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Cooperative Benefit and Cost Games under Fairness Concerns

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ABSTRACT

Solution concepts in cooperative games are based on either cost games or benefit games. Although cost games and benefit games are strategically equivalent, that is not the case in general for solution concepts. Motivated by this important observation, a new property called invariance property with respect to benefit/cost allocation is introduced in this paper. Since such a property can be regarded as a fairness criterion in cooperative games when deciding on choosing the solution concepts in coordination contracts, it is crucially important for players to check if the solution concepts available in contract menu possesses this property. To this end, we showed that some solution concepts such as the Shapley value, and the τ -value satisfy invariance property with respect to benefit/cost allocation but some others such as Equal Cost Saving Method (ECSM) and Master Problem variant I ($MP^I(S)$), do not. Furthermore, a measure for fairness with respect to equitable payoffs and utility is defined and related to invariance property. To validate the proposed approach, a numerical example extracted from the existing literature in benefit/cost cooperative games is solved and analyzed. The results of this research can be generalized for all solution concepts in cooperative games and is applicable for *n*-person games.

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1. INTRODUCTION[1]

The intensified competition, coupled with increasing costs of operation, have resulted in the failure of enterprises to achieve low cost, efficient and effective outcomes. Moreover, acting alone can no longer guarantee success in such a competitive global environment. For instance, in a decentralized supply chain, the order quantity of a buyer is less than that of the optimal quantity in a centralized case [1]. Game theory, as a helpful tool to analyze the benefit/cost allocation [2], deals with situations in which the outcomes of players (e.g., individuals, and coalitions) depend not only on their own decisions and actions but also on those of others [3]. One way of classifying games is according to type of interaction, among others. Accordingly, games can be divided into two broad categories: (a) cooperative games, and (b) non-cooperative games. The former deals with situations in which players are willing to cooperate with each other to benefit from working together, while the latter refer to the lack of any willingness to cooperate. The focus of this paper is on cooperative games which have attracted much attention in both practical applications and academic research since the origin of game theory.

A fundamental question that arises in practice is how to distribute the total benefit of the grand coalition among players. To successfully answer this question, lots of game-theoretic solution concepts have been proposed [2], each with its own advantages and disadvantages. These include, but are not necessarily limited to, the shapley value, core, nucleolus, the owen value, and the τ –value. It is a common practice to break a cooperative game problem into two phases. In the Phase 1, the problem is solved, usually by operations research techniques such as LP- or NLP-based programming, to determine the potential benefit or cost

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saving among players (e.g., customers, companies, supply chains). When complexity of computation increases, especially in complicated structures, using heuristics and meta-heuristic algorithms are required which are beyond the scope of this paper. Subsequently, in phase 2, the question of how to divide benefits among players is answered by some appropriate solution concepts. While academic research on cooperative game theory (CGT) traditionally focus on phase 2 approaching the problem from a mathematical point of view, the practical applications require both phases to be considered. Some recent studies on cooperative games within real-world application contexts include, to name but a few, vehicle routing problem [4-6]; inventory [7-9]; transportation [10-13]; furniture industry [14]; newsvendor game with product substitution [15], network data envelopmnet analysis (DEA) [16], energy supply chain [17, 18], and cooperative advertising [19]. The reader is reffered to Borm et al. [20] and Kogan and Tapiero [21], for more details on operations research games and supply chain games.

Frisk et al. [11] studied the role of cooperation and coordination among forest companies for cost reduction in a wood supply chain. The authors also implemented the shapley value, the nucleolus, a specific proportional allocation rule based on standalone costs, and, some other allocation rules based on separable and non-separable costs [22] such as equal charge method (ECM), Alternative cost avoided method (ACAM), and cost gap method (CGM). The authors reported an average cost saving of 8.6 percent which encourages forest companies to cooperate with each other.

Drechsel and Kimms [9] studied some procurement games with non-empty cores and used few solution concepts. In addition, they presented a computation procedure to find some core elements. The authors first used a LP model, to check if the core is empty. Concerning some fairness criteria, they then proposed two model, namely the master problem variant 1, $MP^{I}(S)$, and the master problem variant 2 , $MP^{II}(S)$.

