of surface elasticity initiated by Gurtin and Murdoch [11] has attracted great interests in nanotechnology. Jiang and Yan [12] employed the surface elasticity theory for static bending of shear deformable nanobeams. They derived the governing equation of the nanobeam and analytically solved the problem. Farshi et al. [13] presented size effects of vibration of the nanobeams taken into account the surface elasticity theory. Assadi and Nazemizadeh [14] presented size-dependent static bending of a Nanobeam based on the surface elasticity theory. They developed Euler nanobeam model to derive the differential equation and used a theoretical solution for the static behavior of the nanobeam.

According to the literature review, there is a need to take into account size-dependent vibration characteristic of the surface parameters. So in this paper, an inverse procedure is proposed to verify the obtained results and calculate size-dependent effective surface modulus of the nanobeam. To do this, the nanobeams are modeled as stepped beams and size-dependent governing vibration equations are derived considering compatibility conditions in stepped sections. Then, an analytical solution is developed to simulate natural frequencies and mode shapes of the nanobeam with various surface properties.

2. PROBLEM FORMULATION

In this section, the governing equation of the size-dependent vibration of the nanobeam is presented. Figure 1 shows a nanobeam with non-uniform cross section and its equivalent stepped beam. For analysis of the problem here it is intended to find the generalized governing equations in the presence of any type of external loadings. For more information, every section

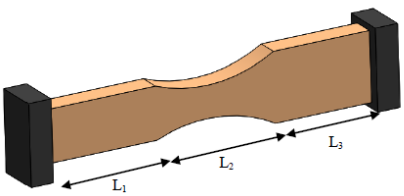

Figure 1. A non-uniform nanobeam

of the beam follows this general equation but we have to satisfy the relating boundary conditions in each case which will be explained in the next section. According to the Euler-Bernoulli beam theory, the strain field of a beam is given as follows:

$$\varepsilon_{xx} = -z\frac{\partial^2 w}{\partial x^2}; \qquad \varepsilon_{xz} = 0 \tag{1}$$

In this relation, w is the transverse deflection of the nanobeam. Also, ε_{xx} and ε_{xz} are the normal and tangential strains. Moreover, x and z are the coordinate selections across the nanobeams' length and thickness, respectfully. It is to be noted that the origin of z is the nanobeams' neutral axes. The stress field of the deflected nanobeams can be given as follows:

$$\sigma_{xx} = \sigma_{xx}^{o} - Ez\frac{\partial^2 w}{\partial x^2} \tag{2}$$

In this equation σ^o is the residual normal stress induced in nanobeams by surface residual stresses. According to generalized Gurtin-Murdoch relation [14], it will be obtained for Euler-Bernoulli beam model:

$$\sigma_{xx}^{s} = \tau^{s} - E^{s} z\frac{\partial^2 w}{\partial x^2}\Big|_{S}$$
$$\sigma_{xz}^{s} = \tau^{s}\frac{\partial w}{\partial x} \tag{3}$$

Accordingly, the bending moment of the cross section will be obtained from the following integral equation:

$$M = \iint_A \sigma_{xx} z dA + \int_z \sigma_{xx}^{s} z dS =$$
$$\left(\iint_A z\sigma_{xx}^{o} dA + \int_z \tau^{s} z dS\right) - EI_{eff}\frac{\partial^2 w}{\partial x^2} \tag{4}$$

While the effective bending rigidity EI^{eff} is obtained from the following relation:

$$EI_{eff} = \iint_A Ez^2 dA + \int_z E^{s} z^2 dS \tag{5}$$

In the above equation, the effective bending rigidity is a combination of bulk and surface elasticity rigidity. In fact, the surface rigidity is originated from the surface effects at nano scales.

On the other hand and according to the self-equilibrating condition, the integral relations in the right hand side of Equation (4) cancel each other and the bending moment is obtained as follows:

$$M = -EI_{eff}\frac{\partial^2 w}{\partial x^2} \tag{6}$$

Finally the governing general differential equation for the deflected nanobeams with consideration of surface effects as follows:

$$EI_{eff}\frac{\partial^4 w}{\partial^4 x}+F\frac{\partial^2 w}{\partial^2 x}+\rho A\frac{\partial^2 w}{\partial t^2}=2\tau^s b\frac{\partial^2 w}{\partial x^2} \quad (7)$$

where b is the length of the nanobeams' neutral axis.

