



Multi-commodity Multimodal Splittable Logistics Hub Location Problem with Stochastic Demands

B. Karimi, M. Bashiri*, E. Nikzad

Department of Industrial Engineering, Shahed University, Tehran, Iran

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ABSTRACT

This study presents a multimodal hub location problem which has the capability to split commodities by limited-capacity hubs and transportation systems, based on the assumption that demands are stochastic for multi-commodity network flows. In the real world cases, demands are random over the planning horizon and those which are partially fulfilled, are lost. Thus, the present study handles demands using a discrete chance constraint programming to make the model one step closer to the reality. On the other hand, commodity splitting makes it possible for the remaining portion of commodity flow to be transported by another hub or transportation system in such a way that demands are completely fulfilled as much as possible. The problem decides on the optimum location of hubs, allocates spokes to established hubs efficiently, adopts and combines transportation systems and then makes a right decision as to whether transportation infrastructure to be built at points lacking a suitable transportation infrastructure and having the potential for infrastructure establishment. A Mixed Integer Linear Programming (MILP) model is formulated with the aim of cost minimization. Also, the proposed sensitivity analysis shows that, the discrete chance constraint programming is a good approximation of the continuous chance constraint programming when an uncertain parameter follows a normal distribution. The results indicate the higher accuracy and efficiency of the proposed model comparing with other models presented in the literature.

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NOMENCLATURE

Sets		Parameters	
F	set of commodities	e_{ihm_k}	A binary parameter is 1 if there is potential for infrastructure construction of the transportation system type k for the m -th combined transportation modality in path i to h , otherwise its value is zero.
K	set of transportation modalities	$w_{ihm_k}^2$	A binary parameter, equal to 1 if the infrastructure of the transportation system type k for the m -th combined transportation modality would have existed in path i to h , otherwise zero.
I	set of nodes of the network including hubs and spokes	$u_{him_k}^2$	A binary parameter, equal to 1 if the infrastructure of the transportation system type k for the m -th combined transportation modality would have existed in path h to i , otherwise zero.
H	set of potential hub nodes ($H \subset I$)	d_{jfs}	Demand of node j from commodity f under scenario s
M	set of available combined transportation modalities	a_f	Maximum occupied space for commodity of type f in the type k transport system
S	set of scenarios	C_h^f	Fixed opening cost of a hub at node h
cc_{m_k}	The cost of building a transportation infrastructure for transportation system type k in combined modality type m	MM	A large positive integer
c_{ijhfm}	Cost of transporting commodity f from node i to node j by hub h with combined modality m	Decision Variables	

* Corresponding Author's Email: bashiri.m@gmail.com (M. Bashiri)

Cu_h	Unit handling cost at hub h	x_{ijhfm}	Is a positive variables that indicate the quantity of flow of commodity f from node i to node j passing through hub h , shipped by the combined modality m
u_h	Maximum handling capacity of hub h	y_h	Is equal to 1, if the hub h is constructed, otherwise 0.
q_{ihm_k}	Flow capacity of arc (i,h) for the k -th transportation modality in combined modality type m	w_{ihm_k}	Isequal to 1 if the infrastructure of the transportation system of type k for the m -th combined transportation modalityin path i -his constructed, otherwise, 0.
q_{jhm_k}	Flow capacity of arc (j,h) for the k -th transportation modality in combined modality type m	AXI_{jfs}	Is equal to 1 if the demand of destination point j for commodity type of f under scenario s has not been met and has been lost, otherwise 0.
N	Total number of times a demand is allowed to be lost under different scenarios	AX_{jf}	Is equal to 1 if the destination point j receives the f product, otherwise 0.

1. INTRODUCTION

Hub location problems deal with transporting goods by intermediate node, instead of a direct connection between supply and demand nodes. In network design problems, direct connection between origins and destinations leads to cost increase, sever traffic congestion and the poor management of commodity flows. However, the related issues can be resolved by considering hubs and spokes connected to them. Generally, hub location problem allows to find the optimum number and location of hubs, and to allocate spokes to them. Another important issue is the good design of transportation systems that has a considerable effect on total network cost. The first mathematical model for hub location problem was first proposed by O'Kelly [1]. Hub network applications are frequently used in fields such as telecommunications, transportation, and fast service delivery [2].