In a different context, Audy et al. [14] considered the transportation collaboration in a furniture supply chain consisting of four Canadian companies. Inspired by two allocation rules including EPM and ACAM, they proposed a modified version of EPM to allocate costs among companies.

Lozano et al. [12] presented a cooperative game on transportation companies (shippers) to reduce costs of operation. To this end, the authors employed five well-known solution concepts including the shapley value, τ –value, nucleolus, core center, and Minmax core. They defined a reliable measure, namely synergy, as the ratio of cost savings of a given coalition to the sum of separate cost of each player of the same coalition. They also defined a metric to measure the satisfaction level of each non-empty coalition which is defined to be the excess of the sum of coalition members' allocated benefit. In order to measure the differences between different solution concepts, they also defined the main absolute deviation (MAD) criterion.

Inspired by core definition, Nguyen [23] introduced two new solution concepts based on core solution concept, namely the fairest core and the fairest least core. The author modeled the fairest core as a LP-based problem in which the objective function is to minimize the Euclidean distance of the solution concept (i.e., the fairest core) and the shapley value, while the constraints are the same as those of core (cf. Definition 5 in Section 2). The optimal solution to this LP gives the fairest core. A similar approach is adopted for computing the fairest least core in a slightly different LP with the same objective function subjected to the efficiency condition and a new inequality set related to the optimal solution to the least core. In fact, Nguyen [23] approached fairness as that of the shapley value.

The so-called vehicle routing problem games (i.e., VRP games, or, equivalently, VRG) were first studied by Göthe-Lundgren et al. [4] in which the authors investigated two well-known solution concepts, namely core and nucleolus. Zibaei et al. [6] studied a cooperative game on multi-depot vehicle routing problem (CoMDVRP) in which each depot belongs to only one distinct owner having only one vehicle. Inspired by EPM, they proposed a new method to fairly divide the joint cost savings among owners (i.e., players) which they called it equal cost saving method (ECSM). The authors compared the results obtained by ECSM with those obtained by three well-known solution concepts including the shapley value, τ –value and least core, which are somehow close to each other in symmetric case, but are not so in asymmetric case.

Very recently, Wu et al. [18] investigated the allocation scheme for cost saving in a cooperative game among three players based on a case study of an energy supply chain consisting of a store, a hotel and a hospital. The authors applied some solution concepts including the Shapley value, core, nucleolus, Propensity to Disrupt (DP) equivalent method, and the Nash-Harsany (N-H) solution. Moreover, in order to investigate the fairness and stability of each solution concept, they employed Shapley-Shubik power index and DP method.

All the above researches treated cooperative games either in terms of cost game [11] or benefit games [6, 12]. Although these two approaches are strategically equivalent, that is not the case for solution concepts. The main contribution of this paper is to fill part of this gap by introducing a new property for solution concepts which is called the invariance property with respect to benefit/cost allocation. The invariance property with respect to benefit/cost allocation implies that the allocation vector of players in a cost game (benefit game) is identical with that indirectly obtained by applying the same solution concept for its associated benefit game (cost game). Since this property can be regarded as a fairness criteria, it is crucially important for players to check if the solution concepts available in contract menu do satisfy this property. The second contribution of this study is to measure the fairness in some selected solution concepts and provide a basis to be able to compare them.

The outline of this paper is as follows. Section 2, deals with some basic definitions and concepts of cooperative games which will be used in this paper. In section 3, some selected solution concepts will be discussed in terms of fairness and stability properties. Section 4 deals with fairness and invariance property with respect to benefit/cost allocations. Section 5 presents a numerical examples to illustrate the proposed model. The paper concludes in section 6.

2. COOPERATIVE GAMES

In this section, a brief review of some fundamental concepts and definitions associated with cooperative games is presented. First, let us introduce some useful notations.

2. 1. Notations The following notations are used in this paper.

i	Index representing the player ($i = 1,2, \dots, n$)
N	Set of players ($N = \{1,2, \dots, n\}$)
n	Number of players ($n = \|N\|$)
S	Coalition of players, as a non-empty subset of N ($S \subseteq N$)
$v(S)$	Characteristic function of the coalition S in a benefit game
$c(S)$	Characteristic function of the coalition S in a cost game
δ	Solution concept
$\psi_i^{\delta}(N, v)$	Benefit allocation of player i through the solution concept δ in a benefit game
$\psi_i^{\delta}(N, c)$	Cost allocation of player i through the solution concept δ in a cost game
x_i	Implicit cost allocation of player i in a benefit game
y_i	Implicit benefit allocation of player i in a cost game

Other notations are introduced according to necessity.

