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Distributed Fuzzy Adaptive Sliding Mode Formation for Nonlinear Multi-quadrotor Systems

P. Manouchehri[a], R. Ghasemi*[b], A. Toloei[c]

[a] *Department of Electrical Engineering, Islamic Azad University, Damavand Branch, Tehran, Iran*
[b] *Department of Electrical Engineering, University of Qom, Qom, Iran*
[c] *Department of Aerospace Engineering, Shahid Beheshti University, Tehran, Iran*

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This paper suggests a decentralized adaptive sliding mode formation procedure for affine nonlinear multi-quadrotor under a fixed directed topology wherever the followers are conquered by dynamical uncertainties. Compared with the previous studies which primarily concentrated on linear single-input single-output (SISO) agents or nonlinear agents with constant control gain, the proposed method is applied on affine nonlinear agents with nonlinear control gain such as the quadrotor. This designing procedure overcomes the problem of unknown nonlinear affine functions of the quadrotors. Fuzzy systems are engaged both to compensate recursively the unknown nonlinear functions and to apply the expert's knowledge on the formation technique. On-line updating the controller parameters, achieving the formation of quadrotor, boundedness of all signals involved in the closed loop of the quadrotor, and chattering reduction are the focal features of the proposed formation methodology. To demonstrate the persistency and efficiency of the methodology, a numerical example of the multi-quadrotor system is considered in this paper.

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NOMENCLATURE

Symbol	Description	Symbol	Description
G	Graph theory	φ	Roll angle
V	node	θ	Pitch angle
E	edge	ψ	Yaw angle
A	Adjecency	U1	Total upward force
D	Degree matrix	U2	Pitch torque
L	Laplacian	U3	Roll torque
x	Position along x-axis	U4	Yaw torque
y	Position along y-axis	z	Position along z-axis

1. INTRODUCTION

In recent years, the formation of multi-agent systems has received influential consideration because of its broad applications, such as UAV formation flying, wireless sensor network and formation of the quadrotor. The poor information about the agents, such as the parameters and the interaction between them, is the main difficulty to achieve the formation in this approach.

Ghasemi settled a fuzzy sliding mode adaptive controller technique for coupled nonlinear large scale systems [1]. Both the leaderless [2] and the leader-follower consensus [3] were comprehensively improved for the first-order and second-order multiagent systems (MAS). Intelligent adaptive back-stepping technique is deliberated for the nonlinear strict-feedback system [4, 5]. The impulsive methodology is used to derive the consensus protocol for the nonlinear MAS [6]. The

*Corresponding Author Institutional Email: *r.ghasemi@qom.ac.ir* (R. Ghasemi)

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leader-follower controller is designated for the single integrator with time delayed communication [7]. The high gain observer based fuzzy adaptive protocol is planned by Chen et al. [2] for the heterogeneous second order MAS without guaranteed stability.

Observer based adaptive back-stepping consensus controller is discussed for nonlinear affine MAS [8]. The fuzzy adaptive sliding mode controller is industrialized for affine nonlinear MAS [9]. Neuro-adaptive consensus procedure is suggested for the nonlinear affine strict-feedback MAS by Shen et al. [10]. In the above literature, it is assumed that the control gains of the agents are constant and equal to 1.

Fuzzy adaptive back-stepping controller is discussed for a class of affine nonlinear systems by Wang et al. [11] based on high gain observer. Neuro-adaptive protocol is suggested by Wang et al. [12] for time-delay affine nonlinear systems. Wang et al. [13], designed observer based TS fuzzy system which is planned for an industrial system. The fault tolerant output predictive controller is improved for industrial processes [14].

A distributed robust leader-follower formation controller methodology is designated by Wang et al. [15] in presence of the mobile obstacles. The sliding mode formation controller is derived by Sanchez and Fierro [16]. Defoort et al. [17] have suggested a sliding mode controller for formation of a multi-robot with limited data accessibility.

A second-order sliding mode controller is presented by Chang et al. [18] to form a prescribed geometry. Guillet et al. [19] have studied a robust adaptive controller to preserve mobile robots' formation considering parametric uncertainties. A distributed robust formation controller has been presented by Shasti et al. [20] to study space-craft flight with six-degree of freedom in the earth orbit.

