IJE TRANSACTIONS A: Basics Vol. 30, No. 10, (October 2017) 1565-1572

International Journal of Engineering

Journal Homepage: www.ije.ir

Variational Iteration Method for Free Vibration Analysis of a Timoshenko Beam under Various Boundary Conditions

K. Torabi*[a], M. Ghassabi[b], M. Heidari-Rarani[a], D. Sharifi[c]

[a] *Department of Mechanical Engineering, Faculty of Engineering, University of Isfahan, Isfahan, Iran*
[b] *School of Mechanical Engineering, Iran University of Science and Technology, Tehran, Iran*
[c] *Department of Mechanical Engineering, University of Kashan, Kashan, Iran*

PAPER INFO

Paper history:
Received 28 April 2017
Received in revised form 22 May 2017
Accepted 07 July 2017

Keywords:
Variational Iteration Method
Boundary Value Problem
Free Vibration
Timoshenko Beam

ABSTRACT

In this paper, a relatively new method, namely variational iteration method (VIM), is developed for free vibration analysis of a Timoshenko beam with different boundary conditions. In the VIM, an appropriate Lagrange multiplier is first chosen according to order of the governing differential equation of the boundary value problem, and then an iteration process is used till the desired accuracy is achieved. Solution of VIM for natural frequencies and mode shapes of a Timoshenko beam is compared to the available exact closed-form solution and numerical results of differential quadrature method (DQM). The accuracy of VIM is approximately the same as exact solution and much better than the DQM for solving the free vibration of a Timoshenko beam. Also, convergence speed and simplicity of this method is more than the other two methods because it works with polynomial at the first iteration. Thus, VIM can be used for solving the complicate engineering problems which do not have analytical solution.

***doi**: 10.5829/ije.2017.30.10a.18*

NOMENCLATURE

BR	v	I	area inertia
$\bar{L}$	linear operator	A	sectional area
$\bar{N}$	nonlinear operator	**Greek Symbols**	
K_s	shear correction factor	γ	Lagrange multiplier
E	Young's modulus	ρ	mass density
G	shear modulus	ω	natural frequency

1. INTRODUCTION

Timoshenko beam vibration problem with different boundary conditions is widely used in different industrial applications. Timoshenko beam theory in comparison to Euler-Bernoulli one takes into account shear deformation and rotational inertia effects [1, 2] Thus, it is closer to reality. For this reason Timoshenko beam has been used in a lot of practical problems. For example, Lin and Hsiao [3], studied vibration of a rotating Timoshenko beam using virtual work principle. Torabi et al. [4] presented an exact closed-form solution for the analysis of the transverse vibration of a Timoshenko beam with multiple concentrated masses. In spite of accuracy of analytical and exact solutions, there are two major problems: 1) solutions are usually long and 2) many complicated engineering problems cannot be solved. Thus, numerical methods are proposed for solving this type of problems. Today, there are many numerical methods in the literature that each has their own advantages and disadvantages. The main feature of all numerical methods is that they are able to solve every problem, but they are not accurate enough. Among the numerical methods, differential quadrature

*Corresponding Author's Email: *k.torabi@eng.ui.ac.ir* (K. Torabi)

Please cite this article as: K. Torabi, M. Ghassabi, M. Heidari-Rarani, D. Sharifi, Variational Iteration Method for Free Vibration Analysis of a Timoshenko Beam under Various Boundary Conditions, International Journal of Engineering (IJE), TRANSACTIONS A: Basics Vol. 30, No. 10, (October 2017) 1565-1572

method (DQM) has a significant attention in the last decade due to its special features in solving complex problems. Zhong and Guo [5] investigated the large-amplitude free vibration of simply supported Timoshenko beams with immovable ends using DQM. Karami et al. [6] used differential quadrature element method (DQEM) for free vibration analysis of arbitrary non-uniform Timoshenko beams resting on elastic supports and carrying concentrated masses.

Variational Iteration Method (VIM) due to its excellent accuracy, high convergence and simple calculations rather than other numerical methods can be used for solving engineering problems. VIM was first introduced by He [7, 8]. Later, it was developed by Wazwaz [9]. He developed a method known as Adomian in this book due to difficulties in determining the Lagrange multipliers in VIM. However, VIM has higher convergence speed and does not include noise terms in comparison to Adomian method. Therefore, most of the available studies in the literature have focused on the superiority of VIM rather than Adomian method [10, 11].

In recent years, different problems have been solved by VIM. For example, Xu [12] solved integral equations (such as Volterra integral equations of the second kind and Fredholm integral equations of the second kind) of payment using VIM. Noor et al. [13] solved the singular fourth-order parabolic partial differential equations using modified VIM. Olayiwola et al. [14] solved the nonlinear partial differential equations such as nonlinear homogeneous gas dynamics equation by this method. Altıntan and Uğur [15] also solved the different initial and boundary value problems using VIM.

