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Finite Time Terminal Synergetic Controller for Nonlinear Helicopter Model

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ABSTRACT

In this paper, an almost new control approach called terminal synergetic control which works based on user defined manifold is applied to a nonlinear helicopter model. Stability analysis is convestigated using Lyapunov stability theory. Synergetic controller is applied to this nonlinear fifth-order helicopter model to control height and angle. Simulation results showed that it has faster and smoother response in tracking reference inputs in comparison to sliding mode control, intelligent autopilot controller and feedback linearization method.

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1. INTRODUCTION

As we know, there are diverse aircrafts having miscellaneous potencies which are used for information gathering, picturing traffic control, military services, emergencies in accidents, mine detection, etc. [1].

Today, helicopter, because of vertical landing, take off and moving forward no need to any runway, has attracted many attention. The word “helicopter” is adapted from a French word, which was attributed by *Gustave Amecourt*, that is composed of Greek words helix/heli meaning twister and petron which means wing [2].

Since these approaches are nonlinear, perturbed by uncertainty, design of controllers for these systems is of challenges. In the current manuscript, the focus is on recently developed approach called, synergetic theory-which was first introduced by Russian scientist, Kolesnikov [3].

Synergetic control theory provides several advantages, such as; it is suitable for digital implementations, and it also operates at constant switching frequency. Furthermore the chattering problem as in sliding mode control is cured [4].

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Because of multivariable nonlinear dynamics of helicopter systems, many of existing results till now is based on model linearization or multiple linearization techniques. These linearized results just provide local stability and will be unstable with deviation or nonstructural uncertainties. Two robust stabilization methods of the helicopter are given in literature [5], which states that Lyapunov based stabilization, has better performance in comparison to H_∞ robust stabilization [6]. Adaptive and linear matrix inequality (LMI) techniques are used in literature to control nonlinear systems. Because of uncertainty specifications in these systems, using robust controllers are essential [7]. Linearized helicopter model has un-modelled dynamics which makes proposed methods unreliable when applied to main nonlinear models. Nonlinear techniques give better response and improve the controlled system performance. Some nonlinear methods like sliding mode and robust control has been applied to this model, earlier [8]. Also intelligent autopilot control has been applied to this model of helicopter in 2008, which is considered in this paper [8]. The intelligent autopilot control method which is based on brain signal processing, had better performance other than previous methods such as feedback linearization; but yet had oscillations in transition from initial state to desired value and need appropriate time to converge.

In this paper synergetic terminal control is applied to nonlinear model of helicopter. This method is almost a new control approach in control science extent and works based on designer defined manifold. There is a complicated fifth-order nonlinear state space model of the system. The system has two inputs and two outputs. The control aim, is tracking desired signal by system's output. Simulation results showed that it has better speed and precision in comparison to previous applied methods.

This paper is organized as follows: Main section describes fifth-order nonlinear helicopter model and includes basics of control synergetic method. The model also apply terminal synergetic method and the controller designed for this helicopter model. Simulation results are presented in the next section. Finally we have conclusion at the end.

2. MAIN SECTION

A helicopter control system is a multi-input, multi-output system. The height of helicopter changes based on pitch angle. Regardless of gravity effect, helicopter moving equation can be described as follows [9]:

$$\dot{x} = f(x) + g_1(x)u_1 + g_2(x)u_2$$
$$y = [y_1 y_2]^T = [x_1 x_4]^T \quad (1)$$

Stated variables are considered as follows:

$$x = [x_1 x_2 x_3 x_4 x_5]^T = [h\ \dot{h}\ \omega\ \theta\ \dot{\theta}]^T \quad (2)$$

That h stands for height, ω, θ represent rotor blade rotary speed and rotor blade collective pitch, respectively. u_1 is considered as gas control input and u_2 is collective control input. Therefore the main equation of helicopter can be stated as follows:

$$\dot{x_1} = x_2$$
$$\dot{x_2} = f_2 = a_0 + a_1 x_2 + a_2 x_2^2 + (a_3 + a_4 x_4 - \sqrt{a_5 + a_6 x_4})x_3^2$$
$$\dot{x_3} = f_3 + u_1 = a_7 + a_8 x_3$$
$$+(a_9 \sin(x_4) + a_{10})x_3^2 + u_1 \quad (3)$$
$$\dot{x_4} = f_4 = x_5$$
$$\dot{x_5} = f_5 + u_2 = a_{11} + a_{12}x_4 + a_{13}x_3^2 \sin(x_4) + a_{14}x_5 + u_2$$

In next section, synergetic controller is applied to the multi input multi output nonlinear model, to control angle and height, without any linearization and results will be discussed.