Multimodal transportation is to transport goods from origin to destination by a combination of two or more transportation modes. Each transportation mode has its own advantages in proportion to price, speed, commodity type, production volume, distance, etc. Therefore, multimodal transportation aims to take advantage of all transportation modes [3]. It necessitates establishing a suitable transportation infrastructure in all transportation system. One of the useful functions served by a hub is to be allowed the switch from one transportation mode to another one [4].

In the real world, because of existing of uncertain parameters such as demand, cost and time during the planning horizon, to make proper decisions at the start of planning horizon [5], stochastic programming should be used. Stochastic parameters can make the model one step closer to the reality and increase its flexibility [6].

Chance constraint programing (CCP) is often encountered applications when there is uncertainty in the data and parameters [7]. The CCP was first introduced by Charnes, Cooper and Symonds [8]. It is well known when the random input has a joint normal distribution, and it can be reduced to a convex problem. Thus, it can be solved efficiently via convex programing techniques [9]. Many problems in various areas, can be

formulated as the CCP. A series of applications has been reviewed in literature [10]. In recent years, more studies considered discrete distribution to define uncertain parameters. In a case when random variables are discretely distributed is extensively studied [11]. In some cases, scenarios are generated by sampling from a distribution with Monte Carlo sampling method. Therefore, these scenarios can be considered as approximation of general distribution [12]. We can find feasible solution and lower bounds for the original problem by such sample approximation methods. Furthermore, the required sample size is determined based on a polynomial function of $1/\alpha$. Therefore, above problem is NP-hard in general [13]. By associating a binary variable to each scenario, the CCP with discrete distribution can be converted to a mixed-integer programing (MIP) formulation. It is clear that the number of binary variables will grow up by increasing the number of scenarios. The size of the resulted MIP reformulation of CCP is usually much larger than the original CCP problem [10]. Hence the difficulty of solving the resulted MIP reformulation will be increased, meanwhile the MIP reformulation is commonly solved by MIP solvers in the framework of Branch-and-Bound [14]. General format of Chance Constraint can be defined as follows;

$P(Ix \geq \varepsilon) \geq 1 - \alpha, x \in O$. Where I is an $m \times d$ random matrix and ε is a random vector taking values in R^m and O is a convex compact set, $\alpha \in (0,1)$ is a prescribed risk level which is given by the decision maker, typically near zero, e.g., $\alpha = 0.01$ or $\alpha = 0.05$. P denotes the probability [14].

Table 1 presents a brief overview of scholarly literature and recent researches on hub and transportation problems, and makes a clear distinction between the present study and the previous works. Based on the literature survey reported in Table 1, the following gap as well as main contributions of this research are indicated :

- Proposing a multimodal splittable hub location for multi commodities.
- Considering discrete chance constraint programming to deal with uncertainty of demands.

Since the existence of transportation infrastructure on the routes between origins and destinations for all transportation systems is impossible and unreal, this study creates conditions under which necessary transportation infrastructures are built at origin-hub and hub-destination points lacking a suitable transportation infrastructure and having the potential for infrastructure establishment.

2. 2. Assumptions

We start itemizing the assumptions that we make:

- The number of hubs to be built is unspecified and it is a part of decision-making process .
- Commodity split is allowed by hubs or transportation systems.
- No direct connection is allowed between non-hub nodes.
- The use of transportation systems necessitates infrastructure existence or establishment.
- The internal capacity of hubs depends on product type.
- The level of demands is considered to be random.