A constrained model predictive controller is derived for linear time varying system via Kautz Parametrization [21]. Li et al. [22] developed a learning methodology for a nonlinear feature collection. Brustad [23] derived a curve geometry for both the reality and the virtual one. A vehicle counting system was developed based on Kalman filter by Espejel-García et al. [24]. Berdnikov and Lokhin [25] proposed a new approach to investigate the stability criteria for nonlinear system based on a fuzzy controller.

Compared with the previous studies which primarily concentrated on linear single-input single-output (SISO) agents or nonlinear agents with constant control gain, the proposed method is applied on affine nonlinear agents with nonlinear control gain such as a quadrotor. Because of 1) unknown nonlinear function of the agents, 2) applying the knowledge of the experts, 3) stability of the overall closed loop system, we emphasis on the policy of stable fuzzy adaptive sliding mode controller for a class of multi-agent affine nonlinear systems. The focal contributions of this methodology are as: 1) dynamics of the agents are all unknown affine nonlinear functions, 2) the robustness against uncertainties and external disturbances is guaranteed, 3) the boundedness of the signals in MAS is satisfied, 4) convergence of the formation error to zero is assured, and 5) stability of the overall MAS is gratified.

The remainder of the paper is prearranged as follows. Section 2 gives preliminaries. Designing fuzzy adaptive sliding mode controllers is proposed in Section 3. Section 4 presents simulation results of the proposed controller, and Section 5 concludes the paper.

2. PRELIMINARIES

This section discusses about the basics of the graph theory, Kronecker mathematics and the problem statement.

2. 1. Graph Theory An undirected graph is denoted as G = (V, E), where V = {1, 2, …, N} is a finite and non-empty set of nodes (each node denotes the follower), there are N followers for i = {1, 2,…, N}, and also $E \subset V \times V$ is a set of edges, each edge denotes an ordered pair of nodes. An edge(v_i, v_j) in an undirected graph shows that the agent i can access to the information of agent j, and it means that the agent j is the neighborhood of agent i. Let's define an adjacency matrix A = $[a_{ij}]$ associated with graph G as follows: $a_{ij} = a_{ji} > 0$ if $(v_i, v_j) \in E$, otherwise $a_{ij} = a_{ji} = 0$. Moreover, it is assumed that $a_{ii} = 0$ for i = {1, 2, …, N}. The set of neighbors of agent i is denoted by $N_i = \{j \mid (v_i, v_i) \in E\}$. Define the degree matrix as D = diag(d_1, …, d_N) with $d_i = \sum_{j\in N_i} a_{ij}$. The symmetric Laplacian matrix corresponding to the undirected graph G is defined as follows: $L = D - A$. The leader agent is represented by vertex 0, and information is exchanged between the leader and the followers that are the neighbors of the leader.

2. 2. Kronecker Mathematics Kronecker multiplication, shown with the symbol $\otimes$, is used in the context of MAS. For two matrices G and F, the operation $G \otimes F$ produces a matrix with dimensions $mp \times nq$ when matrix $G = [a_{ij}]$ with dimensions $m \times n$ and matrix F with dimensions $p \times q$ are available or given:

$$G \otimes F = \begin{bmatrix} a_{11}F & . & . & . & . & a_{1n}F \\ . & & & & & . \\ . & & & & & . \\ . & & & & & . \\ . & & & & & . \\ a_{m1}F & . & . & . & . & a_{mn}F \end{bmatrix}$$

Consider the following nonlinear canonical multi-agent system in Equation (1).

$$\begin{cases} \dot{x}_i = v_i i = 1,2,\ldots,N \\ \dot{v}_i = f_i(x_i) + g(x_i)u_i + d_i(t) \\ y_i = C_i^T \begin{bmatrix} x_i \\ v_i \end{bmatrix} \end{cases} \quad (1)$$

where x_i, v_i are the state variable of the i[th] agent. N is a number of agents and $u_i \in \mathbb{R}$ is the control input and $y_i \in \mathbb{R}$ is the output of i[th] agent and C_i is apropriate mtrice. It should be mentioned that state variables of all agents are accessible. $f_i(x_i)$ and $g(x_i)$ are nonlinear smooth and unknown functions and $d_i(t)$ is bounded external disturbance. Based on $z_i = [x_i \quad v_i]^T$, the above equation can be rewritten as Equation (2).

