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The Integrated Supply Chain of After-sales Services Model: A Multi-objective Scatter Search Optimization Approach

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ABSTRACT

In recent decades, high profits of extended warranty have led third-party firms considering it as a lucrative after-sales service. However, customers' division in terms of risk aversion and effect of offering extended warranty on manufacturers' basic warranty should be investigated through adjusting such services. Risk-averse customers welcome extended warranty, while the customers without taking on risk may remain at the level of basic warranty. In this paper, a multi-objective integer nonlinear programming model is presented for integrating the supply chain of after-sales services. In the suggested model, firstly the strategies used by the manufacturers in the basic warranty period and the third party's policy during the extended warranty period, including the development of a new imperfect maintenance approach, are regulated. The effects of these strategies on the desirability of customers with different levels of risk-taking are then analyzed. In order to optimize the model, the scatter search based approach was introduced for extracting set of non-dominated solutions. The results indicated that increasing level of customers' risk-taking convinces manufacturers to diminish the basic warranty period and the third party can apply less costly preventive maintenance.

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1. INTRODUCTION

In after-sales services, product warranty refers to a contract between manufacturer and customer which shows the manufacturer's responsibility for repairing /replacing services or paying customer compensation due to the existence of a defective product within a specified time period called "warranty period" [1]. In addition to basic warranty (BW) that is issued in the form of a bundle with the product, extended warranty (EW) is proposed for products with a long useful life. EW is usually suggested to customers at the end of BW; therefore, the third-party (3P) companies such as insurance ones are interested in offering it. Extended warranty usually possesses high profit margins. For example, purchasing EW ranges from 30% for products such as cars to 75% for electronic devices and their applications [2].

Generally, there are two classifications in relation to basic and extended warranty: (1) Studies which consider EW independent from BW; (2) Studies which consider EW with regard to the effect of BW and in the form of a supply chain of after-sales services. In the first group, EW is investigated only from the customer, manufacturer, or the third party's viewpoints [3-6]. However, from customers' viewpoints, extended warranty can be complementary and sometimes replace basic warranty. As a consequence, strategies of EW periods affect the manufacturers. In addition, the strategies adopted by the manufacturer during the BW period are effective on EW policies of the third party; hence, the second group of studies is considered.

Jiang and Zhang [7] investigated the effect of EW policies of a 3P on warranty strategies offered by a manufacturer. The results illustrated that for the manufacturer displaying product quality to customers at the time of existence of EW is more possible compared to its absence. Heese [8] indicated that although increasing BW period for a product causes the establishment of its position against competitors'

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products, customers of the product are less interested in EW. This issue is due to the overlap of BW with EW. Esmaeili et al. [9] used a game theory approach to model the contracts between the third party, manufacturer and costumer in two non-cooperative and semi-cooperative states.

Since warranty service costs directly affect the provider's profit, studies [10-17] applied maintenance strategies alongside BW/EW policies to reduce incurred costs. Although the conducted studies in the second group investigate the concurrent effects of BW and EW on each other, they mostly assume that customers have the same interests to the offered warranty. However, in the real world, degrees of customers' risk-taking are different; consequently, negligence of after-sales service providers to this issue, particularly EW providers, can not only increase costs of warranty periods, but also involve the risk of customers' unwillingness to the offered warranty.

In this paper, a multi-objective integer nonlinear programming (MOINLP) model is presented for integrating the supply chain of after-sales services with considering the manufacturer, third party and customer's viewpoints. In the proposed model, BW is supported by one manufacturer and EW is offered by a 3P. Moreover, customers have different degrees of risk-taking and policies adopted by the manufacturer and 3P affect their desirability. Since EW is carried out after the end of BW, a new imperfect maintenance strategy based on virtual age approach will be developed from the third party's viewpoint. As the proposed model is a MOINPL, obtaining optimal solutions for large scale of such problems is practically impossible. So, a multi-objective scatter search approach is developed for extracting a set of non-dominated solutions. According to our knowledge, this is for the first time that an integrated after-sales services model is developed to optimize policies of warranty and extended warranty periods upon maintenance strategies and customers' desirability.

In the following, the problem definition is presented in Section 2. In Section 3, the model components including manufacturer, 3P and customer perspectives are discussed. The integrated supply chain of after-sales services model is introduced in Section 4. The proposed solution approach is presented in Section 5. The numerical examples are discussed in Section 6. Finally, concluding remarks and suggestions for further research are discussed in Section 7.

2. PROBLEM DEFINITION

Consider a repairable product that faces failures over time and as a result of deterioration process. Suppose that the random variable of T shows failure process. $0f(t)$ and $F(t)$ are respectively the probability distribution function and the cumulative distribution function of failure process. Accordingly, the hazard rate function ($h(t)$) will be calculated as follows:

$$h(t) = \frac{f(t)}{1-F(t)} \tag{1}$$

$h(t)$ is an increasing function that enhances over time due to deterioration process and finally lead to product breakdown. The product is sold by a manufacturer to a set of customers as a bundle with non-renewable basic warranty for a period of time BW. After the basic warranty is expired, a non-renewable extended warranty is offered to customers by a third party during the EW. Since the degree of risk-taking is different among customers, there is no same willingness to pay to the proposed extended warranty.