2. 2. Preliminaries In the study of games, it seems reasonable to classify them as either cooperative or non-cooperative, depending on whether or not all players will cooperate with each other. Cooperative games, in turn, can be classified into two subgroups, namely, games with transferable utilities (TU) and those with nontransferable utilities (NTU). A cooperative game with transferable utilities means that the utilities obtained from cooperation can freely be transferred among players involved. A common interpretation of such transferable utilities are money, which can be conveniently distributed among players. Actually, any divisible commodity can be viewed as transferable utility in terms of ability to be transferred among players. For a detailed discussion on NTU games, we refer to Myerson [24] and Peleg and Sudhölter [25].

Definition 1. A coalitional game with transferable utility [25]. Formally, a coalitional game with transferable utility is characterized by a pair $\mathcal{G}: (N, v)$ where N denotes the finite set of players such that $N = \{1,2, \dots, n\}, n = |N|$ and $v \in G^N$ denotes the characteristic function which assigns a real value to each coalition.

In other words, a characteristic function v can be expressed by $v: 2^n \rightarrow \mathrm{R}$. Furthermore, by convention, it is common to assume $v(\phi) = 0$ in all such games. The set of all players also called the grand coalition which is desired and assumed to be formed. Note that, if v denotes a cost function, one can denote cost games by (N, c). When there is no ambiguity, a game can be briefly denoted by v.

Definition 2. Essential Game [26]. A game $\mathcal{G}: (N, v)$ is said to be essential if $\sum_{i \in N} v(i) < v(N)$ and would be inessential otherwise.

Through this paper we restrict our attention to cooperative games with transferable utilities (CGTU) in coalitional form which are essential and often interchangeably use the term games to refer to this class of games.

Definition 3. Solution concept [22]. A solution concept is a map which assigns to each game a worth in R^N. This allocated worth can be either a vector (i.e., a single-valued solution such as the nucleolus and the Shapley value) or a set of vectors (i.e., a set-valued solution such as core and bargaining sets).

Definition 4. Rationality conditions [27]. Rationality can be classified into three categories based on their domain as follows:

(i) Individual rationality (IR): $\psi_i^{\delta}(N, v) \geq v(i), \forall\, i \in N$. The IR condition states that the value allocated given to i-th player should not be less than what he could obtain on his own.

(ii) Coalition rationality (CR): $\sum_{i \in S} \psi_i^{\delta}(N, v) \geq v(S), \forall\, S \subset N$. The CR condition implies that the sum of values allocated to all players involved in a coalition should not be less than the value of the same coalition.

(iii) Group rationality (GR): $\sum_{i \in N} \psi_i^{\delta}(N, v) = v(N)$. The GR condition suggests that the sum of values allocated to all players should be equal to the value of the grand coalition.

All game-theoretic solution concepts satisfy both coalition rationality and group rationality as necessary conditions, but some fails to do individual rationality (e.g., the shapley value in some cases).

Definition 5. Pre-imputation, imputation, and core [25].

(i) Pre-imputation.

$$\mathcal{PI}(N,v) = \{\psi_i^{\delta}(N,v) \in \mathrm{R}^n \mid \sum_{i\in N} \psi_i^{\delta}(N,v) = v(N)\} \quad (1)$$

(ii) Imputation.

$$\mathcal{I}(N,v) = \{\psi_i^{\delta}(N,v) \in \mathcal{PI}(N,v) \mid \psi_i^{\delta}(N,v) \geq v(i)\ for\ \forall\ i \in N\} \quad (2)$$

(iii) Core.

$$\mathcal{C}(N,v) = \{\psi_i^{\delta}(N,v) \in \mathcal{I}(N,v) \mid \sum_{i\in S} \psi_i^{\delta}(N,v) \geq v(S)\ for\ \forall\ S \subseteq N\} \quad (3)$$

In part *(iii)*, the inequality set $\sum_{i\in S} \psi_i^{\delta}(N,v) \geq v(S)$ are called core defining inequality (CDI) in the literature [4]. As can be seen from definition 5, there may not exist a unique answer to the question of dividing benefits or cost savings among players. For instance, pre-imputation set contains infinite allocation schemes which most of them might not satisfy some criteria (e.g., individual rationality, coalitional rationality, group rationality, cf. definition 4 for more details). Imputation set satisfies the IR condition but might fail to satisfy coalition rationality. Core set, however, satisfies all the three conditions but might be empty as is the case in most applications (see, for instance, Göthe-Lundgren et al. [4] for VRP games; and Drechsel and Kimms [9] for inventory games).

