# A New Approach Generating Robust and Stable Schedules in m-Machine Flow Shop Scheduling Problems: A Case Study 

Z. Abtahia, R. Sahraeian*a, D. Rahmanib ${ }^{\text {b }}$<br>${ }^{a}$ Department of Industrial Engineering, College of Engineering, Shahed University, Tehran, Iran<br>${ }^{b}$ Department of Industrial Engineering, K.N. Toosi University of Technology, Tehran, Iran

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#### Abstract

$A B S T R A C T$

This paper considers a scheduling problem with uncertain processing times and machine breakdowns in industriall/office workplaces and solves it via a novel robust optimization method. In the traditional robust optimization, the solution robustness is maintained only for a specific set of scenarios, which may worsen the situation for new scenarios. Thus, a two-stage predictive algorithm is proposed to efficiently handle the uncertainties and find robust and stable solutions. The first stage creates robust solutions and ensures their stability in the new scenarios. The second stage proposes a novel stability measure to proactively offset the effects of the machine breakdowns of the former stage. Moreover, a tri-component measure based on efficiency, robustness, and stability is proposed which aims to create a realistic schedule to satisfy the customers, manufacturers, and the staff. To meet the customer's requirements, the robustness measure is defined based on the tardiness and the delivery dates of jobs. Finally, the proposed algorithm is applied to a case study, and the findings are compared with the empirical data. The results emphasize the superiority of the proposed technique in satisfying the customers, staff, and increasing the profitability and accountability of the company.


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## 1. INTRODUCTION

The shop scheduling literature includes a large number of papers dealing with the classical permutation flowshop scheduling (FSS) problem, in which a set of jobs must be processed on the series of machines in the same order. In the majority of these researches, no disruption is assumed in the production systems, and the classical performance measures are usually optimized under deterministic assumptions [1]. However, in practice, many disruptions may occur e.g. machine breakdowns, uncertain processing times, the arrival of new jobs, etc. Two sources of uncertainty are considered in this paper; the unexpected breakdowns of the first machine and the uncertain processing times of all the jobs. Due to the uncertainties, it is expected to be some deviations between the real schedule (i.e. the schedule that is carried out on the shop floor), and the initial one (i.e. the schedule planned at the beginning of the scheduling horizon). The comparison of the real schedule with the
initial one can be carried out via two measures; robustness and stability. Robustness (quality robustness) refers to the deviation between the performance criterions of the real and the initial schedules [2]. Stability (solution robustness) is concerned with the deviation between the solutions of the real and the initial schedules [3]. Alongside efficiency measure optimization, the uncertainty oriented FSS studies aimed at optimization of robustness, stability, or both. Confronting the deviations resulted from the uncertainties can be performed with predictive (proactive), reactive, or Predictive-reactive strategies [4]. In the predictive scheduling strategy, contrary to the reactive one, future uncertainties are considered upon the setting of the initial schedule [5]. To proactively generate a robust solution in counter with the uncertain processing times, a robust optimization method has been implemented in over 30 percent of the robustness oriented studies. According to literature [6] "adding or removing a scenario from the set of scenarios may lead

[^0]to defining very different solutions as robust". In other words, the schedule robustness is guaranteed only for the set of considered scenarios and may fail when dealing with a new scenario. Therefore it is crucial to identify a robust and stable solution in the face of new scenarios. In this paper, a predictive two-stage algorithm is proposed that in its first stage, a robust schedule is produced and then evaluated for its stability against a new job processing times' scenario. In almost all of the robustness oriented studies, exposure to machine failure has been done with the reactive approach or in the reactive phase of the hybrid approaches. In the reactive scheduling methods (e.g. rescheduling), especially in the case of big problems, it takes a long time to deal with the uncertain conditions. Predictive scheduling methods can overcome this problem by proactively preparing for any possible occurrence of such uncertain conditions [6]. Ergo, in this paper, exposer to machine failure has been done with the predictive approach. Idle time insertion has been a common strategy to handle the effect of breakdown disruptions (e.g. see $[4,5]$ ). This method faces two challenges, which are finding the optimal amount and the appropriate position to insert the buffer times [7]. In the second stage of our proposed algorithm, a linear optimization method is employed to overcome these challenges. A new surrogate measure is proposed to enhance the solution robustness by interfering with the probability of machine breakdown during the job processing times. This linear programming model simultaneously optimizes the proposed surrogate measure of solution robustness, and quality robustness to determine the proper position and the amount of buffer times. One of the essential prerequisites of practical scheduling is to meet customer, manufacturer and staff requirements simultaneously. This is addressed in this paper by defining a tri-component performance measure of robustness, efficiency, and stability. Due to the important role of customer-agreed delivery deadlines in manufacturing systems, tardiness-based performance measures have been the focus of attention. However, tardiness is considered as the performance measure in only 5 percent of the related studies [8]. In this paper, the total real tardiness of jobs is considered a robustness measure. Finally, a case study is presented to compare the performance of the proposed method with the empirical method used in the company based on a tri-criteria objective measure of robustness, stability, and efficiency. Thus, the main contributions of this paper are as follows:

- An m-machine FSS problem under two sources of uncertainty is considered.
- A triple scale is proposed to simultaneously meet the needs of the customer, producer, and worker.
- An appropriate predictive method to handle the uncertainty of the job processing times and machine breakdown is proposed.
- The stability of the robust solution in the face of a new job processing time scenario is ensured.
- An innovative surrogate measure is proposed to enhance solution robustness.
A linear programming model is proposed to determine the proper position and the amount of the buffer times.

This paper is organized as follows. In section 2, the related literature is reviewed. A brief description of the robust optimization approach and the proposed predictive algorithm are presented in section 3 . The case study and the computational results are provided in section 4. Finally, the conclusion and the suggestions for future researches are discussed in section 5 .

## 2. LITERATURE REVIEW

In this paper uncertainty oriented flow shop scheduling problem studies in 2000-2019 are reviewed focusing on the source of uncertainty, The purpose of scheduling, and the approach in counter with uncertainty and the results are summarized in this section. "Flow shop" is considered as a keyword in the title of studies, and "robustness/ robust", "stability/ stable", "uncertainty/ uncertain job processing time/ machine breakdown" are considered as keywords in the body of the studies.