Suppose, random variable of r_i represents the ith customer's risk taking, that is defined in the range of $[0, R]$ and has the $g(r_i)$ probability function. In these circumstances, $r_i = R$ shows a customer is highly risk-averse and $r_i = 0$ indicates a risky customer. The more a customer is risk-taking, the less he/she uses the extended warranty of products [7] and the product failure has less negative effect on him/her [6]. In addition, the duration of the basic and extended warranty for customers with different degrees of risk taking, does not make equal desirability. As a result, third party and the manufacturer should set the warranty policy with the aim of controlling its associated costs along with maximizing the customers' satisfaction.

3. COMPONENTS OF THE MODEL

This section gives the model components, including the customer's perspective, the opinions of third-party and the manufacturer, in addition to provide a base for introducing an integrated supply chain model of after-sales services.

3. 1. Manufacturer Perspective

Manufacturer, only performs *minimal* corrective maintenance (CM) during the basic warranty period and does not apply any preventive maintenance (PM) policy. In this regard, the average number of CM actions during the basic warranty for one product will be in the form of Equation (2):

$$N_{BW} = \int_0^{BW} h(t)dt \tag{2}$$

N_{BW} is a function of the basic warranty period. If C_M^{CM} defines cost of carrying out one CM by the manufacturer, then total cost of basic warranty period for a product from the manufacturer perspective can be obtained as follows:

$$TC_M = C_M^{CM} N_{BW} = C_M^{CM} \int_0^{BW} h(t)dt \tag{3}$$

3. 2. Third-party Perspective During the extended warranty period, the third party applies the *imperfect* preventive maintenance policy in order to reduce cost of product breakdown, under which: (1) at the time of $\tau_1, \tau_2, \dots, \tau_n$ $(n = \left[\frac{EW-BW}{\Delta}\right])$ with fixed time intervals of Δ, the product is sent for inspection and doing PM to the third party. (2) In the case of failure, the ith customer's product is sent to the third party for inspection and conducting minimal CM. Under this policy, there are 4 modes including:

1- The product is broken down due to the deterioration process. In this case, by applying a minimal CM, the product returns to the condition of "as bad as old".
2- At the time of PM inspection, deterioration process of the product is at a lower level or equal to γ^{lower} (i.e, $h(t) \leq \gamma^{lower}$). In such a condition, there is no need to apply PM.
3- At the time of PM inspection, deterioration process of the product is at a level between γ^{lower} and γ^{upper}. In this case, the product is rejuvenated, by applying an imperfect PM at the level of $m \in [m^{min}, m^{max}]$.
4- At the time of PM inspection deterioration process of the product is at a level greater than or equal to γ^{upper} (i.e. $h(t) \geq \gamma^{upper}$). In this case, the product will be replaced by a spare part.

Figure 1 shows the mentioned maintenance policy during extended warranty period on a given product.

In Figure 1, at the time of τ_1, the degree of deterioration process is more than γ^{upper}, which results in a replacement of the product with a spare part. At the time of the second inspection of τ_2, the deterioration process is at a level lower than γ^{lower}, thus no PM is required.

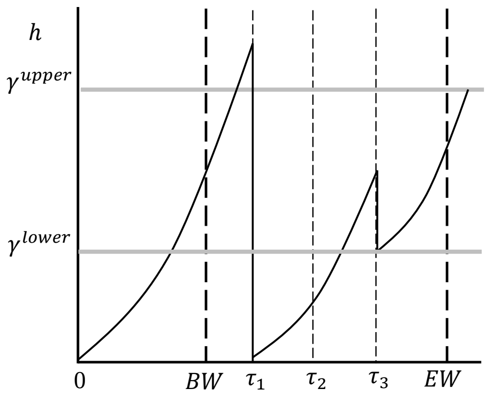

Figure 1. The maintenance policy of third party during the extended warranty on a given product

At the time of τ_3, deterioration process value is in the interval $(\gamma^{lower}, \gamma^{upper})$. As a result, by applying an imperfect PM at level of m, the product becomes rejuvenated.

Suppose that $v(t)$ shows the virtual age of the product at time of t. In such conditions, the virtual and real age are equal to the first time of PM action. As a result, we have:

$$v(t) = t \quad BW \leq t < \tau_1 \tag{4}$$

$$h(v(t)) = h(t) \quad BW \leq t < \tau_1 \tag{5}$$

After applying the first PM, the virtual age of the product is obtained by the following equation.

$$v_1 = h^{-1}(\eta h(\tau_1)) \quad t = \tau_1 \tag{6}$$

In Equation (6), $h^{-1}(.)$ is the inverse function of the failure rate and η is a function that indicates the reduction in the deterioration process and is calculated based on Equation (7):

$$\eta = \begin{cases} 1 & h(a) \leq \gamma^{lower} \\ \delta(m) & \gamma^{lower} < h(a) < \gamma^{upper} \\ 0 & \gamma^{upper} \leq h(a) \end{cases}, \tag{7}$$

According to Equation (7), if the deterioration process at the moment of a $(h(a))$ is less than or equal to γ^{lower}, no PM action is taken by 3P. If $\gamma^{lower} < h(a) < \gamma^{upper}$, the amount of deterioration process reduces by a factor of $\delta(m) \in [0,1]$ and if the deterioration process is greater than or equal to γ_i^{upper}, the product is replaced by a spare part. $\delta(m)$ is a decreasing function of the PM which has chosen by the third party $(m \in [m^{min}, m^{max}])$, in a way that $\delta(m^{min}) = 1$ transfers the product to "as bad as old" condition and $\delta(m^{max}) = 0$ transfers the product to "as good as new" condition.