3. MATHEMATICAL MODEL

In this section, a mathematical model is proposed for the Multi-commodity multimodal splittable logistics hub location problem under uncertain parameters. The sets, parameters, and decision variables in the model are described in the beginning of this article. The mathematical MILP model of this problem is expressed in terms of minimizing the establishment and transportation costs, the operating costs of established hubs and constructing costs of new transportation infrastructures in the network as follows:

$$\begin{aligned} & \min \sum_i \sum_j \sum_h \sum_f \sum_m c_{ijhfm} x_{ijhfm} + \sum_h C f_h y_h \\ & + \sum_i \sum_j \sum_h \sum_f \sum_m C u_h x_{ijhfm} \\ & + \sum_i \sum_j \sum_k \sum_m c c_{m_k} w^1_{ihm_k} \end{aligned} \tag{1}$$

$$\text{Subject to : } \sum_i \sum_j \sum_f \sum_m x_{ijhfm} \leq u_h y_h \quad \forall h \in H \tag{2}$$

$$\begin{aligned} & \sum_{i \neq j} \sum_h \sum_m x_{ijhfm} - d_{jfs} \leq MM \times (1 - AXI_{jfs}) \\ & \forall i \in I, s \in S, f \in F \end{aligned} \tag{3}$$

$$\begin{aligned} & - \sum_{j \neq h} \sum_h \sum_m x_{ijhfm} + d_{jfs} \leq MM \times AXI_{jfs} \\ & \forall i \in I, s \in S, f \in F \end{aligned} \tag{4}$$

$$\begin{aligned} & \sum_j \sum_f \sum_m \sum_k a_f x_{ijhfm_k} \leq 0.5 q_{ihm_k} \times (w^1_{ihm_k} + w^1_{him_k} \\ & + w^2_{ihm_k} + u^2_{him_k}) \quad \forall i \in I, h \in H \end{aligned} \tag{5}$$

$$\begin{aligned} & \sum_i \sum_f \sum_m \sum_k a_f x_{ijhfm_k} \leq 0.5 q_{jhm_k} \times (w^1_{jhm_k} + w^1_{hjm_k} \\ & + w^2_{jhm_k} + u^2_{hjm_k}) \quad \forall j \in I, h \in H \end{aligned} \tag{6}$$

$$\sum_i \sum_h \sum_m x_{ijhfm} \leq MM \times AX_{jf} \quad \forall j \in I, f \in F \tag{7}$$

$$\sum_i \sum_h \sum_m x_{ijhfm} \leq MM \times (1 - AX_{jf}) \quad \forall j \in I, f \in F \tag{8}$$

$$\sum_s \sum_f AXI_{jfs} \leq N \quad \forall j \in I \tag{9}$$

$$w^1_{ihm_k} \leq e_{ihm_k} \quad \forall i \in I, h \in H, k \in K, m \in M \tag{10}$$

$$w^1_{ihm_k} + w^2_{ihm_k} \leq 1 \quad \forall i \in I, h \in H, k \in K, m \in M \tag{11}$$

$$w^1_{ihm_k} + w^2_{him_k} \leq 1 \quad \forall i \in I, h \in H, k \in K, m \in M \tag{12}$$

$$w^1_{him_k} + w^2_{ihm_k} \leq 1 \quad \forall i \in I, h \in H, k \in K, m \in M \tag{13}$$

$$w^1_{ihm_k} = w^2_{him_k} \quad \forall i \in I, h \in H, k \in K, m \in M \tag{14}$$

$$w^1_{ihm_k} = \{0, 1\}, y_h = \{0, 1\}, x_{ijhfm} \geq 0 \tag{15}$$

Objective function (1) computes total costs of hubs establishment and operating, flow of commodities and new transportation infrastructure in which should be minimized. Equation (2) guarantees that the total flow from different sources to the destination for different goods passing through a hub should not exceed its maximum operational capacity. Equations (3) and (4) determine scenarios when delivery is less than demands. Equations (5) and (6) ensure that the flow between a hub and spoke should not exceed the maximum capacity of the transit system. Equations (7) and (8) guarantee that demand points receiving type-f product are no longer able to transport the same type to other points and vice versa. Equation (9) ensures that the total number of unserved demands under different scenarios must not exceed N. Equation (10) states that a new transportation infrastructure can be constructed in case of potentially possibility. Equations (11) to (14) are

rational constraints of establishing a new transportation infrastructure. Equation (15) defines types of variables.