Job processing times are the most frequently studied parameter subjected to uncertainty, while the second most frequently studied parameter is the machine breakdown. In only 19 percent of related studies, two sources of uncertainty are considered. Meanwhile, the share of studies considered job processing times uncertainty and machine breakdowns, has been 10 percent [8]. Interval, probabilistic, and scenario-based description are considered to represent the uncertainty of job processing times [8]. Interval description is applied to state the uncertainty of job processing times where there is insufficient information about the probability distribution of the data but the lower or upper bound (or both) is available [9]. the probabilistic description is used when sufficient data is already collected to estimate the probability (e.g. see [10, 11]. The scenario-based description is used when different scenarios exist for the data [1]. Interval description job processing times studies have focused on the efficiency scale (very often Cmax is considered as an efficiency measure (e.g. see [12-14]). But most recently, Cmax and total completion times are respectively considered as the measures of robustness and efficiency discussed by Liao and Fu [9]. GA is proposed to solve the flow shop scheduling problem with interval job processing times description. Job processing times generally follows from the normal distribution in the related studies with probabilistic job processing times description. Besides, robustness is measured with the
probability of not exceeding from the specified threshold (e.g. [10, 11]). Considering an identical threshold is one of the gaps in this type of researches. Approximately in all of the scenario-based robustness oriented flow shop schedule studies, dual or multiple performance measures of efficiency, robustness and/ or stability were used. In all of these studies, the basis for constructing robustness and stability measures has been Cmax (e. g. see [1, 9, 15, 16]). According to the literature [17], failure is considered for only one machine even in recent papers. However, in some related studies [1], the failure is considered for more than one machine. Also, the repair time can be varied, which is not the case in most related articles [18]. In the face of machine breakdown disruption, predictive $[2,4,6]$ or reactive [1] rescheduling methods have been addressed in the literature. The non-idle time insertion methods such as time-consuming simulation-based methods proposed in some studies in the face of machine breakdown disruption [3, 7]. Buffer time insertion method has been a common predictive strategy to counter the effect of breakdowns [4], but it faces two problems; how to find the optimal amount and the appropriate position to insert the buffer times. Briskorn et al. [18] analyzed the allocation of idle times in a single-machine environment. It can be gathered from the literature that in most of the related studies:

- One source of uncertainty has been considered.
- Efficiency, stability, and robustness were often considered separately except in some recent papers [1, 15].
- The stability of the robust solution in the face of new scenarios has not been investigated.
- None of the measures of robustness and efficiency are defined based on the tardiness of jobs, which includes customer-agreed delivery deadlines.
Following Rahmani [1], Mulvey et al. [19] method is applied in this paper to produce a robust schedule in uncertainty. But in contrast with most of the robustness oriented studies such as Rahmani [1], a predictive approach is proposed to adjust the effect of machine failures. Also, in our proposed algorithm the stability of the robust solution is maintained in the face of new scenarios. Also, contrary to the usual injection buffer time methods [4], by applying the proposed linear programming model the quality of the robust solution is maintained in addition to stability enhancement.


## 3. PROBLEM DEFINITION AND SOLUTION METHOD

In this paper, the uncertain job processing time and the random breakdowns of the first machine are regarded as systems disruption in an m-machine FSS problem. The time between two consecutive failures follows an
exponential distribution with the rate of, and at most one failure is expected on the first machine in each interval $\left(\frac{1}{\theta}\right)$. After each random breakdown, the minimal repair is carried out to restore the first machine to its operating condition, which does not affect the age and breakdown parameter of the machine. Following Chaari et al. [20], $U \in\left[P_{I}-\alpha P_{I}, P_{I}+\alpha P_{I}\right]$ applied to generate new scenarios, which $P_{I}$ is the initial scenario, and $\alpha \in[0,1]$ is the degree of uncertainty of job processing times. $\alpha=$ $\pm 0.1$ is considered as the low, $\alpha= \pm 0.5$ as the medium, and $\alpha= \pm 1$ as a high degree of uncertainty.

When there is significant uncertainty in job processing times that cannot be approximated with a probability distribution, discrete scenarios offer a good representation of uncertainty $[15,18]$. In this situation, the classical FSS model suffers from some weaknesses. The schedules which are optimal concerning the initial scenario might be substantially infeasible or yield poor performance when evaluated relative to the actual job processing times. That is, with each new scenario occurrence, a new optimal schedule is required, which leads to staff confusion and system instability. Many approaches called robust seeks for solutions that optimize a global performance instead of seeking for solutions that optimize a local performance. But the schedule robustness is guaranteed only for the set of considered scenarios and may fail when an encounter a new scenario. Therefore, it is crucial to identify a robust and stable solution in the face of new scenarios.

To handle such a problem, a two-stage predictive algorithm is developed. In the first stage, the processing time uncertainty is regarded as the only source of uncertainty, and the robust partial schedule is determined by applying the robust optimization method. Then the stability of the robust solution is ensured in counter with the new scenarios. In the second stage, the effect of the breakdowns is proactively handled and the appropriate amounts of the buffer times are determined to compose a completely robust and stable schedule. The initial scenario, the number of iterations $(I)$, are the inputs of the algorithm.

## 3. 1. The Proposed Predictive Algorithm

First stage. Robust \& stable partial solution generation.
Step1.1. Initialization:
$I \leftarrow 1$.
$\lambda_{I} \leftarrow\{$ initial scenario $\}$.
$\Omega_{I} \leftarrow \lambda_{I}$.
Generate the robust partial solution from (1)-(11) for $\Omega_{I}$ via the optimization software IBM CPLEX 12.6.
Step1.2.
$I \leftarrow I+1$.
$\Omega_{I} \leftarrow \Omega_{I-1} \cup \lambda_{I}$.
Step1.3. Generate robust partial solution from (1)-(11) for $\Omega_{I}$ via the optimization software IBM CPLEX 12.6.
Step1.4. Evaluation of the stability of the robust solution.

Step 1.4.1. Calculate Solution robustness.
$\sum_{j=1}^{n} T_{j}^{\lambda}$, which is obtained from the step 1.3 , is calculated for $\Omega_{I}, \Omega_{I-1}$.
Step 1.4.2. Calculate the structural robustness.
The completion time of each job in the robust solution is calculated for $\Omega_{I}, \Omega_{I-1}$.
Step 1.5. The stop criteria checking.
If the difference between solution robustness or structural robustness be less than the predefined threshold, or the number of iteration exceeds the predefined number of iteration, go to the second stage, else go to step1.2.
Second stage. Robust \& stable solution generation.
Main Loop: for $s=1 \ldots N_{s}$
Step 2.1. The predictive schedule generation.
Linear programming model (Equations (15)-(27)) is solved via CPLEX 12.6 to obtain the adequate idle times for every job on the first machine per scenario. Then the partial schedule of step 1 is modified to include the adequate idle times.
Step 2.2. Random breakdown generation.
It is assumed that $\lambda^{\prime \prime} \in \Omega$ is a real scenario. Random breakdowns are generated according to $\lambda^{\prime \prime}$.
Step 2.3. Robustness, stability, and efficiency calculation. To obtain the actual schedule, the partial robust schedule (from step1) is shifted to the right once a breakdown occurs. The robustness and stability measures are calculated via Equations (28) and (29), respectively.