According to Equation (6), after failure rate at the moment τ_1 reduced to level $\eta h(\tau_1)$, corresponding virtual age of this level is determined by function $h^{-1}(.)$. After the pth PM, the virtual age (v_p) and the failure rate are calculated as follows:

$$h(v_p) = \eta h(v_{p-1} + \Delta) \qquad p = 2, \dots, n, \tag{8}$$

$$v_p = h^{-1}\left(\eta h(v_{p-1} + \Delta)\right) \quad p = 2, \dots, n, \tag{9}$$

As is evident in relations (8) and (9), the failure rate after applying the pth preventive maintenance is a fraction of failure rate before that. The virtual age and the failure rate in the range of $\tau_p \leq t < \tau_{p+1}$ are as follows:

$$v(t) = v_p + (t - \tau_p), \quad \tau_p \leq t < \tau_{p+1} \quad p = 2, \dots, n-1 \tag{10}$$

$$h(v(t)) = h\left(v_p + (t - \tau_p)\right), \qquad \tau_p \leq t < \tau_{p+1} \quad p = \tag{11}$$

$2, \dots, n-1$

Finally, Equations (12) and (13) show the values of the virtual age and the failure rate in the range of $\tau_n \leq t \leq EW$.

$$v(t) = v_n + (t - \tau_n) \qquad \tau_n \leq t \leq EW \tag{12}$$

$$h\big(v(t)\big) = h\big(v_n + (t - \tau_n)\big), \ \tau_n \leq t \leq EW \tag{13}$$

Since the occurred failures in the product are rectified minimally and with a negligible time, the expected number of failures in each interval can be obtained by integrating the hazard rate function during that interval. As a result, the expected number of failures over an extended warranty period (N_{EW}) is obtained as $N_{EW} = \int_{BW}^{EW} h\big(v(t)\big)dt$. So, we have the following equation (14):

$$N_{EW} = \int_{BW}^{EW} h\big(v(t)\big)dt = \int_{BW}^{\tau_1} h\big(v(BW) + (t - BW)\big)dt + \sum_{p=1}^{n-1} \int_{\tau_p}^{\tau_{p+1}} h\Big(v_p + \big(t - \tau_p\big)\Big)\,dt + \int_{\tau_n}^{EW} h\big(v_n + (t - \tau_n)\big)dt \tag{14}$$

Suppose C_{3P}^{CM} is defined as the cost of operating CM on a product during the extended warranty. Then the expected CM costs for a customer's product during extended warranty period is obtained as follows:

$$ECM_{EW}(\Delta, m) = C_{3P}^{CM} N_{EW} = C_{3P}^{CM} \int_{BW}^{\tau_1} h\big(v(BW) + (t - BW)\big)dt + C_{3P}^{CM} \sum_{p=1}^{n-1} \int_{\tau_p}^{\tau_{p+1}} h\Big(v_p + \big(t - \tau_p\big)\Big)\,dt + C_{3P}^{CM} \int_{\tau_n}^{EW} h\big(v_n + (t - \tau_n)\big)dt \tag{15}$$

If C^{PM} is defined as the cost of doing PM on a product during the extended warranty, then the expected PM costs for a customer's product during extended warranty period is calculated as follows:

$$EPM_{EW}(\Delta, m) = \sum_{p=1}^{n} C^{PM} y_p \tag{16}$$

In Equation (16), y_p is a binary variable. If the value of a deterioration process at pth PM action is placed between upper and lower allowable limit, it will be one and otherwise it will be zero, that is:

$$y_p = \begin{cases} 1 & if \quad \gamma^{lower} < h\big(v_{p-1} + \Delta\big) < \gamma^{upper} \\ 0 & otherwise \end{cases}, p = 1, 2, \dots, n, \tag{17}$$

In Equation (17), the cost of PM actions (i.e. C^{PM}) depends on the level of chosen maintenance (m). The average number of required spare parts during the extended warranty period is obtained as following:

$$ES_{EW}(\Delta, m) = C^S \sum_{p=1}^{n} x_p \tag{18}$$

In Equation (18), x_p is the binary variable and if the deterioration process at time of pth PM inspection is equal to or greater than upper allowed limit, it will be one and the otherwise will be zero, that is:

$$x_p = \begin{cases} 1 & if \quad h\big(v_{p-1} + \Delta\big) \geq \gamma^{upper} \\ 0 & otherwise \end{cases}, \quad p = 1, 2, \dots, n, \tag{19}$$

As it is evident in Equations (15), (16) and (18), the values of ECM_{EW} and EPM_{EW} and ES_{EW} are a function of maintenance polices made by the third party during the extended warranty period, including the values of the distance between two consecutive PM (Δ) and the applied PM level (m). The total cost of the third party among extended warranty period (TC_{3P}) is as follows:

$$TC_{3P} = ECM_{EW}(\Delta, m) + EPM_{EW}(\Delta, m) + ES_{EW}(\Delta, m) \tag{20}$$

