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Stiffness-based Approach for Preliminary Design of Framed Tube Structures

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ABSTRACT

A parametric formulation for preliminary design of tubed-system tall buildings is presented in which some optimality criteria and practical constraints are considered. Here, a minimum compliance optimization formulation, developed by other researchers, is applied to a framed-tube structure. The tube behavior is modeled as a cantilevered box beam. Independent variable in this problem is thickness of the box, and a formulation for its optimal value is proposed. The challenge in this research was treatment of the lower bound constraint on thickness in an analytical manner. To deal with this constraint, a critical height parameter is introduced, and the design domain is divided into two zones of constant thickness and constant curvature. This definition allows for computation of optimal thickness distribution along the structure through an analytic dimensionless equation. Most of the previously published papers in the field of tall structures are suitable for abstract analyses but not for design. In addition, most of them are computer-based. Considering these limitations, the current research presents a hand-calculation method for preliminary design, suitable for sensitivity analyses and parametric studies. As the presented formulations are dimensionless, they are applicable in any dimensional system. Different static loading patterns are considered; including the concentrated, uniform, triangular and quadratic forms. A numerical example is presented to demonstrate the ease of the proposed method in application, and the analysis results are presented by charts to validate the efficiency of it.

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1. INTRODUCTION

Nowadays, tall buildings are a suitable solution for housing in dense cities in addition to being a symbol of technological advancement [1]. High cost of such structures demands optimization at any possible level [2]. In general, the design process of a tall building involves conceptual design and approximate analysis, preliminary design, and finally detailed design [3]. Some engineers skip the preliminary design step, in which a suitable stiffness distribution is calculated, and go straight to the final step after the conceptual design and finalize the structural configuration using some sort of optimization algorithm. However, this way little can be improved upon the original design. Consideration of the preliminary design step allows for assessment of structural performance. Most researchers have presented simple models which are suitable for abstract analyses but not for design [4-11]. Furthermore, most presented design methods are computer-based. Aldwaik and Adeli [2] presented a review of papers on optimization of high-rise structures. Almost all of the mentioned papers are code-based and numerical, while a hand-calculation method, as in the preliminary design step, is a more suitable tool for sensitivity analyses and parametric studies [12].

In the research done by Connor and Pouangare [12], a framed tube structure is modeled by a string-shear-panel system with uniform properties along the structure. To make the method applicable for structures with varying geometric along its height, the structure which is assumed made up by some segments and transfer-matrix is introduced for each segment and finally a transfer matrix for whole structure is obtained. However, due to piecewise characteristic of this method, parametric study is not simple. Some other investigations have been done in this field, but most of them are about diagrid systems [13, 14]. Connor and Laflamme [15] used a cantilever beam to model the tall buildings, and closed-form relations were introduced for

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optimal shear and bending stiffness distribution along the model's length. Objective of the current research effort is to devise this model in an optimization-framework so to make the structural improvement quantifiable. Furthermore, a lower bound on stiffness of the structure is observed in an analytical framework. Although such constraints have been explored by many researchers so far, most of them are numerical [16-18].

The framed-tube system, Figure 1, is considered here, in which axial stress field generated in the closely spaced columns located on structure's perimeter resists the applied lateral loads. Behavior of this system can be modeled as a cantilevered box-beam [19]. The optimization process applied here uses total potential energy as the objective function. It will be shown that this selection results in constant curvature along the structure. In the optimization strategy, thickness of the box is calculated with this objective. As to be expected, thickness value at high elevations becomes relatively small and inapplicable. To deal with this issue, a lower bound constraint is added, and the problem is analyzed by introducing a parameter called critical height (CH). In the optimal state, the points above CH have the same thickness of low limit but different curvature values. Opposite situation exists for thickness and curvature at points lower than CH. Optimum thickness distribution is determined with respect to CH, and its closed-form formulation is presented.

2. LIMITED VOLUME CONSTRAINED OPTIMIZATION

Limited volume constrained is the optimization framework used here. In this part, the model approximating structure's response is first introduced and then the constrained optimization process is applied to this model.

2. 1. Modeling A cantilevered box-beam with bending response only is used to model a tall building, Figure 2.

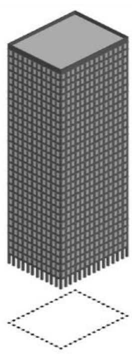

Figure 1. Framed-tube system

In this model, exterior dimensions are invariant in height and remain so throughout the optimization process. In this process, the only independent variable is thickness (t) of the perimeter panels. For convenience, identical thickness is assumed for all panels at each elevation. The selected coordinate system has its origin placed at the center of the rectangular section located at top of the structure with positive z direction pointing downward, Figure 3, and the length is ℓ. In this paper, the structure is modeled using Euler-Bernoulli beam; however, considering shear lag effect would probably better represent structure's actual response [20-22], and is planned for future research efforts.