## 3. 2. Partial Solution Generation

This stage
deals with job processing time uncertainty via a robust partial solution generation model. Indices, parameters, variables, and the robust optimization model of the $m$-machine FSS problem are as follows.

## Indices

$j \quad$ index for jobs $\{1,2, \ldots, n\}$
$k \quad$ index for position $\{1,2, \ldots, n\}$
$i \quad$ index for machine $\{1,2, \ldots, m\}$
$\lambda \quad$ indices for scenarios $\Omega=\{1,2, \ldots, \lambda, \ldots, N\}$
$\lambda^{\prime} \quad$ indices for scenarios $\Omega=\left\{1,2, \ldots, \lambda^{\prime}, \ldots, N\right\}$

## Parameters

$t_{i j}^{\lambda} \quad$ the processing time of job $j$ on machine $i$ under scenario $\lambda$
$P^{\lambda} \quad$ the occurrence probability of scenario $\lambda$

## Variables

$C_{i k}^{\lambda} \quad$ the completion time of the job in the $k^{t h}$ position on $C_{i k} \quad$ machine $i$ under scenario $\lambda$
$T_{k} \quad$ the tardiness of the job in the $k^{t h}$ position
$x_{j k} \quad 1$ if job $J_{j}$ is in the $k^{t h}$ position in the sequence; 0 otherwise
$\theta^{\lambda}$ the non-negative, linearizing variable of the objective function.

$$
\begin{align*}
& \min \sum_{\lambda \in \Omega} P^{\lambda} \sum_{k=1}^{n} T_{k}^{\lambda}+\sum_{\lambda \in \Omega} P^{\lambda} \mid \sum_{k=1}^{n} T_{k}^{\lambda}-  \tag{1}\\
& \sum_{\lambda^{\prime} \in \Omega} P^{\lambda^{\prime}} \sum_{k=1}^{n} T_{k}^{\lambda^{\prime}} \mid \\
& \text { s.t. } \sum_{k=1}^{n} x_{j k}=1, \quad \forall j \in\{1,2, \ldots, n\}  \tag{2}\\
& \sum_{j=1}^{n} x_{j k}=1, \quad \forall k \in\{1,2, \ldots, n\} \tag{3}
\end{align*}
$$

$$
\begin{align*}
& C_{11}^{\lambda}=\sum_{j=1}^{n} t_{1 j}^{\lambda} x_{j 1}  \tag{4}\\
& C_{1 k}^{\lambda}=C_{1 k-1}^{\lambda}+\sum_{j=1}^{n} t_{1 j}^{\lambda} x_{j k}, \forall k \in\{2, \ldots, n\}  \tag{5}\\
& C_{i 1}^{\lambda}=C_{i-11}^{\lambda}+\sum_{j=1}^{n} t_{i j}^{\lambda} x_{j 1}, \forall i \in\{2, \ldots, m\}  \tag{6}\\
& \begin{array}{c}
C_{i k}^{\lambda} \geq C_{i-1 k}^{\lambda}+\sum_{j=1}^{n} t_{i j}^{\lambda} x_{j k}, \forall i \in\{2, \ldots, m\}, \\
\forall k \in\{2, \ldots, n\}
\end{array}  \tag{7}\\
& \begin{array}{c}
C_{i k}^{\lambda} \geq C_{i k-1}^{\lambda}+\sum_{j=1}^{n} t_{i j}^{\lambda} x_{j k}, \forall i \in\{2, \ldots, m\}, \\
\forall k \in\{2, \ldots, n\}
\end{array} \\
& T_{k}^{\lambda} \geq C_{m k}^{\lambda}-\sum_{j=1}^{n} d_{j}^{\lambda} x_{j k}, \forall k \in\{1, \ldots, n\} \tag{8}
\end{align*}
$$

Constraints (2) to (9) guarantee the feasibility of the partial robust schedule. These scenario-based constraints are necessary to calculate the total tardiness in an $m$ machine FSS problem. Constraints (2) and (3) respectively ensure that each particular job is exactly assigned to one position and that each position is exactly assigned to one job. Constraint (4) calculates the completion time of the job in the first position on the first machine. Constraint (5) calculates the completion time of the job in the $k^{t h}$ position on the first machine. Constraint (6) computes the completion time of the job in the first position on all the machines except the first one. Constraints (7) and (8) calculate the departure time of the job in the $k^{t h}$ position on all machines other than the first machine. Constraint (9) calculates the tardiness of all jobs. Following Yu and $\mathrm{Li}[21] \theta^{\lambda}$ is defined to linearize the objective function. Ergo, the objective Function (1) is replaced with Equation (10). Moreover, Constraint (11) is added.

$$
\begin{align*}
& \min \sum_{\lambda \in \Omega} P^{\lambda} \sum_{k=1}^{n} T_{k}^{\lambda}+\sum_{\lambda \in \Omega} P^{\lambda}\left[\left(\sum_{k=1}^{n} T_{k}^{\lambda}-\right.\right. \\
& \left.\left.\sum_{\lambda^{\prime} \in \Omega} P^{\lambda^{\prime}} \sum_{k=1}^{n} T_{k}^{\lambda^{\prime}}\right)+2 \theta^{\lambda}\right]  \tag{10}\\
& -\theta^{\lambda}-\left(\sum_{k=1}^{n} T_{k}^{\lambda}-\sum_{\lambda^{\prime} \in \Omega} P^{\lambda^{\prime}} \sum_{k=1}^{n} T_{k}^{\lambda^{\prime}}\right) \leq 0, \forall \lambda \tag{11}
\end{align*}
$$

Objective (10) and Constraint (11) ensure the conformity of the optimal schedule to the definition of the robust linear model-based schedule.