3. 3. Customers Perspective

As mentioned before, the rate of risk-taking of customers is shown by the random variable of r. For the ith customer with r_i value of risk taking, the function of $\psi(EW|r_i)$ indicates the desirability of extended warranty, which is defined in the following.

$$\psi(EW|r_i) = \chi r_i EW^{\Omega(r_i - \theta)} \tag{21}$$

In Equation (21), values of χ, Ω and θ are the scale, shape and center parameters,According to Equation (21), the ith customer with the risk-taking degree of $r_i \in [0, \theta)$ is assumed risk-taker. In this case, by increasing the length of the extended warranty period, his desirability decreases. For values of $r_i \in [\theta, R]$, the ith customer is assumed risk averse. In this case, by increasing the warranty period, his desirability value increases. Based on the customer desirability, the expected value of the ith customer's desirability of the proposed extended warranty, can be concluded as follows.

$$\psi(EW) = \int_0^R \psi(EW|r_i)\, g(r_i)\, dr = \int_0^R \chi r_i EW^{\Omega(r_i - \theta)}\, g(r_i)\, dr \tag{22}$$

The probability function of risk-taking (i.e., $g(r)$) is triangular distribution and is defined as Equation (23):

$$g(r) = \begin{cases} \frac{2r}{R\theta} & 0 \leq r \leq \theta \\ \frac{2(R-r)}{R(R-\theta)} & \theta < r \leq R \end{cases} \tag{23}$$

In Equation (23), customers are risk-taking in the $r \in [0, \theta)$ and are risk-averse in the $r \in [\theta, R]$. By substituting Equation (23) in (22), we have:

$$\psi(EW) = \int_0^R \chi r_i EW^{\Omega(r_i - \theta)}\, g(r_i)\, dr = \int_0^\theta \frac{2{r_i}^2 \chi EW^{\Omega(r_i - \theta)}}{R\theta} dr + \int_\theta^R \frac{2\chi r_i (R - r_i) EW^{\Omega(r_i - \theta)}}{R(R-\theta)} dr = \frac{2\chi(2-R)}{(R-\theta).EW^{\Omega\alpha}(\Omega \ln EW)^2} e^{(\Omega \ln EW)R} - \frac{4\chi}{R(R-\theta)EW^{\Omega\alpha}(\Omega \ln EW)^3} e^{(\Omega \ln EW)R} + \frac{4\chi\theta}{REW^{\Omega\alpha}\Omega \ln EW} e^{(\Omega \ln EW)\theta} + \tag{24}$$

$$\frac{2\chi(\theta-2)}{(R-\theta)EW^{\Omega\alpha}(\Omega\ln EW)^2}e^{(\Omega\ln EW)\theta}+\frac{4\chi}{(R-\theta)\theta EW^{\Omega\alpha}(\Omega\ln EW)^3}e^{(\Omega\ln EW)\theta}-\frac{4\chi}{R\theta EW^{\Omega\alpha}(\Omega\ln EW)^3}$$

Equation (24) shows the expected customer's desirability from offered extended warranty to the length of EW. Using Equation (25), the customer's desirability function of Extended Warranty converts in the interval [0,1]. In Equation (25) ψ^{min} and ψ^{max} are the minimum and maximum values of customer's desirability from the offered extended warranty. In this regard, if $u = 1$, the shape of $d(EW)$ function is linear, If $u < 1$, is concave and if $u > 1$, is convex [18].

$$d(EW) = \begin{cases} 0 & \psi(EW) < \psi^{min} \\ \left(\frac{\psi(EW)-\psi^{min}}{\psi^{max}-\psi^{min}}\right)^u & \psi^{min} \le \psi(EW) < \psi^{max} \\ 1 & \psi(EW) \ge \psi^{max} \end{cases} \tag{25}$$

Since the basic warranty is presented as a bundle with the product, it is assumed that the customers, either risk-taking or risk averse, will have a higher desirability with the longer basic warranty. As a result, function of customer's desirability from basic warranty ($d(BW)$)) is defined as follows (q parameter is similar to u parameter in Equation (25)).

$$d(BW) = \begin{cases} 0 & BW < BW^{min} \\ \left(\frac{BW-BW^{min}}{BW^{max}-BW^{min}}\right)^q & BW^{min} \le BW < BW^{max} \\ 1 & BW \ge BW^{max} \end{cases} \tag{26}$$

In addition to the extended warranty period, the number of failures occurred during the basic warranty and extended warranty period also affect customer satisfaction. In such conditions, the occurrence of product failure is associated with more dissatisfaction with increasing degree of risk aversion.