2. 1. Synergetic Controller

First, in this section synergetic control basics are specified, and then synergetic controller is designed and simulated for two input two output helicopter system.

Generally, in synergetic control theory optimal controllers are suggested in systems such that controllers are designed based on internal expectations. The produced control system has attracting areas which are corresponded to control aims. In this way, attractors are constructed in nonlinear equations roots, which makes this algorithm to converge and find system`s roots [10].

Synergetic control theory presents an equation which can be used to make dynamical systems with attractors at $\psi_i = 0$ [10]:

$$T\dot{\psi} + \psi = 0 \quad (4)$$

T, presents convergence speed, $\dot{\psi}$ is time derivative of macro variable and ψ is some functions which affect reaching to the attractor.

2. 2. Synergetic Controller Design For Helicopter

Because of helicopter`s general Equation (3), macro variables here are stated as follows:

$$\psi_1 = c_{11}e_1 + c_{12}\dot{e_1} + \ddot{e_1},$$
$$\psi_2 = c_{21}e_2 + \dot{e_2} \quad (5)$$

Which $e_1 = r_1 - x_1$ and, are desired r_1, r_2. $e_2 = r_2 - x_4$ height and angle and $c_{i,j}$s are positive integers. Now, calculating control signals, u_1, u_2, each of chosen macro variables are placed in dynamical Equation (4). Assuming:

$$A = a_1 f_2 + 2a_2 f_2 x_2$$
$$+\left(a_4 x_5 - \frac{1}{2}a_6 x_5 \frac{1}{\sqrt{a_5 + a_6 x_4}}\right)x_3^2$$
$$+2x_3(f_3)\left(a_3 + a_4 x_4 - \sqrt{a_5 + a_6 x_4}\right) \quad (6)$$
$$B = T_1 2x_3\left(a_3 + a_4 x_4 - \sqrt{a_5 + a_6 x_4}\right)$$

Control signal u_1, u_2 is calculated as follows:

$$u_1 = \frac{T_1 c_{11}\dot{e_1} + T_1 c_{12}\ddot{e_1} - T_1 A + c_{11}e_1 + c_{12}\dot{e_1} + \ddot{e_1}}{B}$$
$$u_2 = \frac{(-T_2 c_{21} x_5 - T_2 f_5) + c_{21}e_2 + \dot{e_2}}{T_2} \quad (7)$$

2. 3. Synergetic Terminal Control

As mentioned earlier in Equation (4), T determines convergence rate to attractor, choosing appropriate T, one can increase transient speed of processes but to have finite convergence, some other conditions is needed that can be provided by finite time terminal controller. Because of using linear manifold in synergetic controller, when initial condition of system is far from equilibrium point, convergence time of reaching reference value may not be desired. In terminal synergetic surface, exponential nonlinear phrase $x^{\frac{q}{p}}$ enters the considered surface, which can reach system states from any initial conditions to equilibrium and keep it there by proper adjusting of parameters. The system on the sliding surface has desirable performance such as stability and the ability of disturbance rejection [11]. Also it has a great robustness in presence of disturbances but it lacks the chattering phenomena [12].

Terminal sliding mode control concept first introduced by Venkatarman et al. [13]. Among several control strategies, the sliding mode control (SMC) is one of the most popular techniques in the control research area. A main advantage that lead scientists to study or apply this control method in various fields are robustness and the simplicity of use. Due to occurring of chattering phenomenon which is in cause by the switching control nature, the use of this method is restricted. Several solutions to reduce the chattering problem have been suggested such as a full-order sliding mode control [14], an adaptive sliding mode control [15] and adaptive fuzzy sliding mode control [16]. Now a chattering free technique is developed as synergetic control technique. The synergetic control was developed by Kolesnikov [3]. Both synergetic control and SMC have the similar principle to drive the system to operate towards the designed manifold. The synergetic control is presented as a solution to avoid the chattering problem, whilst provide simplicity and more flexible analytical design approach.

