



## Analytical Investigation of Tire-Road Contact Characteristics for Wheelchair Robots Safely Running

C. Zhou, L. Zhao\*, Y. Yu, X. Li

School of Transportation and Vehicle Engineering, Shandong University of Technology, Zibo, China

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### ABSTRACT

To effectively improve the tire grounding behaviors of wheelchair robots, an analytical method is proposed to analyze and optimize the tire grounding safety. Firstly, taking the cushion and tires as the vibration isolation elements with stiffness and damping, the vertical vibration model of the human-wheelchair robot is established. Then, taking the random excitation as the typical input, the formulae of the TDD (tire dynamic deflection) frequency response function  $H$  and the RMS (root mean square) response are derived and the response coefficient  $\lambda$  is proposed. Moreover, the influence laws of system parameters on  $H$  and  $\lambda$  are revealed. Thirdly, based on  $\lambda$ , the analytical optimization model for the cushion system damping ratio  $\zeta_2$  is established. Finally, a case study and numerical simulation were carried out. The results show that the relative deviation of the cushion optimal damping is about 0.3%.

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## 1. INTRODUCTION

Wheelchair robot is a kind of mobile robots [1, 2]. Wheelchair robot is the advanced product of wheelchairs. Various wheelchair robots belong to medical devices. They are widely used in hospitals and rehabilitation institutions [3].

On relatively flat roads, wheeled mobile mode is quite advantageous [4, 5]. The shape and structure of wheels depend on the nature of the ground and the carrying capacity of wheelchair robots. At present, wheelchair robots are mostly equipped with inflatable tires. The road excitation decreases the recovery rate of rehabilitation patients. More seriously, the excessive vibration reduces the tire grounding safety [6]. A good tire grounding is the guarantee for wheelchair robot running safely. In recent years, the enormous demand for wheelchairs leads to higher performance requirements [7-9]. How to improve effectively the tire grounding safety of wheelchair robots has aroused a growing public attention [10].

To better study the grounding behaviors of tires, scholars focus on the related research, for example, the tire model and the tire characteristics. Geng et al.

proposed a research approach specifically designed for the measurement, identification and modelling of damping in pneumatic tires [11]. Dąbek et al. [12] researched tire models for studies of wheeled mobile robot dynamics. Levratti et al. [13] made an experimental evaluation of a novel tire workshop assistant robot. These studies provide useful references for improving the tire grounding safety of wheelchair robots. However, there are few studies on the analytical safety analysis and optimization of tire grounding behaviors for wheelchair robots undergoing road irregularities.

In this paper, in order to improve effectively the tire grounding behaviors for wheelchair robots under the random excitation, an analytical method is proposed to analyze and optimize the tire grounding safety.

## 2. HUMAN-WHEELCHAIR ROBOT MODEL

A simplified model of the human-wheelchair robot was created, as shown in Figure 1 [14]. In Figure 1,  $m_2$  and  $m_1$  are the human body mass and the wheelchair body mass, respectively;  $C_2$  and  $C_1$  are the damping of the cushion

\*Corresponding Author Email: [zhaoleilei611571@163.com](mailto:zhaoleilei611571@163.com) (L. Zhao)

and the tire, respectively;  $K_2$  and  $K_1$  are stiffness coefficients of the cushion and the tire system, respectively;  $z_2$  and  $z_1$  are the corresponding displacements;  $q$  is the road excitation.

The differential equation set of the vibration model can be expressed as follows:

$$\begin{cases} m_2 \ddot{z}_2 + C_2(\dot{z}_2 - \dot{z}_1) + K_2(z_2 - z_1) = 0 \\ m_1 \ddot{z}_1 + C_2(\dot{z}_1 - \dot{z}_2) + K_2(z_1 - z_2) + C_1(\dot{z}_1 - \dot{q}) \\ + K_1(z_1 - q) = 0 \end{cases} \quad (1)$$

The random road model can be expressed as follows [14]:

$$\dot{q}(t) = -2\pi n_0 u q(t) + 2\pi n_0 \sqrt{G_q(n_0)} u w(t) \quad (2)$$

where,  $n_0$  represents the reference space frequency and  $n_0=0.1 \text{ m}^{-1}$ ;  $u$  represents the running speed;  $n_{00}$  represents the lower cut-off space frequency and  $n_{00}=0.011 \text{ m}^{-1}$ ;  $t$  represents time;  $\dot{q}(t)$  represents the road excitation velocity;  $w(t)$  represents the Gaussian white noise with zero mean;  $G_q(n_0)$  represents the road roughness coefficient.