3. PROBLEM SOLUTION

The general solution of Equation (7) is taken as $w(x,t)=W(x).sin(\omega_n t)$ from simple variable separation method. Therefore, the following equation must be satisfied for each section of the nanobeams for $W(x)$:

$$EI_{eff}\frac{\partial^4 W}{\partial^4 x}-2\tau^s b\frac{\partial^2 W}{\partial x^2}-\rho A\omega_n^2 W=0 \quad (8)$$

For simplicity of the problem, another form of Equation (8) is given in bellow:

$$\frac{\partial^4 W}{\partial^4 x}-\alpha^2\frac{\partial^2 W}{\partial x^2}-\eta_n^4 W=0$$
$$\alpha^2=\frac{2\tau^s b}{EI_{eff}} \qquad \eta_n^4=\frac{\rho A\omega_n^2}{EI_{eff}} \quad (9)$$

Next, Equation (9) is separated as two 2nd order independent differential equations from which the summation of the solutions represents the general solution of Equation (9):

$$\left.\begin{aligned}&\frac{\partial^2 W_I}{\partial^2 x}+\zeta_n^2 W_I=0 \qquad \zeta_n^2=\frac{\alpha^2+\sqrt{\alpha^4+4\eta_n^4}}{2}\\&\frac{\partial^2 W_{II}}{\partial^2 x}+\psi_n^2 W_{II}=0 \qquad \psi_n^2=\frac{\alpha^2-\sqrt{\alpha^4+4\eta_n^4}}{2}\end{aligned}\right\}$$
$$\Rightarrow \quad W=W_I+W_{II} \quad (10)$$

In this equation, W_I is the shape function at the first section of the beam and W_{II} is the shape function at the second section of the nanobeam. Also, it is seen that ζ_2 is a positive definite parameter but ψ_2 is always negative. On the other hand, in order to reduce the number of unknowns of the problem, the following relation governs between these parameters:

$$\zeta_n^2+\psi_n^2=\alpha^2 \quad (11)$$

From solving Equation (10), the general solution for W in the case of free vibration is given by the following equation:

$$W=C_1\sin(\zeta_n x)+C_2\cos(\zeta_n x)+C_3\sinh(\psi_n x)+C_4\cosh(\psi_n x) \quad (12)$$

Next, the boundary conditions of Equation (12) must be satisfied to valid the eigenvalue problem of free vibration for each sets of boundary conditions. Here the whole nanobeam is assumed to be simply supported. Similar to the buckling analysis, the following matrix equation is obtained for the common boundary conditions of *i-th* and *(i+1)-th* sections:

$$\begin{bmatrix} f_n^i & g_n^i & h_n^i & l_n^i & 0 & -1 & 0 & -1\\ \zeta_n^i g_n^i & -\zeta_n^i f_n^i & \psi_n^i l_n^i & \psi_n^i h_n^i & -\zeta_n^{i+1} & 0 & -\psi_n^{i+1} & 0\\ -\zeta_n^{i2} f_n^i & -\zeta_n^{i2} g_n^i & \psi_n^{i2} h_n^i & \psi_n^{i2} l_n^i & 0 & \phi_i(\zeta_n^{i+1})^2 & 0 & -\phi_i(\psi_n^{i+1})^2\\ -\zeta_n^{i3} g_n^i & \zeta_n^{i3} f_n^i & \psi_n^{i3} l_n^i & \psi_n^{i3} h_n^i & \phi_i(\zeta_n^{i+1})^3 & 0 & -\phi_i(\psi_n^{i+1})^3 & 0 \end{bmatrix} \quad (13)$$
$$f_n^i=\sin(\zeta_n^i L_i);\quad g_n^i=\cos(\zeta_n^i L_i);\quad h_n^i=\sinh(\psi_n^i L_i);\quad l_n^i=\cosh(\psi_n^i L_i);\quad \phi_i=\frac{EI_{eff}^{i+1}}{EI_{eff}^i}$$

On the other hand, the boundary conditions for the left side of the first section and the right side of the last section make the problem to follow the given relations in Equation (14):

$$C_2^1+C_4^1=0;$$
$$C_2^1(\zeta_n^1)^2-C_4^1(\psi_n^1)^2=0$$
$$\begin{Bmatrix} C_1^N f_n^N+C_2^N g_n^N+C_3^N h_n^N+C_4^N l_n^N\\ +C_1^N(\zeta_n^N)^2 f_n^N+C_2^N(\zeta_n^N)^2 g_n^N\\ -C_3^N(\psi_n^N)^2 h_n^N-C_4^N(\psi_n^N)^2 l_n^N \end{Bmatrix}=0 \quad (14)$$

Finally, all the matrixes of Equation (13) for i = 2…N together with Equation (14) must be merged to give the general matrix equation of a step-wised nanobeam with N sections. It is seen that if all the material and geometric parameters are given, then the only unknown parameter in the coefficient matrix of Equation (13) is the natural frequency ω_n.