Mathematical model of one-dimensional transverse bending about the x axis is utilized in all analyses. Moment of inertia can be approximated as:

$$I = I(z) = t(z) I_0 \tag{1}$$

where, $t = t(z)$ is thickness of the box, and $I_0 = \left[(4/3)b^3 + 4ab^2\right]$ is the moment of inertia for a unit thickness box, in which $2a$ and $2b$ are dimensions in x and y directions, respectively. It is shown in Figure 2.

2. 2. Minimum Compliance Formulation In structural optimization problems, one attempts to distribute the available construction materials throughout the design domain with the objective of making the structure as stiff as possible. There is no unique measure of stiffness in this process; the so-called compliance measure, which minimize external work, $\ell(u) = \int_0^{\ell} u\, f\, dz$, as the objective function [23]. It is selected here for one crucial reason; roughly speaking, optimization process based on compliance measure tends to create a system with almost constant stress distribution [23].

An equivalent approach is utilization of total potential energy as the objective function. For the system under consideration, the total potential energy is:

$$J = \frac{EI_0}{2}\int_0^{\ell} t\left(\frac{d^2u}{dz^2}\right)^2 dz - \int_0^{\ell} f(z)\, u\, dz \tag{2}$$

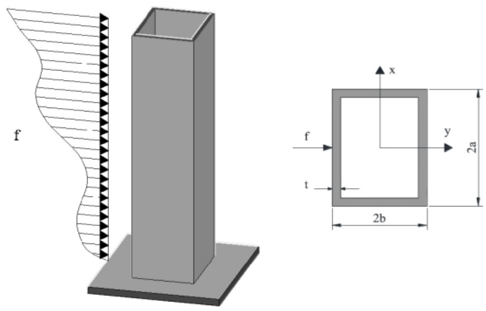

Figure 2. Hollow-box-beam model for framed-tube structure and its cross section

where, J is the total potential energy, E is Young's modulus of elasticity, u denotes the displacement field in *y*-*z* plane, and f is the induced lateral force.

Based on Reference [23]:

$$\ell(u_t) = -2J(u_t) \tag{3}$$

From Equation (3), ℓ and J are proportional, so $J(u_t)$ can be selected as the objective function instead of $\ell(u_t)$, and due to the negative sign, the minimization statement must be converted to maximization one. That is compliance measure which can be replaced by total potential energy one. In preceding relations, subscript t emphasizes that u is a function of t, so $J(u_t)$ can be replaced by $J(t)$. Therefore, the design problem can be stated as follows:

$$\begin{cases} \max\limits_{t} \; J(t) \\ s.t. \{ t \in \mathrm{H} \end{cases} \tag{4}$$

In which, H denotes the design constraints of the amount of available material. Hence, for a given material's volume V, $t \in \mathrm{H}$ must satisfy

$$\int_0^{\ell} t \, dz = A_0 \tag{5}$$

where, $A_0 = V/p$, and p denotes beam's perimeter, i.e. $p=4(a+b)$. Hence, the optimization problem can be stated using Lagrangian method as

$$\mathrm{L}(t,\lambda) = J(t) - \lambda \left[\int_0^{\ell} t \, dz - A_0 \right] \tag{6}$$

where, $\lambda \geq 0$ is the Lagrangian multiplier. Variation of L with respect to t yields

$$\delta \mathrm{L} = \int_0^{\ell} \left[\left(\frac{EI_0}{2} u''^2 - \lambda \right) \delta t \right] dz \tag{7}$$

With optimality condition of $\delta \mathrm{L} = 0$, the following relating is reached:

$$u'' = \pm \sqrt{\frac{2\lambda}{EI_0}} = \pm \chi \tag{8}$$

where, χ denotes the curvature. Equation (8) states that for optimum thickness, absolute value of curvature is constant along the structure, which is in agreement with the general theorem of constant strain energy presented in Reference [23]. This theorem states that for the optimization problem (4), $t^* \in H$ optimizes the objective function if the specific strain energy is constant in the design domain. As the specific strain energy and curvature are proportional [23], optimal thickness is evaluated by enforcing the requirement for the curvature to be constant.

2. 3. Optimal Thickness Distribution Based on previous section, optimal thickness is evaluated by forcing the curvature to be constant. Under such conditions, different loading patterns have been investigated; and for each case, optimal thickness distribution is obtained.