## 3. 3. Second Stage: Dealing with Machine Breakdown Disruption via Linear Programming

 Model Increasing the amount of idle times enhances the schedule stability but degrades the schedule robustness [4]. Here, a linear optimization method is proposed to promote stability without robustness degradation. The idle times $(E B D)$ of the $j^{t h}$ job is obtained from Equation (12) [22], where $t^{r}, t_{[j]}$ are the expected repair and processing time of the $j^{\text {th }}$ job, respectively.$$
\begin{equation*}
E B D_{[j]}=\frac{t_{r} \cdot t_{[j]}}{\theta} \tag{12}
\end{equation*}
$$

In the original insertion method [4], idle times were inserted before each job, but in the proposed linear programming model, the proper positions and amounts of the idle times are determined. Stability is interpreted as the degree of reordering of the job sequence, the completion, or the start-times after any disruption [23]. To control the expected degradation of quality robustness, the proposed surrogate stability measure is defined based on the rationale of minimizing the instability of every job.
3. 3. 1. Surrogate Measure of Stability First, let $p r_{[j]}^{\lambda}$ as the machine breakdown probability during the processing of the job in position $j u n d e r$ the scenario $\lambda$ as follows (Equation 13).

$$
\begin{equation*}
p r_{[j]}^{\lambda}=1-\exp \left(\frac{t_{1 j j}^{\lambda}}{\theta^{\lambda}}\right) \tag{13}
\end{equation*}
$$

Let $E B T_{[j]}^{\lambda}$ be the expected breakdown duration of the job in position $j$ under scenario $\lambda$. Suppose that two consecutive breakdowns have respectively occurred during the processing of the jobs in positions $[k-1]$ and [j]. The amount of adjusted (expected) idle time from $[k]$ to $[j]$ i.e. $A T_{[k][j]}^{\lambda}$ is determined in such a way to be as close as possible to the $E B T_{[j]}^{\lambda}$. Ergo, the stability measure ( $S M$ ) can be defined via Equation (14).

$$
\begin{equation*}
S M=\sum_{j=1}^{n} \sum_{k=2}^{i-1} p r_{[j]}^{\lambda} \max \left\{E B D_{[j]}^{\lambda}-A T_{[k][j]}^{\lambda}, 0\right\} \tag{14}
\end{equation*}
$$

The machine breakdown probability during each job $\left(p r_{[j]}^{\lambda}\right)$ affects the proposed stability measure. In this way, once the breakdown during a job processing time is more probable, the difference between $E B D_{[j]}^{\lambda}$ and the total inserted idle times of job $[j]$ is minimized.
3. 3. 2. Surrogate Measure of Robustness Quality robustness is interpreted as the scheduling performance insensitivity against the disruptions [1]. Following Goren and Sabuncuoglu [2], $R M=\sum_{j=1}^{n} T_{[j]}^{\lambda}$ is adopted as a robustness measure. The following notations are used in the linear programming model.

## Indices

[j], $[k] \quad$ indices for position $j \in\{1,2, \ldots, n\}$
$i \quad$ index for machine $i \in\{1,2, \ldots, m\}$
$\lambda \quad$ index for scenario $\lambda \in \Omega$
Parameters
$t_{i[j]}^{\lambda}$
the processing time of job in position jon machine iunder scenario $\lambda$.

## Parameters

$p r_{[j]}^{\lambda}$
$E B D_{[j]}^{\lambda}$
the breakdown probability of machine one during the process of job junder scenario $\lambda$ the expected breakdown duration if it happens during the process of job $j$ on machine one under scenario $\lambda$

## Variables

| $S_{i[j]}^{\lambda}$ | the planned completion time of the job in the $j^{\text {th }}$ position on machine $i$ under scenario $\lambda$ |
| :---: | :---: |
| $C_{[i][5]}^{\lambda}$ | the planned completion time of the job in the $j^{\text {th }}$ position on machine $i$ under scenario $\lambda$ |
| $T_{[j]}^{\lambda}$ | the planned tardiness of the job in the $j^{\text {th }}$ position under scenario $\lambda$ |
| $A T_{[j]}^{\lambda}$ | the adequate idle time of job $j$ on machine one under scenario $\lambda$ |
| $A T_{[k][j]}^{\lambda}$ | the sum of adequate idle times between the jobs $k$ and $j$ on machine one in scenario $\lambda$ |

The linear programming model is formulated as follows.

$$
\begin{align*}
& \min z=\alpha \sum_{j=1}^{n} \sum_{k=2}^{i-1} p r_{[j]}^{\lambda} \max \left\{E B D_{[j]}^{\lambda}-\right. \\
& \left.A T_{[k][j]}^{\lambda}, 0\right\}+(1-\alpha) \sum_{k=1}^{n} T_{[k]}^{\lambda} \text { s.t. } \\
& S_{1[k]}^{\lambda}=S_{1[k-1]}^{\lambda}+t_{1[k-1]}^{\lambda}+A T_{[k]}^{\lambda} \quad \forall k \geq 2 \\
& C_{1[k]}^{\lambda}=S_{1[k]}^{\lambda}+t_{1[k]}^{\lambda} \\
& C_{i[k]}^{\lambda} \geq C_{i[k-1]}^{\lambda}+t_{i[k]}^{\lambda} \quad \forall i \geq 2, \forall k \geq 2 \\
& C_{i[k]}^{\lambda} \geq C_{i-1[k]}^{\lambda}+t_{i[k]}^{\lambda} \quad \forall i \geq 2, \forall k \\
& T_{[k]}^{\lambda}=\max \left\{C_{m[k]}^{\lambda}-d_{[k]}^{\lambda}, 0\right\}  \tag{20}\\
& A T_{[k][j]}^{\lambda}=\sum_{l=k+1}^{j} A T_{[l]}^{\lambda} \quad \forall j \geq 2, \forall k<j  \tag{21}\\
& S_{1[1]}^{\lambda}=0  \tag{22}\\
& A T_{[1]}^{\lambda}=0  \tag{23}\\
& S_{i[k]}^{\lambda} \geq 0  \tag{24}\\
& C_{i[k]}^{\lambda} \geq 0  \tag{25}\\
& A T_{[k]}^{\lambda} \geq 0  \tag{26}\\
& A T_{[k][j]}^{\lambda} \geq 0 \quad \forall j, k<j \tag{27}
\end{align*}
$$

Constraint (16) indicates that under scenario $\lambda$, the planned start time of the job in position $k$ on the first machine equals to the sum of the planned start time of the $[k-1]^{\text {th }}$ job on the first machine, plus its processing time and its additional time. Scenario-based Constraints (17) to (20) are required in an $m$-machine $F S S P$ to calculate the total tardiness. Constraint (17) gives the completion time of the job in the $k^{\text {th }}$ position on the first machine. Constraints (18) and (19) calculate the completion time of the job in the $k^{\text {th }}$ position on the other machines other than the first one. Constraint (20) calculates the tardiness of all jobs. Constraint (21)
calculates the sum of idle times between job [ $k$, [ $j$ ]. Constraint (22) ensures that under scenario $\lambda$, the start time of the job in the first position on the first machine is zero and no additional time exists before the job in the first position on the first machine. Constraint (23) indicates that there is no additional time before the job in the first position. Constraints (24)-(26) respectively emphasize the positivity of the start time, the completion time, and the additional time of the job in the $k^{t h}$ position under scenario $\lambda$. Also, Constraint (27) indicates the positivity of the additional times between the jobs in positions $[k]$ and $[j]$. This model should be solved for all the possible scenarios to determine the adequate additional times of each job on each machine per scenario.