### 3. ANALYTICAL FORMULAE OF TIRE DYNAMIC DEFLECTION RESPONSES

The characteristic parameters are adopted as follows [14]:

$$r_k = \frac{K_1}{K_2}, r_m = \frac{m_2}{m_1}, \omega_2 = \sqrt{\frac{K_2}{m_2}}, \omega_1 = \sqrt{\frac{K_1}{m_1}}, \xi_2 = \frac{C_2}{2\sqrt{m_2 K_2}},$$

$$\xi_1 = \frac{C_1}{2\sqrt{m_1 K_1}}, \eta = \frac{\omega_1}{\omega_2} = \sqrt{r_k r_m}$$

where,  $\omega_1$  and  $\omega_2$  are the natural circular frequencies;  $\xi_1$  and  $\xi_2$  are the damping ratios of the tire system and the cushion system, respectively;  $r_k$  is the stiffness ratio of the tire stiffness to the cushion stiffness;  $r_m$  is the mass ratio of the human body mass to the wheelchair body mass.

Based on the model in Figure 1, the tire dynamic deflection (TDD)  $f_d$  is defined as follows:

$$f_d = z_1 - q \quad (3)$$

Based on Equations (1) and (3), the frequency response function  $H(j\omega)$  between the TDD  $f_d$  and the road velocity excitation  $\dot{q}$  can be obtained as follows:

$$H(j\omega) = \frac{j\omega^3 \cdot (-1)}{\omega_2^4 \eta^2 + j\omega \cdot 2\omega_2^3 \eta (\xi_1 + \xi_2 \eta) - \omega^2 \cdot (1 + \eta^2 + r_m + 4\xi_2 \xi_1 \eta) \omega_2^2 - j\omega^3 \cdot 2(\xi_2 + \xi_1 \eta + \xi_2 r_m) \omega_2 + \omega^4} \quad (4)$$

The RMS TDD  $f_d$  can be expressed as follows:

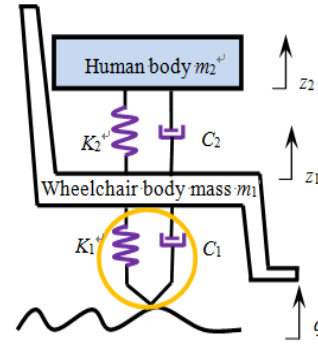


Figure 1. The vibration model of the wheelchair robot

$$\sigma_{f_d} = \sqrt{\frac{1}{2\pi} \int_{-\infty}^{+\infty} 2\pi^2 G_q(n_0) n_0^2 u^2 |H(j\omega)|^2 d\omega} \quad (5)$$

By the integral calculation, Equation (5) can be further expressed as follows:

$$\sigma_{f_d} = \sqrt{G_q(n_0) n_0^2 v \pi^2 \frac{(A_0 A_3 - A_1 A_2) + A_1(-2B_1 - B_2^2) - A_3 B_1^2}{A_0 A_3^2 + A_1^2 - A_1 A_2 A_3}} \quad (6)$$

where,  $A_0 = \omega_2^4 \eta^2$ ,  $A_1 = 2\omega_2^3 \eta (\xi_1 + \xi_2 \eta)$ ,  $A_2 = (1 + \eta^2 + r_m + 4\xi_2 \xi_1 \eta) \omega_2^2$ ,  $A_3 = 2(\xi_2 + \xi_1 \eta + \xi_2 r_m) \omega_2$ ,  $B_1 = -\omega_2^2 (1 + r_m)$ ,  $B_2 = -2(1 + r_m) \xi_2 \omega_2$ ,  $\eta = \sqrt{r_k r_m}$ .

The identical transformation of Equation (6) can be expressed as follows:

$$\sigma_{f_d} = \lambda \sqrt{G_q(n_0) n_0^2 u} \quad (7)$$

where, the coefficient  $\lambda$  is defined as the TDD coefficient of wheelchair robots and its unit is  $s^{0.5}$ .

