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The Simultaneous Effect of Holding Safety Stock and Purchasing Policies on the Economic Production Quantity Model Subject to Random Machine Breakdown

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ABSTRACT

In this paper, we develop an economic production quantity (EPQ) model under machine breakdown and two types of repair (corrective and preventive). also, study the simultaneous effect of holding safety stock and purchasing policy. In order to avoid shortages occurring as a result of the random repair time, in addition to keep safety stock, we suppose that the manufacturer could purchase some quantities from an external supplier. This paper addresses the following question: how the manufacturer determine the optimal values of safety stock, production and purchasing lot sizes, simultaneously, to minimize the expected total cost? The introduced model is then compared with the situations in which the manufacturer only keeps safety stock or just uses an external supplier, respectively. The results through the analysis show that using the simultaneous policy when the system is prone to shortages due to long repair times, have more improvement in the performance of the system rather than using the safety stock or purchasing policies, separately.

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1. INTRODUCTION

Considering the importance that manufacturers place on responding to customers' needs in today's world of trade, and organizations' efforts to keep up their credit and service level in the supply chain, process deterioration during the production run is one of the important challenges facing economic production quantity (EPQ) systems. Process deterioration is manifested as a decrease in production rate, the production of defective items or machine breakdown. In this situation, the production manager is forced to deviate from production planning. The remarkable weakness of the classical EPQ models is the ignorance of the production facilities stoppage during the production run time. This issue makes the inconsistency between models and practical situations. Over the past three decades, researchers considered the unavailability of production facilities in the classical models and addressed different strategies such as preventive maintenance, holding safety stock and inspection/rework operation individually or in combination to reduce any disruption's side effects. Although holding safety stock as an appropriate solution in machine breakdown case is addressed in the part of researches [1, 2]. Sometimes the expensive costs of this strategy are not beneficial for the system due to high product holding costs or storage limitations. In such situations, emergency replenishment may be advantageous to meet the demand when the machine is being repaired. This policy has been used in the inventory management literature to address a perfect EPQ system with stochastic demand [3, 4] and an imperfect EPQ system with constant demand [5, 6]. The issue is not necessarily about the final product. It could be related to a standard piece of a final product manufactured by the main supplier, in which he/she is responsible for supplying the piece according to a long-term contract. In this case, the supplier can purchase from smaller suppliers with lower reliability to prevent shortages in urgent situations. Although purchasing items from an external supplier reduces margins, but it could protect the reputation of the manufacturer, and guarantee future demand [6].

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The contribution of this study is the simultaneous implementation of the holding safety stock and purchasing policy on a failure prone production-inventory system. What distinguishes our model from existing models in the literature is that in the simultaneous policy, the manufacturer has more flexibility to cope with the variation of the production, holding and shortage cost and can reduce the total cost of the system through the trade-offs between the amount of buffer stock, order quantities and production lot size. We also incorporate preventive maintenance, as an option to confront machine failure. Considering all of these issues bring the model closer to real-world situations; however, it will make the model more complicated. The remainder of this paper is structured as follows. Section 2 presents the literature survey; section 3 defines the notation and the basic assumptions of the model. In section 4 the details of the model are described along with the mathematical model under general failure and general repair time distributions. For verification of the model, we presented the purchasing and safety stock policy, that is presented in previous articles, separately and make them comparable to our proposed model. Results of this comparison and sensitivity analysis of the model through a numerical example are illustrated in section 6. The final section concludes the paper and provides areas for future research.

2. LITERATURE REVIEW

Unexpected machine breakdown is a very common incident in a production environment and regarded as a critical reliability factor. This issue is one area of research in imperfect production-inventory systems. Groenevelt et al. [7] as one of the pioneer researchers, introduced two order policies on an EPQ system under stochastic machine breakdown. Under the first policy (no resumption-NR policy), when a breakdown occurs, the new production cycle started after repair operation, once the available inventory is totally depleted; while under the second policy (abort-resume A/R policy), the production process will be instantly started, after the renovation of the production facility. Boone et al. [8] investigated the simultaneous effect of process deterioration and machine breakdown under the NR policy. Abboud [9] developed an imperfect EPQ model by assuming that a shortage occurred due to machine failure, is partially backlogged. Giri and Yun [10] proposed a model where the machine was subject to random failure and limited to at most two failures in a production cycle. In their model, if the shortage occurred because of longer repair time, there was a partial backlog after machine repair. Accordingly, if a failure occurs again during the backlogging, then the shortages accumulated during the second repair time will be lost. Chiu et al. [11] developed an EPQ model with scrap, rework and machine breakdown under the NR policy. Chiu et al. [12] studied rework capability for defective items for an EPQ model under A/R policy.

Preventive maintenance and holding safety stock are two strategies that researchers analyze the performance of an imperfect EPQ system by using either one of two strategies or simultaneously. Cheung and Hausman [13] studied the joint effect of preventive maintenance and safety stock in an unreliable manufacturing system. They assumed that production and demand rate are equal in a normal production phase. Dohi et al. [14] developed the model theoretically by adding different restrictive assumptions. Giri et al. [1] presented an unreliable production-inventory system, which considered exponential failure time while repair time occurs under an NR inventory control policy. In their model, the production cost and failure rate depend on the production rate, and they formulated the problem with and without a policy of holding safety stock. Chakraborty et al. [15] extended Boone et al. [8] model by concurrently considering the impacts of process deterioration, machine failure and two kinds of repairs on the manufacturer optimal decisions. Assuming that non-conforming items are not detectable during the production run time, they fixed the warranty cost for the sold defective items. The joint effects of the safety stock and age-based preventive maintenance were studied by Chelbi and Rezg [16]. El-Ferik [17] examined similar research by assuming that the maintenance activities are incomplete and unable to restore the system to its primary state. Sana and Chaudhuri [2] considered the joint determination of preventive maintenance and safety stock on an EPQ system under two A/R and NR policies for optimizing production rate and lot size. Chakraborty and Giri [18] expanded a similar model by assuming a varying production rate. They also discussed on the optimality of the model and suggested a computational algorithm to solve the problem. Other related researches can be referred to more literature [19–21].

In recent studies, Al-Salamah [22] developed EPQ models for the case when both the production and inspection processes are imperfect. Shah et al. [23] considered an imperfect production system which sales price is depended to stock demand and investigated optimal production rate, cycle time and retail price to maximize the system’s total profit. Ozturk [24] investigate optimal production run time on an EPQ system under A/R machine breakdown policy with inspection and rework capability. Taleizadeh et al. [25] Developed a single-vendor/single-buyer model under the NR policy with random machine breakdown, multiple shipments and keeping safety stock capability. They assumed both batch lot size and distance between two shipments are identical and the buyer pays transportation cost. Sarkar et al. [26] Developed an imperfect production system to obtain the optimal production run and inspection policy. They considered two types' inspection errors to make the model more realistic. Due to inspection error, they assumed that the non-inspected defective items are passes to customers with free minimal repair warranty. Al-Salamah [27] presented an imperfect EPQ system under A/R machine breakdown policy and rework capability. The author assumed that the rework rate is performed after the main manufacturing process and be different from the production rate. He implemented an artificial bee colony algorithm to find the optimal production lot size.