## 4. DISCUSSION AND RESULTS

Here to provide the managerial results, the performance of the most widely used strategies in the literature namely reactive and hybrid are compared with the proposed (predictive) algorithm. In the reactive strategy, the optimal schedule is acquired according to the classical FSS model by ignoring uncertainties. In the hybrid strategy, the robust schedule is acquired from the robust optimization model (section 3.1) and RSH is implemented upon machine breakdown. In Figures 1 and 2 , the results are reported for different problems' sizes and parameters. In these figures, $A O F O P T, A O F R M$, and $A O F P R M$ respectively represent the average objective function of the classical, robust, and proposed prediction methods. It can be concluded from Figures 1 and 2 that PRM outperforms the other two methods, no matter the problem size and parameters. According to Figures 1 and 2 , there is a high difference between $O . F$. of the classical and robust schedules, also the classical schedule suffers from extreme fluctuations in O.F. versus uncertainties, so the scheduling should not be performed regardless of job processing time uncertainty even in a low degree of uncertainty. Also, it can be deduced from Figure 2 that job processing time uncertainty has a more decisive effect on $O . F$. than the machine failure rate. Since the $O . F$. of PRM and RM are close to each other, given the cost of implementing the second step of the proposed algorithm, a manager can use a hybrid or the proposed predictive approach.
A case study. To indicate the applicability of the model it has been implemented on a real case in Petro Tajhiz Sepahan Company in Iran that specializes in designing and manufacturing various types of valves for the petrochemical industry. In summary, the functions of the valves are stopping/starting the fluid flow, varying its amount and controlling the direction of it. They are also used in regulation downstream systems, process pressure, relieving component and piping overpressure.


Figure 1. The comparison of the $O . F$. of $O P T, R M$ and $P R M$ methods for the 2-machine 15 jobs problem, with a low degree of uncertainty, low failure rate, low MTTR, and coefficients $(0.2,0.3,0.5)$ for efficiency, stability, and robustness


Figure 2. Comparison of the O.F. of $O P T, R M$ and $P R M$ methods for the 2-machine 20 jobs problem, with a moderate degree of uncertainty, high failure rate, high MTTR, and coefficients ( $0.2,0.3,0.5$ ) for efficiency, stability, and robustness

According to construction standards, valves are divided into five categories [24]. The case studied in this paper focuses on the production of two types of Forged Steel Valves. The production of the Forged Steel valves is carried out via a flow shop system in six stages; Turning- Drilling- Grinding- Milling- Welding and Polishing. The production volume, the predetermined due dates and the processing times per stage are given in Table 1.

The first type of valves in this category are Needle Valves that can be used as a component for other valves. They are also used in fluid transmission lines, which include pharmaceuticals, foodstuffs, and chemicals. The second type of valves in this category is the Globe valve. In this type of valves, a disk moves perpen. The company receives orders from various oil/gas companies.

At the beginning of 2016, the company was contracted to construct and deliver eight orders. Due to the uncertain nature of the production, the processing times are defined via pessimistic, probable, and optimistic scenarios.

TABLE 1. The processing time of Forged Steel Valves under different scenarios*

|  | Job |  | Processing time per stage (min) |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\boldsymbol{\lambda}$ | $\boldsymbol{P}^{\boldsymbol{\lambda}}$ | Valve | $\mathbf{1}$ | $\mathbf{2}$ | $\mathbf{3}$ | $\mathbf{4}$ | $\mathbf{5}$ | $\mathbf{6}$ |  |
| 1 | 0.2 | Needle | 100 | 85 | 30 | 230 | 40 | 50 |  |
| 2 | 0.6 | Needle | 105 | 90 | 33 | 240 | 45 | 55 |  |
| 3 | 0.3 | Needle | 115 | 100 | 40 | 270 | 50 | 60 |  |
| 1 | 0.2 | Gate | 120 | 95 | 25 | 260 | 50 | 65 |  |
| 2 | 0.6 | Gate | 125 | 100 | 25 | 275 | 55 | 70 |  |
| 3 | 0.3 | Gate | 140 | 110 | 30 | 305 | 60 | 80 |  |
| 1 | 0.2 | Check | 55 | 45 | 25 | 125 | 25 | 35 |  |
| 2 | 0.6 | Check | 55 | 50 | 25 | 130 | 27 | 35 |  |
| 3 | 0.3 | Check | 60 | 55 | 30 | 145 | 30 | 40 |  |
| 1 | 0.2 | Check | 60 | 50 | 25 | 135 | 35 | 40 |  |
| 2 | 0.6 | Check | 65 | 54 | 25 | 145 | 35 | 40 |  |
| 3 | 0.3 | Check | 75 | 60 | 30 | 160 | 40 | 45 |  |
| 1 | 0.2 | Check | 65 | 75 | 25 | 155 | 35 | 35 |  |
| 2 | 0.6 | Check | 70 | 80 | 30 | 165 | 40 | 37 |  |
| 3 | 0.3 | Check | 80 | 90 | 35 | 185 | 45 | 40 |  |
| 1 | 0.2 | Ball | 155 | 125 | 35 | 400 | 50 | 10 |  |
| 2 | 0.6 | Ball | 167 | 135 | 40 | 417 | 55 | 10 |  |
| 3 | 0.3 | Ball | 183.7 | 148.5 | 44 | 458.7 | 60.5 | 11 |  |
| 1 | 0.2 | Ball | 190 | 175 | 55 | 455 | 60 | 20 |  |
| 2 | 0.6 | Ball | 200 | 185 | 58 | 480 | 65 | 20 |  |
| 3 | 0.3 | Ball | 220 | 203.5 | 63.8 | 528 | 71.5 | 22 |  |
| 1 | 0.2 | Ball | 250 | 260 | 60 | 550 | 70 | 25 |  |
| 2 | 0.6 | Ball | 265 | 273 | 65 | 580 | 75 | 30 |  |
| 3 | 0.3 | Ball | 291.5 | 300.3 | 71.5 | 638 | 82.5 | 33 |  |
| $\boldsymbol{2}:$ | Scenario, $P^{\lambda}:$ The probability | of scenario $\lambda$. |  |  |  |  |  |  |  |

## 4. 1. Data Generation

In this section, the obtained schedule from the proposed predictive algorithm and the empirical schedule in the company are compared with each other in terms of robustness, stability, and efficieny.