When Wazwaz [16] could determine the Lagrange multiplier by a new method and developed VIM, it led to solve more practical problems. Ding et al. [17] studied the steady-state responses of a Timoshenko beam of infinite length supported by a nonlinear viscoelastic Pasternak foundation subjected to a moving harmonic load using VIM. Berkani et al. [18] proposed an approach for designing an optimal control law based on the VIM. Baghani et al. [19] also presented an analytical solution for the nonlinear free vibration of a conservative oscillator. Rezazadeh et al. [20] dealt with the study of parametric oscillation of an electrostatically actuated microbeam using VIM. However, there are a few initial and boundary value problems yet to be solved by VIM. In this paper, VIM is used to solve the free vibration of Timoshenko beam with various boundary conditions, while such a solution has not been presented until now. High accuracy and convergence of this method is evaluated by comparing the natural frequencies and mode shapes with available results of exact closed-form and DQM solutions.

2. VARIATIONAL ITERATION METHOD (VIM)

To illustrate the basic concept of VIM, consider the following general nonlinear differential equation:

$$\bar{L}u(\zeta)+\bar{N}u(\zeta)=g(\zeta), \tag{1}$$

where $\bar{L}$, $\bar{N}$ and g(t) are linear operator, nonlinear operator and a definite function, respectively [9]. Now, a correctional function can be defined using VIM:

$$u_{n+1}(\zeta)=u_n(\zeta)+\int_0^{\zeta}\gamma\left\{\bar{L}u_n(\mathrm{z})+\bar{\mathrm{N}}\tilde{u}_n(\mathrm{z})-\mathrm{g(z)}\right\}dz, \tag{2}$$

where γ is the Lagrange multiplier and it is introduced elsewhere [21]. n is the nth approximation and $\tilde{u}_n$ a finite variation that $\delta\tilde{u}_n=0$.

3. FREE VIBRATION ANALYSIS OF A TIMOSHENKO BEAM USING VIM

The differential equations of motion for free vibration of a uniform Timoshenko beam is:

$$\left\{K_sGA\frac{\partial}{\partial x}\left[\frac{\partial v(x,t)}{\partial x}-\psi(x,t)\right]=\rho A\frac{\partial^2 v(x,t)}{\partial t^2},\right. \tag{3}$$

$$\left\{EI\frac{\partial^2\psi(x,t)}{\partial x^2}+K_sGA\left[\frac{\partial v(x,t)}{\partial x}-\psi(x,t)\right]=\rho I\frac{\partial^2\psi(x,t)}{\partial t^2},\right. \tag{4}$$

where K_s, E, G, I, ρ and A are shear correction factor, Young's modulus, shear modulus, area inertia, mass density and sectional area, respectively. Assuming the solutions of Equations (3) and (4) as $\psi(x,t)=\Psi(x)e^{i\omega t}$ and $\mathrm{v}(x,t)=V(x)e^{i\omega t}$, the governing equations are simplified as follow:

$$\left\{K_sGA\frac{d}{dx}\left[\frac{dV(x)}{dx}-\Psi(x)\right]=-\rho\omega^2 ALV(x),\right. \tag{5}$$

$$\left\{EI\frac{d^2\Psi(x)}{dx^2}+K_sGA\left[\frac{dV(x)}{dx}-\Psi(x)\right]=-\rho I\omega^2\Psi(x),\right. \tag{6}$$

where ω is natural frequency of the beam. Defining $\zeta=x/L$ and $Y(\zeta)=V(x)/L$ for $0\le\zeta\le 1$, the dimensionless governing equations are obtained as follow:

$$\left\{\frac{K_sGA}{L}\frac{d}{d\zeta}\left[\frac{dY(\zeta)}{d\zeta}-\Psi(\zeta)\right]=-\rho\omega^2 ALY(\zeta),\right. \tag{7}$$

$$\left\{\frac{EI}{L^2}\frac{d^2\Psi(\zeta)}{d\zeta^2}+K_sGA\left[\frac{dY(\zeta)}{d\zeta}-\Psi(\zeta)\right]=-\rho I\omega^2\Psi(\zeta),\right. \tag{8}$$

where $\lambda^4=\dfrac{\rho AL^4\omega^2}{EI}$, $r^2=\dfrac{I}{AL^2}$, and $s^2=\dfrac{EI}{KGAL^2}$. So, Equations (7) and (8) are rewritten as:

$$\begin{cases}\left[Y'(\zeta)-\Psi(\zeta)\right]'=-s^2\lambda^4Y(\zeta), & (9)\\ s^2\Psi''(\zeta)+\left[Y'(\zeta)-\Psi(\zeta)\right]=-s^2\lambda^4r^2\Psi(\zeta). & (10)\end{cases}$$

Equations (9) and (10) are coupled to each other. They can be represented into one fourth-order ordinary differential equation by some simple mathematical operations [22-24]:

$$Y^{(4)}(\zeta)+\lambda^4\left(r^2+s^2\right)Y''(\zeta)+\lambda^4\left(s^2\lambda^4r^2-1\right)Y(\zeta)=0. \tag{11}$$