As the first step in this process, the governing equation and natural boundary conditions (NBC) for each case are defined exactly. General form of the governing equation, along with NBC, are as follows:

$$\begin{cases} \dfrac{d^2}{dz^2}(EIu'') = f \qquad z \in [0 \quad \ell] \\ NBC \begin{cases} \dfrac{d}{dz}(EIu'')\big|_{z=0} = S \\ (EIu'')\big|_{z=0} = M \end{cases} \end{cases} \tag{9}$$

for which induced lateral load (f) and NBC including shear force (S) and bending moment (M) at the free end need to be defined in each case. Table 1 lists these parameters according to Figure 3.

By integrating governing differential equation twice and taking NBC into account and rearranging the relations in terms of the dimensionless parameter $\bar{z} = z/\ell$, we have:

$$EIu'' = M_{max} \bar{m}(\bar{z}) \tag{10}$$

TABLE 1. List of lateral loads, shear forces and bending moments at the free end, according to Figure 3

	F	S	M
Case 1	0	Q	0
Case 2	q	0	0
Case 3	$q(1 - z/\ell)$	0	0
Case 4	$q(1 - z^2/\ell^2)$	0	0

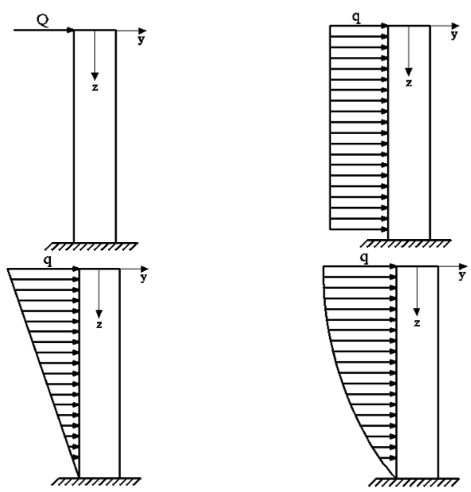

Figure 3. Different loading patterns: case 1: concentrated; case 2: uniform; case 3: triangular; case 4: quadratic

where M_{max} is maximum moment that happens at the base, and we name $\bar{m}(\bar{z})$ relative. Table 2 presents these two parameters for each case. Clearly, $M_{max}, EI \geq 0$ for all $\bar{z} \in [0 \quad 1]$. Figure 4 shows diagrams of $\bar{m}(\bar{z})$ that $\bar{m}(\bar{z}) \geq 0$ for all $\bar{z} \in [0 \quad 1]$ in each case.

Therefore, it is concluded from Equation (10) that $u'' \geq 0$. Hence, by Equation (8) we have:

$$u'' = +\chi \tag{11}$$

Substituting for I and u'' from Equations (1) and (11) respectively into Equation (10), and defining new parameter $\chi_{0max} = M_{max}/EI_0$, we have:

$$t\,\chi = \chi_{0\max}\bar{m}(\bar{z}) \tag{12}$$

Thus, considering χ as a constant parameter, optimal thickness distribution would be:

$$t = \frac{\chi_{0max}}{\chi}\bar{m}(\bar{z}) \tag{13}$$

Indeed, this formulation must satisfy constraint (5). It is needed to restate this constraint with respect to new variable $\bar{z}$, as is done in the following.

$$\int_0^1 t\, d\bar{z} = \bar{t} \tag{14}$$

where:

$$\bar{t} = \frac{A_0}{\ell} \tag{15}$$

TABLE 2. Maximum and relative moments

	$M_{\max}$	$\bar{m}(\bar{z})$
Case 1	$Q\ell$	$\bar{z}$
Case 2	$q\ell^2/2$	$\bar{z}^2$
Case 3	$q\ell^2/3$	$(3/2)\bar{z}^2 - (1/2)\bar{z}^3$
Case 4	$5q\ell^2/12$	$(6/5)\bar{z}^2 - (1/5)\bar{z}^4$

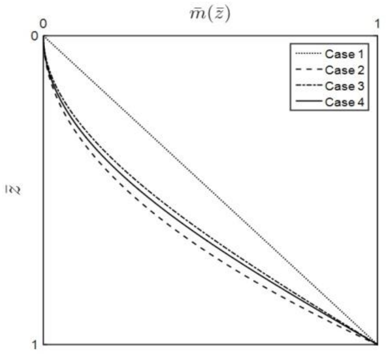

Figure 4. Diagram of relative moment for different cases

This is the limited volume (LV) constraint. $\bar{t}$ is defined for convenience and is named as average thickness. Substituting Equation (13) into constraint (14) dictates the value of χ, which after substitution in Equation (13), the optimal thickness relations are obtained, Table 3.

3. LIMITED VOLUME AND LOWER BOUND CONSTRAINED OPTIMIZATION

In order to make the results more applicable, the obtained relations for thickness in Table 3 are modified considering a lower bound (LB) constraint.