4. SENSIVITY ANALYSIS

The model proposed in previous section has been implemented in GAMS 24.7.3 and solved by CPLEX on an Intel R core (TM) i7 CPU and 4.00 GB RAM. The model has been validated by using randomly generated data and solved, it is assumed that there are three types of products and two transportation modes together to create four combined transportation modes. This section examines the effect of important problem parameters, and verifies the accuracy of the proposed model by showing its high efficiency. For this purpose, four types of sensitivity analysis are conducted. Figures 1 and 2 illustrate the association of hub and route capacities with the total network cost for splittable and un-splittable commodities. At the same capacities, the total network cost for splittable commodities is much less than un-splittable ones. In Figure 3, the relationship between transportation route capacity and transportation infrastructure is investigated for both transportation systems. It shows that transportation infrastructure requires lower investment levels as route capacity increases. As shown in Figure 4, the number of splittable demands is reduced by an increase in the total number of allowable unserved demands. As shown in Figure 5, reduction occurs in the total transportation network cost by increasing of transportation modes. Figure 6 illustrates that total network cost can be decreased by increasing of allowable unserved demands. It is also shown that network cost progressively increases by demand fulfillment, and it reaches its maximum amount when no demand can be lost.

4. 1. Comparison of Discrete and Continuous Chance Constraint Programming

In this section, we show that the proposed discrete chance constraint can be used as an approximation of continuous chance constraint.