According to Equation (7), the coefficient  $\lambda$  can be expressed as follows:

$$\lambda = \pi \sqrt{\frac{(A_0 A_3 - A_1 A_2) + A_1(-2B_1 - B_2^2) - A_3 B_1^2}{A_0 A_3^2 + A_1^2 - A_1 A_2 A_3}} \quad (8)$$

### 4. INFLUENCE ANALYSIS OF SYSTEM PARAMETERS ON THE TIRE GROUNDING SAFETY

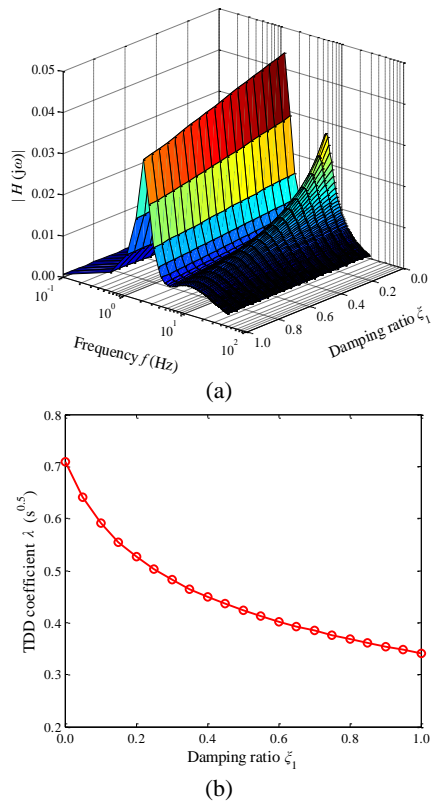
To effectively improve the tire grounding behaviors for wheelchair robots under the random excitation, the influence laws of system parameters on the frequency response function  $H$  and the TDD coefficient  $\lambda$  are revealed in this section. The values of the physical parameters for the human-wheelchair system are shown in Table 1. Based on Table 1, the values of the characteristic parameters were calculated, as shown in Table 2. Based on the values for the human-wheelchair robot system in Table 2, using Equations (4) and (8), the influences of  $\xi_1$ ,  $\xi_2$ ,  $r_m$ ,  $r_k$ , and  $f_2$  on the tire grounding safety were calculated, as shown in Figures 2~6.

**TABLE 1.** The values of the physical parameters

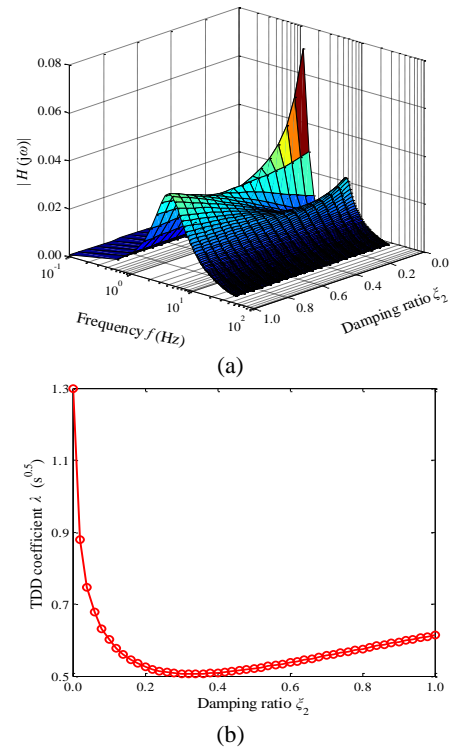
Parameter	Value
$m_2$ (kg)	75
$m_1$ (kg)	25
$C_2$ (Ns/m)	438
$K_2$ (N/mm)	16
$K_1$ (N/mm)	80
$C_1$ (Ns/m)	566

**TABLE 2.** The values of the characteristic parameters

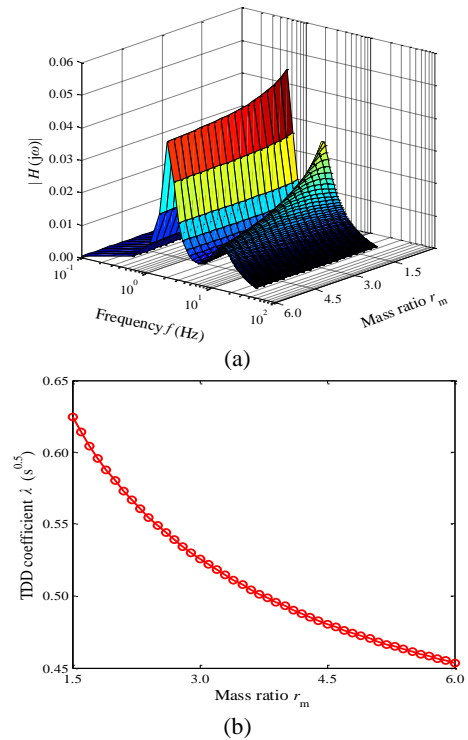
Parameter	Value
$\zeta_2$	0.2
$\zeta_1$	0.2
$r_m$	3
$r_k$	5
$f_2$ (Hz)	2.4
$\eta$	3.9