3. 1. New Optimization Problem Based on Table 3, for the points near the top of the structure, as $\bar{z} \to 0$ thickness approaches zero, which is not right. Thus, based on practical consideration, LB constraint of t_{min} on thickness should be defined in the original optimization problem in addition to LV one:

$$\begin{cases} \max\limits_t \; J(t) \\ s.t. \begin{cases} LV : \int_0^1 t\, d\bar{z} = \bar{t} \\ LB : 0 < t_{min} \leq t \end{cases} \end{cases} \tag{16}$$

Note: t_{min} has an upper bound controlled by LV constraint, i.e. t_{min} cannot be larger than average thickness ($\bar{t}$) introduced by Equation (15). In all parts of this research, it is assumed that this constraint is satisfied. That is, $t_{min} \leq \bar{t}$.

From Table 3, clearly, there exists a height named primary height (PH), denoted by $\bar{z}_p$ in dimensionless relations, such that for $\bar{z} \in [0 \quad \bar{z}_p]$, LB constraint is violated. Referring to Table 3, PH can be calculated by substituting $\bar{z}_p$ for $\bar{z}$ and t_{min} for *t*. Doing so and introducing a parameter named relative minimum thickness (RMT) as:

$$\bar{t}_{\min} = \frac{t_{\min}}{\bar{t}} \tag{17}$$

TABLE 3. Optimal thickness distribution observing LV constraint

	$t(\bar{z})$
Case 1	$2\bar{t}\,\bar{z}$
Case 2	$3\bar{t}\,\bar{z}^2$
Case 3	$4\bar{t}\,\bar{z}^2 - (4/3)\bar{t}\,\bar{z}^3$
Case 4	$(10/3)\bar{t}\,\bar{z}^2 - (5/9)\bar{t}\,\bar{z}^4$

yields polynomials for which an acceptable root would be considered as the PH value. Table 4 presents the relevant polynomials for each case.

3. 2. Intervening Variable There are some approaches to deal with the new optimization statement. It is recommended to solve a min-max problem for such cases [Formulation (16)]. There are many methods to solve a min-max problem such as sequential quadratic programming [24], which has a numerical aspect. In the analytic approach, a Lagrangian function is constructed using the objective function and LV constraint while ignoring LB constraint. Lagrangian function is then minimized with respect to LB constraint [23]. This process will yield a relation between the Lagrangian multiplier λ and variable t through a one-variable function $\varphi(\lambda)$:

$$\varphi(\lambda) = \min_{t \in LB} \mathrm{L}(t, \lambda) \tag{18}$$

Finally, owing to convexity of the problem [23], the optimum solution is obtained by maximizing (or minimizing) $\varphi(\lambda)$ with respect to $\lambda \geq 0$.

Consequently, it is essential to find the relation between λ and t such as Equation (18). However, this process is rather complicated for the current problem. To deal with this complexity, this research strives to construct the problem based on an intervening variable, instead of λ. Parameter χ can be selected for such a pupose. Appropriateness of this selection is supported by two factors: 1) the problem is greatly simplified 2) Since the intervening variable is structure's curvature, the results are more sensible from a structural viewpoint. To find the relation between χ and t, note Equation (13), which states t as a function of two variables χ and $\bar{z}$. Specially, consider case 1.

Figure 5 shows variation of t with respect to χ for different values of $\bar{z}$; with marked positions of minimum thickness t_{min} and the particular unknown χ_c the critical curvature. For other cases, the data on the ordinate is replaced by $t = (\chi_{0max} / \chi)\bar{m}(\bar{z})$. As $\bar{m}(\bar{z})$ is positive and bounded (based on Figure 4), we would have similar schematic diagrams. Considering Figure 5, there is a critical height, $\bar{z}_c$.

Considering Figure 5, there is a critical height, $\bar{z}_c$

TABLE 4. Polynomials associate with PH

Case 1	$2\bar{z}_p - \bar{t}_{\min} = 0$
Case 2	$3\bar{z}_p^2 - \bar{t}_{\min} = 0$
Case 3	$-(4/3)\bar{z}_p^3 + 4\bar{z}_p^2 - \bar{t}_{\min} = 0$
Case 4	$-(5/9)\bar{z}_p^4 + (10/3)\bar{z}_p^2 - \bar{t}_{\min} = 0$

located on the chart at intersection of $t = t_{\min}$ and $\chi = \chi_c$. At elevations higher than the CH value ($0 \leq \bar{z} \leq \bar{z}_c$) LB constraint is violated; hence for this region, uniform thicknesses (t_{min}) with varying curvature, as computed from Equation (12) is considered. We call this region constant thickness (CT) zone, formally specified as:

$$\mathrm{CT}(0 \leq \bar{z} \leq \bar{z}_c): \begin{cases} t_{CT} = t_{min} \\ \chi_{CT} = \dfrac{\chi_{0max}}{t_{\min}} \bar{m}(\bar{z}) \end{cases} \tag{19}$$