$$\begin{cases} \dot{z}_i = Az_i + B(f_i(x_i) + g(x_i)u_i + d_i(t)) \\ y_i = C_i^T z_i \end{cases} \quad (2)$$

where matrix $A$and vector B are defined as follows.

$$A=\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, B = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \quad (3)$$

Purposes of the controller are both to keep signals of the closed loop system bounded, and to obtain a predetermined formation for the agents.

In this section, the following assumptions are considered for the agents of the form (2):

Assumption 1: without loss of generality, it is assumed that smooth functions $g(x_i) \neq 0$ and $f_i(x_i)$ are continuous. In addition, without loss of generality, it is assumed that $g(x_i) > g_{min}$ and $\frac{dg(x_i)}{dt} > g'_{min}$ are satisfied which can be rewritten for $g(x_i) < 0$.

Assumption 2: external disturbances given in Equation (2) satisfies the inequality mentioned in (4).

$$\|d_i(t)\|_\infty \leq d_{max} \quad (4)$$

d_{max} is a known value.

Assumption 3: the agents given in Equation (2) are controllable and observable.

Assumption 4: the graph of the system is undirected with spanning tree.

The closed form of the multi-agent system mentioned in Equation (2) can be written in the form of Equation (5).

$$\begin{cases} \dot{Z} = (I_N \otimes A)Z + (I_N \otimes B)\begin{pmatrix} f_i(x_i) + g(x_i)u_i \\ +d_i(t) \end{pmatrix} \\ Y = (I_N \otimes C_i^T)Z \end{cases} \quad (5)$$

In the above equation, $Y = [y_1, \quad y_2, \quad \cdots \quad y_N]^T \in \mathbb{R}^NandZ = [z_1, \quad z_2, \quad \cdots \quad z_N]^T \in \mathbb{R}^{n.N}$are the output and the state vector and of the general multi-agent system, respectively. $I_N$depicts for Identity matrice with dimention N.

In order to control the mentioned multi-agent system in Equation (2), a formation error of the ith agent can be described as in Equation (6).

$$e_i = k_p(x_0 - x_i - \Delta_i) + \gamma k_v(v_0 - v_i) + c\sum_{j\in N_i} a_{ij}\big((x_j - \Delta_j - x_i + \Delta_i) - \gamma(v_j - v_i)\big) \quad (6)$$

where x_0 is the position of the leader, v_0 is the velocity of the leader, Δ_i is the difference position of the ith agent with the leader and N_i is the neighborhood of the ith agent.

After some mathematical manipulation, the closed form of the multi-agent system in Equation (6) can be written in overall form as Equation (7).

$$E = (I_N \otimes k_Z - cL \otimes BK)\tilde{Z} \quad (7)$$

where $E = [e_1 \quad e_2 \quad \cdots \quad e_N]^T$ is the error vector of the multi-agent system, $k_Z = [k_p \quad k_v]^T$, and L is the Laplacian matrix and $\tilde{Z}$ is defined as follows.

$$\tilde{Z}_i = \begin{bmatrix} x_0 - x_i - \Delta_i \\ v_0 - v_i \end{bmatrix} \quad (8)$$

By defining $Z_{i0} = [x_0 \quad v_0]^T$ and $Z_{id} = [\Delta_i \quad 0]^T$, the above equation can be written in the form of Equation (9).

$$\tilde{Z}_i = Z_{i0} - Z_i + Z_{id} \quad (9)$$

Consider the dynamics of Z_0 and Z_d as follows.

$$\begin{cases} \dot{Z}_d = (I_N \otimes A)Z_d + (I_N \otimes B')r_d \\ Y_d = (I_N \otimes C_i^T)Z_d \end{cases} \quad \begin{cases} \dot{Z}_0 = (I_N \otimes A)Z_0 \\ Y_0 = (I_N \otimes C_i^T)Z_0 \end{cases} \quad (10)$$