4. SIMULATION RESULTS

In this section, a wide range of vibration simulations are presented for aluminum and silicon-100 nanobeams with the following material parameters (see Table 1) [14].

To verify the proposed method, in Figure 2, the obtained results for vibration analysis of a uniform nanobeam are compared with experiment results presented by He and Lilley [15].

From this figure it is observed that, satisfactory agreement is achieved between the results of this work and those of experimental investigations of other references.

For other simulations, the problem is solved once for nanobeams with surface effects, then for nanobeams without surface effects and the results are comprehensively compared. For this purpose the

TABLE 1. Material properties of the nanobeam

Material	E(Gpa)	υ	E^s(N/m)	τ^s(N/m)
Al	68.50	0.35	6.090	0.910
Si [100]	130.0	0.24	-11.50	-0.505

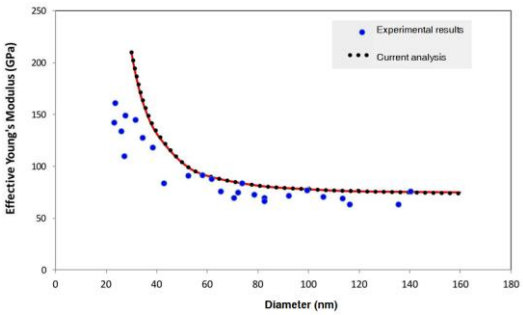

Figure 2. Comparison of effective Young's modulus between the present work and experiment results given by He and Lilley [15]

parameter of NNF (normalized natural frequency) is introduced which the ratio of natural frequency ω_n to that of nanobeams without surface effects. Figure 3 gives the results for size dependent behavior of NNF for one-step nanobeams.

According to Figure 3, it is seen that as D2 gets higher, the surface effects reduce. In addition, the surface effects on the natural frequency are higher at longer nanobeams. The positive surface residual stress increases the natural frequency but the negative one decreases. Moreover, sensitivity of the problem to D2/D1 is higher for longer nanobeams. Also, in the cases of nanobeams with total lengths lower than 30 nm, the whole structure may be modeled as a uniform nanobeam with a simple solution with errors less than %15. In addition, in general the nanobeams with negative surface effects must be modeled more precisely since their natural frequency extremely depend on the magnitude of D2 .

Next, the vibration problem is solved for two-step nanobeams with the numerical results given in Figure 4.

From this figure, it is seen that, for the constant length of the nanobeams, NNF can be taken independent of D3 but with an engineering approximation. Moreover, it is observed that thickening and shortening of one section of the step-wised nanobeams reduces the effect of surface properties on the fundamental natural frequency.

For the next simulation, it is tried to clarify the effect of proper modeling of the non-uniform nanobeam on its vibration analysis. For this purpose, a non-uniform nanobeam is once solved by the developed method in this section and again is considered as a uniform nanobeam with mean diameter. The ratio of the results of this analysis is plotted versus D2 in Figure 5.

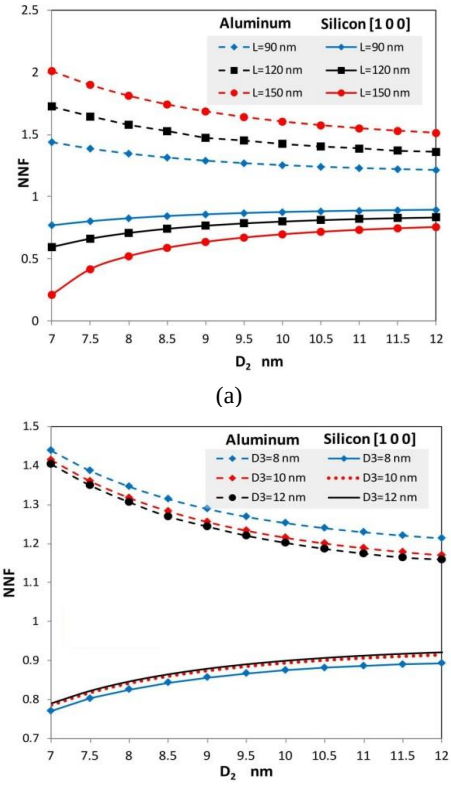

Figure 4. NNF of the two-step nanobeams (a): D3 = 8 nm, D1 = 10 nm and (b): L = 90 nm, D1 = 10 nm