For heights lower than CH ($\bar{z}_c \leq \bar{z} \leq 1$) curvature is kept constant at χ_c with varying thickness computed from Equation (12). This region is referred to as the constant curvature (CC) zone with following specifications:

$$\mathrm{CC}(\bar{z}_c \leq \bar{z} \leq 1): \begin{cases} t_{CC} = \dfrac{\chi_{0max}}{\chi_c} \bar{m}(\bar{z}) \\ \chi_{CC} = \chi_c \end{cases} \tag{20}$$

3. 3. Modified Optimal Thickness and Curvature Distribution Considering relations (19) and (20), optimal thickness formulation while observing LB constraint becomes:

$$t(\bar{z}) = \begin{cases} t_{CT} = t_{min} & 0 \leq \bar{z} \leq \bar{z}_c \\ t_{CC} = \dfrac{\chi_{0max}}{\chi_c} \bar{m}(\bar{z}) & \bar{z}_c \leq \bar{z} \leq 1 \end{cases} \tag{21}$$

Similarly, relations for curvature are as follows:

$$\chi(\bar{z}) = \begin{cases} \chi_{CT} = \dfrac{\chi_{0max}}{t_{\min}} \bar{m}(\bar{z}) & 0 \leq \bar{z} \leq \bar{z}_c \\ \chi_{CC} = \chi_c & \bar{z}_c \leq \bar{z} \leq 1 \end{cases} \tag{22}$$

In the stated optimization problem, χ_c and $\bar{z}_c$ are unknown as yet.

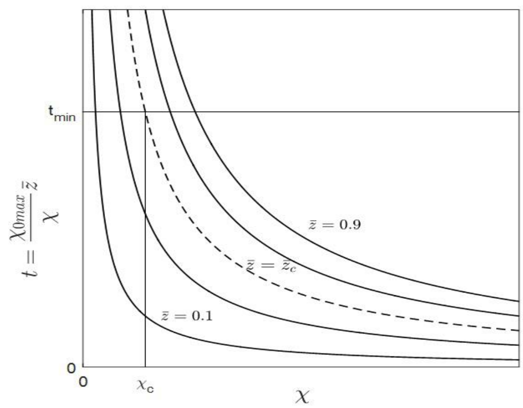

Figure 5. Variation of t with respect to χ for different values of $\bar{z}$ for case 1

Considering Equation (21) and the fact that thickness must remain continuous along structure's height, i.e. $t_{CT}(\bar{z}_c) = t_{CC}(\bar{z}_c)$, yield:

$$\chi_c = \frac{\chi_{0max}}{t_{min}} \bar{m}(\bar{z}_c) \tag{23}$$

Substituting into Equations (21) and (22), results in the following relations for thickness and curvature; for thickness:

$$t(\bar{z}) = \begin{cases} t_{CT} = t_{min} & 0 \le \bar{z} \le \bar{z}_c \\ t_{CC} = t_{min} \dfrac{\bar{m}(\bar{z})}{\bar{m}(\bar{z}_c)} & \bar{z}_c \le \bar{z} \le 1 \end{cases} \tag{24}$$

and for curvature:

$$\chi(\bar{z}) = \begin{cases} \chi_{CT} = \dfrac{\chi_{0max}}{t_{\min}} \bar{m}(\bar{z}) & 0 \le \bar{z} \le \bar{z}_c \\ \chi_{CC} = \dfrac{\chi_{0max}}{t_{min}} \bar{m}(\bar{z}_c) & \bar{z}_c \le \bar{z} \le 1 \end{cases} \tag{25}$$

To evaluate the only unknown $\bar{z}_c$, LV constraint must be satisfied, so Equation (14) is imposed. By substituting the computed thickness value from relation (24) into Equation (14), a polynomial with a root acceptable as the CH value is obtained. Applying to all cases yields the relations presented in Table 5.

Solving the equations presented in Table 5 while observing $0 \le \bar{z}_c \le 1$ and $0 \le \bar{t}_{min} \le 1$ would yield $\bar{z}_c$, which after substitution into (24) and (25), curvature χ and thickness t are obtained. Table 6 contains the values of CH for different values of RMT ($\bar{t}_{\min}$).