The implementation of the purchasing policy, in an imperfect production-inventory system was first introduced by Pirayesh and Yavari [5]. By assuming exponential time to failure and repair time, they obtained the optimal production and purchasing lot sizes. Peymankar et al. [6] developed the model in which they assumed that the external supplier has a known reliability. They also investigated the effects of the revenue sharing and price discount contracts on the optimal design of the production system. In this paper, we developed the worked done by Peymankar et al. [6] in which the manufacturer, in addition to outside purchasing, can benefit from the safety stock policy, simultaneously. We also considered preventive maintenance as an approach that strives to avoid machine failure. Through these assumptions, we have formulated a cost function, and minimize it by considering production, purchasing, and safety stock lot sizes as decision variables.

3. PROBLEM NOTATION AND ASSUMPTION

3. 1. Notations

The notations for the proposed model are presented as follows:

t_w: random variable denoting time to failure

$F(t_w), f(t_w)$: Cumulative distribution function and probability density function of t_w

D: demand rate (units/time)

P>D: production rate (units/time)

t_r: Random variable denoting corrective maintenance time

$G(t_r), g(t_r)$: Cumulative distribution function and probability density function of t_r

t_m: Random variable denoting preventive maintenance time

$H(t_m), h(t_m)$: Cumulative distribution function and probability density function of t_m

A: Fixed setup cost for each production run time

A': Fixed ordering cost for the purchased item from an external supplier

θ: Reliability of external supplier

c_h: Inventory holding cost per produced/purchased item (units/time)

c_p: Production cost per unit item r

c': Unit purchase price from external supplier

c_r: Corrective repair cost (units/time)

$c_m < c_r$: Preventive repair cost (units/time)

c_s: Shortage cost (units/time)

T: Expected total cycle time

Q: Production lot size per cycle (decision variable)

S_f: Safety stock lot size (decision variable)

Q': Order quantity lot size to external supplier per cycle (decision variable)

3. 2. Assumptions

The assumptions for the proposed model are as follows:

1. The planning horizon is infinite.
2. The problem concerns a single-machine single-product environment.
3. Setup time is negligible and equals to zero.
4. The demand rate and production rate are known constants and the production rate is greater than the demand rate.
5. The machine breakdown may occur at any random time during the production run.
6. If a machine failure occurs during the production run, the corrective repair is started immediately; thereafter, the machine is restored to its primary state.
7. If a machine failure does not occur during the production run, the preventive maintenance action renews the production system at the end of each production run.
8. The demand during repair operations is met first from the accumulated inventory. Safety stock in the system is used to avoid possible shortages during the machine repair.
9. If the safety stock is not able to meet the demand during repair operation, the manufacturer can obtain the required items from the market.
10. To avoid the complexity of the model, it is assumed that supplier lead time is negligible and equals to zero
11. The Shortages that may occur due to longer repair times will be lost.

4. MODEL DESCRIPTION

We consider a production-inventory system, which may stop, at any random time during the production run. We suppose that the manufacturer holds a safety stock S_f at the beginning of each production cycle to protect against possible shortages during the period of machine repair. If a machine failure occurs, the corrective repair is started immediately; otherwise, the preventive maintenance starts after the production runs at time $t_P = Q/P$. During the corrective and preventive maintenance, the accumulated on-hand inventory is reduced at a constant D to response the demand during the machine idle time (in the case of no machine breakdown $t_{IP} = Q/D - Q/P$ and in the case of machine breakdown $t_{IC} = Pt_w/D - t_w$). If the maintenance activity is completed before the termination of the accumulated on-hand inventory, then the new production cycle is not started until the inventory level decreases to the safety stock level. If the inventory level drops below the safety stock level then the production process is started immediately after repair operation to increase the inventory level to the safety stock level. Lost sales are incurred when the stock level drops to zero before the repair is completed. To avoid shortages in this situation, we assume that the manufacturer has the option to fulfill the demand using an external supplier with a determined service level. Configuration of the model is presented in Figure 1 and 2. Based on this assumption and given that different situation that the manufacturer may face during one inventory cycle, we present the formulation of the model in section 5.

5. FORMULATION OF THE MODEL

As shown in Figure 1 and 2, based on whether theproduction facility is corrupted or not during the production run, the manufacturer may face two cases and in each case based on repair time, five different. situations maybe take place during one inventory cycle. In sections 5.1 and 5.2, we present a mathematical model for each condition.

5. 1. The Model with Machine Breakdown

Suppose that the machine is corrupted at random time t_w before producing the planned lot size. If the corrective repair operation completed before the inventory level reaches to S_f, the new cycle begins after the finishing of the on-hand inventory (Figure 1(a)). The cycle time and the total cost for the system for this situation are:

$$T_1{}^r = \frac{P.t_w}{D} \quad (1)$$

$$TC_1^r = A + c_h\left(\frac{(P-D)P.t_w^2}{2D} + \frac{S_f.P.t_w}{D}\right) + c_P.P.t_w + c_r.t_r \quad (2)$$

If t_r gets longer, it may cause the manufacturer using safety stock to meet the demand. In this situation, if corrective repair completed before the inventory level reaches zero, the manufacturer immediately starts the production process to increase inventory level to S_f (Figure 1(b)). So, we have:

$$T_2^r = t_w + t_r + \left(t_w + t_r - \frac{P.t_w}{D}\right)\left(\frac{D}{P-D}\right) = \left(\frac{P}{P-D}\right)t_r \quad (3)$$

$$TC_2^r = A + c_h\left(\frac{(P-D)P.t_w^2}{2D} + \frac{S_f.P.t_r}{D} - \frac{P(D.t_r-(P-D)t_w)^2}{2D(P-D)}\right) + c_P\frac{P.D.t_r}{P-D} + c_r.t_r \quad (4)$$