Despite common sliding mode control, terminal sliding manifold is defined as: $S = \dot{x} + \beta x^{\frac{a}{b}}$. which S is terminal sliding function and x is system`s state. β is a positive constant, a, b, are two positive integers which satisfies $1 < \frac{a}{b} < 2$.

Terminal synergetic method adds nonlinear functions to macro variable design and a terminal synergetic manifold is created and tracking errors on manifold will converge to zero in finite time. Consider a nonlinear system as follows [17]:

$$x^{(n)} = f(x) + g(x)u\,,\; y = x \tag{8}$$

Terminal synergetic method can force system states to converge to zero in finite time. Asymptotic convergence of states under normal synergetic method is obtained. When initial states are far from origin, in order to increase convergence rate of states to origin, necessity of terminal synergetic controller appears.

Theorem: A system with following structure using terminal synergetic control law (10) is stable.

Proof: consider a system as:

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= f(x) + g(x)u \end{aligned} \tag{9}$$

and the control synergetic terminal control is considered as:

$$u = (-Tg(x))^{-1}[Tf(x) + T\beta\frac{a}{b}x_1^{\frac{a}{b}-1}x_2 + x_2 + \beta x_1^{\frac{a}{b}}] \tag{10}$$

The macro variable ψ in this way is chosen as:

$$\psi = x_2 + \beta x_1^{\frac{a}{b}} \tag{11}$$

Lyapunov candidate function is considered as:

$$\begin{aligned} V &= \frac{1}{2}\psi^2 \;\rightarrow \dot{V} = \psi\dot{\psi} \\ &= \psi[f + \left(-f - \beta\frac{a}{b}x_1^{\frac{a}{b}-1}x_2 - \frac{1}{T}\psi\right) + \beta\frac{a}{b}x_1^{\frac{a}{b}-1}x_2] \end{aligned} \tag{12}$$

As a result derivative of Lyapunov function is as follows:

$$\dot{V} = \psi\dot{\psi} = -\frac{1}{T}\psi^2 < 0 \tag{13}$$

Therefore, terminal synergetic stability because of negative derivative of Lyapunov function is guaranteed.

The Lyapunov candidate is expressed in term of ψ. The Lyapunov stability criteria provides global stability (of course in term of the macro variable, i.e. ψ, here). However, the proposed finite time technique provides an upper convergence time, which is shown as follows:

The concept of Terminal Synergetic Control can be described by the following first order dynamics:

$$\psi = \dot{x}_1 + \beta x_1^{\frac{q}{p}} \tag{14}$$

where x_1 is a scalar variable and $p > q$ and $\beta > 0$ are positive integers. Parameters p and q must be prime odd numbers.

$$\dot{x}_1 = -\beta x_1^{\frac{q}{p}} \tag{15}$$

With the initial state $x_1(0) \neq 0$ the system dynamics in the finite time will reach $x_1 = 0$. The required time to reach the initial state $x_1(0)$ to zero, t_s, is specified by the following equation:

$$\begin{aligned} dt &= -\frac{1}{\beta}x_1^{\frac{-q}{p}}dx_1 \rightarrow \int_0^t dt = \\ &\int_{x_0}^{0} -\frac{1}{\beta}x_1^{\frac{-q}{p}}dx_1 \\ t_s &= \frac{p}{\beta(p-q)}(x_1(0))^{\frac{p-q}{p}} \end{aligned} \tag{16}$$

It is also proved that the zero equilibrium point is an attractor, i.e. when the state x_1 reaches 0, it remains in zero forever. This can be demonstrated by assuming a Lyapunov function $V = \frac{1}{2}x_1^2$ where the time derivative of V:is as:

$$\dot{V} = x_1\dot{x}_1 = -\beta x_1 \dot{x}_1^{\frac{q}{p}} = -\beta x_1^{\frac{p+q}{p}} \tag{17}$$

Since $p + q$ is even, $\dot{V}$ is negative definite and thus $x_1 = 0$ is terminally stable. The arrival time, t_s depends on the parameters p, q and β and the initial value $x_1(0)$. Since $x_1(0)$ is fixed or known in a limited area, it is possible to choose β such that t_s is too small.