TABLE 5. Polynomials associate with CH

Case 1	$\bar{t}_{\min}\bar{z}_c^2 - 2\bar{z}_c + \bar{t}_{\min} = 0$
Case 2	$2\bar{t}_{\min}\bar{z}_c^3 - 3\bar{z}_c^2 + \bar{t}_{\min} = 0$
Case 3	$3\bar{t}_{\min}\bar{z}_c^4 - (8\bar{t}_{\min} + 4)\bar{z}_c^3 + 12\bar{z}_c^2 - 3\bar{t}_{\min} = 0$
Case 4	$4\bar{t}_{\min}\bar{z}_c^5 - 5\bar{z}_c^4 - 20\bar{t}_{\min}\bar{z}_c^3 + 30\bar{z}_c^2 - 9\bar{t}_{\min} = 0$

TABLE 6. CH for different values of RMT

RMT	Case 1	Case 2	Case 3	Case 4
0	0	0	0	0
0.1	0.0501	0.1837	0.1635	0.1747
0.2	0.1010	0.2628	0.2367	0.2504
0.3	0.1535	0.3271	0.2974	0.3123
0.4	0.2087	0.3855	0.3535	0.3689
0.5	0.2679	0.4421	0.4087	0.4242
0.6	0.3333	0.5000	0.4660	0.4810
0.7	0.4084	0.5625	0.5288	0.5429
0.8	0.5000	0.6350	0.6029	0.6155
0.9	0.6268	0.7310	0.7033	0.7132
1	1.0000	1.0000	1.0000	1.0000

Structural designers can refer to Tables 6, find the critical height and substituting in Equations (24) easily determine thickness value. The most prominent merit of the presented method in practice, in addition to be convenience, is its analytical characteristic; there by making the method as a fast and reliable approach in design process of tall buildings.

4. ILLUSTRATIVE EXAMPLE

This section illustrates method's application to preliminary design of a tube system. Efficiency of the proposed method is evaluated by adopting the example given by Kwan [4] as the reference point. From this example, the structure with uniform stiffness distribution subjected to uniform loading is selected as the basic model. This structure is redesigned (with identical amount of material) using hand-calculation approach based on Equations (17) and (24) as well as Tables 6 and 2. The models are then analysed using a standard program, and the results are graphically compared to those of the reference model.

Structural definition: The reference structure is a 40-story reinforced concrete framed tube building with typical height story of 3*m*. All columns and beams have cross sectional area of $0.8\,m \times 0.8\,m$; and center-to-center column spacing is 2.5*m*. Equivalent properties of the orthotropic membrane tube model of this structure are presented as followings:

$$\begin{array}{ll} a = 17.5\,m & t = 0.256\,m \\ b = 15\,m & E = 20\,GPa \\ \ell = 120\,m & q = 120\,kN/m \end{array} \tag{26}$$

where a, b, ℓ and t are geometric characteristics, Figure 2; E is the modulus of elasticity; and q is the uniform load intensity.

Redesign of this structure based on relations presented here, requires two more parameters; total available material (V), and the minimum allowable thickness ($t_{\min}$). We have approximately $V = \ell t\,[4(a+b)]$, which can be evaluated using geometric properties given in Equation (26), hence:

$$V = 3993\,m^3 \tag{27}$$

According to relation for equivalent thickness presented by Kwan [4], $t = A_c/s$ in which A_c is columns' cross sectional area and s is the column spacing, which is 2.5*m* here. Supposing cross sectional of $0.5\,m \times 0.5\,m$ as the accepted minimum for columns and beams, then minimum thickness for the equivalent tube model would be:

$$t_{\min} = 0.1m \tag{28}$$

At this point, all basic inputs are prepared and the proposed preliminary design process can be illustrated through the following five steps.

Step 1 (RMT): Firstly, it is needed to calculate RMT [Equation (17)]. To that end, we need an estimate of $\bar{t}$ (average thickness). As thickness in the first structure is uniformly distributed, It is concluded intuitively that $\bar{t} = 0.256\,m$. However, using Equation (14) and substituting t=0.256 [from Equation (26)], the same result can be obtained. Therefore, approximately:

$$\bar{t}_{\min} = 0.39 \tag{29}$$

Step 2 (CH): Referring to Table 6, one can determine CH. To that end, we select case 2 row and 0.4 (approximate value instead of 0.39) for RMT. The CH relative amount would be:

$$\bar{z}_c = 0.3855 \tag{30}$$

To know the absolute amount of CH (z_c), it is enough to multiply the obtained value by $\ell = 120$, which yields 46.26 m. Thus, the points above this elevation ($0 \le z \le 46.26$) are assigned the minimum thickness value, i.e. 0.1 m.