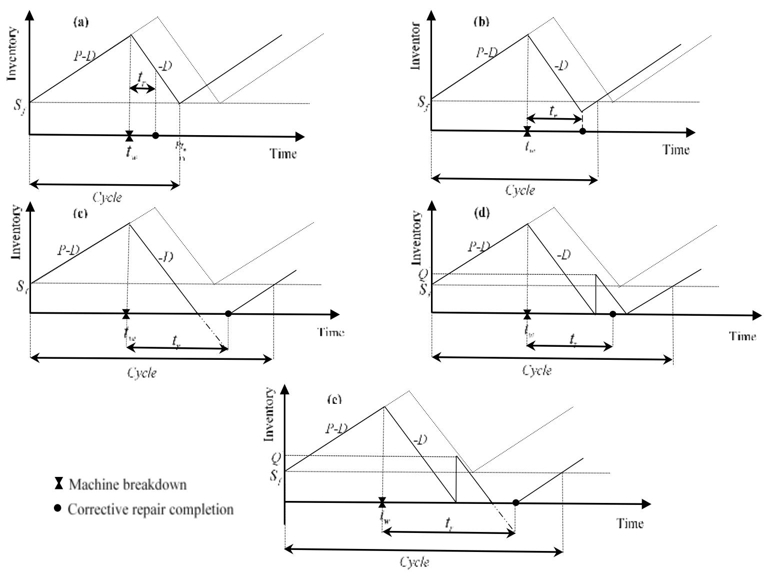

Figure 1. Configuration of model with machine breakdown when (a) on-hand inventory are sufficient to meet up demand during corrective repair operation; (b) safety stocks are sufficient to meet up demand during corrective repair operation; (c) orders not satisfied with probability *1-θ* and shortages occurs due to prolonged corrective repair time; (d) orders satisfied with probability *θ* and corrective repair operation done before finishing purchasing items; (e) orders satisfied, but shortages occurs due to prolonged corrective repair time

It may happen that the safety stock neither can cover the demand during the repair time. In this situation, the manufacturer can meet the demand by using an external supplier. However, with the probability of $1-\theta$, the necessary items cannot be delivered by the supplier and the system faces shortage until the machine is fixed (Figure 1(c)). So, the system's cycle time and total cost are calculated as follows:

$$T_3^r = t_w + t_r + \frac{S_f}{P-D} \quad (5)$$

$$TC_3^r = A + c_h\left(\frac{(P-D)P.t_w^2}{2D} + \frac{S_f.P.t_w}{D} + \frac{P.S_f^2}{2D(P-D)}\right) + c_P\left(P.t_w + \frac{P.S_f}{P-D}\right) + c_r.t_r + c_s\left(D.t_r - (P-D)t_w - S_f\right) \quad (6)$$

With the probability of θ supplier can provide necessary items. If the order quantity Q' was sufficient to cover the demand and the corrective repair operation completed before the purchasing lot size reaches to zero, then shortages would not occur and the new cycle starts after the inventory level reaches to S_f (Figure 1(d)). In this situation, we have:

$$T_4^r = \frac{P.t_w}{D} + t_r + \frac{P.S_f}{D(P-D)} + \frac{Q'}{D} \quad (7)$$

$$TC_4^r = A + c_h\left(\frac{(P-D)P.t_w^2}{2D} + \frac{S_f.P.t_w}{D} + \frac{P.S_f^2}{2D(P-D)} + \frac{Q'^2}{2D}\right) + A' + c_P\left(P.t_w + \frac{P.S_f}{P-D}\right) + c'.Q' + c_r.t_r \quad (8)$$

Figure 1(e) shows that the prolonged corrective operation causes the purchasing items is finished before the machine is repaired and the system faces shortages. The cycle time and total cost for this situation are:

$$T_5^r = t_w + t_r + \frac{S_f}{P-D} \quad (9)$$

$$TC_5^r = A + c_h\left(\frac{(P-D)P.t_w^2}{2D} + \frac{S_f.P.t_w}{D} + \frac{P.S_f^2}{2D(P-D)} + \frac{Q'^2}{2D}\right) + A' + c_P\left(P.t_w + \frac{P.S_f}{P-D}\right) + c'.Q' + c_r.t_r + c_s\left(D.t_r - (P-D)t_w - S_f\right) \quad (10)$$

Now we can obtain the expected cycle time and the expected total cost per cycle in the machine breakdown case as follows:

$$E(T^r) = \int_0^{\frac{(P-D)t_w}{D}} T_1^r dG(t_r) + \int_{\frac{(P-D)t_w}{D}}^{\frac{(P-D)t_w}{D}+\frac{S_f}{D}} T_2^r dG(t_r) + \theta\left(\int_{\frac{(P-D)t_w}{D}+\frac{S_f}{D}}^{\frac{(P-D)t_w}{D}+\frac{S_f+Q'}{D}} T_4^r dG(t_r) + \int_{\frac{(P-D)t_w}{D}+\frac{S_f+Q'}{D}}^{\infty} T_5^r dG(t_r)\right) + (1-\theta)\int_{\frac{(P-D)t_w}{D}+\frac{S_f}{D}}^{\infty} T_3^r dG(t_r) \quad (11)$$

$$E(TC^r) = \int_0^{\frac{(P-D)t_w}{D}} TC_1^r dG(t_r) + \int_{\frac{(P-D)t_w}{D}}^{\frac{(P-D)t_w}{D}+\frac{S_f}{D}} TC_2^r dG(t_r) + \theta\left(\int_{\frac{(P-D)t_w}{D}+\frac{S_f}{D}}^{\frac{(P-D)t_w}{D}+\frac{S_f+Q'}{D}} TC_4^r dG(t_r) + \int_{\frac{(P-D)t_w}{D}+\frac{S_f+Q'}{D}}^{\infty} TC_5^r dG(t_r)\right) + (1-\theta)\int_{\frac{(P-D)t_w}{D}+\frac{S_f}{D}}^{\infty} TC_3^r dG(t_r) \quad (12)$$

5. 2. Model without Machine Breakdown In this case, the manufacturer faces no machine breakdowns and the planned lot size are produced at the production run t_P. After the completion of the production process, the preventive repair starts to return the machine to the primary state efficiency before the beginning of the new production run. Similar to machine breakdown case, based on the preventive repair time t_m, five different modes may occur. If the preventive repair completed before the inventory level reaches to S_f, the new cycle starts when all the produced items delivered to the customer (Figure 2(a)). So, the length of one cycle and total cost are:

$$T_1^m = \frac{P.t_P}{D} = \frac{Q}{D} \quad (13)$$

$$TC_1^m = A + c_h\left(\frac{(P-D)Q^2}{2P.D} + \frac{S_f.Q}{D}\right) + c_P.Q + c_m.t_m \quad (14)$$

The inventory level may go down below the safety stock if $t_m > \frac{Q}{D} - \frac{Q}{P}$; But shortages do not occur when the preventive operation ended before the inventory level reaches zero. For this situation, the manufacturer immediately starts the production process to increase inventory level to S_f (Figure 2(b)). So, we have:

$$T_2^m = \frac{Q}{P} + t_m + \left(\frac{Q}{P} + t_m - \frac{Q}{D}\right)\left(\frac{D}{P-D}\right) = \left(\frac{P}{P-D}\right)t_m \quad (15)$$

$$TC_2^m = A + c_h\left(\frac{(P-D)Q^2}{2P.D} + \frac{S_f.P.t_m}{P-D} - \frac{P(D.t_m-(P-D)Q/P)^2}{2D(P-D)}\right) + c_P\frac{P.D.t_m}{P-D} + c_m.t_m \quad (16)$$

If the preventive repair takes longer, as the inventory cannot cover the demand meet up, purchasing strategy is the only solution for the manufacturer to avoid shortages.