Since, in helicopter system, time has great importance and smooth control and also fast and precise transition from initial state to desired state is very attractive, height and angle tracking of helicopter is calculated and simulated using terminal synergetic method.

Calculating $u_1$for first manifold as a combination of state variables 1, 3 is considered as follows:

$$\psi_1 = x_1 + \beta x_3^{\frac{q_1}{p_1}} \tag{18}$$

Now replacing in main synergetic equation, u_1 is calculated as follows:

$$u_1 = -\frac{p_1}{\beta q_1} x_2 x_3^{1-\frac{q_1}{p_1}} - f_3 - \frac{p_1}{T_1 \beta q_1} x_2 x_3^{1-\frac{q_1}{p_1}} - \frac{p_1}{T_1 q_1} x_3 \tag{19}$$

Similarly, choosing second manifold as a combination of x_4, x_5 states and replacing in main equation, we have:

$$u_2 = -\frac{p_2}{\alpha q_2} x_5^{2-\frac{q_1}{p_1}} - f_5 - \frac{p_2}{T_2 \alpha q_2} x_4 x_5^{1-\frac{q_2}{p_2}} - \frac{p_2}{T_2 q_2} x_5 \tag{20}$$

3. SIMULATION

In following, in order to evaluate synergetic controller in comparison to BELBIC controller expressed in [8], in tracking sinusoidal height reference signal, is considered. As it is obvious in Figure 1 synergetic controller almost instantly is able to track this reference from the first point. Similarly, system`s response with BELBIC controller and feedback linearization technique is shown in Figure 2.

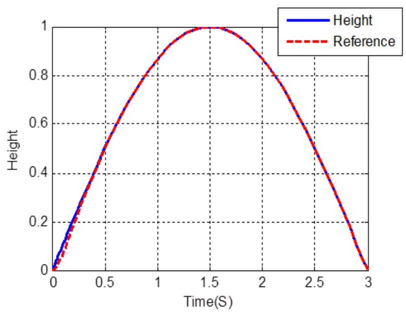

Figure 1. Synergetic controller response in tracking height sinusoidal reference signal

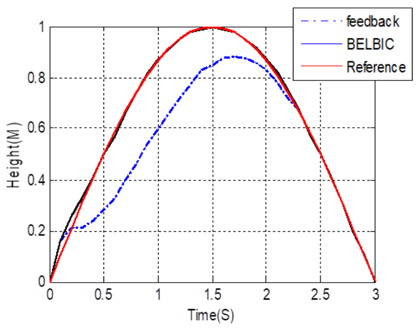

Figure 2. Sinusoidal reference signal tracking using BELBIC and feedback linearization [8]

Figures 1 and 2 obviously represents better performance of synergetic controller in comparison to two mentioned controllers in [8]. Considering height reference signal of constant 2 and angle reference signal equal to 0.2, system's response using synergetic controller will be as shown in Figures 3 and 4.

As it can be seen from the above figures in both cases, system needs less than 0.5 seconds for tracking constant height and angle reference signals. Also, comparing Figures 4 and 6, with Figure 5 represents better performance of synergetic and terminal synergetic controller in tracking angle reference signal which is considered as a constant, in comparison to two previous controllers reported in literature [8].

Now, using terminal synergetic controller in tracking angle, the resulted figure is as shown in Figure 6.

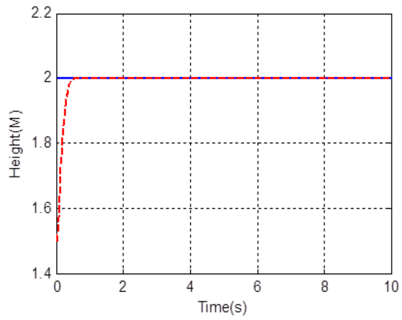

Figure 3. Height reference signal tracking

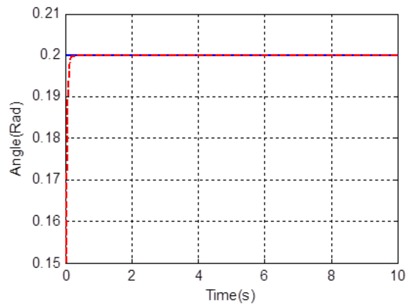

Figure 4. Angle reference signal tracking

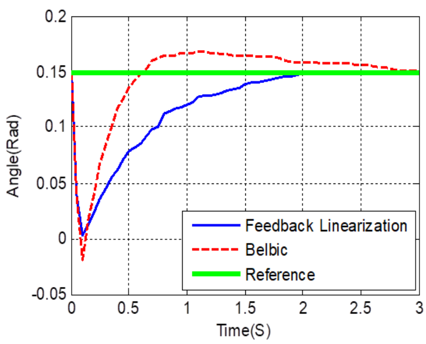

Figure 5. Angle reference signal tracking using proposed controller [8]