Step 3 (relative moment): The other quantity which must be specified is $\bar{m}(\bar{z})$. Based on Table 2 and the fact that case 2 is considered, we have:

$$\bar{m}(\bar{z}) = \bar{z}^2 \tag{31}$$

Step 4 (optimal thickness): Now, optimal thickness can be evaluated using Equation (24) as follows:

$$t(\bar{z}) = \begin{cases} t_{CT} = 0.1m & 0 \le \bar{z} \le 0.3855 \\ t_{CC} = 0.1\dfrac{\bar{z}^2}{(0.3855)^2} & 0.3855 \le \bar{z} \le 1 \end{cases} \tag{32}$$

To have optimal thickness in terms of z instead of $\bar{z}$, it is enough to substitute z/ℓ for $\bar{z}$. Doing so and considering $\ell = 120\,m$, will result in:

$$t(z) = \begin{cases} t_{CT} = 0.1 & 0 \le z \le 46.26 \\ t_{CC} = 0.1\left(\dfrac{z}{46.26}\right)^2 & 46.26 \le z \le 120 \end{cases} \tag{33}$$

where all values are in meters.

Step 5 (making the results practical): According to Equation (33), $t(z)$ is a continuously varying parameter, the solid curve in Figure 6.

However, it is not practical and further modification is required. In order to handle this issue, we can suppose that thickness would remain constant for every 5-levels, shown with dashed lines in Figure 6. The only constraint that must be observed while computing the equivalent thickness over a given 5-level span is to keep the material volume unchanged by selecting the average thickness over that span, e.g. two filled areas in the first region in Figure 6 are the same. All that remains is to calculate the dimensions of columns in accordance to their equivalent thickness value. This is done here using the relation $t = A_c/s$, Table 7.

Since the proposed method is based on bending deformation, it is expected that normal stresses due to bending would decrease in the proposed model, as compared to the reference model. Outputs from analyses using ETABS 9.7.4 [25] show that the maximum axial stress in perimeter columns decreases by 47 percent. Hence, it validates the optimization process to some extent. In order to get a better understanding of enhancements, some new measures are introduced here, and the results are presented through charts in Figure 7. For instance, consider the first chart; data on the abscissa shows the average axial stress in the compressed columns and the ordinate denotes the story number.

Similar diagrams are presented as *web average stress*: the average axial stress of compressed columns of the web panels, *flange average stress*: the average axial stress of compressed columns of the flange panels, and *maximum stress*: the maximum axial stress of compressed columns.

From diagrams in Figure 7, it can be deduced that in addition to decrease in magnitude of normal stresses in the proposed structure, as compared to the reference structure, stress dispersion has been decreased as well.

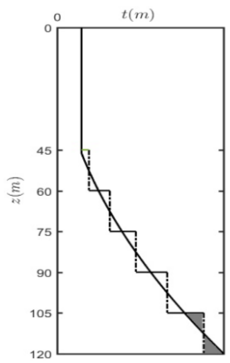

Figure 6. Schematic design diagram of example

TABLE 7. Design information of example

Region (m)	Optimal thickness (m)	Theoretical values of column dimension (m)	Practical values for dimension of columns (m)
0-45	0.1000	0.5000	0.5
45-60	0.1299	0.5699	0.55
60-75	0.2138	0.7311	0.75
75-90	0.3189	0.8929	0.9
90-105	0.4451	1.0549	1
105-120	0.5923	1.2169	1.2

This characteristic is due to imposition uniformity condition on curvature in the CC zone (1-25 stories here).

For a qualitative assessment of improvements to the new structure, two parameters are presented in each chart ; 1) *stress decrease*: percentage decrease in euclidean norm of (40-dimensional) the stress field, and 2) standard *deviation decrease*: percentage decrease in standard deviation of stresses as related to levels 45*m* through 120*m* (CC zone). Note that the stress values include average, web average, flange average and maximum stresses. These diagrams confirm improvements in the structure designed based on the proposed method as compared to the reference structure, in which the material is uniformly distributed along its height.