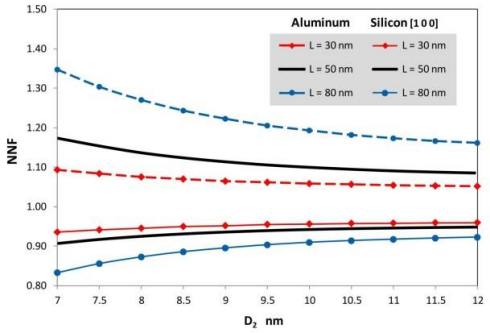

Figure 3. *NNF* of the one-step nanobeams based on different beam theories (D1 = 10 nm, L1 = L2)

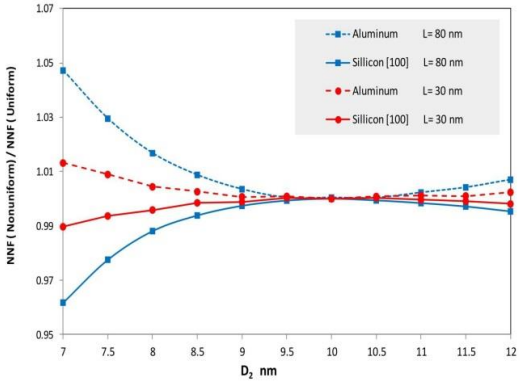

Figure 5. Significance of modeling for proper prediction of NNF for non-uniform nanobeams (D1 = 10 nm)

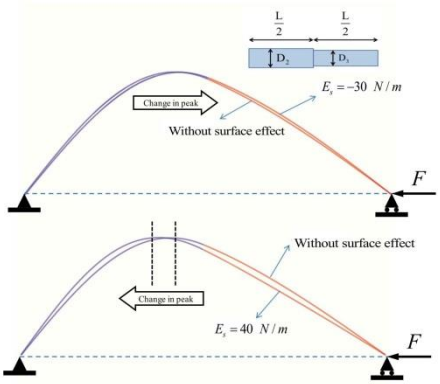

Figure 6. First mode shape of the one-step nanobeams

From this figure, it is observed that in the vibration analyses when the diameters of two adjacent sections differ only in %20, the non-uniform nanobeam can be treated as uniform ones. Otherwise, it is preferred to solve the problem properly.

Furthermore, it is tried to investigate size effects on the mode shapes of the nanobeam. Figure 6 shows the surface effects on the first mode shapes of the nanobeam.

From Figure 6 it is seen that even though large values of surface parameters are examined, but the mode shape is not dependent on their value either the surface parameters be positive or negative. However, it is observed that location of the peak of the principal mode shape depends on the value of surface effects but its magnitude is not changed by variation of these parameters. Also, it should be mentioned that force F is an axial force.

Also, Figure 7 shows the surface effects on the second mode shape of the one-step nanobeam:

In spite of the principal mode shape, Figure 6 illustrates extreme dependence of the second mode shape of a non-uniform nanobeam on the magnitude of surface properties. Generally it is observed that the thinner section is more affected in this case but it is to be noted that the mode shape must be interpreted for the whole structure. From Figure 7 it is seen that location of nodes and peaks of the mode shape changes considerably with changes of surface parameters. It can be intrepreted that the surface parameter causes considerable changing of the beam rigidity. In addition, the amplitude of the mode shape in this case is highly dependent on the surface properties. It is to be noted, more variation in the nanobeam's cross section, more changes in mode shapes by surface effects. In addition, the location of peaks and nodes is pushed toward the thinner section for negative values of surface effects and conversely toward the thicker section for positive ones.

Furthermore, Figure 8 shows the effect of surface properties on the second mode shape of the two-step nanobeam.