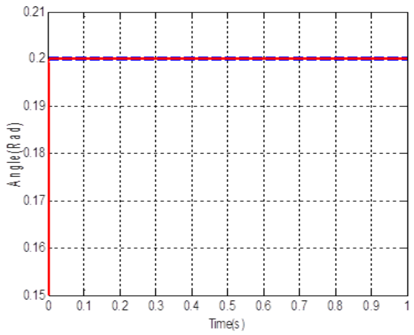

Figure 6. Angle reference signal tracking using terminal synergetic controller

As its clear in above figure, in this case the state variable is able to track angle reference signal with more proper speed in comparison to all previous mentioned methods. As it is mentioned earlier, one of synergetic terminal controller advantages is obtaining convergence possibility with proper speed and precision even when the initial condition is so far from reference value. In comparison to data reported in literature [18], where simulation is done with the same nonlinear helicopter model with initial condition $\theta_0 = 0.125, h_0 = 0.75.$ Produced control signal by synergetic controller is smoother. Figure 7, shows obtained control signal by synergetic controller, while the control signal in literature [18] is so oscillatory and has chattering.

Sometime main rotor speed trip, working in joint with other flight traits of helicopters, may result in unbalanced torque causing the nose of the aircraft to deviate from the desired point. This undesirable deviation may cause an increase in both, pilot workload or saturation of the aircraft stability system.

Similarly, when turning right rapidly, the tail rotor blades are positioned to a zero or negative pitch position so fast, and consequently the power demand of the tail rotor is suddenly reduced. Hence, a reduction in the accuracy of controlling the heading of the aircraft will be resulted [19].

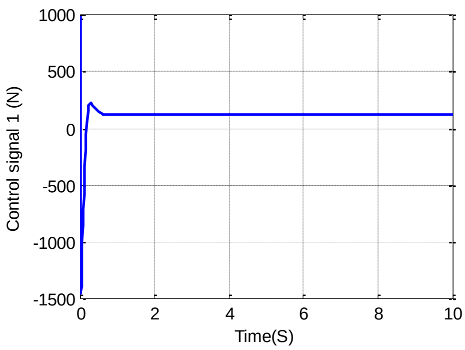

Figure 7. Control signal 1 using synergetic controller

4. CONCLUSION

In this paper basics and algorithm of synergetic control method, which can be considered as one of almost new branches of control science, is investigated. After that introducing multi input multi output model of helicopter, the terminal synergetic control method with a new manifold is applied to the helicopter model. Simulation results are compared to results from other control methods on the same helicopter model to evaluate responses. Comparing responses shows that synergetic controller has better and faster response, meanwhile gives more desired control signal and at the same time it does not need any linearization or transformation in system.

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چکیده

در این مقاله، یک تحقق کنترلی تقریباً جدید به نام کنترل ترمینال سینرجتیک که براساس مانیفولد تعریف شده توسط کاربر کار می‌کند به یک مدل غیرخطی هلی‌کوپتر اعمال می‌شود. تحلیل پایداری با استفاده از تئوری پایداری لیاپانوف مورد بررسی قرار گرفته است. کنترل‌کننده سینرجتیک ترمینال به این مدل غیرخطی مرتبه پنج هلی‌کوپتر برای کنترل ارتفاع و زاویه اعمال می‌شود. نتایج شبیه‌سازی نشان می‌دهد که این روش در مقایسه با روش کنترل مد لغزشی و کنترل‌کننده خلبان توماتیک هوشمند و روش خطی‌سازی فیدبک در ردیابی ورودی مرجع، پاسخ هموارتر و سریعتری ارائه می‌دهد.

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