Remark 1. In cooperative VRP games, the core might be possibly empty and thus the core variants (e.g., least core) should be taken into account in cases a core-related solution concept is required to have a stable solution concept.

Definition 6. Efficiency, Branzei et al. [28]: $\sum_{i\in N} \psi_i^{\delta}(N,v) = v(N)$

Efficiency tells us that the sum of elements of solution concepts is equal to the worth of the grand coalition. In other words, solution concepts satisfying efficiency will distribute all benefits among players. The terms efficiency, feasibility and group rationality (see definition 4 (iii)) often are used interchangeably in the literature [29].

2. 3. Cost Saving Games Alternatively, of course, a cost game can be converted into a benefit game (i.e., cost savings game) in two ways [29]. The first technique is as follows [25, 29]:

$$v(S) = \sum_{i\in S} c(i) - c(S) \geq 0 \quad (4)$$

This converting technique, as will be shown in the paper, results in zero values for 1-person coalitions, that is, $v(i) = 0$ for every $i \in N$. This naturally implies that each player has no cost saving when acting alone. The second converting technique [29] is given by:

$$c(S) = v(N) - v(N\backslash S)\ for\ \forall\ S \subseteq N \quad (5)$$

Nevertheless, in what follows we will concentrate on the first converting technique, because it seems to be more realistic than the former, as adopted by Young et al. [30], Lemair [31], Lozano et al. [12], Elomri et al. [32], and Zibaei et al. [6], to name but a few.

Definition 7. Implicit benefit/cost allocations.

When the underlying problem is modeled as a benefit game,(N,v), the implicit cost allocation of player $i, i \in N$, which is denoted by x_i can be obtained by:

$$x_i = \mathrm{c}(i) - \psi_i^{\delta}(N,v) \quad (6)$$

Similarly, in a cost game,(N,c), the implicit benefit allocation of player $i \in N$ which is denoted by y_i can be obtained by:

$$y_i = \mathrm{c}(i) - \psi_i^{\delta}(N,c) \quad (7)$$

3. SOLUTION CONCEPTS

For the sake of brevity, we restrict our attention to some selected solution concepts in this section. The results of this paper can be generalized to other solution concepts, as will be discussed later.

3. 1. Shapley Value. The Shapley value was brilliantly introduced and axiomatized by Shapley which is fundamentally based on marginal contribution of each player [33]. For a game (N,v), the Shapley value is the function $\psi_{(N,v)}^{Shapley}(i) : (N,v) \to R^n$, which is given by:

$$\psi_i^{Shapley}(N,v) = \sum_{S\subseteq N} \frac{(s-1)!(n-s)!}{n!} [v(S) - v(S - \{i\})], \forall i \in N \quad (8)$$

Owen [26] presented an interesting interpretation of the shapley value, namely multilinear extension (MLE) which is defined as follows:

$$\psi_i^{Shapley}(N,v) = \int_0^1 f^i\left(\underbrace{t,t,\dots,t}_{n}\right) dt \quad (9)$$

Where, f^i is the *i*-th partial derivative of function f, that is, $f^i = \frac{\partial f}{\partial z_i}$, and function f is given by:

$$f(z_1, z_2, \dots, z_n) = \sum_{S\subseteq N} \left(\prod_{i\in S} z_i \prod_{j\notin S} (1 - z_j)\right) v(S) \quad (10)$$