Suppose that $\pi(N|r_i)$ represents the ith customer satisfaction with a degree of r_i for risk-taking, at the time of existence N failures during the warranty period. Then we have:

$$\pi(N|r_i) = \frac{a}{(1+N)^{r_i}} \tag{27}$$

The expected customer's satisfaction when there are N failures in a product will be obtained as follows:

$$d(N) = \int_0^R \pi(N|r_i)\, g(r_i)\, dr = \int_0^\theta \frac{2ar_i}{R\theta(1+N)^{r_i}}dr + \int_\theta^R \frac{2a(R-r_i)}{R(R-\theta)(1+N)^{r_i}}dr = \frac{2a(R-1)e^{-R\ln(1+N)}}{R(R-\theta)\ln(1+N)} + \frac{2ae^{-R\ln(1+N)}}{R(R-\theta)(\ln(1+N))^2} - \frac{2a(\theta-1)e^{-\theta\ln(1+N)}}{R(R-\theta)\ln(1+N)} - \frac{2ae^{-\theta\ln(1+N)}}{R(R-\theta)(\ln(1+N))^2} \tag{28}$$

According to Equations (26)-(28) and through considering $a = 1$, the total expected desirability function for a customer (D) is obtained as of Equation (29):

$$D = \sqrt[4]{d(BW).d(EW).d(N_{EW}).d(N_{BW})} \tag{29}$$

4. THE INTEGRATED SUPPLY CHAIN OF AFTER-SALES SERVICES MODEL

After the introduction of the model components, including viewpoints of manufacturer, third party and customer, the integrated supply chain of after-sales services model can be presented as follows:

$$\max D = \sqrt[4]{d(BW).d(EW).d(N_{EW}).d(N_{BW})} \tag{29}$$

$$\min TC_{3P} = ECM_{EW}(\Delta,m) + EPM_{EW}(\Delta,m) + ES_{EW}(\Delta,m) \tag{30}$$

$$\min TC_M = C_M^{CM}\int_0^{BW} h(t)dt \tag{31}$$

Subject to:

(15), (16), (18), (21), (25), (26), (28)

$$m^{min} \le m \le m^{max}\ integer \tag{32}$$

$$\Delta^{min} \le \Delta \le \Delta^{max}\ integer \tag{33}$$

$$BW^{min} \le BW \le BW^{max}\ integer \tag{34}$$

$$EW^{min} \le EW \le EW^{max}\ integer \tag{35}$$

The proposed model is an integer non-linear multi-objective problem. Achieving the optimal solutions for this type of models is practically impossible in large-size problems. So, in the next section, to optimize the proposed model, a multi-objective solution approach develops based on scatter search.

5. MULTI-OBJECTIVE SCATTER SEARCH ALGORITHM

In general, two approaches are used to optimize multi-objective problems. First, the model objectives are combined in a single objective, while in the second method a set of non-dominated solutions (Pareto-set) are extracted by algorithms such as NSGA-II and MOPSO [19]. In this paper, a multi-objective scatter search algorithm (MOSS) are developed for extracting Pareto-set. Scatter search is an exact strategy that was presented for the first time by F. Glover [20] and is applied well to solve combinatorial optimization problems [21]. MOSS steps are as follows:

Step 0 (solution representation): In the proposed MOSS algorithm the values of $\{BW, EW, m, \Delta\}$ are displayed in the form of a four-component vector according to Figure 2.