The processing times in the proposed method are uncertain, and they are estimated via a finite number of scenarios (Table 1). The processing times in the empirical method are the expected value of the processing time of all scenarios, i.e. $t_{i j}=\sum t_{i j}^{\lambda} P^{\lambda}$ where $t_{i j}^{\lambda}$ is the processing time of $j^{t h}$ job on machine $M_{i}$ under scenario $\lambda$ and $P^{\lambda}$ is the occurrence probability of scenario $\lambda$. In the proposed method, the time between consecutive breakdowns on the first machine is assumed to respectively follow an exponential distribution with the rates of $0.02,0.0166$ and 0.0125 for the optimistic, probable and pessimistic scenarios. Similar to Nouiri et
al., [7], the duration of the repair times follows an exponential distribution based on the meantime to repair value (MTTR) at two-level. The repair time duration is calculated via $t r=\exp r n d(M T T R)$. The MTTR is calculated based on the machine busy time ( $M B$ ); low level $M T T R_{l} \in[0.01 M B, 0.05 M B]$ and high level $M T T R_{h} \in[0.05 M B, 0.1 M B]$. In the empirical method, the reaction to breakdown is done by implementing the right shift rescheduling (RSH) policy to the affected jobs.
4. 2. The Empirical Schedule Procedure The company uses the following procedure to achieve an empirical schedule:
Step 1. The initial schedule generation.
The initial sequence of the jobs is determined according to the earliest due date rule $(E D D)$ to minimize the total tardiness as an efficiency measure.
Step 2. Main Loop: for $s=1 \ldots N_{s}$
Step 2.1. Random breakdown generation.
It is assumed that $\lambda^{\prime \prime} \in \Omega$ is a real scenario. The random breakdowns are generated according to the rate of breakdown in $\lambda^{\prime \prime}$.
Step 2.2. Robustness, stability and efficiency.
The RSH is implemented upon a breakdown occurrence to obtain the actual (real) schedule. The robustness and stability measures are calculated with Equations (28) and (29), respectively.
4. 3. Robustness, Stability, Efficiency, and the Objective Function Suppose that $\lambda^{\prime \prime} \in \Omega$ is the scenario that has actually happened. The robustness measure ( $R M$ ) is defined as an absolute deviation of an efficiency measure (total tardiness) of the actual schedule from the initial one. It can be calculated via Equation (28), where $\sum T^{\lambda^{\prime \prime}}$ is the total tardiness of the actual schedule under scenario $\lambda^{\prime \prime} \in \Omega$, and $\sum T^{\lambda}$ is the total tardiness of the predictive schedule under scenario $\lambda \in \Omega$.

$$
\begin{equation*}
R M=\left|\sum_{j} T^{\lambda^{\prime \prime}}-\sum_{j} T^{\lambda}\right| \tag{28}
\end{equation*}
$$

Moreover, stability measure ( $S M$ ) is defined as an absolute deviation in job completion times (Equation 29), where $C_{m[k]}^{\lambda^{\prime \prime}}$ is the completion time of the job in position $[k]$ in the actual schedule under scenario $\lambda^{\prime \prime} \in \Omega$, and $C_{m[k]}^{\lambda}$ is the completion time of the job in position $[k]$ in the predictive schedule under scenario $\lambda \in \Omega$.

$$
\begin{equation*}
S M=\sum_{k=1}^{n}\left|C_{m[k]}^{\lambda^{\prime \prime}}-C_{m[k]}^{\lambda}\right| \tag{29}
\end{equation*}
$$

Efficiency (Eff) is the measure of the optimality of the schedule. Here, the total completion time of the actual schedule is considered as an Efficiency measure.

$$
\begin{equation*}
E f f=\sum_{j} c^{\lambda^{\prime \prime}} \tag{30}
\end{equation*}
$$

The objective function is a multi-component measure based on the predefined measures of robustness, stability, and efficiency as follows (Equation 31).

$$
\begin{equation*}
O . F .=\alpha(R M)+\beta(S M)+\gamma(E f f) \tag{31}
\end{equation*}
$$

where $\alpha+\beta+\gamma=1$ and $\alpha, \beta, \gamma$ respectively indicate the degrees of importance of their corresponding objective. These parameters can be determined using methods such as sensitivity analysis, eigenvector, entropy, or the least-square method [1]. The proposed method ( $P M$ ) and the empirical method ( $E M$ ) can be compared concerning the value of O.F (Table 2). The calculations are made for different values of the coefficients at two levels of MTTR.
4. 4. Sensivity Analysis This section provides additional tests on the parameters of the model to gauge their effects on the value of the O.F.
4. 2. 1. Testing on the Time Interval Between Concequtive Breakdowns Figure 3 depicts the effect of different $\frac{1}{\theta}$ on the value of the $O . F$. in the proposed method $(P M)$. The dataset in the Figure 3 is derived from Table 2. From the problems with the low level of $M T T R$, instances 1 to 10,21 to 30 and 41 to 50 are chosen that respectively correspond to values of 80 , 60 and 50 for the interval between two consecutive failures.

In Figure 4, the notation $O F-P M-80-L$ corresponds to the objective values found for the proposed method when $\frac{1}{\theta}=80$ and the $M T T R$ is set to the low level. As expected, in all the 10 instances of different categories, an increase in the average time between two failures improves the objective function, since the minimum values of the objective are achieved for $\frac{1}{\theta}=80$.
4. 2. 2. Testing the Effect of MTTR Level In this section, the MTTR is first set to the low level (for the instances 1-10 in Table 2) and then set to the high level (instances 11-20) to see the effect of its increment on the $O . F$. The results are demonstrated in Figure 4. It can be seen in Figure 4 that an increase in $M T T R$, in turn, worsens the O.F. of the proposed method; ergo lower MTTR levels are preferred.