The closed form of Equation (9) is as:

$$\dot{\tilde{Z}} = \dot{Z}_0 - \dot{Z}_i + \dot{Z}_d \quad (11)$$

Using Equations (10) and (5), Equation (11) can be designated in Equation (12).

$$\dot{\tilde{Z}} = (I_N \otimes A)Z_0 - (I_N \otimes A)Z + (I_N \otimes A)Z_d + (I_N \otimes B)\begin{pmatrix} f_i(x_i) \\ +g(x_i)u_i + d_i(t) \end{pmatrix} + (I_N \otimes B')r_d \quad (12)$$

Considering the formation error mentioned in Equation (7), the dynamics of the formation error can be derived in Equation (13).

$$\dot{E} = (I_N \otimes k_Z - cL \otimes BK)(I_N \otimes A)\tilde{Z} + (I_N \otimes k_Z - cL \otimes BK)(I_N \otimes B)\begin{pmatrix} f_i(x_i) \\ +g(x_i)u_i \\ +d_i(t) \end{pmatrix} + (I_N \otimes k_Z - cL \otimes BK)(I_N \otimes B')r_d \quad (13)$$

Theorem 1: Assume that the function $f_i(x_i)$ and $g(x_i)$ are both continuously differentiable for any arbitariry $(\boldsymbol{x}_i, u_i) \in \mathbb{R}^n \times \mathbb{R}$ and there exists a constant value g_{min} which satisfies the assumption 1. Then, there exists a continuous function like $u_i{}^* = u_i(\boldsymbol{x}_i)$ such that $f_i(x_i) + g(x_i)\, u_i^*(\boldsymbol{x}_i) = 0$.

Considering Theorem 1, it is clear that equation $f_i(x_i) + g(x_i)u_i - v_i = 0$ can be solved locally with respect to control input u_i for any arbitrary $(\underline{x}_i, v_i)$; therefore, an ideal control input $u_i^*(\underline{x}_i, v_i)$ for any arbitrary $(\underline{x}_i, v_i) \in \mathbb{R}^n \times \mathbb{R}$, should satisfy the following equation.

$$f_i(x_i) + g(x_i)u_i^*(\underline{x}_i, v_i) - v_i = 0 \tag{14}$$

Using the mean value theorem, Equation (15) can be attained.

$$f_i(x_i) + g(x_i)u_i = f_i(x_i) + g(x_i)u_i^*(\underline{x}_i, v_i) + g(x_i)(u_i - u_i^*(\underline{x}_i, v_i)) = f_i(x_i) + g(x_i)u_i^*(\underline{x}_i, v_i) + g(x_i)e_u \tag{15}$$

Considering Equation (15), Equation (13) can be written as (16).

$$\dot{E} = (I_N \otimes k_Z - cL \otimes BK)(I_N \otimes A)\tilde{Z} + (I_N \otimes k_Z - cL \otimes BK)(I_N \otimes B) * (f_i(x_i) + g(x_i)u_i^* + g(x_i)e_u + d_i(t) + v_i - v_i) + (I_N \otimes k_Z - cL \otimes BK)(I_N \otimes B^{'})r_d \tag{16}$$

In order to facilitate the formulation, the following variables are defined.

$$\begin{aligned} \xi &= (I_N \otimes k_Z - cL \otimes BK) \\ A^{'} &= (I_N \otimes A) \\ B^{'} &= (I_N \otimes B) \\ B^{''} &= (I_N \otimes B^{'}) \end{aligned} \tag{17}$$

using Equations (13) and (17), Equation (16) can be rewritten as:

$$\dot{E} = \xi A^{'}\tilde{Z} + \xi B^{'}(g(x_i)e_u + d_i(t) + v_i) + \xi B^{''}r_d \tag{18}$$

The ideal control input of Equation (14) is suggested below.

$$u_i^* = f_i(\underline{\Omega}_i) \tag{19}$$

In the above equation, $\underline{\Omega}_i = [x_i, v_i]^T$ and $f_i(\underline{\Omega}_i)$ is also approximated using fuzzy systems as $f_i(\underline{\Omega}_i) = \theta^* w_i(\underline{\Omega}) + \varepsilon$. where θ^* and $w_i(\underline{\Omega}_i)$ are fuzzy parameters and basis function; $\varepsilon$is also estimation error which satisfies $|\varepsilon| \le \varepsilon_{max}$. Parameter θ^* is obtained through the following optimization.

$$\theta^* = \underset{\theta}{argmin}\left[sup\left|\theta^T w_i(\underline{z}_i) - f_i(\underline{z}_i)\right|\right] \tag{20}$$

where θ is an estimation of θ^* .