In this situation, depended on the supplier reliability and the preventive repair time, three scenarios may occur (Figures 2(c)-2(e)). The corresponding inventory period and total cost for each scenario are stated as follows:

$$T_3^m = \frac{Q}{P} + t_m + \frac{S_f}{P-D} \quad (17)$$

$$TC_3^m = A + c_h\left(\frac{(P-D)Q^2}{2P.D} + \frac{S_f.Q}{D} + \frac{P.S_f^2}{2D(P-D)}\right) + c_P\left(Q + \frac{P.S_f}{P-D}\right) + c_m.t_m + c_s\left(D.t_m - (P-D)\frac{Q}{P} - S_f\right) \quad (18)$$

and

$$T_4^m = \frac{Q}{D} + \frac{P.S_f}{D(P-D)} + \frac{Q'}{D} \quad (19)$$

$$TC_4^m = A + c_h\left(\frac{(P-D)Q^2}{2P.D} + \frac{S_f.Q}{D} + \frac{P.S_f^2}{2D(P-D)} + \frac{Q'^2}{2D}\right) + A' + c_P\left(Q + \frac{P.S_f}{P-D}\right) + c'.Q' + c_m.t_m \quad (20)$$

and

$$T_5^m = \frac{Q}{P} + t_m + \frac{S_f}{P-D} \quad (21)$$

$$TC_5^m = A + c_h\left(\frac{(P-D)Q^2}{2P.D} + \frac{S_f.Q}{D} + \frac{P.S_f^2}{2D(P-D)} + \frac{Q'^2}{2D}\right) + A' + c_P\left(Q + \frac{P.S_f}{P-D}\right) + c'.Q' + c_m.t_m + c_s\left(D.t_m - (P-D)\frac{Q}{P} - S_f\right) \quad (22)$$

By the conditional probability, the expected cycle time in no machine breakdown case is:

$$E(T^m) = \int_0^{\frac{(P-D)Q}{P.D}} T_1^m dH(t_m) + \int_{\frac{(P-D)Q}{P.D}}^{\frac{(P-D)Q}{P.D}+\frac{S_f}{D}} T_2^m dH(t_m) + \theta\left(\int_{\frac{(P-D)Q}{P.D}+\frac{S_f}{D}}^{\frac{(P-D)Q}{P.D}+\frac{S_f+Q'}{D}} T_4^m dH(t_m) + \int_{\frac{(P-D)Q}{P.D}+\frac{S_f+Q'}{D}}^{\infty} T_5^m dH(t_m)\right) + (1-\theta)\int_{\frac{(P-D)Q}{P.D}+\frac{S_f}{D}}^{\infty} T_3^m dH(t_m) \quad (23)$$

Similarly, the expected total cost for this case is given by the following expression:

$$E(TC^m) = \int_0^{\frac{(P-D)Q}{P.D}} TC_1^m dH(t_m) + \int_{\frac{(P-D)Q}{P.D}}^{\frac{(P-D)Q}{P.D}+\frac{S_f}{D}} TC_2^m dH(t_m) + \theta\left(\int_{\frac{(P-D)Q}{P.D}+\frac{S_f}{D}}^{\frac{(P-D)Q}{P.D}+\frac{S_f+Q'}{D}} TC_4^m dH(t_m) + \int_{\frac{(P-D)Q}{P.D}+\frac{S_f+Q'}{D}}^{\infty} TC_5^m dH(t_m)\right) + (1-\theta)\int_{\frac{(P-D)Q}{P.D}+\frac{S_f}{D}}^{\infty} TC_3^m dH(t_m) \quad (24)$$

5. 3. Integrated Model Now, according to the probability of occurrence of two conditions described in sections 5.1 and 5.2, the integrated model for the expected inventory cycle and the expected total cost is obtained as follows:

$$E(T) = \int_0^{\frac{Q}{P}} E(T^r)dF(t_w) + \int_{\frac{Q}{P}}^{\infty} E(T^m)dF(t_w) \quad (25)$$

$$E(TC) = \int_0^{\frac{Q}{P}} E(TC^r)dF(t_w) + \int_{\frac{Q}{P}}^{\infty} E(TC^m)dF(t_w) \quad (26)$$

We employ the renewal reward theorem [28] to optimize the expected total cost per unit time:

$$C(Q, Q', S_f) = \frac{E(TC)}{E(T)} \quad (27)$$

6. NUMERICAL EXPERIMENT

In this section, using a numerical example, we obtain the optimal value of decision variables and investigate the joint effects of safety stock, purchasing policy and repair operations on the optimal lot sizing decisions. We will also perform a sensitivity analysis on some important parameters related to the purchasing, failure and the repair mechanisms. Furthermore, we compare the simultaneous policy, the safety stock policy and the purchasing policy through sensitivity analysis. For this issue, we present the model with the safety stock policy and the purchasing policy, separately. The results of these policies are used as an index to evaluate the performance of the simultaneous policy. The model with the purchasing policy is similar to the model introduced by Peymankar et al. [6]. The difference is that we considered the effect of preventive maintenance. The corresponding functions related to the cycle time and the cycle cost for this policy are presented in Appendix A.

The safety stock policy has also been considered in numerous studies. We adopt the model proposed by Sana and Chaudhuri [2] and relax some minor assumption to be comparable with our model (e.g. the shortages will be backlogged after repair operation, whereas in our paper, shortages will be lost). The corresponding expected inventory period and expected total cost are formulated in Appendix B.

We first suppose that the random time to failure as well as repair times (corrective and preventive) have the exponential distributions with the probability density functions as follows:

$$f(t_w) = \lambda e^{-\lambda t_w},\ g(t_r) = \mu_r e^{-\mu_r t_r},\ h(t_m) = \mu_m e^{-\mu_m t_m}$$

where λ is the machine failure rate, $\frac{1}{\mu_r}$ and $\frac{1}{\mu_m}$ are respectively the meantime for corrective and preventive repair.