Figure 3. The effect of different breakdown intervals on the objective value

TABLE 2. The comparison between the $O . F$. of $P M \& E M$

| NO | $\alpha$ | $\beta$ | $\gamma$ | $\frac{1}{\theta}$ | PM |  | EM |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  | O.F. | CPU(s) | O.F. | CPU(s) |
| 1 | 0.2 | 0.4 | 0.4 | 80 | 871.21 | 1.81 | 1242.0 | 0.6 |
| 2 | 0.2 | 0.3 | 0.5 | 80 | 837.74 | 1.66 | 1000.5 | 0.54 |
| 3 | 0.2 | 0.5 | 0.3 | 80 | 890.76 | 1.54 | 1472.4 | 0.48 |
| 4 | 0.3 | 0.3 | 0.4 | 80 | 830.56 | 1.89 | 970.54 | 0.68 |
| 5 | 0.3 | 0.4 | 0.3 | 80 | 858.49 | 1.65 | 1209.4 | 0.51 |
| 6 | 0.4 | 0.3 | 0.3 | 80 | 835.98 | 1.65 | 938.77 | 0.48 |
| 7 | 0.4 | 0.1 | 0.5 | 80 | 792.5 | 1.79 | 468.50 | 0.39 |
| 8 | 0.4 | 0.5 | 0.1 | 80 | 890.44 | 1.66 | 1410.5 | 0.52 |
| 9 | 0.5 | 0.3 | 0.2 | 80 | 839.13 | 1.7 | 910.0 | 0.57 |
| 10 | 0.5 | 0.2 | 0.3 | 80 | 815.25 | 1.38 | 674.73 | 0.6 |
| 11 | 0.2 | 0.4 | 0.4 | 80 | 902.96 | 1.86 | 1240.7 | 0.67 |
| 12 | 0.2 | 0.3 | 0.5 | 80 | 870.96 | 1.81 | 1006.5 | 0.64 |
| 13 | 0.2 | 0.5 | 0.3 | 80 | 931.06 | 1.78 | 1481.3 | 0.61 |
| 14 | 0.3 | 0.3 | 0.4 | 80 | 872.25 | 1.79 | 976.46 | 0.66 |
| 15 | 0.3 | 0.4 | 0.3 | 80 | 902.41 | 1.72 | 1212.4 | 0.64 |
| 16 | 0.4 | 0.3 | 0.3 | 80 | 868.91 | 1.83 | 942.36 | 0.68 |
| 17 | 0.4 | 0.1 | 0.5 | 80 | 790.00 | 1.6 | 468.75 | 0.5 |
| 18 | 0.4 | 0.5 | 0.1 | 80 | 897.85 | 1.79 | 1419.0 | 0.65 |
| 19 | 0.5 | 0.3 | 0.2 | 80 | 871.29 | 1.78 | 912.86 | 0.58 |
| 20 | 0.5 | 0.2 | 0.3 | 80 | 838.83 | 1.72 | 675.96 | 0.61 |
| 40 | 0.5 | 0.2 | 0.3 | 60 | 841.24 | 1.79 | 675.77 | 0.68 |
| 41 | 0.2 | 0.4 | 0.4 | 50 | 877.44 | 1.75 | 1234.8 | 0.68 |
| 42 | 0.2 | 0.3 | 0.5 | 50 | 848.05 | 1.34 | 1003.9 | 0.61 |
| 43 | 0.2 | 0.5 | 0.3 | 50 | 903.34 | 1.35 | 1478.0 | 0.66 |
| 44 | 0.3 | 0.3 | 0.4 | 50 | 847.85 | 1.45 | 972.16 | 0.63 |
| 45 | 0.3 | 0.4 | 0.3 | 50 | 878.26 | 1.7 | 1211.8 | 0.65 |
| 46 | 0.4 | 0.3 | 0.3 | 50 | 849.74 | 1.73 | 942.3 | 0.66 |
| 47 | 0.4 | 0.1 | 0.5 | 50 | 790.81 | 1.29 | 468.00 | 0.51 |
| 48 | 0.4 | 0.5 | 0.1 | 50 | 899.34 | 1.29 | 1416.7 | 0.69 |
| 49 | 0.5 | 0.3 | 0.2 | 50 | 849.64 | 1.39 | 913.01 | 0.67 |
| 50 | 0.5 | 0.2 | 0.3 | 50 | 818.73 | 1.35 | 673.46 | 0.52 |
| 51 | 0.2 | 0.4 | 0.4 | 50 | 914.74 | 1.59 | 1242.9 | 0.62 |
| 52 | 0.2 | 0.3 | 0.5 | 50 | 878.25 | 1.74 | 1002.1 | 0.69 |
| 53 | 0.2 | 0.5 | 0.3 | 50 | 950.15 | 2.02 | 1475.8 | 0.89 |
| 54 | 0.3 | 0.3 | 0.4 | 50 | 877.23 | 1.74 | 973.45 | 0.65 |
| 55 | 0.3 | 0.4 | 0.3 | 50 | 917.48 | 1.9 | 1212.4 | 0.68 |
| 56 | 0.4 | 0.3 | 0.3 | 50 | 878.95 | 1.83 | 940.44 | 0.67 |
| 57 | 0.4 | 0.1 | 0.5 | 50 | 807.19 | 1.94 | 468.49 | 0.68 |
| 58 | 0.4 | 0.5 | 0.1 | 50 | 944.40 | 1.82 | 1418.4 | 0.65 |
| 59 | 0.5 | 0.3 | 0.2 | 50 | 879.71 | 1.88 | 913.33 | 0.64 |
| 60 | 0.5 | 0.2 | 0.3 | 50 | 841.87 | 1.86 | 675.35 | 0.66 |



Figure 4. The effect of the MTTR level on the O.F.
4. 2. 3. Testing on the Stability and Robustness Coefficients

In this section, different values of the robustness and the stability coefficients (respectively $\alpha, \beta$ ) are used to achieve the objective values of both $P M$ and $E M$. The results for the low level of $M T T R, \theta=50$ and $\gamma=0$ are summarized in Table 3.

The effects of the varying coefficients on O.F. are also depicted in Figure 5. According to Table 3 and Figure 5, the two methods perform similarly once $\alpha=$ 0.7 and $\beta=0.3$. Before these values, PM is superior to the EM. Any more increase in the value of $\alpha$ and any more decrease in the value of $\beta$ worsen the $O . F$. of $P M$.