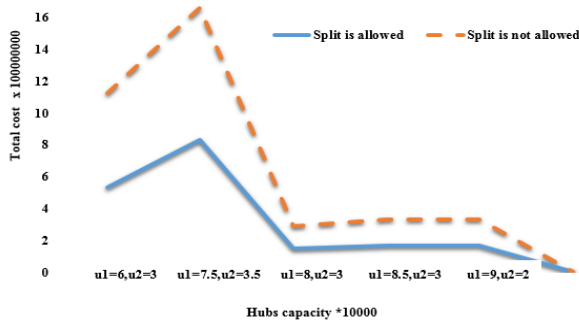


Figure 1. Total costs with split and without split through hub

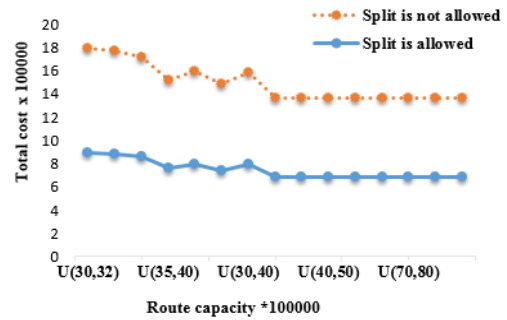


Figure 2. Total costs with spilt and without spilt through different modes

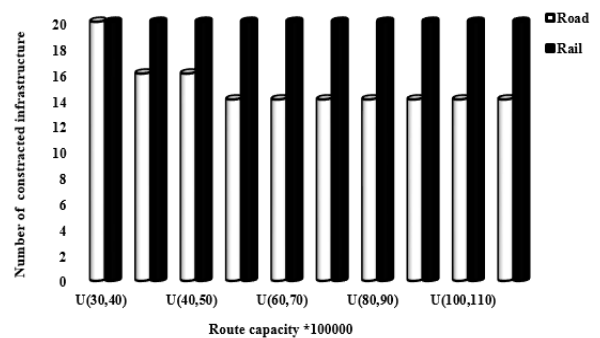


Figure 3. The number of constructed infrastructures vs. route capacity

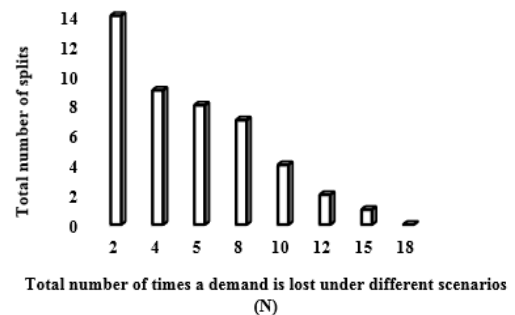


Figure 4. The number of the lost demands vs. the number of splits

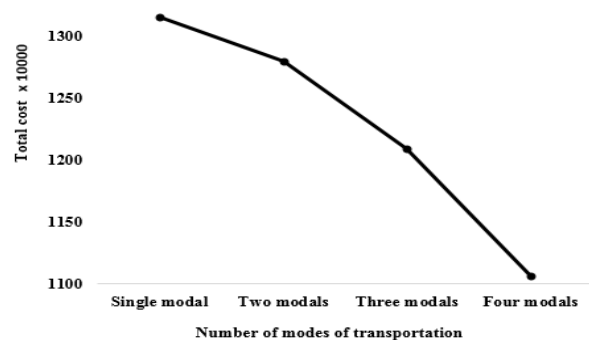


Figure 5. The number of transportation modes vs. total network cost

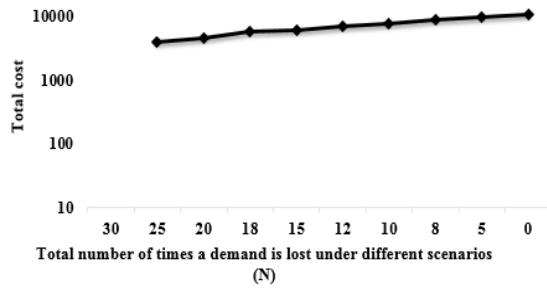


Figure 6. The number of the lost demands vs. total network cost

For this purpose, two parameters mean and variance of normal distribution are estimated based on the scenarios. Equations (16) and (17) are continuous chance constraint and the linearization of this constraint, respectively. In Equation (17), z_α represents the point of standard normal distribution, so that: $p(z > z_\alpha) = 1 - \alpha$.

$$p\left(\sum_{i \neq j} \sum_h \sum_m x_{ijhfm} \leq d_{jfs}\right) \leq \alpha \quad (16)$$

$\forall i \in I, f \in F$

$$\frac{\sum_{i \neq j} \sum_h \sum_m x_{ijhfm} - \text{mean}}{\delta} > z_\alpha \quad (17)$$

$\forall i \in I, f \in F$

Figures 7 and 8 show the comparison between the results from the use of discrete and continuous chance constraints. It has been shown that the results of a discrete and continuous random variable on the variable of the construction of the hubs and the cost of constructing the hubs are the same. In other words, if using a continuous distribution as normal, the same result can be obtained from the use of discrete distribution on the number, type and the cost of constructing the hub, and in some way a discrete chance constraint approach is a good approximation of continuous distributions.

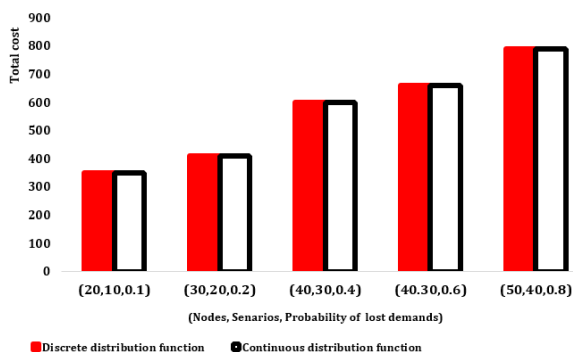


Figure 7. Comparison of the effect of using discrete or continuous chance constraint on the cost of constructing a hub

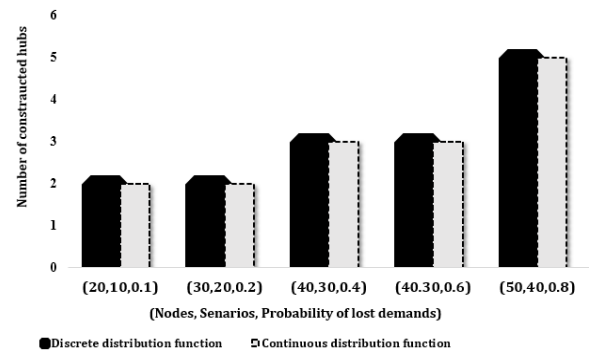


Figure 8. Comparison of the effect of using discrete or continuous chance constraint on the number of hubs constructed