Given that we have three decision variables in the proposed model, we have used a comprehensive search algorithm with numerical computational software MATHEMATICA to find the optimal values. To consider the performance of the model, the following relevant data with appropriate units are set on the parameters:

$P = 600, D = 400, A = 100, A' = 120, c_P = 20, c' = 25, c_h = 4, c_s = 40, c_r = 80, \mu_m = 2.4$

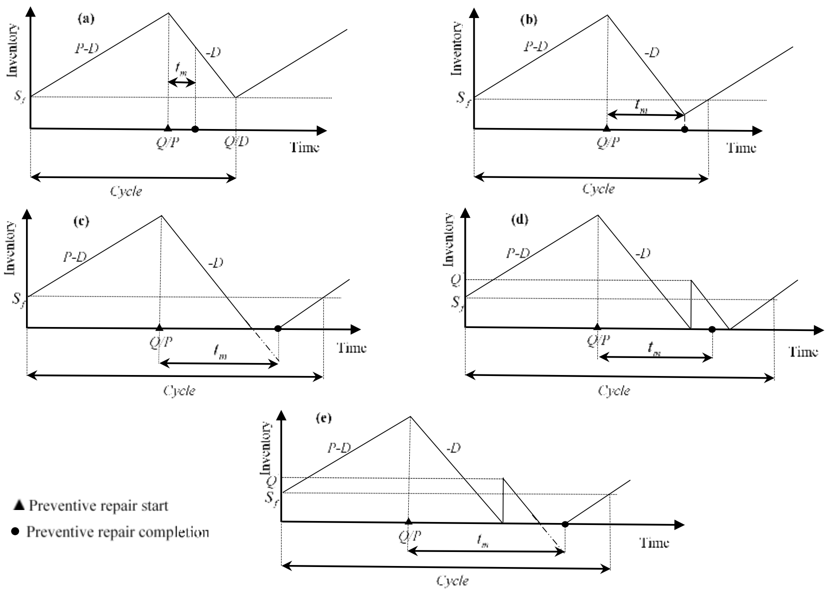

Figure 2. Configuration of model without machine breakdown case when (a) on-hand inventory are sufficient to meet up demand during preventive repair operation; (b) safety stocks are sufficient to meet up demand during preventive repair operation; (c) orders not satisfied with probability *1-θ* and shortages occurs due to prolonged preventive repair time; (d) orders satisfied with probability *θ* and preventive repair operation done before finishing purchasing items; (e) orders satisfied, but shortages occurs due to prolonged preventive repair time

By optimizing the proposed model, the optimal value of decision variables is $Q^* = 615.9$, $S_f^* = 90.9$, $Q'^* = 273.9$ and the minimum value of $C^*(Q, Q', S_f)$ is obtained as 9217.1. Comparison between policies demonstrates that the result of the simultaneous policy is better than two other policies (in the safety stock policy $C^*(Q, S_f) = 9501.1$ and in the purchasing policy $C^*(Q, Q') = 9266.7$).

Now, we examine the sensitivity of optimal values of decision variables against changes in some important parameters. As shown in Figure 3 when the holding cost increases, the manufacturer prefers to respond demand met up through the produced and the purchased quantity; so that, for $c_h > 15$ the simultaneous policy is equivalent to the purchasing policy. This can be justified from the managerial point of view. Higher holding cost increases the cost of holding safety stock to deal with potential lost sales during a cycle time. In this situation, the production manager prefers to consider an external supplier with acceptable reliability to meet the demand during the shortage period.

The supplier parameters (θ and c') have an important role on the manufacture decisions for the optimal design of the system. In the following, we have numerous analysis with regard to the supplier parameters and the manufacturer parameters. The variation of θ illustrated in Table 1. The table shows that the production batch size and the order quantity in simultaneous policy is lower than the purchasing policy, because of the safety stock usage in the simultaneous policy. When the supplier is less reliable, in addition to increasing production lot size, the model raises the order quantity to confront uncertainties. In other words, by accumulating more inventory and make a balance between holding cost and shortage cost, the model tries to avoid lost sales. As Table 1 depicts, the increasing rate of the production and purchasing lot in simultaneous policy is slower. Indeed, the existence of the safety stock can help the production manager to attain a lower expected cost with fewer changes in production lot size.

A comparison between three policies by variation of c_P when the supplier is more reliable is shown in Table 2. It is obvious that the keeping safety stock is preferable

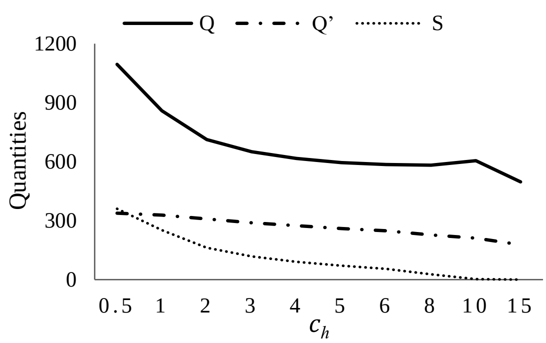

Figure 3. Optimal value of decision variables verses holding cost

TABLE 1. Q'^* and C^* when θ changes in simultaneous and purchasing policy

θ	*Simultaneous policy*				*Purchasing policy*		
	Q^*	Q'^*	S_f^*	$C^*(Q, Q', S_f)$	Q^*	Q'^*	$C^*(Q, Q')$
0.1	633.8	288.7	131.2	9434.1	1177.4	309.9	9566.8
0.2	631.4	286.8	126.5	9408.6	1151.8	307.1	9528.9
0.3	629.1	285	121.8	9382.7	1126.4	304.3	9491.2
0.4	626.7	283.3	116.9	9356.4	1101	301.5	9453.6
0.5	624.5	281.4	112	9329.6	1075.6	298.7	9416.12
0.6	622.2	279.6	107	9302.3	1050.2	296	9378.7
0.7	620	277.7	101.7	9274.4	1024.7	293.2	9341.3
0.8	617.9	275.8	96.4	9246	999.2	290.4	9304.2
0.9	615.9	273.9	90.9	9217.1	973.5	287.7	9266.7
1	614.2	272	85.2	9187.4	947.6	284.9	9229.4

TABLE 2. A comparison of performance of three policies under the variation of c_P when $\theta=0.9$

c_P	*Simultaneous policy*				*Safety stock policy*			*Purchasing policy*		
	Q^*	Q'^*	S_f^*	$C^*(Q, Q', S_f)$	Q^*	S_f^*	$C^*(Q, S_f)$	Q^*	Q'^*	$C^*(Q, Q')$
5	662.6	129	195	3783.5	661.2	208.6	3853.6	1567	142.8	4140.9
10	653.2	157.7	168.6	5640.4	653.9	187.3	5738.5	1394.6	172.6	5886.5
15	639.6	201.4	135.9	7462.1	645.6	163.5	7608.6	1204.6	217.3	7603.1
20	615.9	273.9	90.9	9217.07	636.2	135.7	9459.1	973.5	287.7	9266.7
25	575.6	397.2	8.1	10801.7	627.3	102.2	11281.7	606.9	398.7	10801.8
30	0	437	0	11415.3	623.3	56.8	13060.2	0	437	11415.3

for the manufacturer for low values of c_P. As c_P decreases, the optimal value of Q^* and S_f^* increases and the manufacturer ordered a smaller quantity to the external supplier whereas by increasing c_P, for higher reliability of the supplier when the production unit cost becomes greater than the purchasing unit cost, the optimal quantity of Q^* and S_f^* are zero and the system act as an inventory system. This is completely justifiable from the managerial insight because when the production unit cost exceeds the sales price, the production system is not economic; otherwise, when the reliability of the supplier decreases, despite increasing the production unit cost, the model suggests the manufacturer has a production lot size to avoid facing lost sales. Figure 4 confirms this issue. In fact, the superiority of the simultaneous policy comparing to other two policies are that by decreasing of c_P, the manufacturer can choose the production as the basis of the inventory decision, whereas, for high values of c_P, he/she switches to outside purchasing option. This is why the expected cost in the simultaneous policy is always less than two other policies (see Table 2).