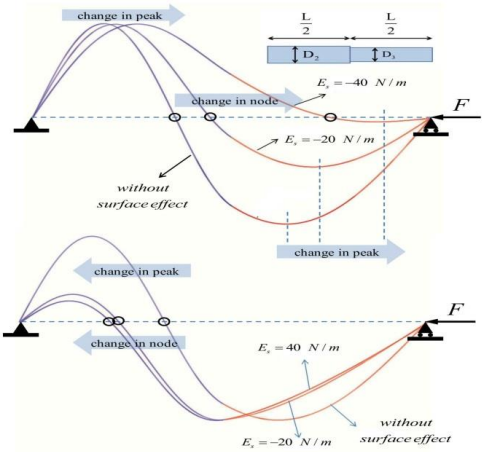

Figure 7. Second mode shape of the one-step nanobeams

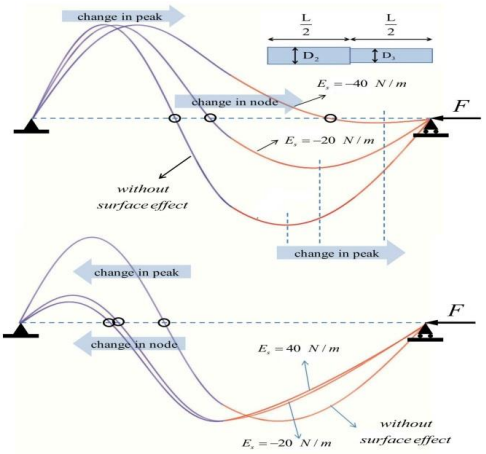

Figure 8. Second mode shape of the two-step nanobeams

It is seen that the surface effects are less than the previous case of Figure 7 since the relative length of the thinner section is reduced but generally the shape of the thinner section is changed considerably. As an important conclusion, it is observed, positive surface parameters make the mode shape more uniform and reduce its amplitude. In this case, it is observed that the location of peaks is not so dependent on the magnitude of surface properties.

5. CONCLUSION

In this paper, size-dependent vibrations of stepped nanobeams have been studied with consideration of the surface elasticity theory. The nanobeams are modeled as stepped beams and size-dependent governing vibration equations have been derived considering compatibility conditions in stepped sections. Then, an analytical solution has been developed to simulate natural frequencies and mode shapes of the nanobeam with

various surface properties. The obtained results indicate that as one section of a step-wised nanobeam gets shorter; the effect of surface properties on the natural frequency of the nanobeam reduces considerably. In addition, the nanobeams with negative surface effects must be modeled more precisely since their natural frequency extremely depend on the diameter magnitude. In fact, proper modeling and choosing the number of steps plays important roles in proper determination of the vibration behavior of non-uniform nanobeams. Moreover, it is seen that the corresponding mode shapes of the nanobeam in some case are extremely dependent on the magnitude of surface effects while their amplitude and locations of nodes and peaks varies a lot by changes of surface effects. It is observed that the thinner section is more affected in this case but it is to be noted that the mode shape must be interpreted for the whole structure. Finally, the obtained results illustrate the capability of the proposed method to model and analyze the size-dependent vibration of the nanobeams