5. CONCLUSIONS

This study presents a Mixed Integer Linear Programming (MILP) model to solve capacitated hub location problems where demands are stochastic for multi-commodity network flows based on the assumption that commodity split is allowed. Discrete chance constraint was considered in the problem. Also, we showed that the discrete chance constraint programming is a good approximation of the continuous chance constraint programming when an uncertain parameter follows a normal distribution. In this study, we consider that only one transportation infrastructure should be existed between nodes. If transportation infrastructure is available, it can be used. However, in case of absence of an infrastructure the model may decide for construction. Also, we considered that some infrastructures are not possible for some kinds of transportation types. It was shown that total transportation cost is sharply reduced by an increase in the number of combined transportation modes. Thus, transportation infrastructure establishment plays an important and influential role in creating diverse transportation systems. In the real world cases, the implementation of the discrete chance constraint method that presented in this paper for modeling of uncertain parameters, helps to planner managers to manage well the effects of key factors such as demand, cost and capacity at the beginning of the planning and analyze results, even without having a probability distribution function. As a managerial insight, it should be noted that logistic managers may develop necessary logistic infrastructures like multimodal transportation possibility and splittable transportation modes to reduce total network transportation costs significantly. In fact, managers by providing required infrastructures to create splittable transportations modes can deliver goods to customers, with significant saving in the cost and time. It should be noted that other related changes is planned during providing required infrastructures. As an

example, type of packaging may be changed to implement the splittable multimodal network. The results obtained from computational experiments show that an increase in the number of the network nodes, scenarios or allowable unserved demands under different scenarios make the problem hard to be accurately solved by commercial solvers in a short time. Therefore, valid inequalities, relaxation methods or heuristic algorithms can be a suggestion to continue the present study for future research directions. Also, a multi-objective problem and multi-capacity hubs can be used to keep a balance between time, costs and environmental conditions for different transportation modes, and to prevent demands being lost as much as possible.

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B. Karimi, M. Bashiri, E. Nikzad

Department of Industrial Engineering, Shahed University, Tehran, Iran

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Splittable

Discrete Chance Constraint

در این مقاله، مساله مکان یابی محور با در نظر گرفتن سیستم حمل و نقل چندوجهی و قابلیت جداسازی کالاها از طریق هاب ها و سیستم های حمل و نقل، با فرض تصادفی بودن تقاضا در حالت چندکالایی ارائه شده است. همچنین ظرفیت هاب ها و سیستم های حمل و نقل محدود در نظر گرفته شده است. در مسائل واقعی، تقاضاها در طی افق زمانی، تصادفی هستند و به طور قطع برخی از تقاضاها به طور کامل برآورده نمی شوند و اصطلاحاً تقاضاها از دست می روند. بنابراین در این مقاله، برای هر چه نزدیک تر کردن مدل به واقعیت، تقاضا به صورت تصادفی و با استفاده از روش محدودیت تصادفی گسسته مورد بررسی قرار گرفته است. از طرفی در نظر گرفتن فرض مجاز بودن جداپذیری کالاها، باعث می شود که در صورت تکمیل ظرفیت هر هاب و یا یک سیستم حمل و نقل، بخش باقی مانده جریان کالا از طریق یک هاب یا سیستم حمل و نقل دیگر ارسال گردد و به این ترتیب، تا حد امکان از عدم برآورده شدن تقاضاها جلوگیری نمود. مسئله حاضر به دنبال انتخاب بهترین مکان برای احداث نقاط هاب، تعیین تخصیص بهینه نقاط غیر هاب به هاب، انتخاب و ترکیب سیستم های حمل و نقل و تصمیم گیری در مورد احداث یا عدم احداث زیرساخت برای سیستم های حمل و نقلی که در حال حاضر زیرساختی برای آنها وجود ندارد و پتانسیل احداث را دارا هستند، می باشد. این مسئله به صورت برنامه ریزی خطی عدد صحیح مختلط با هدف کمینه کردن کل هزینه شبکه، مدل و حل گردیده است. همچنین تحلیل حساسیت های انجام شده، نشان می دهد که روش محدودیت تصادفی گسسته تخمین خوبی از روش محدودیت تصادفی پیوسته هنگامی که پارامترهای غیر قطعی از توزیع نرمال پیروی میکنند، می باشد. نتایج حاصله بیانگر صحت و کارایی بالاتر مدل در مقایسه با مدل های ارائه شده در ادبیات موضوع می باشد.

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