However, the implicit function theory only guarantees the existence of the ideal controller $u_i{}^*$, and does not recommend a technique for constructing solution even if the dynamics of the system are well known and $u_i$is used as an estimation of u_i^*.

3. FUZZY ADAPTIVE SLIDING MODE CONTROLLER DESIGN

In previous section, the existence of an ideal controller for achieving control objectives is presented. We show how to derive a fuzzy system to adaptively approximate the unknown ideal controller.

In order to design the sliding mode controller for the system described in Equation (18), the sliding surface is proposed in Equation (21).

$$s = \Lambda^T E \tag{21}$$

where $\Lambda = [\lambda_1 \quad \lambda_2 \quad \cdots \quad \lambda_N]^T$. Considering the sliding surface given in the above equation, the control input of the system in Equation (19) can be described as in Equation (22).

$$u = \theta^T w_i(\underline{z}_i) + u_{ieq} + u_{ir} \tag{22}$$

where u_{eq} is equivalent input and u_r is the input to reach the sliding surface; where u_{eq} and u_r can be described as Equation (23).

$$\begin{aligned} u_{eq} &= -\hat{\theta}^T w_i(\underline{z}_i) - \hat{\varepsilon} - \left(\Lambda^T \xi B^{'}\right)^+ \left\|\Lambda^T \xi B^{'}\right\| \hat{v}_i \, sign(s) \\ u_r &= -k^{'} sign(s) \end{aligned} \tag{23}$$

The adaptive updating of the controller parameters are presented in (24).

$$\begin{aligned} \dot{\theta}_i &= \gamma_1 w_i(\underline{z}_i) B^{'T} \xi^T \Lambda s \\ \dot{\hat{\varepsilon}} &= -\gamma_1 s \Lambda^T \xi B^{'} \\ \dot{\hat{v}}_i &\frac{\gamma_3 |s|}{f_{min}|\Lambda^T \xi B^{'}|} \end{aligned} \tag{24}$$

where $\gamma_3 > 0, \gamma_2 > 0, \gamma_1 > 0$ are positive constant parameters.

Theorem: Consider the multi-agent nonlinear system mentioned in Equation (2) which satisfies Assumptions 1 and 3, the graph of the system satisfies Assumption 4 and the external disturbances gratify Assumption 2. Consider the sliding surface as cited in Equation (21), the controller input mentioned in Equation (22) with equivalent and reaching terms in Equation (23) and the updating terms in Equation (24) makes the dynamics of the formation error in Equation (18) uniformly ultimately bounded. Furthermore all closed loop system signals also remain bounded.

Proof: In order to analyze the Lyapunov stability of the closed loop system, the following Lyapunov function is candidate.

$$V = \frac{1}{2g(x_i)} s^2 + \frac{1}{2\gamma_1} \tilde{\theta}^T \tilde{\theta} + \frac{1}{2\gamma_2} \tilde{\varepsilon}^T \tilde{\varepsilon} + \frac{1}{2\gamma_3} \tilde{v}_i{}^T \tilde{v}_i \tag{25}$$

Taking the time derivative of the Lyapunov function as with respect to time is derived in (26).

$$\dot{V} = \frac{1}{g} s\dot{s} - \frac{\dot{g}}{g^2} s^2 + \frac{1}{2\gamma_1} \dot{\hat{\theta}}^T \tilde{\theta} + \frac{1}{2\gamma_2} \dot{\hat{\varepsilon}}^T \tilde{\varepsilon} + \frac{1}{2\gamma_3} \dot{\hat{v}}_i{}^T \tilde{v}_i \le -\eta |s| \tag{26}$$