3. 2. Master Problem Variant I. Drechsel and Kimms [9] proposed a solution concept, based on the master problem discussed in subsection 3.1 which they called it master problem variant I (henceforth $MP^I(S)$). Since the focus of Drechsel and Kimms [9] is on cost games, all notations are in terms of costs but, as already stated in this paper, a cost game simply can be converted to a benefit game. In order to make this paper self-contained, we begin with a brief overview of $MP^I(S)$ method. However, we prefer to present $MP^I(S)$ for benefit game which can be given by:

$$\begin{aligned} &\text{Min } \overline{\theta} - \underline{\theta} \\ &\text{s.t.} \\ &e_i \le \overline{\theta}, \quad \forall\, i \in N \\ &e_i \ge \underline{\theta}, \quad \forall\, i \in N \\ &v(S) \le \textstyle\sum_{i\in S} e_i, \qquad \forall\, S \subset N, S \ne N \\ &v(N) = \textstyle\sum_{i\in N} e_i \\ &e_i \in R, \forall\, i \in N \\ &\overline{\theta}, \underline{\theta} \in R \end{aligned} \tag{11}$$

where, e_i denotes the allocation of player i, and $\overline{\theta}$ and $\underline{\theta}$ denote the upper and lower bounds of possible allocations, respectively.

3. 3. Equal Cost Saving Method (ECSM). Zibaei et al. [6] proposed the ECSM as a new allocation scheme without any theoretical support as well as without assessment of its relative merits. Motivated by these observations, we address some aspects of this new method. To make this paper more self-contained, a brief overview of ECSM model is also presented as follows:

$$\begin{aligned} &\text{Min } \lambda \\ &\text{s.t.} \\ &|e_i - e_j| \le \lambda, \quad \text{for } \forall\, i \text{ and } j \in N \\ &v(S) \le \textstyle\sum_{i\in S} e_i, \quad \text{for } \forall\, S \subset N, S \ne N \\ &v(N) = \textstyle\sum_{i\in N} e_i \\ &e_i, e_j \ge 0, \qquad \text{for } \forall\, i \text{ and } j \in N \end{aligned} \tag{12}$$

where, λ denotes the maximum difference between pairwise payoffs, and $v(S)$ and $v(N)$ denote the value of nonempty coalitions S and grand coalition N, respectively.

Zibaei et al. [6] proposed ECSM as a stable and uniform allocation scheme. We respectfully disagree with the authors and provide justification for our claims that stability and uniformity of ECSM is not guaranteed at all but rather for some specific cases only. Proposition 2 deals with uniformity aspect of ECSM. The fairness aspect will be discussed in Proposition 8 and corollary 2.

3. 4. $\tau - value$. Tijs [34] introduced the $\tau - value$ for quasi-balanced games, as a compromise solution concept based on upper- and lower-payoff of each player. Furthermore, an axiomatization of the $\tau - value$ presented by Tijs [35]. A game $\mathcal{G}: (N, v)$ is called quasi-balanced if and only if:

(i) $m(N,v) \le M(N,v)$

(ii) $\sum_{i\in N} m^i(N,v) \le v(N) \le \sum_{i\in N} M^i(N,v)$

The $\tau - value$ is given by:

$$\psi_i^{\tau}(N,v) = \alpha . m(N,v) + (1-\alpha). M(N,v) \tag{13}$$

where, $\sum_{i\in N} \tau^i = v(N)$, and $m(N,v)$ and $M(N,v)$ are given by:

$$M^i(N,v) = v(N) - v(N\backslash i) \tag{14}$$

$$R^i(N,v) := v(S) - \textstyle\sum_{j\in S\backslash\{i\}} M^j(N,v) \tag{15}$$

$$\begin{aligned} m^i(N,v) &:= \underbrace{\max}_{S: i\in S} R(N,v) \\ &= \underbrace{\max}_{S: i\in S} \left(v(S) - \textstyle\sum_{j\in S\backslash\{i\}} M^j(N,v)\right) \end{aligned} \tag{16}$$

Proposition 1. The $MP^I(S)$ model and ECSM one are equivalent.

Proof. Consider the $MP^I(S)$ model as shown in section 3.2. According to the $MP^I(S)$ model, for any i and j belonging to N, we have $e_i \le \overline{\theta}$ and $e_j \ge \underline{\theta}$ which clearly leads to $e_i - e_j \le \overline{\theta} - \underline{\theta}$. In a similar manner, from $e_i \ge \underline{\theta}$ and $e_j \le \overline{\theta}$, we have $e_j - e_i \le \overline{\theta} - \underline{\theta}$. Combining these two inequalities produces $|e_i - e_j| \le \overline{\theta} - \underline{\theta}$. Changing the variable $\overline{\theta} - \underline{\theta}$ into λ gives the desired results.