BW	EW	m	Δ

Figure 2. Solution representation of MOSS algorithm

```
Step 1: Consider Pareto-set: $Pset \leftarrow \emptyset$
Step 2: Create the initial population with the size of POP randomly and for each member of the population, calculate the values of $TC_{3P}, TC_M, D$.
Step 3: Repeat the steps 4 to 19, $IT_{max}$ time.
Step 4: For each solution ($S$) of POP, set $k \leftarrow 0$ and $y \leftarrow 0$ and repeat the 5th and 6th steps.
Step 5: Compare $S$ with each solution ($j$) of Pset.
Step 5-1: if $D(S) \geq D^{Pset}(j)$ and $TC_{3P}(S) \leq {TC_{3P}}^{Pset}(j)$ and $TC_M(S) \leq {TC_M}^{Pset}(j)$ then
        $Pset \leftarrow Pset/j$
Step 5-2: elseif $D(S) > D^{Pset}(j)$ or $TC_{3P}(S) < {TC_{3P}}^{Pset}(j)$ or $TC_M(S) < {TC_M}^{Pset}(j)$ then
        $k \leftarrow k + 1$
Step 5-3: elseif $D(S) \leq D^{Pset}(j)$ or $TC_{3P}(S) \geq {TC_{3P}}^{Pset}(j)$ or $TC_M(S) \geq {TC_M}^{Pset}(j)$ then
    $y \leftarrow y + 1$
Step 6: if $k > 0$ and $y == 0$ then
        $Pset \leftarrow Pset \cup S$
Step 7: For solutions of $S_i = \{BW_i, EW_i, m_i, \Delta_i\}$ and $S_j = \{BW_j, EW_j, m_j, \Delta_j\}$of $Pset$, repeat steps 8-19.
Step 8: Using crossover operator calculate new solution of $S^{new}$: $S^{new} \leftarrow \left\{\langle (BW_i * BW_j)^{0.5} \rangle \cup \langle (EW_i * EW_j)^{0.5} \rangle \cup \langle (m_i * m_j)^{0.5} \rangle \cup \langle (\Delta_i * \Delta_j)^{0.5} \rangle \right\}$, /statement of $\langle . \rangle$ shows the rounding up/*
Step 9: Repeat steps 10-18 for $M$ iterations.
Step 10: Create the random number of $a$ in the interval $\{1,2,\ldots,8\}$.
Step 11: if $a == 1$, then
Step 11-1: Create the random number of $b$ in the range of $[BW^{min}, BW^{max}]$
Step 11-2: if $S^{new}\{BW\} + b \leq BW^{max}$ then
        $S^{new}\{BW\} + b \leq BW^{max}$
Step 12: if $a == 2$, then
Step 12-1: Create the random number of $b$ in the range of $[BW^{min}, BW^{max}]$
Step 12-2: if $S^{new}\{BW\} - b \geq BW^{min}$ then
        $S^{new}\{BW\} \leftarrow S^{new}\{BW\} - b$
Step 13: if $a == 3$, then
Step 13-1: Create the random number of $b$ in the range of $[EW^{min}, EW^{max}]$
Step 13-2: if $S^{new}\{EW\} + b \leq EW^{max}$ then
        $S^{new}\{EW\} \leftarrow S^{new}\{EW\} + b$
Step 14: if $a == 4$, then
Step 14-1: Create the random number of $b$ in the range of $[EW^{min}, EW^{max}]$
Step 14-2: if $S^{new}\{EW\} - b \geq EW^{min}$ then
        $S^{new}\{EW\} \leftarrow S^{new}\{EW\} - b$
Step 15: if $a == 5$, then
Step 15-1: Create the random number of $b$ in the range of $b \in [m^{min}, m^{max}]$
Step 15-2: if $S^{new}\{m\} + b \leq m^{max}$ then
        $S^{new}\{m\} \leftarrow S^{new}\{m\} + b$
Step 16: if $a == 6$, then
Step 16-1: Create the random number of $b$ in the range of $[m^{min}, m^{max}]$
Step 16-2: if $S^{new}\{m\} - b \geq m^{min}$ then
        $S^{new}\{m\} \leftarrow S^{new}\{m\} - b$
Step 17: if $a == 7$, then
Step 17-1: Create the random number of $b$ in the range of $[\Delta^{min}, \Delta^{max}]$
Step 17-2: if $S^{new}\{\Delta\} + b \leq \Delta^{max}$ then
        $S^{new}\{\Delta\} \leftarrow S^{new}\{\Delta\} + b$
Step 18: if $a == 8$, then
Step 18-1: Create the random number of $b$ in the range of $[\Delta^{min}, \Delta^{max}]$
Step 18-2: if $S^{new}\{\Delta\} - b \geq \Delta^{min}$ then
        $S^{new}\{\Delta\} \leftarrow S^{new}\{\Delta\} - b$
Step 19: $NewPOP \leftarrow NewPOP \cup S^{new}$
Step 20: Put $POP \leftarrow NewPOP$ and go to step 4.
Step 21: Show the $Pset$.
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6. NUMERICAL EXAMPLE

In this section, for evaluating the integrated supply chain of after-sales services model via MOSS algorithm, a set of numerical examples is presented. To investigate the validation of the proposed problem-solving algorithm, the exhaustive-search technique was employed. Both approaches were coded in MTALAB R2013a, and all calculations were implemented on a system with the following configuration: Core i5/CPU 2.4 GHz/RAM 4GB.

It is assumed that the probability distribution function of product failure process is a two-parameter Weibull distribution (according to Equation (36)) with the shape parameter β and scale parameter α.

$$f(t) = \frac{\beta}{\alpha^\beta} t^{\beta-1} e^{-\left(\frac{t}{\alpha}\right)^\beta}, \quad t \geq 0, \beta > 0 \tag{36}$$

Moreover, function $\delta(m)$ is defined as follows:

$$\delta(m) = 1 - \xi m \tag{37}$$

In Equation (37), the parameter ξ is a number in the range [0,1] which regulates the PM level. Table 1 shows the data of numerical examples, including parameters of third-party's maintenance strategy, product lifetime, risk and desirability functions related to customers and the MOSS approach.