Comparing Equation (11) with Equation (1), the correctional function in Equation (2) is introduced as below:

$$Y_{n+1}(\zeta)=Y_n(\zeta)+\int_0^{\zeta}\gamma\left\{\begin{array}{l}Y_n^{(4)}(z)+\lambda^4\left(r^2+s^2\right)Y_n''(z)+\\ +\lambda^4\left(s^2\lambda^4r^2-1\right)Y_n(z)\end{array}\right\}dz. \tag{12}$$

Now, to determine the Lagrange multipliers, Equation (12) is first integrated part by part:

$$\begin{aligned}Y_{n+1}(\zeta)=&Y_n(\zeta)+\left[\begin{array}{l}\gamma Y_n'''(\zeta)-\gamma' Y_n''(\zeta)+\gamma'' Y_n'(\zeta)\\ -\gamma''' Y_n(\zeta)+\int_0^{\zeta}\gamma^{(4)}Y_n(z)\,dz\end{array}\right]+\\ &+\lambda^4\left(r^2+s^2\right)\left[\gamma Y_n'(\zeta)-\gamma' Y_n(\zeta)+\int_0^{\zeta}\gamma'' Y_n(z)\,dz\right]+\\ &+\lambda^4\left(s^2\lambda^4r^2-1\right)\int_0^{\zeta}\gamma Y_n(z)\,dz.\end{aligned} \tag{13}$$

Then variational operator is applied to Equation (13):

$$\begin{aligned}\delta Y_{n+1}(\zeta)=&\delta Y_n(\zeta)+\left[\begin{array}{l}\gamma\delta Y_n'''(\zeta)-\gamma'\delta Y_n''(\zeta)+\gamma''\delta Y_n'(\zeta)\\ -\gamma'''\delta Y_n(\zeta)+\int_0^{\zeta}\gamma^{(4)}\delta Y_n(z)\,dz\end{array}\right]+\\ &+\lambda^4\left(r^2+s^2\right)\left[\gamma\delta Y_n'(\zeta)-\gamma'\delta Y_n(\zeta)+\int_0^{\zeta}\gamma''\delta Y_n(z)\,dz\right]+\\ &+\lambda^4\left(s^2\lambda^4r^2-1\right)\int_0^{\zeta}\gamma\delta Y_n(z)\,dz.\end{aligned} \tag{14}$$

Stationary conditions are satisfied in Equation (14) when:

$$\delta Y_n:\ \ 1-\gamma'''-\lambda^4\left(r^2+s^2\right)\gamma'\big|_{z=\zeta}=0, \tag{15}$$

$$\delta Y_n':\ \ \gamma''+\lambda^4\left(r^2+s^2\right)\gamma\big|_{z=\zeta}=0, \tag{16}$$

$$\delta Y_n'':\ \ -\gamma'\big|_{z=\zeta}=0, \tag{17}$$

$$\delta Y_n''':\ \ \gamma\big|_{z=\zeta}=0, \tag{18}$$

$$\int\delta Y_n dt:\ \ \gamma^{(4)}+\lambda^4\left(r^2+s^2\right)\gamma''+\lambda^4\left(s^2\lambda^4r^2-1\right)=0. \tag{19}$$

Finally, the Lagrange multiplier is introduced as the following relation to satisfy Equations (15)-(19):

$$\gamma=\frac{(z-\zeta)^3}{3!}. \tag{20}$$

The correctional function is obtained by replacing Equation (20) into Equation (12):

$$\begin{aligned}Y_{n+1}(\zeta)=&Y_n(\zeta)+\\ &+\int_0^{\zeta}\frac{(z-\zeta)^3}{6}\left[\begin{array}{l}Y_n^{(4)}(z)+\lambda^4\left(r^2+s^2\right)Y_n''(z)+\\ +\lambda^4\left(s^2\lambda^4r^2-1\right)Y_n(z)\end{array}\right]dz.\end{aligned} \tag{21}$$

In this paper, a single equation and higher Lagrange multipliers is used while in the literature [25], the two coupled equation and lower order Lagrange multipliers are used.

To start the iteration in Equation (21), $Y_0(\zeta)$ should be defined. The primary function ($Y_0(\zeta)$) can be selected any arbitrary continuous function that satisfies boundary conditions. Type of selected polynomial has no significant effect on the final results but it affects the convergence process. Thus, $Y_0(\zeta)$ is expanded by Maclaurin series [10]:

$$\begin{aligned}Y_0(\zeta)=&\sum_{m=0}^{3}\frac{\zeta^m}{m!}Y^{(m)}(0)=\\ =&Y(0)+Y'(0)\zeta+\frac{Y''(0)}{2!}\zeta^2+\frac{Y'''(0)}{3!}\zeta^3.\end{aligned} \tag{22}$$