Table 3 illustrated the effect of c' variation on production and inventory decisions. As the table shows, changes in c' value in the opposite direction of c_P variations, affect the expected total cost. As the purchasing unit cost decrease, the manufacturer decreases production lot size and buys a bigger lot size from the market.

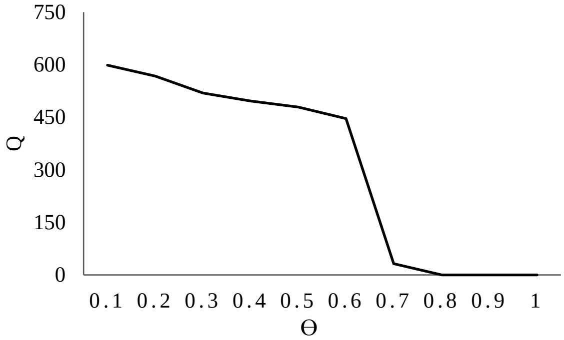

Figure 4. Manufacturer decision for using production by variation of θ when $c_P = 30$ and $c' = 25$

TABLE 3. A comparison of performance of simultaneous policy and purchasing policy under the variation of c'

c'	Simultaneous policy				Purchasing policy		
	Q^*	Q'^*	S_f^*	$C^*(Q,Q',S_f)$	Q^*	Q'^*	$C^*(Q,Q')$
15	0	508.7	0	7640	0	508.7	7640
20	526.9	436.8	33.2	8868.3	657	444.2	8871.3
25	615.9	273.9	90.9	9217.07	973.5	287.7	9266.7
30	636.5	153.2	120.2	9379.7	1125.7	164.5	9485.2
35	642.2	65.7	134	9454	1199	71.5	9595.3
40	643.2	0	137.9	9475	1220	0	9627.5

When the purchasing unit cost becomes smaller than the production unit cost, Q^* and S_f^* will be null and both policies will yield the same result; while, by increasing sales price, the purchasing quantity decreases. When sales price becomes bigger than the shortage cost, purchasing from the external supplier is not beneficial. in this situation, for the simultaneous policy, the manufacturer can optimize the system total cost by determining the values of S_f^* and Q^*, but in purchasing policy he/she does not have the advantage of using safety stock and is forced to increase the production lot size.

Sensitivity analysis based on the change in unit shortage cost is depicted in Table 4 and Figure 5. Although the purchasing strategy raising the manufacturer service level through the reducing lost sales; but applying this strategy completely depend on the importance of the customer retention for the manufacturer. This value is determined through the cost of lost sale. Theoretically, by decreasing the unit shortage cost, the model prefers lost sales. In simultaneous policy, as c_s decreases, the model at first reduces the amount of Q'^* and S_f^*, in return, increases Q^*. By continuing drop rate, if the purchasing lot size and stock level reach to zero, the production lot size will be decreased. The behavior of the model can be interpreted as follows: By decreasing the lost sale cost, the accumulated inventory required to meet the demand is reduced. In this situation, the manufacturer decides to reduce the implementation of purchasing and keeping safety stocks and respond to demand by using more production. Therefore, he/she reduces the amount of Q'^* and S_f^* at higher rates and instead increases Q^* with a slight rate. It is obvious that this strategy has a better performance on reducing the expected total cost than the concurrent reduction of all of the decision variables in the purchasing policy (see Table 4).

Moreover, as illustrated in Figure 5 by decreasing the value of shortage cost, the model prefers to face more lost sale. Therefore, the expected shortage cost will be increased. Due to the expected total cost reduction (Table 4), the percentage of shortage cost in total cost, increases in both policies. From the managerial insight, based on the comparison between the lost sale cost and production, purchasing and holding costs, the less the lost sale cost, the more beneficial for the manufacturer to reduce the accumulated inventory level.

TABLE 4. A comparison of performance of simultaneous policy and purchasing policy under the variation of c_s

c_s	Simultaneous policy				Purchasing policy		
	Q^*	Q'^*	S_f^*	$C^*(Q,Q',S_f)$	Q^*	Q'^*	$C^*(Q,Q')$
20	232	0	0	8320.96	232	0	8320.96
25	635.7	0.017	0.018	8790.38	635.7	0.0002	8790.38
30	636.1	139.6	46	9016.8	809.5	143.2	9027.4
35	621.2	218.6	73.4	9135.06	904.8	227.8	9165.42
40	615.9	273.9	90.9	9217.07	973.5	287.7	9266.7
45	613.9	316.7	103.8	9280.84	1028.4	344.4	9348.57
50	613.4	351.8	114.3	9333.6	1075.1	372.7	9418.31
60	614.4	407.3	130.8	9418.9	1152.8	433.6	9535.1
80	618.9	486.5	154.7	9546.13	1275	520.4	9718.7

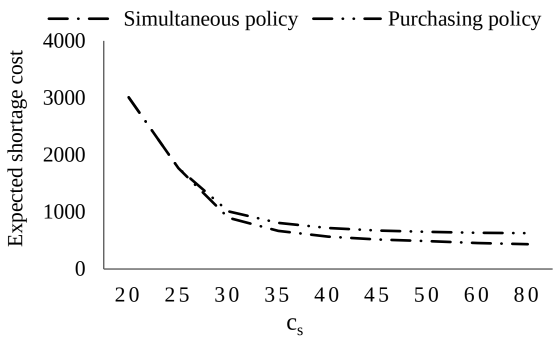

Figure. 5. Variation of expected shortage cost per unit time with respect to c_s

The impact of λ variations on the optimal value of decision variables is shown in Table 5. Studying this table gives remarkable results. As failure rate increases, due to the rising probability of machine failure with increasing production quantity, the model decreases Q^* and instead increases Q'^* and S_f^*. Whereas, when the failure rate decreases, the model first reduces safety stock (due to its high holding cost) and replace it by growth in the production quantity. If the safety stock level reaches zero, with decreasing failure rate, the model reduces the optimal quantity of Q^* and Q'^* to optimize systems expected cost. The performance of the safety stock policy and the purchasing policy, are similar to the simultaneous policy in this situation. From the manufacturer point of view, although keeping the safety sock is more reliable than outside purchasing, decreasing in the safety stock level is more beneficial than the reduction in purchasing quantity. This is because of the fact by decreasing the probability of machine failure, the risk of shortages is decreased and holding safety stocks incur more cost.