It should be noted that the parameter setting of MOSS algorithm was performed based on primitive experiments.

For appropriate evaluation of the proposed model, the numerical example is investigated when it is put in $C^S \in \{400,500,..,1000\}$ ‘$C_{3P}^{CM} \in \{200,300,\ldots,800\}$ ‘

$C_M^{CM} \in \{400,500 \ldots,800\}$ and $\theta \in \{1.5,2,\ldots,4.5\}$. Hence, numerical example includes 119 scenarios. In addition, for accessing exact solutions through exhaustive search method, variables of the problem were considered in ranges $BW \in \{0,0.5,\ldots,3\}$, $EW \in \{0,0.5,\ldots,7\}$, $\Delta \in \{1,2\ldots,52\}$ and $m \in \{1,2,\ldots,10\}$. Table 2 presents the obtained results from optimization of the integrated supply chain of after-sales services model via the MOSS algorithm and the exhaustive search algorithm for the 119 scenarios.

TABLE 1. Information of the numerical examples

Parameter	Value	Parameter	Value
γ^{lower}	0.00001	∂	50
γ^{upper}	0.00020	β	3
ξ	0.1	α	1000
u	1	POP	100
q	3	IT_{max}	50
C^{PM}	$\partial \times m$		

TABLE 2. Results of optimizing the proposed model based on MOSS approach and exhaustive search algorithm

No	θ	C^S	C_{3P}^{CM}	C_M^{CM}	MOSS		Exhaustive Search		MS	No	θ	C^S	C_{3P}^{CM}	C_M^{CM}	MOSS		Exhaustive Search		MS
					NPS	CPU (S)	NPS	CPU (S)							NPS	CPU (S)	NPS	CPU (S)	
1	1.5	600	400	800	240	150	233	777	0.944	61	3	600	700	800	144	121	146	795	0.986
2	1.5	700	400	800	230	139	224	779	1.000	62	3	600	800	800	145	129	145	799	1.000
3	1.5	800	400	800	212	153	220	785	0.932	63	3	600	300	800	152	131	147	795	0.925
4	1.5	900	400	800	223	143	215	792	0.949	64	3	600	200	800	151	127	148	798	0.939
5	1.5	1000	400	800	222	157	218	793	1.000	65	3	600	400	700	151	126	147	786	1.000
6	1.5	500	400	800	227	158	237	791	0.928	66	3	600	400	600	150	132	147	793	0.980
7	1.5	400	400	800	233	148	236	795	0.924	67	3	600	400	500	151	129	147	796	0.972
8	1.5	600	500	800	237	150	232	809	0.897	68	3	600	400	400	150	124	147	798	0.980
9	1.5	600	600	800	225	136	231	800	0.909	69	3.5	600	400	800	122	127	122	790	1.000
10	1.5	600	700	800	237	159	231	801	0.909	70	3.5	700	400	800	122	126	122	792	1.000
11	1.5	600	800	800	237	148	231	801	0.913	71	3.5	800	400	800	122	126	122	795	1.000
12	1.5	600	300	800	241	140	236	804	0.979	72	3.5	900	400	800	122	122	122	793	1.000
13	1.5	600	200	800	228	148	236	807	0.898	73	3.5	1000	400	800	122	127	122	797	1.000
14	1.5	600	400	700	241	143	233	805	0.944	74	3.5	500	400	800	122	125	122	803	0.984
15	1.5	600	400	600	237	148	233	797	0.965	75	3.5	400	400	800	122	122	122	803	1.000
16	1.5	600	400	500	220	144	233	802	0.906	76	3.5	600	500	800	122	127	122	809	1.000
17	1.5	600	400	400	236	152	233	807	0.974	77	3.5	600	600	800	121	124	121	808	1.000
18	2	600	400	800	209	138	204	804	0.946	78	3.5	600	700	800	121	133	121	803	1.000
19	2	700	400	800	207	155	203	811	0.970	79	3.5	600	800	800	121	127	121	785	1.000
20	2	800	400	800	194	148	192	811	0.958	80	3.5	600	300	800	122	131	122	786	1.000
21	2	900	400	800	191	159	194	809	0.979	81	3.5	600	200	800	123	134	123	788	1.000
22	2	1000	400	800	187	144	191	810	0.916	82	3.5	600	400	700	123	137	122	799	0.984
23	2	500	400	800	216	146	209	809	1.000	83	3.5	600	400	600	122	139	122	803	1.000
24	2	400	400	800	230	151	221	791	0.937	84	3.5	600	400	500	122	138	122	799	1.000
25	2	600	500	800	209	150	204	803	0.922	85	3.5	600	400	400	122	132	122	801	1.000
26	2	600	600	800	194	160	203	821	0.946	86	4	600	400	800	98	118	97	800	0.990
27	2	600	700	800	198	142	203	807	0.927	87	4	700	400	800	97	117	97	802	1.000
28	2	600	800	800	198	156	203	791	0.946	88	4	800	400	800	97	116	97	809	1.000

29	2	600	300	800	200	152	205	798	0.975	89	4	900	400	800	98	117	97	792	0.990
30	2	600	200	800	203	156	206	800	0.942	90	4	1000	400	800	98	115	97	782	0.990
31	2	600	400	700	204	149	204	811	1.000	91	4	500	400	800	97	115	97	787	1.000
32	2	600	400	600	201	137	204	802	0.946	92	4	400	400	800	97	111	97	796	1.000
33	2	600	400	500	201	139	204	809	0.985	93	4	600	500	800	97	114	97	798	1.000
34	2	600	400	400	200	142	204	815	0.980	94	4	600	600	800	97	112	96	803	0.990
35	2.5	600	400	800	172	158	171	780	0.953	95	4	600	700	800	95	117	96	807	0.958
36	2.5	700	400	800	164	138	163	784	1.000	96	4	600	800	800	95	116	96	808	0.958
37	2.5	800	400	800	164	157	166	794	0.946	97	4	600	300	800	98	115	97	810	0.990
38	2.5	900	400	800	165	136	165	803	0.945	98	4	600	200	800	95	115	98	809	0.918
39	2.5	1000	400	800	163	155	162	809	0.994	99	4	600	400	700	98	116	97	804	0.969
40	2.5	500	400	800	179	148	175	815	0.949	100	4	600	400	600	98	119	97	802	0.969
41	2.5	400	400	800	173	141	175	870	0.966	101	4	600	400	500	96	121	97	807	0.990
42	2.5	600	500	800	170	157	170	819	0.971	102	4	600	400	400	96	116	97	808	0.990
43	2.5	600	600	800	175	150	168	822	0.946	103	4.5	600	400	800	45	118	45	777	1.000