Therefore, Equation (21) is rewritten using $Y_0(\zeta)$ in Equation (22):

$$\begin{aligned}Y_1(\zeta)=&Y_0(\zeta)+\\ &+\int_0^{\zeta}\frac{(z-\zeta)^3}{6}\left[\begin{array}{l}Y_0^{(4)}(z)+\lambda^4\left(r^2+s^2\right)Y_0''(z)+\\ +\lambda^4\left(s^2\lambda^4r^2-1\right)Y_0(z)\end{array}\right]dz,\\ Y_2(\zeta)=&Y_1(\zeta)+\\ &+\int_0^{\zeta}\frac{(z-\zeta)^3}{6}\left[\begin{array}{l}Y_1^{(4)}(z)+\lambda^4\left(r^2+s^2\right)Y_1''(z)+\\ +\lambda^4\left(s^2\lambda^4r^2-1\right)Y_1(z)\end{array}\right]dz,\\ &\vdots\\ Y_k(\zeta)=&Y_{k-1}(\zeta)+\\ &+\int_0^{\zeta}\frac{(z-\zeta)^3}{6}\left[\begin{array}{l}Y_{k-1}^{(4)}(z)+\lambda^4\left(r^2+s^2\right)Y_{k-1}''(z)+\\ +\lambda^4\left(s^2\lambda^4r^2-1\right)Y_{k-1}(z)\end{array}\right]dz.\end{aligned} \tag{23}$$

The solution of Equation (11) is $Y(\zeta)=\lim_{k\to\infty}Y_k$. Since the iteration process is not possible to continue till infinity, it continues to the required accuracy for natural frequency, e.g., $\left|\lambda_i^{[n]}-\lambda_i^{[n-1]}\right|\le\varepsilon$, where ε determines the accuracy of natural frequency.

Now, applying the boundary condition at $\zeta = 0$ gives

$$\sum_{j=0}^{3} f_{rj}^{[k]}(\lambda) Y^{(j)}(0) = 0, \quad r = 1,2 \tag{24}$$

where $f_{rj}^{[k]}$ is polynomial in terms of λ. Finally, a two-equation homogenous system is obtained that determinant of coefficients should be equal to zero for non-trivial solution.

$$\begin{vmatrix} f_{10}^{[k]}(\lambda) & f_{11}^{[k]}(\lambda) \\ f_{20}^{[k]}(\lambda) & f_{21}^{[k]}(\lambda) \end{vmatrix} = 0. \tag{25}$$

To clearly show how the boundary conditions are applied, a polynomial like Equation (26) which is obtained from Equation (22) is considered. Equation (26) has four unknowns because there are two physical or geometrical conditions at each end of the beam. Also, one term is added to Equation (26) for increasing the accuracy of approximation:

$$Y_0(\zeta) = \zeta^4 + a\zeta^3 + b\zeta^2 + c\zeta + d, \tag{26}$$

where a, b, c and d are unknown coefficients which are determined by applying the boundary conditions. Two unknowns are determined from the boundary conditions at $\zeta = 0$ and applying the other two boundary conditions at $\zeta = 1$ forms the determinant in Equation (25).

All natural and geometrical boundary conditions for a Timoshenko beam are as follow:

Simply-support: $\mathrm{Y}(0) = \Psi'(0) = 0,$ (27a)

Free: $\Psi'(0) = \left[\Psi(0) - Y'(0)\right] = 0,$ (27b)

Clamped: $\mathrm{Y}(0) = \Psi(0) = 0.$ (27c)

Substituting Equation (27) into Equation (26), unknowns can be expressed as follow:

Simply-support: $b = -\dfrac{24s^2}{2(1+s^4\lambda^4)}, \quad d = 0,$ (28a)

Free: $b = -\dfrac{24s^2}{2(1+s^4\lambda^4)}, \quad a = -\dfrac{c}{6}\lambda^4\left(r^2+s^2\right),$ (28b)

Clamped: $a = -\dfrac{c(1+s^4\lambda^4)}{6s^2}, \quad d = 0.$ (28c)

4. NUMERICAL EXAMPLES FOR DIFFERENT BOUNDARY CONDITIONS

In this section, the formulations developed for the free vibration of a Timoshenko beam is numerically studied for four different boundary conditions: SS, CC, CS and CF. S, C and F stand for simply-supported, clamped, and free, respectively.

After applying the boundary conditions in Equation (28), a matrix of coefficients is obtained. Eigenvalues of this matrix gives the natural frequencies of each beam with a special boundary condition. However, it should be noted for applying the boundary conditions, $\psi(\zeta)$ is necessary. $\psi(\zeta)$ can be obtained knowing $Y(\zeta)$ as follows [4]:

$$\Psi(\zeta) = \frac{1}{1 - r^2 s^2 \lambda^4}\left[s^2 Y'''(\zeta) + (1 + s^4\lambda^4) Y'(\zeta)\right]. \tag{29}$$

Table 1 shows the three eigenvalues of a Timoshenko beam with different boundary conditions obtained by three different solution methods, i.e., exact solution [26], differential quadrature method (DQM) [27] and VIM. r and s are selected based on the chosen physical problem. According to the literature [4, 27], r and s are selected as 0.03 and 0.05, respectively.