Using Equations (18) and (21), the above equation can be rewritten as Equation (27).

$$\dot{V} = \frac{1}{g}E^T\Lambda\Lambda^T(\xi A'\tilde{Z} + \xi B'(g(x_i)e_u + d_i(t) + v_i) + \xi B''r_d) - \frac{\dot{g}}{g^2}s^2 + \frac{1}{2\gamma_1}\dot{\hat{\theta}}^T\tilde{\theta} + \frac{1}{2\gamma_2}\dot{\hat{\varepsilon}}^T\tilde{\varepsilon} + \frac{1}{2\gamma_3}\dot{\hat{v}}_i^{\,T}\tilde{v}_i \le -\eta|s| \quad (27)$$

After some mathematical manipulations and the updating laws in Equation (24) can be designated as follows

$$\dot{V} \le -\left\{\begin{array}{l}\left(\begin{array}{l}\frac{\dot{g}}{g^2}\lambda(\xi^T\Lambda\Lambda^T\xi)_{min} \\ -\frac{1}{g_{min}(\xi^T\Lambda\Lambda^T\xi A')_{max}}()\|\tilde{Z}\|\|\tilde{Z}\| \\ -\frac{1}{g_{min}\|\xi^T\Lambda\Lambda^T\xi B''\|\|r_d\|}\end{array}\right) \\ -E^T\Lambda\Lambda^T\xi B'ksign(s) \\ +\frac{1}{g_{min}|s|\|\Lambda^T\xi B'\|\|\tilde{Z}\|^2|s|_{max}}\end{array}\right\} \quad (28)$$

By a proper selection of k in the above equation, the closed loop system remains ultimate uniform bounded in the closed area Ω.

$$\Omega = \left\{\tilde{Z}\left|\|\tilde{Z}\| \ge \frac{1}{g_{\min}}\frac{\left\|\xi^T\Lambda\Lambda^T\xi B''\right\|\|r_d\|}{\left(\begin{array}{l}\frac{\dot{g}}{g_{\min}{}^2}\lambda_{\min}\left(\xi^T\Lambda\Lambda^T\xi\right) \\ -\frac{1}{g_{\min}}\lambda_{\max}\left(\xi^T\Lambda\Lambda^T\xi A'\right)\end{array}\right)}\right.\right\} \quad (29)$$

The proof is complete.

One of the main disadvantages of this method is unwanted chattering of the control input; in order to eliminate these oscillations and to stabilize the agents that tracking error converges to the neighborhood of zero.

The design of decentralized sliding mode super twisting controller for the non-affine nonlinear multi-agent system is studied in the next section.

4. SIMULATION RESULTS

In order to investigate the proposed controller, the proposed methodology is applied on the multi-quadrotor. The E-frame and the B-frame of the quadrotor is shown in Figure 1.

The topology of the MAS is proposed in Figure 2. As shown in Figure 2, the system has three agents.

The quadrotor has two coordinate systems, as the Earth Fixed Frame (E) and the Body Fixed Frame (B). The roll, pitch and yaw angles, angular velocities of the quadrotor are stated in earth fixed frame (E), while the linear accelerations are presented in body fixed frame (B). The nonlinear dynamics of the quadrotor mentioned in Figure 1 is proposed as bellows:

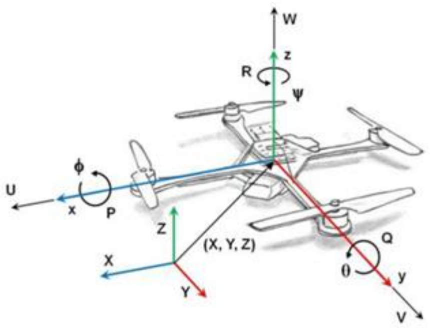

Figure 1. The inertial, body and quadrotor frames of reference

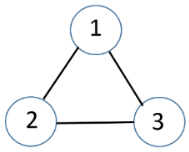

Figure 2. The communication graph among the agents

$$\begin{cases}\ddot{X} = (\sin\Psi\sin\phi + \cos\Psi\sin\theta\cos\phi)\frac{U_1}{m} \\ \ddot{Y} = (-\cos\Psi\sin\phi + \sin\Psi\sin\theta\cos\phi)\frac{U_1}{m} \\ \ddot{Z} = -g + (\cos\theta\cos\phi)\frac{U_1}{m} \\ \ddot{\varphi} = \frac{I_{YY}-I_{ZZ}}{I_{XX}}\dot{\theta}\,\dot{\psi} - \frac{J_{Tp}}{I_{XX}}\dot{\theta}\,\Omega + \frac{U_2}{I_{XX}} \\ \ddot{\theta} = \frac{I_{ZZ}-I_{XX}}{I_{YY}}\dot{\varphi}\,\dot{\psi} + \frac{J_{Tp}}{I_{YY}}\dot{\varphi}\,\Omega + \frac{U_3}{I_{YY}} \\ \ddot{\psi} = \frac{I_{XX}-I_{YY}}{I_{ZZ}}\dot{\varphi}\,\dot{\theta} + \frac{U_4}{I_{ZZ}}\end{cases} \quad (33)$$

$[x, y, z, \varphi, \theta, \psi]$ is the vector of the linear and angular position of the quadrotor in the earth frame and $[\dot{x}, \dot{y}, \dot{z}, \dot{\varphi}, \dot{\theta}, \dot{\psi}]$ shows the vector containing the linear and angular velocities in the body frame. $[U_1 \quad U_2 \quad U_3 \quad U_4]$is the control input vector. The parameters for simulation are shown in Table 1.

In this section, the proposed methodology is applied on the multi-quadrotor. The quadrotors should form a triangle shape. The path formation of the leader and the followers is shown in Figure 3. The control inputs of the first agent is shown in Figure 4. The control inputs of the second agent is demonstrated in Figure 5.

TABLE 1. the value of the parameters

Parameters	value	Parameters	value
g	9.81	J_{TP}	6×10^{-5}
m	3.2	l	0.2
I_{xx}	11×10^{-2}	I_{yy}	19×10^{-2}
I_{zz}	1.3×10^{-2}		

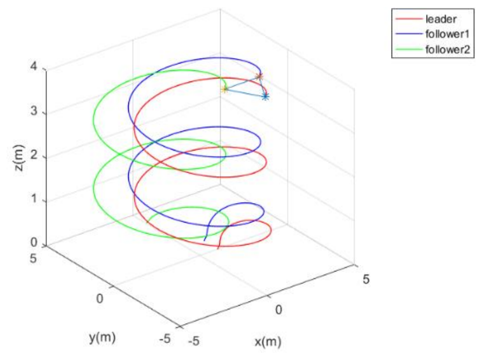

Figure 3. The path formation of the MAS in x-y-z plane

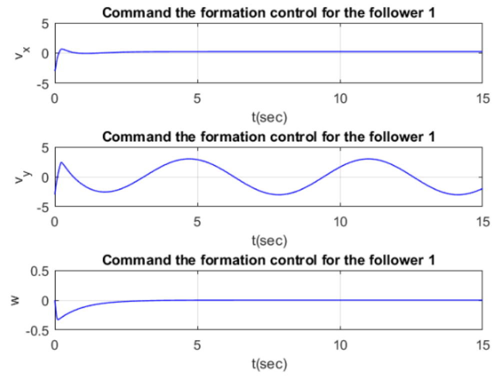

Figure 4. Control inputs of the first agent

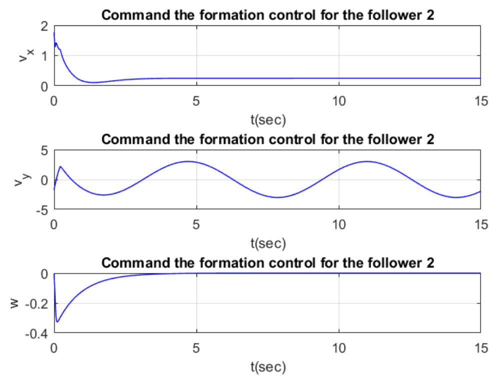

Figure 5. The control inputs of the second agent

The convergence of the sliding surface of the followers to zero is clear in the above figures. The position of the followers and the leader is demonstrated in Figure 6. The angles of the followers and the leader are verified in Figure 7.