Proposition 2. Under specific conditions, both the $MP^I(S)$ and ECSM models would behave like the so-called egalitarian method, which assign an equal share for each player. That is,

(i) for a given cost game: $x_i = \frac{c(N)}{|N|}$,

if and only if

$\frac{|S|}{|N|} \le \frac{c(S)}{c(N)}, \quad \forall\, S \subset N\backslash\emptyset$

(ii) for a given benefit game: $y_i = \frac{v(N)}{|N|}$,

if and only if

$\frac{|S|}{|N|} \ge \frac{v(S)}{v(N)}, \quad \forall\, S \subset N\backslash\emptyset$

Proof. The proof is omitted for brevity and is available on request from the authors.

Proposition 3. The ECSM model (as shown in Section 3. 3) with a new constraint set $\left|\frac{e_i}{c(\{i\})} - \frac{e_j}{c(\{j\})}\right| \le \lambda$, instead of the constraint set $|e_i - e_j| \le \lambda$ leads to another model which would be equivalent to $MP^{II}(S)$. Similarly, $MP^I(S)$ model, with the constraint sets $\frac{e_i}{c(\{i\})} \le \overline{\theta}$ and $\frac{e_i}{c(\{i\})} \ge \underline{\theta}$, instead of the constraint sets $e_i \le \overline{\theta}$ and $e_i \ge \underline{\theta}$, leads to $MP^{II}(S)$.

Proof. The proof is straightforward and thus omitted here.

Proposition 4. Both the $MP^I(S)$ and ECSM models are monotonic if some specific onditions are met as described in Proposition 2.

Proof. Suppose two different cooperative games, namely $\mathcal{G}_1$ and $\mathcal{G}_2$. Let $\mathcal{G}_1$ be the original problem having $v_{\mathcal{G}_1}(N) = \pi$ and $\mathcal{G}_2$ be the same problem except that $v_{\mathcal{G}_2}(N) = \pi + \Delta$. According to proposition 2, if the specific conditions are met, we get values in $\mathcal{G}_1$ and $\mathcal{G}_2$ be $v_{\mathcal{G}_1}(S) = \frac{v_{\mathcal{G}_1}(N)}{|N|} = \frac{\pi}{|N|}$ and $v_{\mathcal{G}_2}(S) = \frac{v_{\mathcal{G}_2}(N)}{|N|} = \frac{\pi+\Delta}{|N|}$, respectively, for any coalition S such that $\emptyset \neq S \subseteq N$. Clearly, these two games satisfy the condition $v_{\mathcal{G}_2}(S) = \frac{\pi+\Delta}{|N|} \geq \frac{\pi}{|N|} = v_{\mathcal{G}_1}(S)$, when $\Delta \geq 0$ and $v_{\mathcal{G}_2}(S) = \frac{\pi+\Delta}{|N|} \leq \frac{\pi}{|N|} = v_{\mathcal{G}_1}(S)$ otherwise. Therefore, the proof is complete.

Corollary 1. In general, nither the $MP^I(S)$ model nor the ECSM model is monotonic.

Proof. The proof is straightforward and thus omitted here.

Proposition 5. Neither the $MP^I(S)$ model nor the ECSM model may have feasible solution.

Proof. The proof immediately follows from the fact that the core might be empty. This is why that Drechsel and Kimms [9] assumed in advance that the core is non-empty in respective games.

Proposition 6. $MP^I(S)$ and ECSM, in general, might be neither fair nor stable.

Proof. The proof can be done easily by induction on the allocation values for the $MP^I(S)$ and ECSM models.

Remark 2. A fair solution concept is expected to make the grand coalition stable, and consequently an unfair solution concept can lead to instability of the grand coalition. Conversely, a stable solution concept (such as core and its variants) is not essentially fair, but an unstable situation in the grand coalition might be unfair as well.