TABLE 1. Three first eigenvalues for free vibration of a Timoshenko beam using exact solution, DQM and VIM (r = 0.03, s =0.05)

Boundary conditions	Exact solution [26]	DQM [27]	VIM	$\frac{\text{DQM} - \text{Exact}}{\text{Exact}} \times 100$	$\frac{\text{VIM} - \text{Exact}}{\text{Exact}} \times 100$
	3.1157	3.1159	3.1159	0.006419	0.006419
SS	6.0922	6.0955	6.0926	0.054168	0.006566
	8.8465	8.8669	8.8465	0.2306	0
	4.5835	4.5838	4.5837	0.006545	0.004363
CC	7.3422	7.3428	7.3427	0.008172	0.006810
	9.8766	9.8828	9.8770	0.062775	0.004050
	3.8535	3.8534	3.8535	0.002595	0
CS	6.7367	6.7377	6.7358	0.014844	0.013360
	9.3790	9.4741	9.3784	1.01	0.006397
	1.8677	1.8678	1.8678	0.005354	0.005354
CF	4.5734	4.5732	4.5736	0.004373	0.004373
	7.4189	7.4432	7.4195	0.327542	0.008087

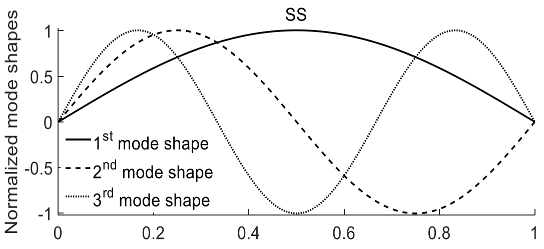
Table 1 shows that eigenvalues obtained by VIM for free vibration of Timoshenko beam are in excellent accuracy with exact and DQM solutions available in the literature. The main feature of VIM rather than the other solution methods is that it converges only by eight iterations and using a polynomial for primary function. Also, comparison of VIM and DQM results represents that the accuracy and convergence of VIM increases at higher frequencies.

Table 2 shows first three eigenvalues for different values of iteration (up to 9 iterations) and various boundary conditions. It is found that the numbers of iterations should be increased at higher frequencies. There is no change between eighth and ninth iterations, so eighth iteration is sufficient to solve the problem.

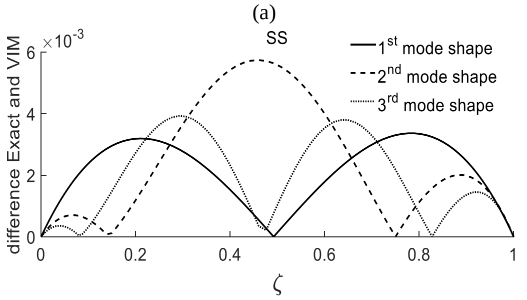
Figure 1(*a*) shows the first three shape modes of a simply-supported Timoshenko beam obtained by VIM. Figure 1(*b*) represents the absolute relative difference between the exact solution and VIM.

TABLE 2. First three eigenvalues for different values of iteration (*k*)

B.C.	k	2	3	4	5	6	7	8	9
	λ_1	3.1175	3.1159	3.1159	3.1159	3.1159	3.1159	3.1159	3.1159
SS	λ_2	5.4044	6.1240	6.0923	6.0926	6.0926	6.0926	6.0926	6.0926
	λ_3	N.A.	7.7305	N.A.	8.8303	8.8481	8.8464	8.8465	8.8465
	λ_1	4.5901	4.5837	4.5838	4.5838	4.5838	4.5838	4.5838	4.5838
CC	λ_2	6.5299	7.7588	7.3373	7.3428	7.3427	7.3427	7.3427	7.3427
	λ_3	N.A.	15.3486	N.A.	9.7306	9.8939	9.8761	9.8770	9.8770
	λ_1	3.8563	3.8535	3.8535	3.8535	3.8535	3.8535	3.8535	3.8535
CS	λ_2	6.0512	6.8317	6.7348	6.7368	6.7367	6.7367	6.7367	6.7367
	λ_3	N.A.	7.8669	N.A.	9.3037	9.3858	9.3787	9.3791	9.3790
	λ_1	1.8678	1.8678	1.8678	1.8678	1.8678	1.8678	1.8678	1.8678
CF	λ_2	4.5324	4.5739	4.5736	4.5736	4.5736	4.5736	4.5736	4.5736
	λ_3	11.2165	7.2339	7.4303	7.4192	7.4196	7.4195	7.4195	7.4195



(a)



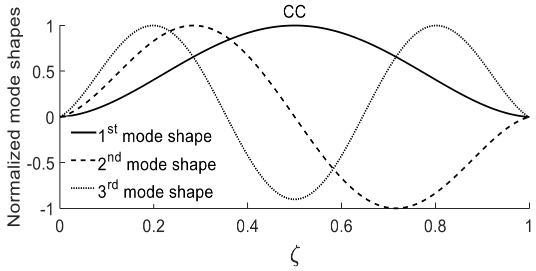
(b)

Figure 1. Simply-supported Timoshenko beam: (*a*) first three mode shapes and (*b*) absolute relative differences of first three mode shapes in exact solution and VIM

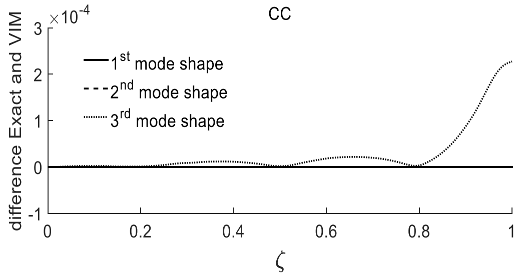
Mode shapes of exact solution are provided in the literature [27]. The difference is approximately negligible and it proved the accuracy of VIM.