44	2.5	600	700	800	174	159	169	794	0.959	104	4.5	700	400	800	45	115	45	785	1.000
45	2.5	600	800	800	174	145	169	807	0.923	105	4.5	800	400	800	45	120	45	787	1.000
46	2.5	600	300	800	171	153	171	801	0.947	106	4.5	900	400	800	45	114	45	792	1.000
47	2.5	600	200	800	171	136	172	798	0.982	107	4.5	1000	400	800	45	116	45	800	1.000
48	2.5	600	400	700	175	154	171	797	0.924	108	4.5	500	400	800	45	115	45	799	1.000
49	2.5	600	400	600	175	141	171	807	0.953	109	4.5	400	400	800	45	116	45	796	1.000
50	2.5	600	400	500	173	156	171	806	0.988	110	4.5	600	500	800	45	112	45	802	1.000
51	2.5	600	400	400	175	143	171	802	0.976	111	4.5	600	600	800	44	115	44	788	1.000
52	3	600	400	800	138	160	147	785	0.891	112	4.5	600	700	800	44	114	44	783	1.000
53	3	700	400	800	141	141	146	797	0.952	113	4.5	600	800	800	44	117	44	787	1.000
54	3	800	400	800	141	148	145	789	0.924	114	4.5	600	300	800	45	115	45	796	1.000
55	3	900	400	800	140	158	146	791	0.910	115	4.5	600	200	800	46	117	46	797	1.000
56	3	1000	400	800	140	158	146	804	0.910	116	4.5	600	400	700	45	118	45	799	1.000
57	3	500	400	800	142	157	147	793	0.946	117	4.5	600	400	600	45	115	45	802	1.000
58	3	400	400	800	150	151	150	802	1.000	118	4.5	600	400	500	45	114	45	803	1.000
59	3	600	500	800	148	145	147	794	0.993	119	4.5	600	400	400	45	117	45	798	1.000
60	3	600	600	800	148	157	146	792	0.973										

The NPS column shows the number of obtained Pareto-set solutions (NPS). The MS column shows the rate adaptation of the obtained NPS of the MOSS algorithm with the NPS of the exhaustive search algorithm. The MS indicator is calculated as follows:

$$MS = \frac{SNPS}{NPS_{Exhaustive\ search}} \qquad (38)$$

In the above equation, SNPS indicates the number of Pareto-set solutions of the MOSS algorithm found also in Pareto-set of the exhaustive search algorithm ($NPS_{Exhaustive\ search}$). Additionally, the CPU column indicates the solving time per seconds.

7. CONCLUSION

The integrated supply chain of after-sales services model was developed for supporting manufacturer, third-party and customer in the present study. To optimize the model, a multi-objective scatter search approach was developed. The results indicated that with the increase in costs of corrective maintenance, the third-party has to reduce the time interval between preventive maintenance actions in order to prevent increasing of product failure rate. When the majority of customers are risk-averse, product failure has remarkable effects on their desirability. In this condition, the manufacturer should prolong the basic warranty period for enhancing the customers' desirability level, and the third-party for keeping this level and also reducing extended warranty costs should apply PM actions with higher levels. When the majority of customers are risk-taking, there is possibility for the manufacturer and the third-party to reduce their costs and keep customer desirability at an appropriate level by reducing the length of the warranty period. The results also indicated that the MOSS algorithm has high efficiency in extracting non-dominated solutions so that in the worst case, 89.1% of the obtained solutions were consistent with exact solutions of the exhaustive search method. In the present study, the maintenance logistics and spare parts management were not discussed, which can be investigated for the further research.

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The Integrated Supply Chain of After-sales Services Model: A Multi-objective Scatter Search Optimization Approach

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چکیده

در طی دهه‌های اخیر سود بالای وارانتی (ضمانت) تمدیدشده باعث شده است که شرکت‌های شخص ثالث آن را به‌عنوان یک خدمت پس از فروش پردرآمد مدنظر قرار دهند. با این حال تقسیم‌بندی مشتریان از دیدگاه ریسک‌گریزی و تاثیر پیشنهاد وارانتی تمدیدشده بر روی وارانتی پایه تولیدکننده بایستی در طی تضمین چنین سیاست‌هایی لحاظ شود. از آن جا که مشتریان ریسک‌گریز از وارانتی تمدیدشده استقبال می‌نمایند و در مقابل مشتریان بدون ریسک ممکن است در سطح وارانتی پایه باقی بمانند. در این مقاله یک مدل چندهدفه غیرخطی عددصحیح برای یک‌پارچه‌سازی زنجیره تامین خدمات پس ازفروش ارائه می‌شود. در مدل پیشنهادی ابتدا استراتژی‌های تولیدکننده در دوره وارانتی پایه و سیاست‌گذاری‌های شخص ثالث در طی دوره وارانتی تمدیدشده، از جمله توسعه یک رویکرد نگهداری و تعمیرات ناقص جدید، تنظیم می‌شود. سپس تاثیر این استراتژی‌ها بر مطلوبیت مشتریان با دارای درجه ریسک‌پذیری متفاوت بررسی خواهد شد. به‌منظور بهینه‌سازی مدل، روش حلی مبتنی بر رویکرد جست‌وجو پراکندگی برای استخراج مجموعه جواب‌های غالب معرفی می‌گردد. نتایج نشان داد که افزایش سطح ریسک‌پذیری مشتریان تولیدکننده را مجاب به‌کاهش طول دوره وارنتی پایه می‌کند و شخص ثالث می‌تواند از نگهداری و تعمیرات پیشگیرانه کم‌هزینه‌تری استفاده نماید.

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