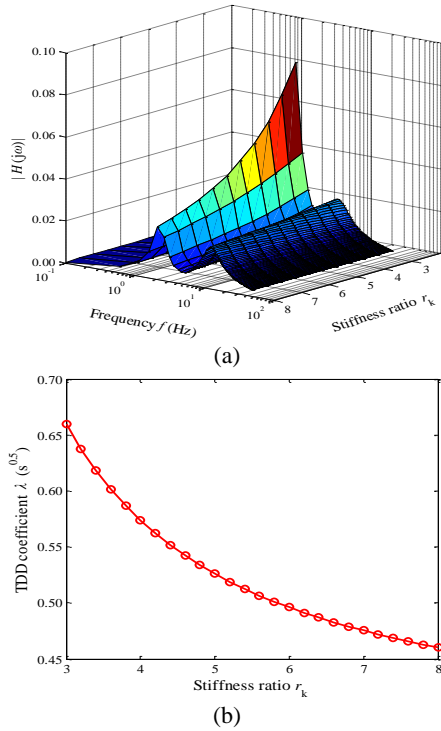
**Figure 2.** The influence of the tire system damping ratio  $\zeta_1$  on the tire grounding safety: (a) the surface of  $|H(j\omega)|$  versus the frequency  $f$  and the damping ratio  $\zeta_1$ , (b) the curve of the response coefficient  $\lambda$  versus the damping ratio  $\zeta_1$



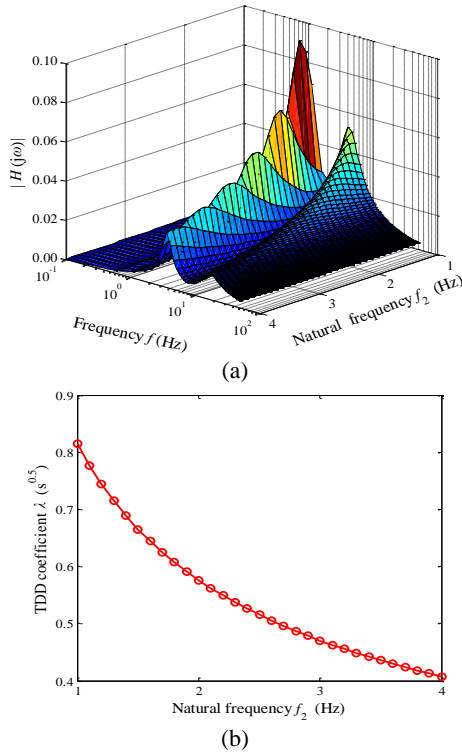
**Figure 3.** The influence of the cushion system damping ratio  $\zeta_2$  on the tire grounding safety: (a) the surface of  $|H(j\omega)|$  versus the frequency  $f$  and the damping ratio  $\zeta_2$ , (b) the curve of the response coefficient  $\lambda$  versus the damping ratio  $\zeta_2$



**Figure 4.** The influence of the mass ratio  $r_m$  on the tire grounding safety: (a) the surface of  $|H(j\omega)|$  versus the frequency  $f$  and the mass ratio  $r_m$ , (b) the curve of the response coefficient  $\lambda$  versus the mass ratio  $r_m$



**Figure 5.** The influence of the stiffness ratio  $r_k$  on the tire grounding safety: (a) the surface of  $|H(j\omega)|$  versus the frequency  $f$  and the stiffness ratio  $r_k$ , (b) the curve of the response coefficient  $\lambda$  versus the stiffness ratio  $r_k$



**Figure 6.** The influence of the natural frequency  $f_2$  on the tire grounding safety: (a) the surface of  $|H(j\omega)|$  versus the frequency  $f$  and the natural frequency  $f_2$ , (b) the curve of the response coefficient  $\lambda$  versus the natural frequency  $f_2$