Figures 2(*a*), 3(*a*) and 4(*a*) show the first three mode shapes of a clamped-clamped, clamped-simply and clamped-free Timoshenko beam obtained by VIM, respectively. Also, Figures 2(*b*), 3(*b*) and 4(*b*) represent absolute relative difference between the exact-solution and VIM for clamped-clamped, clamped-simply and clamped-free, respectively. In these cases, the differences between the results are negligible.

From Figures 1(*b*), 2(*b*), 3(*b*) and 4(*b*), it is seen that the difference between exact solution and VIM is more at higher mode shapes. The reason is that VIM is an iterative method and it usually gives the first frequency at the first iteration. This difference can be reduced by increasing the numbers of iterations. This issue is shown in Figures 5-8. According to Figures 5-8, the difference between exact solution and VIM for simply-supported Timoshenko beam is more than the clamped-clamped, clamped-simply and clamped-free cases.

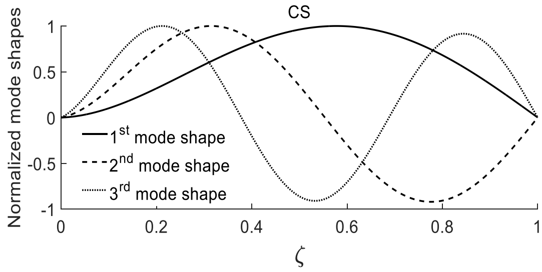


(a)

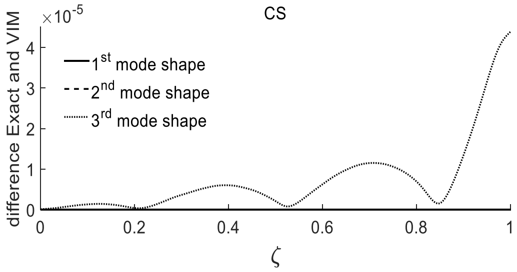


(b)

Figure 2. Clamped-clamped Timoshenko beam: (*a*) first three mode shapes and (*b*) absolute relative differences of first three mode shapes in exact solution and VIM

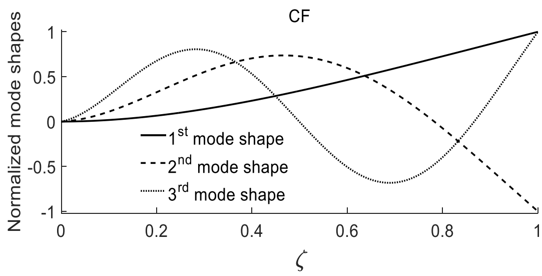


(a)

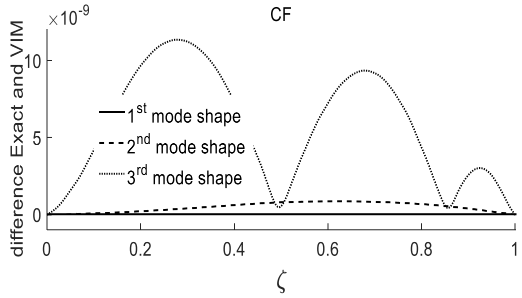


(b)

Figure 3. Clamped-simply supported Timoshenko beam: (*a*) first three mode shapes and (*b*) absolute relative differences of first three mode shapes in exact solution and VI



(a)



(b)

Figure 4. Clamped-free Timoshenko beam: (*a*) first three mode shapes and (*b*) absolute relative differences of first three mode shapes in exact solution and VIM

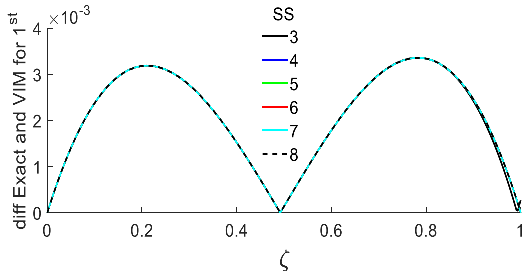


Figure 5. Absolute relative difference of mode shapes between VIM and exact solution for a simply-supported beam with different numbers of iterations

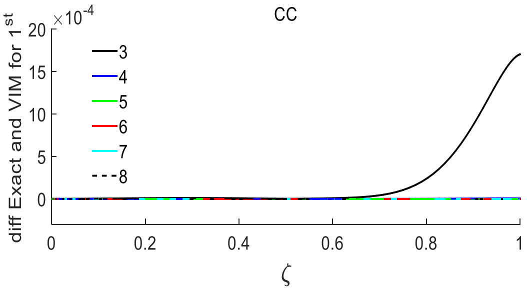


Figure 6. Absolute relative difference of mode shapes between VIM and exact solution for a clamped-clamped beam with different numbers of iterations