6. REFERENCES

1. Akbari, G., Nikravesh, M., Poladi, A. "Mechanical Properties and Microstructural Evolution of Ta/TaNx Double Layer Thin Films Deposited by Magnetron Sputtering." ***International Journal of Engineering, Transactions B: Applications***, (2017), Vol. 30, No. 2, 288-293. DOI: 10.5829/idosi.ije.2017.30.02b.16.
2. Mofidian, R., Barati, A., Jahanshahi, M., Shahavi, M. H. "Generation Process and Performance Evaluation of Engineered Microsphere Agarose Adsorbent for Application in Fluidized-bed Systems." ***International Journal of Engineering, Transactions B: Applications***, (2020), Vol. 33, No. 8, 1450-1458. DOI: 10.5829/ije.2020.33.08b.02
3. Bagheri, M., Sangpour, P., Badiei, E., Pazouki, M. "Graphene Oxide Antibacterial Sheets: Synthesis and Characterization (RESEARCH NOTE)." ***International Journal of Engineering Transactions C: Aspects***, (2014), Vol. 27, No. 12, 1803-1808. DOI: 0.5829/idosi.ije.2014.27.12c.01
4. Bakhtiari-Nejad, F., Nazemizadeh, M. "Size-dependent dynamic modeling and vibration analysis of MEMS/NEMS-based nanomechanical beam based on the nonlocal elasticity theory." ***Acta Mechanica***, Vol. 227, No. 5, 1363-1379. DOI: 10.1007/s00707-015-1556-3
5. Korayem, M. H., Taheri, M., Korayem, A. H. "Manipulation with atomic force microscopy: DNA and yeast micro/nanoparticles in biological environments", ***Proceedings of the Institution of Mechanical Engineers, Part K: Journal of Multi-body Dynamics,*** (2014), Vol. 228, No. 4, 414-425. DOI: 10.1177/1464419314542544.
6. Nazemizadeh, M., Bakhtiari-Nejad, F. "Size-dependent free vibration of nano/microbeams with piezo-layered actuators", ***Micro & Nano Letters***, (2015), Vol. 10, No. 2, 93-98. DOI: 10.1049/mnl.2014.0317
7. Nazemizadeh, M., Bakhtiari-Nejad, F., "A general formulation of quality factor for composite micro/nano beams in the air environment based on the nonlocal elasticity theory", ***Composite Structures***, (2015), Vol. 132, 772-783. DOI: 10.1016/j.compstruct.2015.05.070
8. Beni, Y. T. "A nonlinear electro-mechanical analysis of nanobeams based on the size-dependent piezoelectricity theory". ***Journal of Mechanics***, (2017), Vol. 33, No. 3, 289-301. DOI: 10.1017/jmech.2016.65.
9. Nazemizadeh, M., Bakhtiari-Nejad, F., Assadi, A., Shahriari, B., "Size-dependent nonlinear dynamic modeling and vibration analysis of piezo-laminated nanomechanical resonators using perturbation method", ***Archive of Applied Mechanics***, (2020), 1-14. DOI: 10.1007/s00419-020-01678-3
10. Nazemizadeh, M., Bakhtiari-Nejad, F., Assadi, A. Shahriari, B., "Nonlinear vibration of piezoelectric laminated nanobeams at higher modes based on nonlocal piezoelectric theory" ***Acta Mechanica***, (2020), Vol. 231, No. 10, 4259-4274. DOI: 10.1007/s00707-020-02736-1
11. Gurtin, M. E., Murdoch, A. I. "A continuum theory of elastic material surfaces". ***Archive for Rational Mechanics and Analysis***, (1975), Vol. 57, No. 4, 291-323. DOI: 10.1007/BF00261375
12. Jiang, L. Y., Yan, Z. "Timoshenko beam model for static bending of nanowires with surface effects", ***Physica E: Low-dimensional Systems and Nanostructures***, (2010), Vol. 42, No. 9, 2274-2279. DOI: 10.1016/j.physe.2010.05.007
13. Farshi, B., Assadi, A., Alinia-Ziazi, A. "Frequency analysis of nanotubes with consideration of surface effects", ***Applied Physics Letters***, (2010), Vol. 96. No. 9, 093105. DOI: 10.1063/1.3332579
14. Assadi, A., Nazemizadeh, M. "Size effects on stability and self-instability of non-uniform nanobeams with consideration of surface effects" ***Micro & Nano Letters***, (2020), Vol. 15, No. 12, 858-862. DOI: 10.1049/mnl.2020.0262
15. He, J., Lilley, C. M. "Surface effect on the elastic behavior of static bending nanowires". ***Nano Letters***, (2008), Vol. 8, No. 7, 1798-1802. DOI: 10.1021/nl0733233

Persian Abstract

چکیده

این مقاله به بررسی ارتعاش وابسته به ابعاد نانوتیرهای پله ای با در نظر گرفتن تئوری الاستیسیته سطحی می‌پردازد. برای این منظور، نانوتیر به عنوان تیر پله‌ای مدلسازی شده و معادلات حاکم ارتعاشی آن با در نظر گرفتن شرایط سازگاری بدست می‌آید. سپس یک حل تحلیلی برای شبیه سازی فرکانسهای طبیعی و شکل مودهای نانوتیر با در نظر گرفتن اثرات سطح، گسترش داده می‌شود. همچنین، یک رویکرد بازگشتی برای صحت سنجی نتایج و محاسبه مدول الاستیسیته وابسته به ابعاد پیشنهاد می‌شود. نتایج نشان می‌دهد که اثرات سطح و انتخاب مناسب پله‌ها، اثر چشمگیری بر فرکانس طبیعی نانوتیر غیریکنواخت دارد. همچنین، برخلاف تیرهای یکنواخت، شکل مودهای نانوتیرهای غیریکنواخت به شدت وابسته به اثرات سطح است.