Variation of corrective repair time effects on the optimal values of decision variables is depicted in Figure 6. According to this figure, as μ_r decreases (the mean time for the correcting repair increases), the manufacturer decreases the optimal production lot size and prefers to order a more quantity of Q' from the supplier as well as rises the safety stock level. In return, when the repair time decreases, the production lot size increases and by declining the safety stock level to zero, the model changes to the purchasing policy. Therefore, when the corrective repair time decreases, the possibility to meet the demand using on-hand inventory, increases. So, the manufacturer prefers to use purchasing to avoid possible shortages, instead of keeping the safety stock with higher costs in the long term.

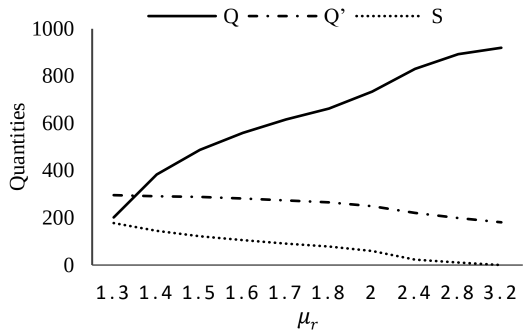

Figure 6. Variation of optimal values of decision variables with respect to μ_r

TABLE 5. Behavior of decision variables by changing machine failure rate in all policies

λ	***Simultaneous policy***				***Safety stock policy***			***Purchasing policy***		
	Q^*	Q'^*	S_f^*	$C^*(Q,Q',S_f)$	Q^*	S_f^*	$C^*(Q,S_f)$	Q^*	Q'^*	$C^*(Q,Q')$
0.1	839.8	232.1	0	8871.3	964.8	0	8991.7	839.8	232.1	8871.3
0.3	890.4	261.5	0	9036.6	1048.5	0	9239.8	890.4	261.5	9036.6
0.4	916.5	271.7	0.2	9115.5	877.9	59.9	9343.6	917.2	271.7	9115.5
0.5	733	272.9	56.7	9176.3	734.2	105.8	9410.6	944.8	280.3	9192.1
0.6	615.9	273.9	90.9	9217.1	636.2	135.7	9459.1	973.5	287.7	9266.7
0.7	533	274.8	113.9	9247.4	562.8	157.1	9497	1003	294.2	9339.3
0.8	469.5	275.5	130.6	9271.3	503.7	173.3	9527.9	1033.5	300.2	9410
0.9	417.3	275.9	143.6	9291	453.3	186.2	9553.8	1064.9	305.7	9478.8
1.2	292.5	278.1	170	9334	324.9	213.5	9611.8	1163.2	320.6	9674.1

6. CONCLUTION

The classical economic production models assume that the production facilities always are failure-free. However, in practical situations, they usually are failure-prone. Since failures are unavoidable, the production manager should have practical solutions to deal with such disruptions. Proactive measures such as inspection, keeping safety stock and recently outside supplying have been carried out in this field to mitigate machine breakdowns consequences. In this study, we presented a new strategy and considered the joint implementation of keeping safety stock and outside supplying on a failure prone production-inventory system. We also incorporate preventive maintenance, as an option to confront machine failure. The superiority of our model to the existing models in the literature is the manufacturer's capability to simultaneous use of the safety stock and outside purchasing. Actually, by changing the production conditions and increasing or decreasing the model parameters, the manufacturer has the flexibility to choose the optimal values of safety stock and order quantity to optimize the expected total cost of the system. For evaluating the performance of the proposed model, we have compared our simultaneous policy with the model with just purchasing policy or the safety stock policy, separately. The analysis conducted shows that the simultaneous policy always imposes a less expected cost to the system than two other policies. For example, the manufacturer can benefit from the safety stock for less expensive items; while for more expensive items he/she can choose the purchasing policy as the basis to cope with shortages during production disruption. As a future research, studying on the effects of the supplier reliability or lead-time on the purchasing price could be developed. Furthermore, effect of carbon emission costs on the expected total cost and the effect of collaborating with an external supplier to reduce it could be interesting for the future research.

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APPENDIX

Appendix A. Formulation of *E (T)* and *E (TC)* for purchasing policy

$$E(T)=\int_0^{\frac{Q}{P}}\left\{\int_0^{\frac{(P-D)t_w}{D}}\frac{P.t_w}{D}dG(t_r)+(1-\theta)\int_{\frac{(P-D)t_w}{D}}^{\infty}(t_w+t_r)dG(t_r)+\theta\left(\int_{\frac{(P-D)t_w}{D}}^{\frac{(P-D)t_w+Q'}{D}}\left(\frac{P.t_w+Q'}{D}\right)dG(t_r)+\int_{\frac{(P-D)t_w+Q'}{D}}^{\infty}(t_w+t_r)dG(t_r)\right)\right\}dF(t_w)+\int_{\frac{Q}{P}}^{\infty}\left\{\int_0^{\frac{(P-D)Q}{P.D}}\frac{Q}{D}dH(t_m)+(1-\theta)\int_{\frac{(P-D)Q}{P.D}}^{\infty}\left(\frac{Q}{P}+t_m\right)dH(t_m)+\theta\left(\int_{\frac{(P-D)Q}{P.D}}^{\frac{(P-D)Q}{P.D}+\frac{Q'}{D}}\left(\frac{Q+Q'}{D}\right)dH(t_m)+\int_{\frac{(P-D)Q}{P.D}+\frac{Q'}{D}}^{\infty}\left(\frac{Q}{P}+t_m\right)dH(t_m)\right)\right\}dF(t_w) \quad \text{(A-1)}$$