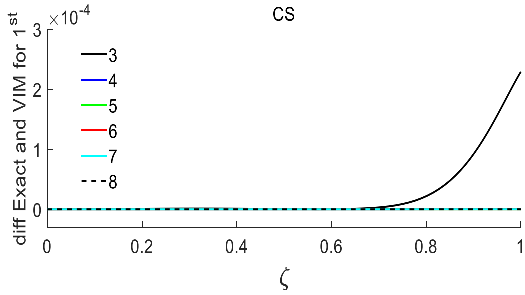


Figure 7. Absolute relative difference of mode shapes between VIM and exact solution for a clamped-simply beam with different numbers of iterations

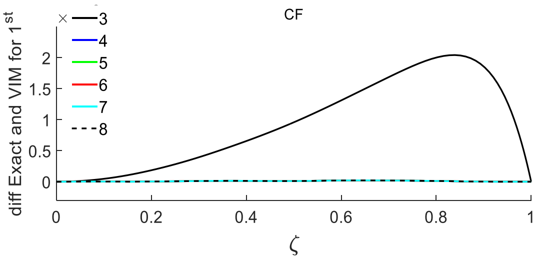


Figure 8. Absolute relative difference of mode shapes between VIM and exact solution for a clamped-free beam with different numbers of iterations

In the last three ones, one clamped condition is at least available and it helps to apply the actual condition. Also, because after a specified number of iterations, accuracy does not change, thus lines are completely overlapped on each other.

5. CONCLUSION

Vibration of Timoshenko beam is an important engineering problem that it is previously solved by exact-solution and some numerical methods. In this paper, VIM is developed for solving the free vibration of Timoshenko beam while such a solution has not been presented until now. Comparison of natural frequencies obtained by VIM, exact-solution and DQM showed that VIM has three advantages: 1) good accuracy in prediction of natural frequencies, 2) simplicity due to working with polynomial at the first iteration and 3) efficient stability with high convergence for less numbers of iterations. Therefore, VIM can be introduced as an efficient, fast and accurate method for solving the complicated engineering problems.

6. REFERENCES

1. Nesterenko, V., "A theory for transverse vibrations of the timoshenko beam", ***Journal of Applied Mathematics and Mechanics***, Vol. 57, No. 4, (1993), 669-677.
2. Horr, A. and Schmidt, L., "Closed-form solution for the timoshenko beam theory using a computer-based mathematical package", ***Computers & Structures***, Vol. 55, No. 3, (1995), 405-412.
3. Lin, S. and Hsiao, K., "Vibration analysis of a rotating timoshenko beam", ***Journal of Sound and Vibration***, Vol. 240, No. 2, (2001), 303-322.
4. Torabi, K., Jazi, A.J. and Zafari, E., "Exact closed form solution for the analysis of the transverse vibration modes of a timoshenko beam with multiple concentrated masses", ***Applied Mathematics and Computation***, Vol. 238, (2014), 342-357.
5. Zhong, H. and Guo, Q., "Nonlinear vibration analysis of timoshenko beams using the differential quadrature method", ***Nonlinear Dynamics***, Vol. 32, No. 3, (2003), 223-234.
6. Karami, G., Malekzadeh, P. and Shahpari, S., "A dqem for vibration of shear deformable nonuniform beams with general boundary conditions", ***Engineering Structures***, Vol. 25, No. 9, (2003), 1169-1178.
7. He, J., "Variational iteration method for delay differential equations", ***Communications in Nonlinear Science and Numerical Simulation***, Vol. 2, No. 4, (1997), 235-236.
8. He, J.-H., "Variational iteration method—some recent results and new interpretations", ***Journal of Computational and Applied Mathematics***, Vol. 207, No. 1, (2007), 3-17.
9. Wazwaz, A.-M., "Partial differential equations and solitary waves theory, Springer Science & Business Media, (2010).
10. Liu, Y. and Gurram, C.S., "The use of he's variational iteration method for obtaining the free vibration of an euler–bernoulli beam", ***Mathematical and Computer Modelling***, Vol. 50, No. 11, (2009), 1545-1552.
11. Hasseine, A., Barhoum, Z., Attarakih, M. and Bart, H.-J., "Analytical solutions of the particle breakage equation by the adomian decomposition and the variational iteration methods", ***Advanced Powder Technology***, Vol. 26, No. 1, (2015), 105-112.
12. Xu, L., "Variational iteration method for solving integral equations", ***Computers & Mathematics with Applications***, Vol. 54, No. 7, (2007), 1071-1078.
13. Noor, M.A., Noor, K.I. and Mohyud-Din, S.T., "Modified variational iteration technique for solving singular fourth-order parabolic partial differential equations", ***Nonlinear Analysis: Theory, Methods & Applications***, Vol. 71, No. 12, (2009), e630-e640.
14. Olayiwola, M., Akinpelu, F. and Gbolagade, A., "Modified variational iteration method for the solution of a class of differential equations", ***American Journal of Computational and Applied Mathematics***, Vol. 2, No. 5, (2012), 228-231.
15. Altıntan, D. and Ugur, O., "Solution of initial and boundary value problems by the variational iteration method", ***Journal of Computational and Applied Mathematics***, Vol. 259, (2014), 790-797.
16. Wazwaz, A.-M., "Linear and nonlinear integral equations", Springer, Vol. 639, ISBN 978-3-642-21449-3, (2011).
17. Ding, H., Shi, K., Chen, L. and Yang, S., "Adomian polynomials for nonlinear response of supported timoshenko beams subjected to a moving harmonic load", ***Acta Mechanica Solida Sinica***, Vol. 27, No. 4, (2014), 383-393.
18. Berkani, S., Manseur, F. and Maidi, A., "Optimal control based on the variational iteration method", ***Computers & Mathematics with Applications***, Vol. 64, No. 4, (2012), 604-610.
19. Baghani, M., Fattahi, M. and Amjadian, A., "Application of the variational iteration method for nonlinear free vibration of conservative oscillators", ***Scientia Iranica***, Vol. 19, No. 3, (2012), 513-518.
20. Rezazadeh, G., Madinei, H. and Shabani, R., "Study of parametric oscillation of an electrostatically actuated microbeam using variational iteration method", ***Applied Mathematical Modelling***, Vol. 36, No. 1, (2012), 430-443.
21. He, J.-H. and Wu, X.-H., "Variational iteration method: New development and applications", ***Computers & Mathematics with Applications***, Vol. 54, No. 7, (2007), 881-894.
22. Sherafatnia, K., Farrahi, G. and Faghidian, S.A., "Analytic approach to free vibration and buckling analysis of functionally graded beams with edge cracks using four engineering beam theories", ***International Journal of Engineering-Transactions C: Aspects***, Vol. 27, No. 6, (2013), 979-990.
23. Rakideh, M., Dardel, M. and Pashaei, M., "Crack detection of timoshenko beams using vibration behavior and neural network", ***International Journal of Engineering-Transactions C: Aspects***, Vol. 26, No. 12, (2013), 1433-1441.