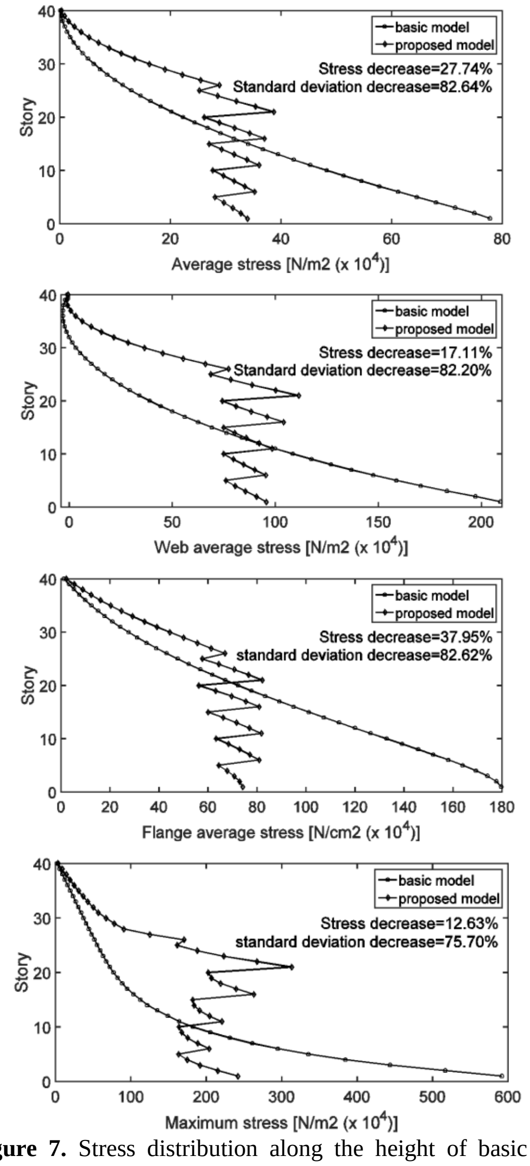

Figure 7. Stress distribution along the height of basic and proposed structures

5. SUMMARY AND CONCLUSIONS

A simple procedure for preliminary design of framed-tube systems of tall buildings has been presented. To takes the practical constraints into account, the optimization problem was constructed in terms of some new parameters, including critical curvature and critical height. This approach contributes to derivation of a nondimensional formulation for optimal thickness distribution, applicable in any consistent unit systems. Different static loading patterns have been considered, and the results related to all patterns are presented in tables, for reference to the design process. The proposed method was validated by redesigning a framed-tube structure, keeping the total amount of material constant. Stress analysis of the reference and optimized structures show a more uniformly distributed stress field with lower stress norm for the optimized structure.

Obviously, there is no guarantee that the proposed methodology would lessen shear stresses. Nonetheless, since in tall-enough buildings, the dominant deformation is due to bending, hence importance of shear stresses reduces as building's design height is increased.

Since tube systems tend to act like thin-walled beams, shear lag phenomenon may occur in a particular region, leading to large values of shear deformations which ultimately complicate normal stress distribution in that region. As a future research topic, it would be valuable to account for shear lag phenomenon in the optimization process in order to further improve on proper distribution of the stress fields.

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Stiffness-based Approach for Preliminary Design of Framed Tube Structures

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چکیده

یک فرمولبندی پارامتریک جهت طراحی سیستمهای لوله‌ای در سازه‌های بلند، با در نظر گرفتن بعضی معیارهای بهینه‌سازی و قیود کاربردی، ارائه شده است. مساله مینیمم‌سازی نرمی، که قبلاً توسط بعضی محققین ارائه شده است، بر یک سیستم لوله‌ای اعمال می‌شود. رفتار سازه به صورت یک کنسول با مقطع قوطی مدل خواهد شد. متغییر مستقل در این فرمولبندی ضخامت قوطی معادل انتخاب شده است و مقدار بهینه آن ارائه خواهد شد. چالش پیش‌رو در این تحقیق حل مساله مقید به قید حداقل ضخامت، با یک رویکرد تحلیلی، بوده است. در برخورد با این قید، پارامتری به نام ارتفاع بحرانی معرفی شده است که محدوده طراحی را به دو ناحیه ضخامت یکنواخت و انحنا یکنواخت تقسیم می‌کند. استفاده از این پارامترها امکان محاسبه ضخامت بهینه در ارتفاع سازه را در قالب یک فرمولبندی تحلیلی و بی‌بعد فراهم خواهد کرد. بیشتر مقالات گذشته در زمینه سازه‌های بلند برای تحلیل مناسب هستند نه طراحی. به علاوه بیشتر آنها مبنای کامپیوتری دارند. با در نظر گرفتن این محدودیت‌ها در این تحقیق یک روش دستی جهت طراحی اولیه ارایه می‌گردد که تحلیل حساسیت و مطالعه پارامتریک را نیز میسر می‌کند. از آنجا که فرمول‌های ارایه شده بی‌بعد هستند، استفاده از آنها در هر سیستم ابعادی امکان‌پذیر است. الگوهای بارگذاری استاتیکی مختلفی شامل بارگذاری متمرکز، یکنواخت، مثلثی و درجه دو مدنظر قرار گرفته‌اند. جهت نشان دادن سادگی روش پیشنهاد شده، یک مثال عددی ارائه شده است و برای صحت سنجی روش، نتایج تحلیل توسط چند نمودار ارائه خواهد شد.

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