4. FAIRNESS AND INVARIANCE PROPERTY

Since, by assumption, players are risk neutral, they are profit maximizer, and thus they believe that a higher benefit allocation would be more equitable than a lower one, or equivalently that a lower cost allocation would be more equitable than a higher one. In this regard, utility of the player $i, i \in N$, in a benefit game can be defined by

$$U_i^{\delta}(N,v) = max(\psi_i^{\delta}(N,v), y_i) \tag{17}$$

Similarly utility of the player $i, i \in N$, in a cost game can be defined by

$$U_i^{\delta}(N,c) = min(\psi_i^{\delta}(N,c), x_i) \tag{18}$$

Therefore it is reasonable to assume that utility of a player, or k times utility of a player, can be regarded as the equitable payoff for him/her. In the sequel, without loss of generality, we assume that $k = 1$.

Definition 8. Invariance property of a solution concept, δ, w.r.t. benefit/cost allocation. A solution concept, δ, is said to have invariance property w.r.t. benefit/cost allocation if and only if $\psi_i^{\delta}(N,c) = x_i$, or, equivalently, if and only if $\psi_i^{\delta}(N,v) = y_i$. In other words, the necessary and sufficient conditions for a solution concept, δ, is said to have invariance property w.r.t. benefit/cost allocation if $\psi_i^{\delta}(N,v) = c(i) - \psi_i^{\delta}(N,c)$, or, equivalently, if $\psi_i^{\delta}(N,c) = c(i) - \psi_i^{\delta}(N,v)$.

Definition 8 tells us that if the benefit/cost allocation of player $i, i \in N$, is independent from the type of the respected game (whether cost game or benefit game), then the solution concept under consideration is said to have invariance property w.r.t. benefit/cost allocations.

Proposition 7. The shapley value satisfies invariance property w.r.t. benefit/cost allocation scheme, that is:

$$\psi_i^{Shapley}(N,v) = c(i) - \psi_i^{Shapley}(N,c) \tag{17}$$

Proof. The proof is omitted for brevity and is available on request from the authors. Also, the reader may refer to González-Díaz et al. [36] for a beautiful proof of this result by using the classic formula of the Shapley value. However, our proof differs from that of González-Díaz et al. [36] in such a way that we use the multi-linear extension as the main basis of the proof.

Proposition 8. Neither the $MP^I(S)$ model nor the $ECSM$ model satisfies invariance property w.r.t. benefit/cost allocation.

Proof. The proof is omitted for brevity, and is available on request from the authors (see also the numerical examle in section 5).

Corollary 2. The $MP^I(S)$ and $ECSM$ are not, in general, fair allocation schemes w.r.t. invariance property.

Proof. The proof follows from Proposition 8.

Remark 3. Note that the concept of the covariant under the strategic equivalence (COV property) [25], and the concept of the relative invariance with respect to the strategic equivalence [28] do not differ from each other, but both differ from the invariance property w.r.t. benefit/cost allocation. For instance, the core satisfies COV property [25]; however, solutions of the $MP^I(S)$ and $ECSM$ models as elements of core do not satisfy invariance property w.r.t. benefit/cost allocation as shown in proposition 8.

Proposition 9. The $\tau - value$ satisfies invariance property w.r.t. benefit/cost allocation.

Proof. The proof is omitted for brevity and is available on request from the authors.

Remark 4. The Shapley value and $\tau - value$ are fair w.r.t. invariance property.

5. NUMERICAL EXAMPLE

The data set for this example is taken from Drechsel and Kimms in the context of procurement game. A three-player cost game is given by characteristic functions as c(1)=644, c(2)=511, c(3)=483, c(1,2)=1029, c(1,3)=1004, c(2,3)=869, c(1,2,3)=1393. The allocation vectors for players according to Drechsel and Kimms [9], obtained by employing $MP^I(S)$, are reported in the second column of Table 1, while the results of the same game according to $ECSM$ are reported in the third column.

As can be seen from Table 1, the solutions obtained by two methods are exactly identical, which is consistent with proposition 1. Furthermore, Table 2 presents benefit/cost allocations for $MP^I(S)$.

The results presented in Table 2 coincide with those of proposition 9, demonstrating that neither the $MP^I(S)$ model nor the $ECSM$ model satisfies the invariance property w.r.t. benefit/cost allocation scheme.