Ergo, if the emphasis is on the robustness of the schedule ( $\alpha=0, \beta=1$ ), the proposed method is better. On the other hand, if a schedule with maximum stability is desired $(\alpha=1, \beta=0)$, the empirical method is preferred. If a robust and stable schedule is required, then the proposed method should be picked since its range of superior performance is larger ( $\alpha \leq 0.7, \beta \geq 0.3$ ). This shows the major impact of the coefficient on the performance, ergo the setting of these parameters should be carried out with care. There is a logical contradiction between stability and robustness since to enhance the schedule robustness, sequence manipulation may be necessary, which leads to stability degradation. To illustrate this conflict, the normalized data from Table 3 is used to plot Figure 6 to compare the stability and the robustness values of the proposed method.

## 4. 2. 4. Comparing the Effectiveness of the Proposed and the Empirical Method

 According to the results of Table 2 and Figure 6, for both levels of $M T T R$, and all values of $\frac{1}{\theta}$, the proposed method is more effective than the empirical one except in $(\alpha, \beta, \gamma)=(0.4,0.1,0.5), \quad \operatorname{and}(\alpha, \beta, \gamma)=(0.5,0.2,0.3)$. hese results are confirmation of the major impact of the coefficient on the performance. As concluded from Figure 5, whenever a robust and stable schedule is required, then the proposed method should be selected since its range of superior performance is larger ( $\alpha \leq 0.7, \beta \geq 0.3$ ). Here, this conclusion becomes more complete. That is, by choosing values more than 2 for the ratio of robustness to stability $\left(\frac{\alpha}{\beta}>2\right), E M$ is thepreferred method and vice versa. Moreover, the trend of the objective values in the proposed method is smoother than its empirical counterpart. This difference is due to the robust optimization method used in the generation of the partial robust schedule.

TABLE 3. Comparison of the objective measures of the proposed and empirical methods

|  | PROPOSED METHOD |  |  |  |
| :--- | :---: | :---: | :---: | :---: |
| $(\alpha, \beta)$ | $R M$ | $S M$ | $E f f$ | $O . F$. |
| $(0,1)$ | 768.78 | 1040.88 | 755.97 | 1040.88 |
| $(0.1,0.9)$ | 768.11 | 1043.79 | 754.63 | 1015.6 |
| $(0.2,0.8)$ | 770.5 | 1045.05 | 756.16 | 990.14 |
| $(0.3,0.7)$ | 770.97 | 1057.35 | 755.91 | 971.44 |
| $(0.4,0.6)$ | 769.24 | 1041.86 | 756.76 | 932.82 |
| $(0.5,0.5)$ | 766.83 | 1030.49 | 755.26 | 898.48 |
| $(0.6,0.4)$ | 769.87 | 1058.75 | 753.1 | 885.42 |
| $(0.7,0.3)$ | 770.72 | 1045.62 | 754.3 | 854.67 |
| $(0.8,0.2)$ | 773.44 | 1053.56 | 756.39 | 829.41 |
| $(0.9,0.1)$ | 772.74 | 1041.51 | 755.99 | 799.61 |
| $(1,0)$ | 772.66 | 1054.2 | 756.3 | 772.66 |
|  |  | EMPIRICAL METHOD |  |  |
| $(\alpha, \beta)$ | $R M$ | $S M$ | $E f f$ | $0 . F$. |
| $(0,1)$ | 49.32 | 2716.89 | 352.8 | 2716.89 |
| $(0.1,0.9)$ | 49.53 | 2715.22 | 352.8 | 2448.65 |
| $(0.2,0.8)$ | 48.69 | 2721.85 | 352.8 | 2187.22 |
| $(0.3,0.7)$ | 48.64 | 2722.27 | 352.8 | 1920.18 |
| $(0.4,0.6)$ | 49.08 | 2718.64 | 352.8 | 1650.82 |
| $(0.5,0.5)$ | 50 | 2713.5 | 352.8 | 1381.75 |
| $(0.6,0.4)$ | 49.13 | 2719.8 | 352.8 | 1117.4 |
| $(0.7,0.3)$ | 48.65 | 2722 | 352.8 | 852.36 |
| $(0.8,0.2)$ | 47.52 | 2728.85 | 352.8 | 583.79 |
| $(0.9,0.1)$ | 47.64 | 2726.98 | 352.8 | 315.58 |
| $(1,0)$ | 48.027 | 2727.4 | 352.8 | 48.027 |
|  |  |  |  |  |
|  |  |  |  |  |



Figure 5. Effect of different stability and robustness coefficients on the objectives


Figure 6. The conflict between robustness and stability

## 5. CONCLUSION

In this paper, a robust and stable approach for scheduling the manufacturing lines in the valve production industry are presented. The problem is modeled as an uncertain $m$-machine FSS system with machine breakdowns to optimize three performance measures; stability, robustness and efficiency, simultaneously. In the proposed approach, the problem is solved in two-stages. The first stage uses robust optimization to create a partial schedule by taking into account the uncertain processing times. Then the stability of the robust schedule is guaranteed faced with the new scenarios of job processing time. In the second stage, the appropriate buffer times were calculated via a linear programming model based on the defined performance measures in case of breakdowns of the first machine. The proposed predictive method is compared with reactive and hybrid approaches. In addition, the proposed predictive algorithm is applied to a real case from a valve company in Iran to investigate the superiority of this method over the empirical method currently used. The results showed that the proposed method is more adaptable to the occurrence of random events so that the variances in the objective values due to this change are much smoother than the empirical method. Ergo, this stable and robust schedule can increase the company's accountability to customers. The proposed model is formulated as an FSS system in the valve-production industry.

In future researches, this model can be applied to other systems such as flexible flow shop or job shop systems, across similar industries. Moreover, our model is limited to the breakdown of the first machine. Depending on the corresponding industry, this model can be generalized to include the breakdowns of other machines as well. Another possibility for extending this work is to consider different probability distributions for the breakdown intervals or variable repair time.

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# A New Approach Generating Robust and Stable Schedules in $m$-Machine Flow Shop Scheduling Problems: A Case Study 

Z. Abtahia, R. Sahraeian ${ }^{\text {a }}$, D. Rahmani ${ }^{\text {b }}$<br>${ }^{a}$ Department of Industrial Engineering, College of Engineering, Shahed University, Tehran, Iran<br>${ }^{b}$ Department of Industrial Engineering, K.N. Toosi University of Technology, Tehran, Iran

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[^0]:    *Corresponding Author Email: sahraeian@shahed.ac.ir (R.Sahraeian)