States of the agents are convergenced to zero, and formation of the MAS are all illustrated based on the above figures.

As shown in above figures, the formation achievement, convergence of the sliding surface to zero, convergence of the formation error to zero, and stability of the closed loop system are all feasible.

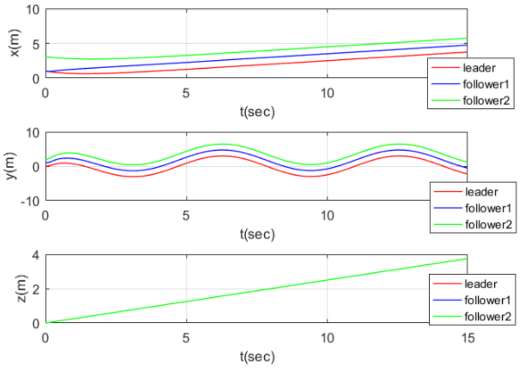

Figure 6. The position of the leader and the followers

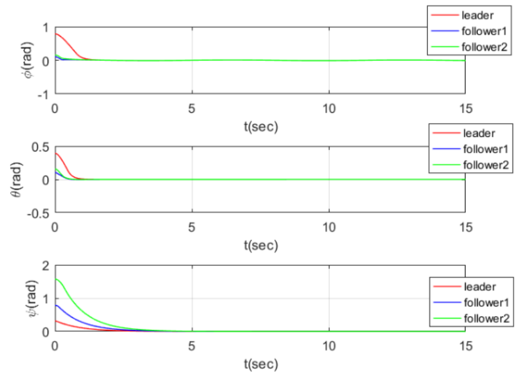

Figure 7. The angle of the sliding surface of the second follower

5. CONCLUSION

In this paper, a decentralized formation design for a class of affine nonlinear multi agent systems is investigated under the fixed directed topology. The proposed method in this research discussed on an affine nonlinear quadrotor with a nonlinear control gain. It is assumed that the functions of the agent are all unknown. To challenge both the uncertainties and the unknown function of the agent, a fuzzy adaptive sliding mode controller was derived for this class of MAS. Among the advantages of the method presented in this paper, the followings can be mentioned:

1) Ultimate uniform boundedness of formation error
2) Robustness against approximation error and bounded external disturbances, and
3) Boundedness of internal signals of the closed loop system.

The proposed methodology can be applied on a wide class of nonlinear affine system with unknown function of the agents. The simulation results show promising performance of the proposed methodology. An extension of this method to nonlinear non-affine multi-agent systems and practical implementations of our approach can be considered in future studies.

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Persian Abstract

چکیده

در این مقاله طراحی کنترل‌کننده آرایش‌بندی تطبیقی مد لغزشی غیرمتمرکز برای سیستم چند عاملی غیرخطی افاین چند کوادکوپتری تحت توپولوژی ثابت بی‌جهت پیشنهاد می‌شود درحالی که عامل‌های پیرو داری عدم قطعیت‌های دینامیکی می‌باشد. با توجه به بررسی مطالعات قبلی که بر روی سیستم‌های چند عاملی با عامل‌های خطی تک ورودی و تک خروجی و یا سیستم‌های غیرخطی با ضریب کنترلی ثابت متمرکز شده بودند، روش ارئه شده بر روی سیستم‌های چند عاملی با عامل‌های غیرخطی با ضریب کنترلی غیرخطی مانند کوادکوپتر متمرکز شده است. پروسه طراحی پیشنهاد شده مشکل نامعین بودن توابع غیرخطی کوادکوپترها را حل می‌کند. سیستم‌های فازی برای اعمال اطلاعات فرد خبره و یادگیری توابع غیرخطی بکار می‌رود. به روزرسانی آنلاین پارامترهای کنترل‌کننده، آرایش‌بندی کوادکوپترها، محدود بودن تمامی سیگنال‌های حلقه بسته و کاهش چترینگ از ویژگی‌های اساسی روش ارائه شده می‌باشد. کارایی روش مطروحه، یک نمونه عددی از سیستم کوادکوپتر در این مقاله در نظر گرفته شده است.