From Figure 2(a), it can be seen that the resonance frequencies of the human body and the wheelchair body are 2.4 Hz and 9.0 Hz, respectively. Moreover, increasing the tire system damping ratio  $\zeta_1$  is beneficial to the attenuation of the resonance peak of  $|H(j\omega)|$ . Figure 2(b) shows that with the increase of the damping ratio  $\zeta_1$ , the TDD coefficient  $\lambda$  nonlinearly decreases. The results prove that using the larger damping tire is beneficial to improve the tire grounding safety. Figure 3(a) depicts that increasing the cushion system damping ratio  $\zeta_2$  helps to attenuate the resonance peak of  $|H(j\omega)|$ , however, it causes the amplitude increase in the non-resonant region between 2.4 Hz and 9.0 Hz. Figure 3(b) illustrates that with the increase of the damping ratio  $\zeta_2$ , the coefficient  $\lambda$  sharply decreases first and then slowly increases. Thus, when the value of  $\zeta_2$  is too large or too small, it is not conducive to improving the tire grounding safety. Figure 4 proves that increasing the mass ratio  $r_m$  is conducive to improving the grounding safety. From Figure 5(a), it can be seen that increasing the stiffness ratio  $r_k$  can effectively suppress the resonance peak at the resonance frequency of the human body. Figure 5(b) shows that with the increase of the stiffness ratio  $r_k$ , the TDD coefficient  $\lambda$  nonlinearly decreases. Thus, increasing the stiffness ratio  $r_k$  can help to improve the tire grounding safety. From Figure 6(a), it can be seen that with the increase of the natural frequency  $f_2$ , the low frequency resonance peak moves to the right and sharply decrease. Figure 6(b) illustrates that is the TDD coefficient  $\lambda$  is nonlinear with the natural frequency  $f_2$  and the greater the natural frequency, the smaller the coefficient  $\lambda$  is. Thus, increasing the natural frequency  $f_2$  can also help to improve the tire grounding safety.

### 5. ANALYTICAL OPTIMIZATION METHOD

This study selects  $\zeta_2$  as the optimization variable. The optimization objective is to minimize  $\lambda$ . This optimization problem can be expressed as follows:

$$\min \{J(\zeta_2)\} = \min \{\lambda\} \tag{9}$$

The following constraint must be satisfied:

$$0 < \zeta_2 < 1 \tag{10}$$

Take the derivative of the TDD coefficient  $\lambda$  with respect to  $\zeta_2$  and let the derivative equal zero, obtain a 4 order equation about  $\zeta_2$ :

$$T_4 \zeta_2^4 + T_3 \zeta_2^3 + T_2 \zeta_2^2 + T_1 \zeta_2 + T_0 = 0 \tag{11}$$

where,  $T_0 = -\zeta_1^2 r_m (4\zeta_1^2 + r_k r_m)$ ,  $T_1 = -2\zeta_1 r_m \eta (4\zeta_1^2 + r_m + 1)$ ,

$$T_2 = 4\zeta_1^2 (4\zeta_1^2 r_m + 4\zeta_1^2 + r_m^2 r_k + 2r_m r_k - r_m^2 - 2r_m - 1) + (2r_m r_k - r_m^2 r_k^2 + 2r_m^2 r_k - r_m^3 - 3r_m^2 - 3r_m - 1)$$

$$T_3 = 8\zeta_1 \eta (r_m + 1) (4\zeta_1^2 + r_m r_k),$$

$$T_4 = 4r_m r_k (r_m + 1)(4\zeta_1^2 + r_m + 1), \eta = \sqrt{r_k r_m}$$

The optimum damping  $C_{2op}$  of the cushion system can be calculated by following expression:

$$C_{2op} = 2\zeta_{2op} \sqrt{K_2 m_2} \quad (12)$$

## 6. CASE STUDY AND NUMERICAL SIMULATION VERIFICATION

A case study and numerical simulation verification were carried out in this section. The system parameters of a commercially available wheelchair robot are as follows:  $K_2=16$  N/mm,  $K_1=80$  N/mm,  $m_1=25$  kg,  $C_1=566$  Ns/m. According to the average weight of adults, selecting  $m_2=75$  kg. The running condition is as follows:  $u=1.6$  m/s, the cement road roughness coefficient  $G_q(n_0)=256 \times 10^{-6} \text{ m}^{-3}$ .