24. Sadeghian, M. and Ekhteraei Toussi, H., "Frequency analysis for a timoshenko beam located on an elastic foundation", ***International Journal of Engineering***, Vol. 24, (2011), 87-105.

25. Chen, Y., Zhang, J. and Zhang, H., "Free vibration analysis of rotating tapered timoshenko beams via variational iteration method", ***Journal of Vibration and Control***, Vol. 23, No. 2, (2017), 220-234.

26. Khaji, N., Shafiei, M. and Jalalpour, M., "Closed-form solutions for crack detection problem of timoshenko beams with various boundary conditions", ***International Journal of Mechanical Sciences***, Vol. 51, No. 9, (2009), 667-681.

27. Torabi, K., Afshari, H. and Aboutalebi, F.H., "A dqem for transverse vibration analysis of multiple cracked non-uniform timoshenko beams with general boundary conditions", ***Computers & Mathematics with Applications***, Vol. 67, No. 3, (2014), 527-541.

Variational Iteration Method for Free Vibration Analysis of a Timoshenko Beam under Various Boundary Conditions

K. Torabi[a], M. Ghassabi[b], M. Heidari-Rarani[a], D, Sharifi[c]

[a] *Department of Mechanical Engineering, Faculty of Engineering, University of Isfahan, Isfahan, Iran*
[b] *School of Mechanical Engineering, Iran University of Science and Technology, Tehran, Iran*
[c] *Department of Mechanical Engineering, University of Kashan, Kashan, Iran*

P A P E R I N F O

Paper history:
Received 28 April 2017
Received in revised form 22 May 2017
Accepted 07 July 2017

Keywords:
Variational Iteration Method
Boundary Value Problem
Free Vibration
Timoshenko Beam

چکیده

در این مقاله از یک روش نسبتاً جدید به نام روش تکرار تغییرات (VIM) برای تحلیل ارتعاشات آزاد تیر تیموشنکو با شرایط مرزی مختلف استفاده شده است. در VIM، ابتدا بر اساس مرتبه معادله دیفرانسیل مسئله شرط مرزی داده شده، ضریب لاگرانژ مناسب انتخاب می شود و سپس فرآیند تکرار تا رسیدن به دقت مطلوب ادامه دارد. پاسخ‌های VIM برای فرکانس های طبیعی و شکل مودهای یک تیر تیموشنکو با پاسخ حل دقیق و نتایج عددی روش DQM مقایسه شده است. دقت VIM تقریباً همان دقت حل دقیق بوده و بسیار از حل DQM برای حل ارتعاشات آزاد تیر تیموشنکو دقیق‌تر است. از طرفی، سرعت همگرایی و سادگی این روش از دو روش دیگر بسیار بهتر است چرا که از یک چند جمله‌ایی در تکرار اول استفاده می شود. بنابراین VIM می‌تواند برای مسائل پیچیده مهندسی که نمی توان از روش تحلیلی بهره برد، استفاده شود.

doi: *10.5829/ije.2017.30.10a.18*