$$E(TC)=A+\int_0^{\frac{Q}{P}}\left\{\int_0^{\frac{(P-D)t_w}{D}}\left(c_h\left(\frac{(P-D)P.t_w^2}{2D}\right)+c_P.P.t_w+c_r.t_r\right)dG(t_r)+(1-\theta)\int_{\frac{(P-D)t_w}{D}}^{\infty}\left(c_h\left(\frac{(P-D)P.t_w^2}{2D}\right)+c_P.P.t_w+c_r.t_r+c_s(D.t_r-(P-D)t_w)\right)dG(t_r)+\theta\left(\int_{\frac{(P-D)t_w}{D}}^{\frac{(P-D)t_w+Q'}{D}}\left(c_h\left(\frac{(P-D)P.t_w^2}{2D}+\frac{Q'^2}{2D}\right)+c_P.P.t_w+c_r.t_r+A'+c'.Q'\right)dG(t_r)+\int_{\frac{(P-D)t_w+Q'}{D}}^{\infty}\left(c_h\left(\frac{(P-D)P.t_w^2}{2D}+\frac{Q'^2}{2D}\right)+c_P.P.t_w+c_r.t_r+A'+c'.Q'+c_s(D.t_r-(P-D)t_w-Q')\right)dG(t_r)\right)\right\}dF(t_w)+\int_{\frac{Q}{P}}^{\infty}\left\{\int_0^{\frac{(P-D)Q}{P.D}}\left(c_h\left(\frac{(P-D)Q^2}{2P.D}\right)+c_P.Q+c_m.t_m\right)dH(t_m)+(1-\theta)\int_{\frac{(P-D)Q}{P.D}}^{\infty}\left(c_h\left(\frac{(P-D)Q^2}{2P.D}\right)+c_P.Q+c_m.t_m+c_s\left(D.t_m-(P-D)\frac{Q}{P}\right)\right)dH(t_m)+\theta\left(\int_{\frac{(P-D)Q}{P.D}}^{\frac{(P-D)Q}{P.D}+\frac{Q'}{D}}\left(c_h\left(\frac{(P-D)Q^2}{2P.D}+\frac{Q'^2}{2D}\right)+c_P.Q+c_m.t_m+A'+c'.Q'\right)dH(t_m)+\int_{\frac{(P-D)Q}{P.D}+\frac{Q'}{D}}^{\infty}\left(c_h\left(\frac{(P-D)Q^2}{2P.D}+\frac{Q'^2}{2D}\right)+c_P.Q+c_m.t_m+A'+c'.Q'+c_s\left(D.t_m-(P-D)\frac{Q}{P}-Q'\right)\right)dH(t_m)\right)\right\}dF(t_w) \quad \text{(A-2)}$$

Appendix B. Formulation of E (T) and E (TC) for safety stock policy

$$E(T)=\int_0^{\frac{Q}{P}}\left\{\int_0^{\frac{(P-D)t_w}{D}}\frac{P.t_w}{D}dG(t_r)+\left(\frac{P}{P-D}\right)\int_{\frac{(P-D)t_w}{D}}^{\frac{(P-D)t_w+S_f}{D}}t_r dG(t_r)+\int_{\frac{(P-D)t_w+S_f}{D}}^{\infty}\left(t_w+t_r+\frac{S_f}{P-D}\right)dG(t_r)\right\}dF(t_w)+\int_{\frac{Q}{P}}^{\infty}\left\{\int_0^{\frac{(P-D)Q}{P.D}}\frac{Q}{D}dH(t_m)+\left(\frac{P}{P-D}\right)\int_{\frac{(P-D)Q}{P.D}}^{\frac{(P-D)Q}{P.D}+\frac{S_f}{D}}t_m dH(t_m)+\int_{\frac{(P-D)Q}{P.D}+\frac{S_f}{D}}^{\infty}\left(\frac{Q}{P}+t_m+\frac{S_f}{P-D}\right)dH(t_m)\right\}dF(t_w) \quad \text{(B-1)}$$

$$
\begin{aligned}
E(TC) = {} & A + \int_0^{\frac{Q}{P}} \Bigg\{ \int_0^{\frac{(P-D)t_w}{D}} \left(c_h \left(\frac{(P-D)P.t_w^2}{2D} + \frac{S_f.P.t_w}{D} \right) + c_P.P.t_w + c_r.t_r \right) dG(t_r) + \int_{\frac{(P-D)t_w}{D}}^{\frac{(P-D)t_w+S_f}{D}} \Bigg(c_h \Bigg(\frac{(P-D)P.t_w^2}{2D} + \frac{S_f.P.t_r}{D} + \\
& \frac{P(D.t_r-(P-D)t_w)^2}{2D(P-D)} \Bigg) + \frac{c_P.P.D.t_r}{P-D} + c_r.t_r \Bigg) dG(t_r) + \int_{\frac{(P-D)t_w+S_f}{D}}^{\infty} \left(c_h \left(\frac{(P-D)P.t_w^2}{2D} + \frac{S_f.P.t_w}{D} + \frac{S_f^2.P}{2D(P-D)} \right) + c_P \left(P.t_w + \frac{S_f.P}{P-D} \right) + \right. \\
& \left. c_r.t_r + c_s(D.t_r - (P-D)t_w - S_f) \right) dG(t_r) \Bigg\} dF(t_w) + \int_{\frac{Q}{P}}^{\infty} \Bigg\{ \int_0^{\frac{(P-D)Q}{P.D}} \left(c_h \left(\frac{(P-D)Q^2}{2P.D} + \frac{S_f.Q}{D} \right) + c_P.Q + c_m.t_m \right) dH(t_m) + \\
& \int_{\frac{(P-D)Q}{P.D}}^{\frac{(P-D)Q}{P.D}+\frac{S_f}{D}} \left(c_h \left(\frac{(P-D)Q^2}{2P.D} + \frac{S_f.P.t_m}{D} + \frac{(P.D.t_m-(P-D)Q)^2}{2P.D(P-D)} \right) + \frac{c_P.P.D.t_m}{P-D} + c_m.t_m \right) dH(t_m) + \int_{\frac{(P-D)Q}{P.D}+\frac{S_f}{D}}^{\infty} \Bigg(c_h \Bigg(\frac{(P-D)Q^2}{2P.D} + \frac{S_f.Q}{D} + \\
& \frac{S_f^2.P}{2D(P-D)} \Bigg) + c_P \left(Q + \frac{S_f.P}{P-D} \right) + c_m.t_m + c_s\left(D.t_m - (P-D)\frac{Q}{P} - S_f\right) \Bigg) dH(t_m) \Bigg\} dF(t_w)
\end{aligned}
\tag{B-2}
$$

The Simultaneous Effect of Holding Safety Stock and Purchasing Policies on the Economic Production Quantity Model Subject to Random Machine Breakdown

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چکیده

در این مقاله، یک مدل تولید اقتصادی تحت شرایط خرابی ماشین و دو نوع تعمیر اضطراری و پیشگیرانه ارائه شده و تاثیر همزمان نگهداری موجودی اطمینان و تامین از بیرون بر روی مسئله مورد مطالعه قرار گرفته است. جهت مقابله با کمبود منتج از طولانی شدن زمان تعمیر فرض شده است که علاوه بر نگهداری موجودی اطمینان، تولیدد کننده، شرایط خرید از یک تامین‌کننده خارجی را نیز دارا می‌باشد. مطالعه پیش رو به این سوال پاسخ می‌دهد که: تولیدکننده چطور می‌تواند با تعیین مقادیر بهینه انباشته تولیدی، سفارش خرید و موجودی اطمینان، هزینه مورد انتظار سیستم را کمینه کند. مدل معرفی شده با شرایطی که تولید کننده تنها قادر به استفاده از موجودی اطمینان یا تامین از بیرون می‌باشد، بطور جداگانه مقایسه شده است. نتایج آنالیز حساسیت نشان می‌دهد که استفاده از سیاست همزمان هنگامی که سیستم مستعد مواجهه با کمبود است، بهبود بیشتری بر عملکرد سیستم نسبت به دو سیاست دیگر دارد.

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