**6.1. Case Study** Based on the above analytical optimization method in section 5, the optimal damping ratio  $\zeta_{2op}=0.33$  and the optimal damping  $C_{2op}=727$  Ns/m for the cushion system. According to Equation (11), when the wheelchair robot runs on the cement road with  $G_q(n_0)=256 \times 10^{-6} \text{ m}^{-3}$  and  $u=1.6$  m/s, the RMS TDD values under different values of  $\zeta_2$  can be calculated. The RMS TDD reaches the minimum when the damping ratio  $\zeta_2=0.33$  and the minimum is  $[\sigma_{fd}]_{\min}=1.0$  mm. In addition, according to the analytical optimization results, it can be seen that the optimal damping ratio of the cushion system is about 0.3, which avoids the blindness of traditionally choosing the damping ratio between 0~1. It is worth noting that the range of the cushion system damping ratio for ride comfort is about 0.25~0.45. Thus, the cushion with  $\zeta_{2op}=0.33$  does not reduce ride comfort.

**6.2. Numerical Simulation Verification** The time domain displacement  $q$  generated by the random road model is used as the simulation input. For the numerical simulation optimization, the optimization objective is to minimize the RMS TDD  $\sigma_{fd}$ . The cushion system damping  $C_2$  is taken as the optimization variable. The genetic algorithm in Matlab software was selected as the optimization algorithm. When  $\sigma_{fd}$  reaches the minimum 1.0 mm,  $C_2=725$  Ns/m. Thus, the optimal damping is  $C_{2op}=725$  Ns/m. The relative deviation between the value of  $C_{2op}$  calculated from the analytical method and that from the numerical simulation method is 0.3%. The comparison result shows that the analytical optimization method is effective. The analytical optimization model has nothing to do with  $u$  and  $G_q(n_0)$ , so it is not necessary to establish a numerical simulation model for multiple simulation and optimization. In addition, compared with taking the specific parameter  $C_2$  as the optimization variable, the analytical optimization method takes the

dimensionless damping ratio  $\zeta_2$  as the optimization variable, which has more theoretical guidance value.

## 7. CONCLUSIONS

This paper proposed an analytical method to analyze and optimize the tire grounding safety. The results of this study demonstrate the following: The TDD coefficient  $\lambda$  is only determined by the parameters of the human-wheelchair robot system. For wheelchair robots running on random roads, the smaller the  $\lambda$  value is, the better the tire grounding safety is. When the value of  $\zeta_2$  is too large or too small, it is not conducive to improving the tire grounding safety. An analytical optimization method for the cushion system damping ratio  $\zeta_2$  was established. The comparison result shows that the analytical optimization method is effective.

## 8. ACKNOWLEDGMENT

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## Analytical Investigation of Tire-Road Contact Characteristics for Wheelchair Robots Safely Running

C. Zhou, L. Zhao, Y. Yu, X. Li

School of Transportation and Vehicle Engineering, Shandong University of Technology, Zibo, China

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برای بهبود موثر و بهبود رفتارهای پایه ریش تراش روبات‌های صندلی چرخدار، یک روش تحلیلی برای تحلیل و بهینه‌سازی ایمنی پایه‌ی تایر صندلی چرخدار پیشنهاد شده است. اولاً، استفاده از کوسن و لاستیک‌ها را به عنوان عناصر جداسازی لرزش با سفتی و سقوط، مدل ارتعاش عمودی از روبات چرخشی انسان ساخته شده است. سپس، با توجه به تحرک تصادفی به عنوان ورودی معمولی، فرمول‌های پاسخ TDD (تابع نفوذ پویای تایر) عملکرد پاسخ H و RMS (میانگین مربع مربع) پاسخ می‌گیرند و ضریب پاسخ  $\lambda$  پیشنهاد می‌شود. علاوه بر این، قوانین نفوذ پارامترهای سیستم در H و  $\lambda$  مشخص می‌شود. سوم، بر اساس  $\lambda$  مدل بهینه‌سازی تحلیلی برای نسبت ساییدگی سیستم  $\lambda$  ایجاد شده است. در نهایت، یک مطالعه موردی و شبیه‌سازی عددی انجام شد. نتایج نشان می‌دهد که انحراف نسبی محدوده مطلوب کاهش حدود ۰/۳٪ داشته است.

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