TABLE 1. Allocation vectors for players

Players	Drechsel and Kimms (2010)	Our Result (using ECSM)
Player 1	524.0	524.0
Player 2	434.5	434.5
Player 3	434.5	434.5
Obj. function	$\overline{\theta} - \underline{\theta} = 524 - 434.5 = 89.5$	λ=89.5

TABLE 2. $MP^I(S)$ and its invariant property

Player	$MP^I(S)$ (cost)	Benefit Allocation	$MP^I(S)$ *(benefit)*	Cost Allocation
Player 1	524.0	120.0	81.67	562.33
Player 2	434.5	76.5	81.67	429.33
Player 3	434.5	48.5	81.67	401.33

6. CONCLUSION

The question of how to divide the total benefits among participants of a game plays a pivotal role in cooperative games with transferrable utility. Since any benefit (cost) game can be theoretically converted to a cost (benefit) game, studies on solution concepts have traditionally concentrated on either benefit- or cost games. Consequently, it is implicitly assumed that having a solution concept for a benefit game can lead to a cost allocation vector which is equivalent to the result of employing the same solution concept for associated cost game. But such an assumption, in most applications, is not valid. In this regard, a general lack of distinguishing between solution concepts in benefit/cost games is identified and highlighted as a research gap. In this study, this property, which is called invariance property with respect to benefit/cost allocations, is introduced and investigated for some selected solution concepts including the Shapley value, the $\tau - value$, the $MP^I(S)$, and the ECSM. In addition, we interpret invariance property as a fairness criterion, by defining an appropriate measure, whenever the respective problem is related to a benefit/cost game.

To summarize, the main contributions of this study are: (i) introducing the invariance property of solution concepts with respect to benefit/cost allocations in n-person cooperative games; (ii) presenting a criterion to measure the fairness of solution concepts for respective problem and relating it to the invariance property.

Future research needs to include risk-averse and risk-seeking attitude in modeling the problem, especially when dealing with fairness measure. Other extensions can then investigate both fairness and stability of solution concepts to provide a trade-off between them. It is also intereseting to take other aspects of fairness into consideration.

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Cooperative Benefit and Cost Games under Fairness Concerns

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چکیده

راهکارهای تخصیص در بازی‌های همکارانه، مبتنی بر بازی های منفعت یا بازی‌های هزینه است. اگرچه بازی‌های هزینه و بازی‌های منفعت از نظر استراتژیک معادل یکدیگر هستند ولی این موضوع در مورد تمام راهکارهای تخصیص صدق نمی‌کند. به‌همین دلیل یک ویژگی جدید تحت عنوان ویژگی بی‌تفاوتی نسبت به تخصیص هزینه/منفعت در این مقاله تعریف گردیده است. راهکارهای تخصیص دارای این ویژگی باعث می شود که چنانچه تخصیص منفعت(هزینه) یک بازیکن مشخص باشد، تخصیص ضمنی هزینه (منفعت) آن بازیکن مساوی با تخصیص هزینه (منفعت) بدست آمده توسط همان راهکار تخصیص است. با توجه به اینکه چنین ویژگی را می‌توان بعنوان معیار منصفانه بودن تخصیص در بازی‌های همکارانه هنگام تصمیم‌گیری در مورد انتخاب راهکارهای تخصیص قلمداد کرد، برای بازیکنان، کنترل اینکه آیا راهکار تخصیص مورد نظر دارای این ویژگی باشد از اهمیت خاصی برخوردار است. بدین منظور، در این مقاله نشان داده می شود که برخی راهکارهای تخصیص منجمله عدد شاپلی و عدد تاو دارای ویژگی بی تفاوتی نسبت به تخصیص هزینه/منفعت هستند، اما برخی دیگر منجمله روش صرفه جویی یکسان و روش مسئله اصلی نوع I دارای این ویژگی نیستند. علاوه بر این، معیاری برای اندازه‌گیری منصفانه بودن با توجه به سود تخصیص یافته و مطلوبیت، تعریف شده و رابطه آن با ویژگی بی‌تفاوتی بررسی شده است. برای اعتبارسنجی رویکرد پیشنهادی، یک مثال عددی که داده‌های آن برگرفته از ادبیات تحقیق زنجیره تامین مربوط به بازی‌های هزینه منفعت می باشد، حل و مورد تجزیه و تحلیل قرار گرفته است. نتایج این تحقیق را می توان برای سایر راهکارهای تخصیص بازی‌های همکارانه نیز تعمیم داد و حتی در بازی های n نفره بکار بست.

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